literal: x or $\bar{x}$ Clause: $C = x_1 \vee \bar{x}_2 \vee x_3$ ↳ or

Boolean in Conjugative Normal Form: $\Phi = C_1 \wedge C_2 \wedge C_3 \wedge C_4$ ↳ ands

→ All of the clauses must be true

3 SAT → 3 literals

$(\bar{x}_1 \vee x_2 \vee x_3) \wedge (x_1 \vee \bar{x}_2 \vee x_3) \wedge (x_1 \vee x_2 \vee x_3) \wedge (\bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3)$

→ It is unconstrained

→ How to represent and / or / not , 1, -1 is first choice

True $\wedge$ True = True
True $\wedge$ False = False
False $\wedge$ True = False
False $\wedge$ False = False

$\overline{\text{True}}$ = False
$\overline{\text{False}}$ = True

True $\vee$ True = True
T $\vee$ F = True
F $\vee$ T = True
F $\vee$ F = False

→ Clause satisfied → -1

Binary 0 - False 1 - True

Not: $(1-x)$ $x_1, \bar{x}_2, x_3$

Make it go to 0

$x_1 + (1-x_2) + x_3$

minimize $x_1(1-x_2)x_3$

000 → 0	✓ 000 → 1
001 → 0	✓ 001 → 2
010 → 0	$\boxed{010 \to 0}$
011 → 0	✓ ~~011 → 1~~
100 → 0	✓ 100 → 2
And gate 101 → 1	✓ 101 → 3
110 → 0	✓ 110 → 1
111 → 0	✓ 111 → 2

not sat if is F, T, F

Clause = 0 → not satisfied by chosen condition

maximize $((1-x_1)+x_2+x_3) \cdot (x_1 + (1-x_2)+x_3) \cdot ($

0 iff $C_1 \wedge C_2 \wedge C_3 \wedge C_4$ condition: $x_1, \bar{x}_2, x_3$

Clause = 0 → is satisfied by chosen?

True - 1
False - 0

$x_1 + (1-x_2) + x_3$

$x_1 \vee \bar{x}_2 \vee x_3$

anyone but 010 needs to go to 0
↳ $(1-x_1) \cdot$

logicals $x_1 \vee x_2 \vee x_3$

$x_1 \vee x_2 \vee x_3 \to$ up to 3

or is typically addition

if (condition true $\to$ 1)

$\to$ we need to scale from 0/3 to 0/1

$x_1 \vee x_2 \vee x_3$

1 of $x_1, x_2, x_3 = 1 \to$ expression $= 1$

$\Longleftrightarrow$ expression $= 0 \Rightarrow x_1 = x_2 = x_3 = 0$

AOTC $\exists \begin{pmatrix} x_1' \\ x_2' \\ x_3' \end{pmatrix}$ s.t. $x_1' = 1$ and

expression $= 0$

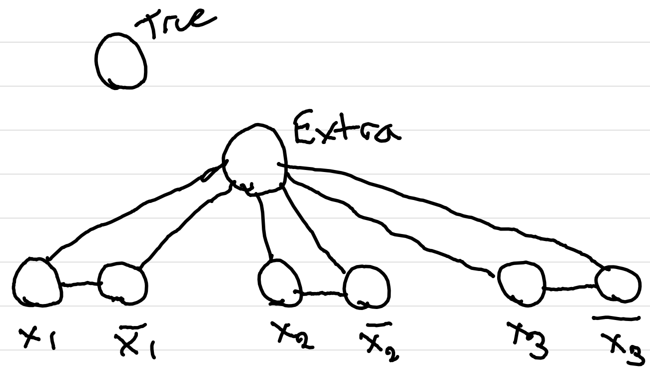
$$\text{expression} = x_1' + \left(x_2' + x_3' - \underbrace{x_1'}_{1}x_2' - \underbrace{x_1'}_{1}x_3' - x_2'x_3' + \underbrace{x_1'}_{1}x_2'x_3'\right)$$

3 CNF → 3 colorability

Clause $x_1 \vee x_2 \vee x_3$

Graph coloring iff 3-SAT
↳ connected vertices have different colors

Attempt for 3SAT with graphs as opposed to algebra. Shortly after, I switch over to Max 3SAT which made a lot more sense for me



