

NUFEB User Manual

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1 Introduction

This document provides information on how to download, compile, and start using NUFEB. NUFEB is a three-dimensional, open-source, and massively parallel Individual based Model (IbM) simulator, distributed under the terms of the GNU Public License. The purpose of NUFEB is to offer a flexible and efficient framework for simulating the growth and dynamics of microbial communities at the micro-scale. A comprehensive IbM is implemented in the solver which explicitly models biological, chemical and physical processes, as well as individual cells. The present solver supports parallel computing and the flexibility of extension and customisation.

NUFEB is implemented as a user package within LAMMPS (package USER-NUFEB). LAMMPS is a classical molecular dynamics simulator and primarily solves particle physics, including a wide range of inter-particle interactions and potentials. NUFEB aims to improve those capabilities with the goal to apply it to microbial community modelling.

In order to use NUFEB, it's important to have some familiarity with LAMMPS. The following links will be helpful:

1. LAMMPS homepage: <https://lammps.sandia.gov/>
2. LAMMPS user manual: <https://lammps.sandia.gov/doc/Manual.html>
3. LAMMPS developer guide: <https://lammps.sandia.gov/doc/Developer.pdf>

1.1 NUFEB features

The list below highlights NUFEB features, with pointers to specific commands which give more details.

Microbes and nutrients

(`atom_style`, `read_data_bio` commands)

- spherical microbes
- microbial species
- nutrients

Biological features

(`fix kinetics/growth/monod`, `fix kinetics/growth/energy`, `fix divide`, `fix eps_extract`, `fix death` commands)

- monod-based microbe growth (Heterotrophs, AOB, NOB, Comammox, EPS, DEAD)
- energy-based microbe growth
- microbe division
- EPS production
- microbe death

Chemical features

(`fix kinetics/ph`, `fix kinetics/diffusion`, `fix kinetics/thermo` commands)

- pH
- nutrient mass balance
- thermodynamics

- gas-liquid transfer

Physical features

(`fix epsadh`, `fix walladh`, `fix shear`, `fix cfddem`, `fix cohesive` commands),

- EPS adhesion
- wall adhesion
- shear force
- hydrodynamics
- cohesive force

Apart from the forces above, the LAMMPS framework offers a wide range of mechanical interactions that one can apply directly to microbial systems, for example, contact force `pair_style gran/hooke/history`, gravity `fix gravity`, and viscous force `fix viscous`.

Output

(`dump`, `dump vtk`, `dump bio`, `dump grid`, `dump bio/hdf5`, `thermo_style`, `compute ntypes`, `compute biomass`, `compute diameter`, `compute dimension`, `compute diversity`, `compute avg_height`, `compute avg_con`, `compute roughness`, `compute segregation` commands)

- output nutrient, pH and energy fields
- output microbe information
- compute and output characteristics of microbial communities
- output data in plain, VTK, HDF5 formats

2 NUFEB Compilation Instructions

This section covers instructions on how to compile NUFEB and get started. NUFEB can be compiled and run on almost any Linux or Mac OS. At the moment, the simulator does not support installation on Windows. However, there are a few ways to use NUFEB on Windows via compatibility layer or virtual machine. If you are using Windows 10 - consider to use Windows subsystem for Linux. Alternatively, you can use a virtual machine software and compile and run NUFEB in a Linux virtual machine

2.1 Pre-compilation instructions

Before compiling NUFEB, please make sure you have installed with the following packages depending upon the operating system used:

- gcc/g++ (<https://gcc.gnu.org/>)
- openmpi (<https://www.open-mpi.org/>)
- git (<https://git-scm.com/>)
- cmake (<https://cmake.org/>)

For example, you can run the following commands to install the packages, on Ubuntu:

```
$ sudo apt-get update
$ sudo apt-get install git-core g++ openmpi-bin openmpi-common libopenmpi-dev cmake
```

Some Ubuntu systems also requires OpenGL to be installed manually:

```
$ sudo apt-get install libglu1-mesa-dev freeglut3-dev mesa-common-dev
```

