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→ Creating Expectation Values and Hamiltonians
↳ problem 3.4 → H_{-cut} minimization in qiskit

105 - Moving from QUBO and back

Subset Sum Problem $S = \{1, 3, 4, 7, -4\}$ $T = 6$
↳ Can elements of S add up to T?
↳ Problems like this can reduce to models like Ising models

→ Rewrite using binary variables x_j, $j \in [0, m]$

$$C(x_0, x_1, \ldots, x_m) = (a_0 x_0 + a_1 x_1 + \ldots + a_m x_m - T)^2$$

→ This is a reduction, but I am confused why we use binary values and not 0/1

↳ Example: $C(x_0, x_1, x_2, x_3, x_4) = (1x_0 + 3x_1 + 4x_2 + 7x_3 - 4x_4 - 6)^2$

↳ all zero values cancel and 1 contribute to sum
↳ if $\sum a_j x_j = T$, $C(x_0 \ldots x_4) = 0$
↳ since we are squaring our formulation, this is the minimum value

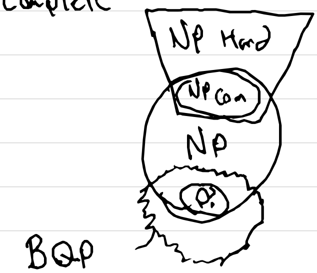
→ Instead of one set or other $(-1, 1)$, is in bag or not $(0, 1)$

QUBO: Quadratic Unconstrained Binary Optimization
↳ Quadratic since the expression is squared (don't want to go negative)
↳ Unconstrained: no restrictions on 0/1 selection
↳ Binary: Binary values are used

Form: $q(x_0 \ldots x_m)$

↳ at least as difficult as problems in NP

NP Hard - God Algorithm which can solve all problems in NP
↳ NP: Non-deterministic Polynomial Time
NP Complete: In both NP Hard and NP Complete



→ BQP: Bounded-error quantum polynomial time
↳ relationship to P is unknown

→ QUBO Problems are NP-Hard $\quad x: 0,1$
$\qquad z: -1,1$

Transformations

Ising → QUBO

$$z_j = 1 - 2x_j$$

↳ makes sense

QUBO → Ising

$$x_j = \frac{1-z_j}{2}$$

→ Squared values are always 1 in the Ising model

Exercise 3.5

Subset Sum → QUBO → Ising $\quad S = \{1,-2,3,-4\} \quad T = 0$

$$q(x_0 \dots x_3) = (x_0 - 2x_1 + 3x_2 - 4x_3 - 0)^2 = (x_0 - 2x_1 + 3x_2 - 4x_3)^2$$

$$= x_0^2 + 4x_1^2 + 9x_2^2 + 16x_3^2 - 4x_0x_1 + 6x_0x_2 - 8x_0x_3 - 12x_1x_2 + 16x_1x_3 - 24x_2x_3$$

$$= 30 - 4x_0x_1 + 6x_0x_2 - 8x_0x_3 - 12x_1x_2 + 16x_1x_3 - 24x_2x_3$$

$$x_0x_1 = \frac{(1-z_0)}{2} \cdot \frac{(1-z_1)}{2} = \frac{1}{4}(1 - z_1 - z_0 + z_0z_1)$$

$$= 30 - 4 + 2z_1 + 2z_0 - 2z_0z_1 + \frac{3}{2} - \frac{3}{2}z_0 - \frac{3}{2}z_2 + \frac{3}{2}z_0z_2 - 2 + 2z_0 + 2z_3 - 2z_0z_3$$
$$- 3 + 3z_1 + 3z_2 - 3z_1z_2 + 4 - 4z_1 - 4z_3 + 4z_1z_3 - 6 + 6z_2 + 6z_3 - 6z_2z_3$$

$$= \frac{47}{2} + \frac{3}{2}z_0 + 0z_1 + \frac{15}{2}z_2 + 4z_3 - z_0z_1 + \frac{3}{2}z_0z_2 - 2z_0z_3 - 3z_1z_2 + 4z_1z_3 - 6z_2z_3$$

$$= \frac{1}{2}(47 + 3z_0 + 15z_2 + 8z_3 - 2z_0z_1 + 3z_0z_2 - 4z_0z_3 - 6z_1z_2 + 8z_1z_3 - 12z_2z_3)$$

↑ minimize

→ Essentially, any QUBO → Ising

Other Problems

→ To me this appears like adding two subset sum problems

Binary Linear Programming

General: Minimize $c_0x_0 + c_1x_1 + \dots + c_mx_m$
subject to $Ax \leq b$
$x \in \{0,1\}$

→ Need to add slack variables to go from $\geq$ expression to an equality

Example in Book: $-5x_0 + 3x_1 - 2x_2$ $\quad x_0 + x_2 \leq 1$ range: $\{0,1\}$
$3x_0 - x_1 + 3x_2 \leq 4$ range: $\{-1, 4\}$

$y_0 \in \{0,1\}$
$x_0 + x_2 + y_0 = 1$ $\quad 3x_0 - x_1 + 3x_2 + y_1 + 2y_2 + 4y_3 = 4$
3 bits (0-7)

→ To achieve QUBO, we incorporate the constraints as penalty terms
↳ Minimize $-5x_0 + 3x_1 - 2x_2 + B(x_0 + x_2 + y_0 - 1)^2 + B(3x_0 - x_1 + 3x_2 + y_1 + 2y_2 + 4y_3 - 4)^2$

↳ QUBO form, B is selected to make sure answers which do not adhere to the conditions are not selected. This makes it unconstrained.

Famous Binary Linear Problem - Knapsack

each object has a weight and value. Goal is to maximize value without going over the weight

Knapsack → BLP $\quad$ Excercise 3.6: values: 3, 1, 7, 7
weights: 2, 1, 5, 4 $\quad$ max is 8

minimize $-3x_0 - x_1 - 7x_2 - 7x_3$
subject to $2x_0 + x_1 + 5x_2 + 4x_3 \leq 8$

QUBO
Range: $\{0, 12\}$ 4 bits needed
$2x_0 + x_1 + 5x_2 + 4x_3 + y_0 + 2y_1 + 4y_2 + 8y_3 = 8$

Minimize $-3x_0 - x_1 - 7x_2 - 7x_3 + B(2x_0 + x_1 + 5x_2 + 4x_3 + y_0 + 2y_1 + 4y_2 + 8y_3 - 8)^2$
↳ B needs to be selected based on the range of the value condition

min: -18 $\quad$ max: 0 $\quad$ $B = 19$ would work

Graphs

One color: $\sum x_{j,l} = 1$ $\quad$ 1 - l^{th} color $\quad$ 0 - else

No adjacent colors: $\sum x_{j,l}\, x_{n,l} = 0$

$$\text{Minimize } \sum_{j=0}^{m}\left(\sum_{l=0}^{k-1} x_{j,l} - 1\right)^2 + \sum_{(j,n)\in E}\sum_{l=0}^{k-1} x_{j,l}\, x_{n,l}$$

$x_{j,l} \in \{0,1\}$ $\quad j \in [0, m]^T$ $\quad l \in [0, k-1]$ $\quad$ k colors
j vertices

Instead