→ All literals connected to Extra so can now only be T/F

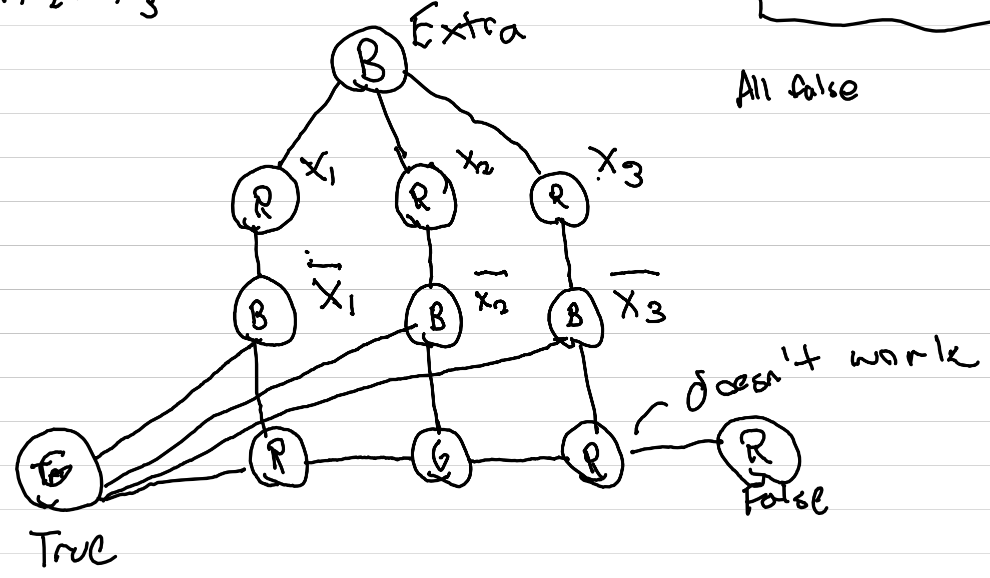
→ Variables and $\overline{\text{Variables}}$ connected to show they are not same

Gadgets If all false, can't 3 color
If 1 is true you can

$x_1 \wedge x_2 \wedge x_3$

General

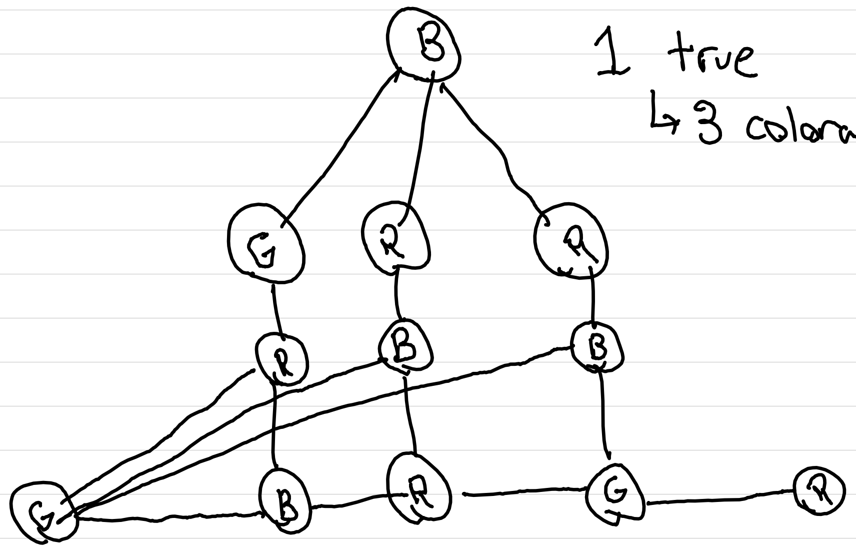
All false



→ Since not literals are attached to G (True), they can only be R/B

→ If all false(R), all not literals are Extra(B)

1 true
↳ 3 colorable



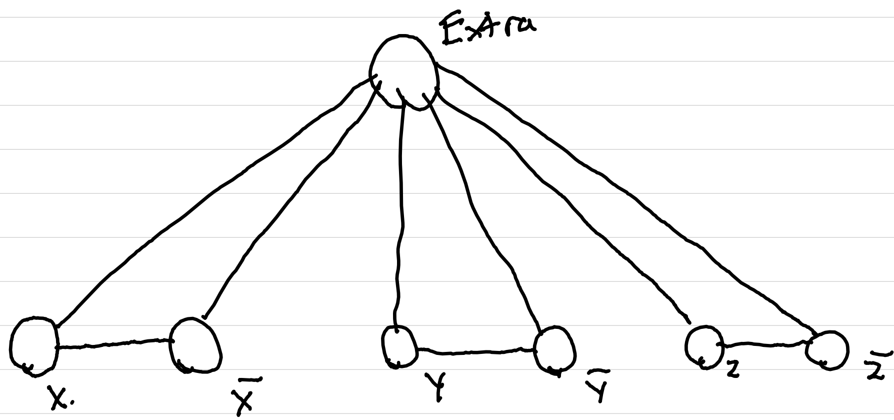
$(x \vee y \vee \bar{z}) \wedge (\bar{x} \vee \bar{y} \vee z) \wedge (\bar{x} \vee y \vee \bar{z})$

Reduction: make a graph that is three colorable iff some 3-SAT problem is satisfiable

→ $x, \bar{x}$ cannot be the same: edge between $x, \bar{x}$

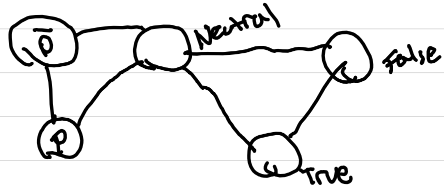
→ edge to extra so each x can only be T/F
↳ 2 variables → 3 variables