On Centos:

```
$ sudo yum update
$ sudo yum install git gcc-c++ openmpi openmpi-devel cmake
```

On MacOS, you need to use Homebrew to install the required libraries:

```
$ ruby -e "$(curl -fsSL https://raw.githubusercontent.com/Homebrew/
install/master/install)" < /dev/null 2> /dev/null
```

Then type the following command to install the libraries:

```
$ brew install gcc open-mpi cmake git
```

2.2 Downloading NUFEB

1. If you have GIT installed on your machine, you can use checkout and update commands to get the NUFEB files once and then stay current. To do this, use the clone command to create a local copy of NUFEB Github repository:

```
$ git clone --recursive https://github.com/nufeb/NUFEB.git
```

Once the command completes, a new directory named "NUFEB" will be created on your machine which contains the latest NUFEB source code, examples, LAMMPS and other useful thirdparty libraries.

After initial cloning, as bug fixes and new features are added to NUFEB, you can stay up-to-date by typing the following GIT commands from within the NUFEB/ directory:

```
$ git checkout master
$ git pull
```

2. You can also download NUFEB source tarball from NUFEB Github repository. Please note that this will not download LAMMPS source code and other thirdparty libraries. You will need to manually download:

- LAMMPS source code from LAMMPS Github repository (version-20Nov2019)
- VTK library from VTK Github repository (version-8.0.0);
- SediFOAM library from SediFOAM Github repository
- HDF5 library from HDF5 Github repository (version-1.10.5)

Then locate the tools in NUFEB/ or NUFEB/thirdparty/ directories.

2.3 Compiling NUFEB

This section gives instruction for compiling NUFEB. Before doing, please make sure you have gcc/g++, cmake (for building VTK library) and openmpi (for parallel run) installed on your system.

You can use the script file `install.sh` provided in `NUFEB/` directory to compile the code:

```
$ ./install.sh [--options]
```

The `--options` is the additional settings of the compilation. The table below lists all available options:

Option	Description
default	mpi version, no VTK, no HDF5
--serial	serial version
--enable-vtk	compile with VTK library
--enable-hdf5	compile with HDF5 library
--static	compile as a static library
--shared	compile as a shared library

Note that in order to compile NUFEB with VTK or HDF5 library, you will need to install the library provided in `thirdparty/` directory. Then compile the NUFEB with the corresponding argument. For example, to compile NUFEB with VTK library:

```
$ cd thirdparty
$ ./install-vtk
$ cd ..
$ ./install.sh --enable-vtk
```

When finished, you should have an executable “`lmp_mpi`” or “`lmp_serial`” in `NUFEB/lammps/src/` directory, and its path will be automatically added to the system.

2.4 Compiling NUFEB with fluid dynamics solver (CFD-DEM)

NUFEB employs and extends CFD-DEM solver SediFoam for the simulation of fluid dynamics. The current version of SediFoam is supported by OpenFoam 2.3.0, 2.3.x, 2.4.0. We recommend you refer the website <https://openfoamwiki.net/index.php/Installation/Linux/OpenFOAM-2.4.0> for the OpenFoam installation.

Note: If you are using Ubuntu 19.x or Ubuntu 20.x, follow the OpenFoam installation guide for Ubuntu 18.04. The only change is in the Steps 8 and 9, set the values of `WM_CC` and `WM_CXX` with `gcc-9` and `g++-9` respectively, as the default c/c++ compilers installed in the new Ubuntu releases is no longer 5.x:

```
$ export WM_CC='gcc-9'
$ export WM_CXX='g++-9'
$ echo "alias of240='source \$HOME/OpenFOAM/OpenFOAM-2.4.0/etc/bashrc $FOAM_SETTINGS;
export WM_CC=gcc-9; export WM_CXX=g++-9'" >> $HOME/.bashrc
```

Then use the script `install-sedifoam.sh` in `thirdparty/` directory to compile the code:

```
$ cd thirdparty
$ ./install-sedifoam.sh
```

Then choose the system you are using, and press enter.