$x \vee \bar{y} \vee z$



Meyer Video

→ Attaching to N makes T/F

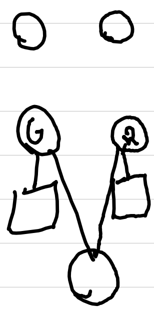


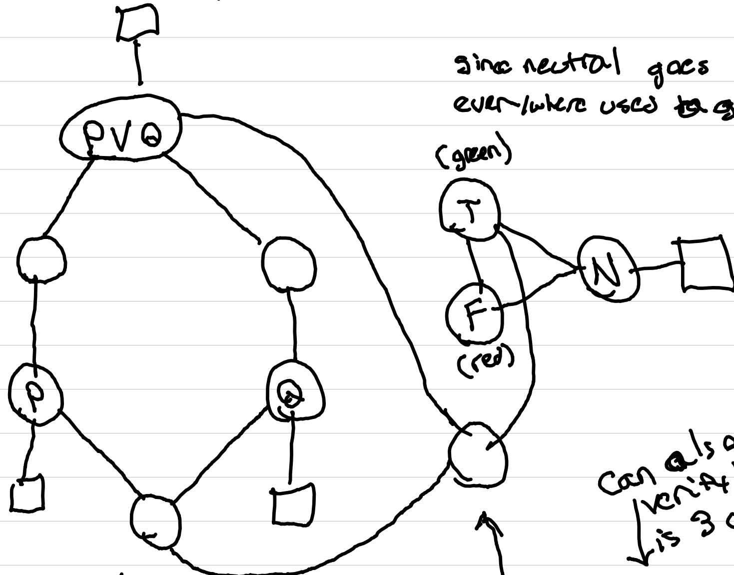
NOT

→ Attach to a vertex x to flip T/F since can't be neutral

OR

since neutral goes everywhere used to gauge

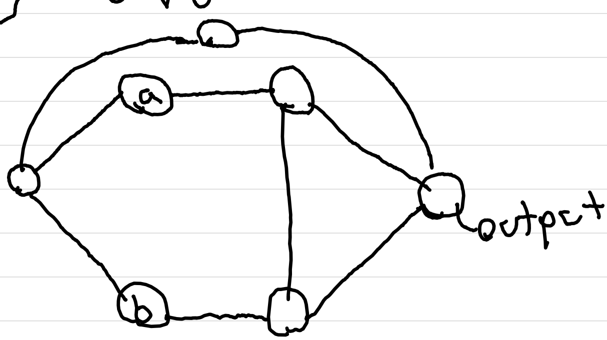




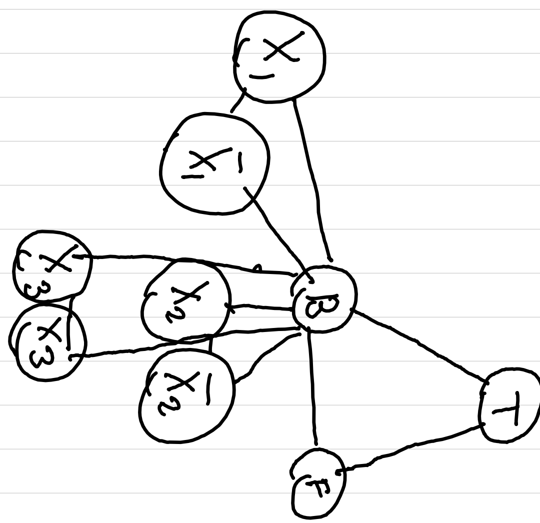
can also verify that it is 3 colorable

→ If they are different 1 is neutral. This means 2 is false and PVQ is true

→ If F/F the top two are G/N. This forces PVQ to be false
→ If T/T same logic applies

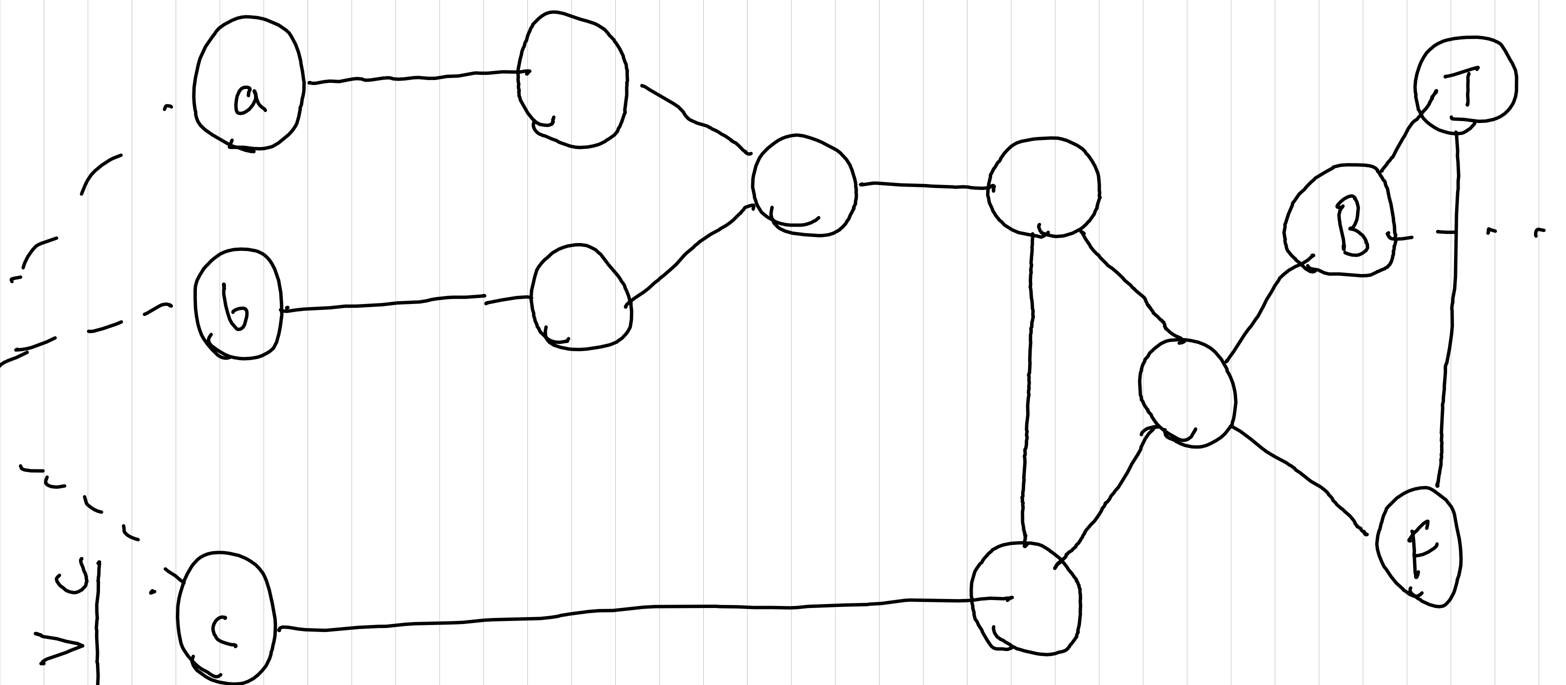


NOT



1 Clause (Can see the two or gates)

$a \vee b \vee c$



This approach would be so big....

Given (for now)

3SAT → MIS (maximum independent set)

· MIS

ILP $x_1 \vee \overline{x_2} \vee x_3$

let y be variables for IP: $0 \le y_i \le 1$

condition $y_i = 1 \rightarrow x_i = \text{true}$

$y_i = 0 \rightarrow y_i = \text{false}$

$y_1 + (1 - y_2) + y_3 > 0$

Example

$$(x_1 \vee x_2 \vee \overline{x_3}) \wedge (x_2 \vee x_3 \vee \overline{x_4}) \wedge (x_1 \vee \overline{x_2} \vee x_4)$$

Conditions

$y_1 + y_2 + (1 - y_3) > 0$

$y_2 + y_3 + (1 - y_4) > 0$

$y_1 + (1 - y_2) + y_4 > 0$

Short foray into linear programming which yielded little (although I think it could be formatted like this). Invevitably, the max trick is a trick to use this same strategy.

Max 3SAT

(# of satisfied or conditions) (inclusion/exclusion for 3 elements

$$C_1 = x_1 + x_2 + x_3 - x_1x_2 - x_1x_3 - x_2x_3 + x_1x_2x_3$$

→ Can get up to 3 conditions
→ Second terms cancels out the double counting
→ If all conditions then needs to be 1. This is the last term

(# of satisfied and conditions) $= \sum_{i=0} C_i$

minimize $-\left(\sum C_i\right)$ QUBO minimize $\left(\sum C_i - \text{\# of clauses}\right)^2$

One clause example $x_1 \vee \overline{x_2} \vee x_3$

$$C_i = x_1 + (1-x_2) + x_3 - x_1(1-x_2) - x_1x_3 - (1-x_2)x_3 + x_1(1-x_2)x_3$$

$$= \cancel{x_1} + 1 - x_2 + x_3 - \cancel{x_1} + x_1x_2 - \cancel{x_1x_3} + \cancel{x_1x_3} - x_1x_2x_3$$

$$= 1 - x_2 + x_3 + x_1x_2 - x_1x_2x_3$$

$$x_j = \frac{1-z_j}{2} \qquad x_ix_jx_k = \frac{1}{8}(1-z_i)^3$$

$$= 1 - \left(\frac{1-z_2}{2}\right) + \left(\frac{1-z_3}{2}\right) + \frac{1}{4}(1 - z_2 - z_1 + z_1z_2)$$
$$- \frac{1}{8}(1 - z_1 - z_2 - z_3 + z_1z_2 + z_1z_3 + z_2z_3 - z_1z_2z_3)$$