2.5 Running NUFEB

Before running a NUFEB simulation, you may need to type the following command to make sure the VTK and/or HDF5 paths are added to `$LD_LIBRARY_PATH`.

```
$ bash
```

By default, NUFEB runs by reading commands from standard input. Thus if you run the NUFEB executable by itself, e.g.

```
$ mpirun -np 4 lmp_mpi
```

it will simply wait, expecting commands from the keyboard. Note that for parallel environments you also need to specify number of processors by using ‘-np X’ command-line switch.

Typically you should put commands in an input script and use I/O redirection, e.g.

```
$ mpirun -np 4 lmp_mpi -in Inputscript.lammps
```

If NUFEB is compiled as serial version, you can simply pass the input file into the executable “lmp_serial” in order to run a simulation:

```
$ lmp_serial < Inputscript.lammps
```

NUFEB provides numbers of examples in the `NUFEB/examples/` directory. To run the examples, go to one of the subdirectories and run NUFEB executable by passing in the input file, for example:

```
$ cd NUFEB/examples/biofilm-het/
$ mpirun -np 4 lmp_mpi -in Inputscript.lammps
```

or

```
$ mpirun -np 4 lmp_mpi -in Inputscript-vtk.lammps
```

if you enabled vtk library. Output files will be generated and saved in the same directory during the simulation.

2.6 Running NUFEB with fluid dynamics

Running hydrodynamic NUFEB is using `lammpFoam` executable compiled in Section 2.4. The file is located in `$FOAM_USER_APPBIN` directory.

NUFEB provides several hydrodynamic biofilm cases in `examples/PAPER-SOFTWARE` directory. To run the examples, go to one of the subdirectories and use `Allrun.sh` file:

```
$ cd NUFEB/examples/PAPER-NUFEB-TOOL/c1-deform-fixwall-0.6u
$ ./Allrun.sh
```


2.7 Post-processing

During the course of a simulation, NUFEB will regularly write and save data describing the system state (e.g, particle and field information) to files.

If you compile the NUFEB with VTK library, results can be in VTK format and readable by Paraview. See `dump vtk` and `dump grid` for more details. The NUFEB online tutorials provide instructions for visualising simulation results with Paraview.

NUFEB also provides routines for post-processing plain simulation outputs. The routine, which is located in `NUFEB/post-processing/` directory, supports for reading in and visualising the output file dumped by `dump custom` command. (Note that the routine does not support to visualise field information.) In order to use the routine, you will need to have the following software packages:

- POVray (<http://www.povray.org/>)
- MATLAB (<http://uk.mathworks.com/products/matlab/>)

To visualise output file, copy the output file produced by `dump custom` to `NUFEB/post-processing/` directory, then change to this directory and execute the “`run.sh`” script. This script will process the output file to generate a collection of images for each time point as well as a time-lapse video of the simulation in `0_images/` directory.

3 Input Script

In order to run NUFEB simulation, an inputscript (text file) is usually prepared with certain commands and parameters list. NUFEB executes by reading those commands and parameters, one line at a time. When the input script ends, NUFEB exits. Each command causes NUFEB to take some actions. It may set an internal variable, read in a file, or run a simulation.

This section explains the commands used for IBM simulation. We will focus on the newer capabilities and commands in NUFEB package. For the pre-existing LAMMPS commands, features and documentation, please refer to the LAMMPS user manual for more details.

3.1 Input script structure

This section describes the structure of a typical NUFEB input script. We will take the scripts in `NUFEB/examples/` directory as examples for the explanation.