$$= 1 - \cancel{\tfrac{1}{2}} + \frac{z_2}{2} + \cancel{\tfrac{1}{2}} - \frac{z_3}{2} + \frac{1}{4} - \frac{z_2}{4} - \frac{z_1}{4} + \frac{z_1z_2}{4} - \frac{1}{8} + \frac{z_1}{8} + \frac{z_2}{8} + \frac{z_3}{8} - \frac{z_1z_2}{8} - \frac{z_2z_3}{8} - \frac{z_1z_3}{8} + \frac{z_1z_2z_3}{8}$$

$$= \frac{1}{8}(9 - z_1 + 3z_2 - 3z_3 + z_1z_2 - z_2z_3 - z_1z_3 + z_1z_2z_3)$$

First attempt --> without decomposing degree 3 terms. There was a cancellation which allowed them to not be present in QUBO, but the calculation still produced inaccurate results

$x_1 \wedge x_2 \wedge x_3$ $\qquad 1-(1-x)(1-y)(1-z)$

$C_\ell = x_1 + x_2 + x_3 - x_1x_2 - x_2x_3 - x_1x_3 + x_1x_2x_3$ $\qquad$ True - 1, False - 0

$$= \frac{(1-z_1)}{2} + \frac{1-z_2}{2} + \frac{1-z_3}{2} - \frac{1}{4}(z_1z_2 - z_1 - z_2 + 1)$$

$$-\frac{1}{4}(z_2z_3 - z_2 - z_3 + 1) - \frac{1}{4}(z_1z_3 - z_1 - z_3 + 1) + \frac{1}{8}(1 - z_1 - z_2 - z_3 + z_1z_2 + z_1z_3 + z_2z_3 - z_1z_2z_3)$$

$$= \frac{3}{2} - \frac{z_1}{2} - \frac{z_2}{2} - \frac{z_3}{2} - \frac{z_1z_2}{4} + \frac{z_1}{4} + \frac{z_2}{4} - \frac{1}{4} - \frac{z_2z_3}{4} + \frac{z_2}{4} + \frac{z_3}{4} - \frac{1}{4} - \frac{z_1z_3}{4} + \frac{z_1}{4}$$
$$+ \frac{z_3}{4} - \frac{1}{4} + \frac{1}{8} - \frac{z_1}{8} - \frac{z_2}{8} - \frac{z_3}{8} + \frac{z_1z_2}{8} + \frac{z_1z_3}{8} + \frac{z_2z_3}{8} - \frac{z_1z_2z_3}{8}$$

$$= \frac{1}{8}(7 - z_1 - z_2 - z_3 - z_1z_2 - z_1z_3 - z_2z_3 - z_1z_2z_3)$$

$(x_1 \vee x_2 \vee x_3) \wedge (x_1 \vee \bar{x}_2 \vee x_3)$

minimize $\left(\sum C_i - \#C\right)^2 = \left(\sum C_i - 2\right)^2$

$$= \left(\frac{1}{8}(16 - 2z_1 + 2z_2 - 4z_3 - 2z_1z_3 - 2z_2z_3) - 2\right)^2$$

$$= \left(-\frac{z_1}{4} + \frac{z_2}{4} - \frac{z_3}{2} - \frac{z_1z_3}{4} - \frac{z_2z_3}{4}\right)^2$$

minimize $\left(\frac{1}{4}(-z_1 + z_2 - 2z_3 - z_1z_3 - z_2z_3)\right)^2$

solutions

Yes	No
100	000
101	010
110	
111	
001	
011	

$x'_j = \frac{1-z'_j}{2}$ $\qquad z'_j = 1 - 2x_j$

x'_j $\qquad z'_j$

1 - true → -1

0 - false → 1

All combinations

$(1,0,0) \to (-1,1,1) \to 0$ $\qquad 000 \to 1$

$(1,0,1) \to (-1,1,-1) \to 1?$ $\qquad 010 \to 1$

$(110) \to (-1,-1,1) \to 0$

$(111) \to (-1,-1,-1) \to 0$

$(001) \to (1,1,-1) \to 1?$

$011 \to (1,-1,-1) \to 0$

did I just mess up the math? lets try again

Let's try again

$(x_1 \vee \bar{x}_2 \vee x_3) \wedge (\bar{x}_1 \vee x_2 \vee \bar{x}_3) \wedge (x_1 \vee x_2 \vee x_3)$

$C_1 = x_1 + (1-x_2) + x_3 - x_1(1-x_2) - (1-x_2)x_3 - x_1x_3 + x_1(1-x_2)x_3$

$= \cancel{x_1} + 1 - x_2 + \cancel{x_3} - \cancel{x_1} + x_1x_2 - \cancel{x_3} + x_2x_3 - \cancel{x_1x_3} + \cancel{x_1x_3} - x_1x_2x_3$

$= 1 - x_2 + x_1x_2 + x_2x_3 - x_1x_2x_3$

$C_2 = (1-x_1) + x_2 + (1-x_3) - (1-x_1)(1-x_3) - (1-x_1)(x_2)$

$-(1-x_3)x_2 + (1-x_1)(1-x_3)(x_2)$

$= 1 - x_1 + x_2 + 1 - x_3 - (1 - x_1 - x_3 + x_1x_3) - x_2 + x_1x_2$

$-x_2 + x_2x_3 + x_2(1 - x_1 - x_3 + x_1x_3)$

$= \cancel{1} - \cancel{x_1} + \cancel{x_2} + \cancel{1} - \cancel{x_3} - \cancel{1} + \cancel{x_1} + \cancel{x_3} - x_1x_3 - \cancel{x_2} + \cancel{x_1x_2}$

$-\cancel{x_2} + \cancel{x_2x_3} + \cancel{x_2} - \cancel{x_1x_2} - \cancel{x_2x_3} + x_1x_2x_3$

$= 1 - x_1x_3 + x_1x_2x_3$

$C_3 = x + x_2 + x_3 \cdot x_1x_2 - x_2x_3 - x_1x_3 + x_1x_2x_3$

$\sum C_i = 1 - \cancel{x_2} + \cancel{x_1x_2} + \cancel{x_2x_3} - x_1\cancel{x_2}x_3 + 1 - x_1x_3$

$+ \cancel{x_1x_2x_3} + x_1 + \cancel{x_2} + x_3 - \cancel{x_1x_2} - \cancel{x_2x_3} - x_1x_3 + x_1x_2x_3$

$= 2 + x_1 + x_3 - 2x_1x_3 + x_1x_2x_3$

$\underline{C_1}$ $\quad 1 - x_2 + x_1x_2 + x_2x_3 - x_1x_2x_3 \qquad x_1 \wedge \bar{x}_2 \wedge x_3$

	Yes	No
	else	010

000 - 1
001 - 1
010 - 0
011 - 1
100 - 1
101 - 1
110 - 1
111 - 1

$\underline{C_2}$ $\quad 1 - x_1x_3 + x_1x_2x_3 \qquad \bar{x}_1 \wedge x_2 \wedge \bar{x}_3$

	Yes	No
	else	101

000 - 1 ?
001 - 1
010 - 0
011 - 1

DAMN Cubics

$x_1x_2x_3 = \max\ w_i(x_1 + x_2 + x_3 - 2)$

if $x_1 \vee x_2 \vee x_3 = 0$, $w_i = 0$ else $w_i = 1$

$$\boxed{C_i = (1+w_i)(x_1+x_2+x_3) - x_1x_2 - x_1x_3 - x_2x_3 - 2w_i}$$

use this

$\underline{C_1}$ $x_1 \wedge \overline{x_2} \wedge x_3$

$$C_i = \left((1+w_i)(x_1+(1-x_2)+x_3) - x_1(1-x_2) - x_1x_3 - (1-x_2)x_3 - 2w_i\right)$$

$$= \cancel{x_1} + (1-x_2) + \cancel{x_3} + w_i x_1 + w_i - w_i x_2 + w_i x_3 - \cancel{x_1} + x_1x_2 - x_1x_3 - \cancel{x_3} + x_2x_3 - 2w_i$$

w_1

$$= 1 + w_i - x_2 + w_i x_1 - w_i x_2 + w_i x_3 + x_1x_2 - x_1x_3 + x_2x_3 - 2w_i$$

$$\rightarrow \quad = 1 - x_2 + x_1x_2 - x_1x_3 + x_2x_3 + w_1(x_1 - x_2 + x_3 - 1)$$

$\underline{C_2}$

$$C_2 = \left((1+w_2)((1-x_1)+x_2+(1-x_3)) - (1-x_1)x_2 - (1-x_1)x_3 - x_2x_3 - 2w_i\right)$$

$$= (1-x_1) + x_2 + (1-x_3) + w_2(1-x_1) + w_2x_2 + w_2(1-x_3) - x_2(1-x_1) - x_3(1-x_1) - x_2x_3 - 2w_i$$

$$= 1 - x_1 + \cancel{x_2} + 1 - x_3 + \cancel{w_2} - w_2x_1 + w_2x_2 + \cancel{w_2} - w_2x_3 - \cancel{x_2} + x_1x_2 - x_3 + x_1x_3 - x_2x_3 - \cancel{2w_i}$$

$$\rightarrow \quad = 2 - x_1 - 2x_3 + x_1x_2 + x_1x_3 - x_2x_3 + w_2(-x_1 + x_2 - x_3)$$

$\underline{C_3}$

$$C_3 = \left((1+w_3)(x_1+x_2+x_3) - x_1x_2 - x_2x_3 - x_1x_3 - 2w_3\right)$$

$$= x_1 + x_2 + x_3 + w_3x_1 + w_3x_2 + w_3x_3 - x_1x_2 - x_2x_3 - x_1x_3 - 2w_3$$

$$\rightarrow \quad = x_1 + x_2 + x_3 - x_1x_2 - x_1x_3 - x_2x_3 + w_3(x_1 + x_2 + x_3 - 2)$$

$$\sum C = 3 + 0x_1 + 0x_2 - 1x_3 + x_1x_2 - x_1x_3 - x_2x_3$$

$$\max\left(3 - x_3 + x_1x_2 - x_1x_3 - x_2x_3 + w_1(x_1 - x_2 + x_3 - 1) + w_2(-x_1 + x_2 - x_3) + w_3(x_1 + x_2 + x_3 - 2)\right)$$

↑ QUBO

$$= \begin{array}{c} x_1 \\ x_2 \\ x_3 \\ w_1 \\ w_2 \\ w_3 \end{array} \begin{bmatrix} 0 & 1 & -1 & 1 & -1 & 1 \\ 1 & 0 & -1 & -1 & 1 & 1 \\ -1 & -1 & -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 & 0 & 0 \\ -1 & 1 & -1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & -2 \end{bmatrix}$$

→ take out 3, add back later

I think I may have messed up my math here as this should theoretically work. I then tried an example I found online to make sure that the steps I took were correct. The decomposition for this is tough, a method should probably be produced to fully create the QUBO matrix, but it would have to be specific to the type of optimization problem.

Only 1 w variable since it relies on all 3

$$3 - x_3 + x_1x_2 - x_1x_3 - x_2x_3 + w(x_1 + x_2 + x_3 - 3)$$

$$Q = \begin{bmatrix} 0 & 1 & -1 & 1 \\ & 0 & -1 & 1 \\ & & -1 & 1 \\ & & & -3 \end{bmatrix}$$

System and QUBO from slides

$$(x_1 \vee x_2 \vee x_3) \wedge (\bar{x}_1 \vee x_2 \vee x_3) \wedge (x_1 \vee \bar{x}_2 \vee x_3) \wedge (\bar{x}_1 \vee x_2 \vee \bar{x}_3)$$

$$Q = \begin{bmatrix} 0 & -2 & 2 \\ & & \\ & & \\ & & \end{bmatrix}$$

Yes	No
001	000
011	010
110	100
111	101

$\underline{C_1}$ $x_1 \vee x_2 \vee x_3$ — crude method

CNOT, CCNOT

$$C_1 = x_1 + x_2 + x_3 - x_1x_2 - x_1x_3 - x_2x_3 + x_1x_2x_3$$

$$= x_1 + x_2 + x_3 - x_1x_2 - x_1x_3 - x_2x_3 + \omega_1(x_1 + x_2 + x_3 - 2)$$

$\underline{C_2}$ $\bar{x}_1 \vee x_2 \vee x_3$

$$C_2 = (1-x_1) + x_2 + x_3 - (1-x_1)x_2 - (1-x_1)x_3 - x_2x_3 + \omega_2((1-x_1) + x_2 + x_3 - 2)$$

$$= 1 - x_1 + \cancel{x_2} + \cancel{x_3} - \cancel{x_2} + x_1x_2 - \cancel{x_3} + x_1x_3 - x_2x_3 + \omega_2(-x_1 + x_2 + x_3 - 1)$$

$$= 1 - x_1 + x_1x_2 + x_1x_3 - x_2x_3 + \omega_2(-x_1 + x_2 + x_3 - 1)$$

$\underline{C_3}$ $x_1 \vee \bar{x}_2 \vee x_3$

$$C_3 = x_1 + (1-x_2) + x_3 - x_1(1-x_2) - x_3(1-x_2) - x_1x_3 + \omega_3(x_1 + (1-x_2) + x_3 - 2)$$

$$= \cancel{x_1} + 1 - x_2 + \cancel{x_3} - \cancel{x_1} + x_1x_2 - \cancel{x_3} + x_2x_3 - x_1x_3 + \ldots$$

$$= 1 - x_2 + x_1x_2 + x_2x_3 - x_1x_3 + \omega_3(x_1 - x_2 + x_3 - 1)$$

$\underline{C_4}$ $\bar{x}_1 \wedge x_2 \wedge \bar{x}_3$

$$C_4 = (1-x_1) + x_2 + (1-x_3) - (1-x_1)(1-x_3) - (1-x_1)x_2$$
$$- (1-x_3)x_2 + \omega_4((1-x_1) + x_2 + (1-x_3) - 2)$$

$$= 1 \cancel{- x_1} + \cancel{x_2} + 1 - \cancel{x_3} - 1 + \cancel{x_3} + \cancel{x_1} - x_1x_3 - \cancel{x_2} + x_1x_2 - x_2$$
$$+ x_2x_3 + \omega_4(-x_1 + x_2 - x_3)$$

$$= 1 - x_2 + x_1x_2 - x_1x_3 + x_2x_3 + \omega_4(-x_1 + x_2 - x_3)$$

Decomposition for Q_3 included in Jupyter Notebook ignore constant

$x_1: 0$ $x_1x_2: 2$ $x_1x_3: -2$ $x_1w_1: 1$ $x_1w_2: -1$ $x_1w_3: 1$ $x_1w_4: -1$

$x_2: -1$ $x_2x_3: 0$ $x_2w_1: 1$ $x_2w_2: 1$ $x_2w_3: -1$ $x_2w_4: 1$

$x_3: 1$ $x_3w_1: 1$ $x_3w_2: 1$ $x_3w_3: 1$ $x_3w_4: -1$

$w_1: -2$ $w_2: -1$ $w_3: -1$ $w_4: 0$

→ Flip all values to minimize, not maximize

$$Q = \begin{bmatrix} 0 & -2 & 2 & -1 & 1 & -1 & 1 \\ 0 & 1 & 0 & -1 & -1 & 1 & -1 \\ 0 & 0 & -1 & -1 & -1 & -1 & 1 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

QUBO

Summary

not producing optimal

MAX 3SAT: returns the maximum amount of satisfied clauses

Problem example: $C_1 \wedge C_2 \wedge C_3 \wedge C_4$

$C_1: x_1 \vee x_2 \vee x_3$ $\quad$ $C_3: x_1 \vee \bar{x}_2 \vee x_3$

$C_2: \bar{x}_1 \vee x_2 \vee x_3$ $\quad$ $C_4: \bar{x}_1 \vee x_2 \vee \bar{x}_3$

$x_i \in \{0, 1\}$ 1 - true
0 - false $\quad \bar{x}_i = (1 - x_i)$

inclusion - exclusion principle: $A \cup B \cup C$ = singletons - pairs + triples

$$A \cup B \cup C = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

$\cup \to \vee$ (or) $\cap \to \wedge$ (and) $\quad$ and ($\times$) or (+)

$$A \vee B \vee C = |A| + |B| + |C| - |A \wedge B| - |A \wedge C| - |B \wedge C| + |A \wedge B \wedge C|$$
$$= x_1 + x_2 + x_3 - x_1 x_2 - x_1 x_3 - x_2 x_3 + x_1 x_2 x_3$$

→ Can't have cubic terms (I tried it)

$$x_1 x_2 x_3 = \max w_i (x_1 + x_2 + x_3 - 2) \quad w_i \in \{0, 1\}$$

$w_i = 1$ if the triple is active

↑ can drop max since we will be maximizing later

# of clauses: $C_i = x_1 + x_2 + x_3 - x_1 x_2 - x_1 x_3 - x_2 x_3 + w_i (x_1 + x_2 + x_3 - 2)$

$$C_i = (1 + w_i)(x_1 + x_2 + x_3) - x_1 x_2 - x_1 x_3 - x_2 x_3 - 2 w_i$$

↳ if not, $x_1 \to (1 - x_1)$

Summary of steps I took to get Q_3 as I felt as the earlier pages were extremely messy

Max(# of satisfied clauses) = max($\sum C_i$) <u>simplification</u>

$$C_1 = x_1+x_2+x_3-x_1x_2-x_2x_3-x_1x_3+w_1(x_1+x_2+x_3-2)$$

$$C_2 = 1-x_1+x_1x_2+x_1x_3-x_2x_3+w_2(-x_1+x_2+x_3-1)$$

$$C_3 = 1-x_2+x_1x_2+x_2x_3-x_1x_3+w_3(x_1-x_2+x_3-1)$$

$$C_4 = 1-x_2+x_1x_2-x_1x_3+x_2x_3+w_4(-x_1+x_2-x_3)$$

$$\sum C_i = 0x_1+2x_1x_2-2x_1x_3+x_1w_1-x_1w_2+x_1w_3-x_1w_4$$
$$-x_2+0x_2x_3+x_2w_1+x_2w_2-x_2w_3+x_2w_4+x_3+x_3w_1+x_3w_2$$
$$+x_3w_3-x_3w_4-2w_1-w_2-w_3+0w_4$$

$$-\sum C_i = \begin{array}{c c} & \begin{array}{ccccccc} x_1 & x_2 & x_3 & w_1 & w_2 & w_3 & w_4 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \\ x_3 \\ w_1 \\ w_2 \\ w_3 \\ w_4 \end{array} & \begin{bmatrix} 0 & -2 & 2 & -1 & 1 & -1 & 1 \\ 0 & 1 & 0 & -1 & -1 & 1 & -1 \\ 0 & 0 & -1 & -1 & -1 & -1 & 1 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$$\max\left(\sum C_i\right) \to \min\left(-\sum C_i\right)$$

Solution Set

000 - No	110
001 - Yes	111
010 - No	011
011 - Yes	001
100 - No	
101 - No	
110 - Yes	
111 - Yes	

- Clause used n twice

- bar instead of not

- reference where i got inclusion exclusion

- equation 4 is not equality

$-3x_1 + 2x_1x_2$

- quadratic programming overview

- classical solutions

- tuples square brackets

111

110

011

$$
\begin{array}{c|cccccc}
 & x_1 & x_2 & x_3 & w_1 & w_2 & \cdots \\
\hline
x_1 & -3 & 2 & & & & \\
x_2 & & & & & &
\end{array}
$$