A NUFEB input script typically has 4 parts:

- Initialisation
- Microbe and simulation domain definition
- Settings
- Run a simulation

3.1.1 Initialisation

Example:

```
atom_style    bio
atom_modify   map array sort 1000 5.0e-7
boundary      pp pp ff
newton        off
processors    * * *
comm_modify   vel yes
...
```

Set parameters that need to be defined before microbes are created or read-in from a file. Most of the commands for the initialisation are the pre-existing LAMMPS commands:

- `atom_style` command: define what style and attributes of atoms to use in the simulation.
- `atom_modify` commands: modify certain attributes of atoms defined and saved within NUFEB, in addition to what is specified by the `atom_style` command.
- `boundary` command: set the style of boundaries for the simulation domain in each dimension.
- `newton` command: turn Newton's 3rd law on or off for pairwise and bonded interactions.
- `processors` command: specify how processors are mapped as a regular 3d grid to the global simulation box.
- `comm_modify` command: sets parameters that affect the inter-processor communication of atom information.

3.1.2 Microbe, nutrient and simulation domain definition

Example:

```
...
read_data_bio atom.in
...
```

In NUFEB, `read_data_bio` command is used to initialise microbes, nutrients and simulation domain. The command reads in a data file containing information that NUFEB needs for running a simulation.

3.1.3 Settings

Example:

```
...
group HET type 1
...
neighbor      5.0e-7 bin
neigh_modify  delay 0 one 5000
pair_style    gran/hooke/history 1.e-4 NULL 1.e-5 NULL 0.0 1

timestep 10

variable EPSdens equal 30

fix fnl all nve/limit 1e-8
fix fv all viscous 1e-5
fix d1 all divide 100 v_EPSdens v_divMass 31231
...
dump id all custom 10 output.lammps id type diameter x y z
...
```

Once initial microbes, nutrients and simulation domain are defined, a variety of settings can be specified: force field, biological processes, chemical processes, output options, etc. The list below is the summary of the commands that can be used for a NUFEB simulation.

Pre-existing LAMMPS commands

Almost all of the LAMMPS commands are compatible with NUFEB package. Here we show some of them which are commonly used for modelling and simulation of microbial systems.

- **group** command: identify a collection of atoms (microbes) as belonging to a group. The group ID can then be used in other commands such as **fix**, **compute**, or **dump** to act on those atoms together.
- **lattice** command: define a lattice for use by other commands.
- **region** command: define a geometric region of space in computational domain.
- **create_atoms** command: create atoms on a lattice, or a single atom (or molecule), or a random collection of atoms in a region.
- **neighbor** command: set parameters that affect the building of pairwise neighbor lists.
- **neigh_modify** command: set parameters that affect the building and use of pairwise neighbor lists.
- **timestep** command: set the timestep size for subsequent molecular dynamics simulations (units: s).
- **pair_style gran/hooke/history** command: set formulas for the contact force between two granular particles.
- **variable** command: assign one or more strings to a variable name for evaluation later in the input script or during a simulation.
- **fix** command: set a fix that will be applied to a group of atoms. In LAMMPS, a “fix” is any operation that is applied to the system during timestepping.

- `fix nve/limit` command: perform constant NVE (Number, Volume and Energy) updates of position and velocity for atoms in the group each timestep.
- `fix nve/sphere` command: perform constant NVE integration to update position, velocity, and angular velocity for finite-size spherical particles in the group each timestep.
- `fix viscous` command: add a viscous damping force to atoms in the group that is proportional to the velocity of the atom.
- `fix wall/gran` command: bound the simulation domain of a granular system with a frictional wall.
- `compute` command: define a computation that will be performed on a group of atoms.
- `dump custom` command: dump a snapshot of atom quantities to one or more files every N timesteps.
- `dump vtk` command: dump a snapshot of atom quantities to one or more files every N timesteps in a format readable by the VTK visualisation toolkit.
- `thermo` command: compute and print thermodynamic info (e.g. temperature, energy, pressure) on timesteps at the beginning and end of a simulation.
- `thermo_style` command: set the style and content for printing thermodynamic data to the screen and log file.
- `restart` command: write out a binary restart file with the current state of the simulation every so many timesteps.

NUFEB commands

- `fix kinetics` command: set parameters for a variety of kinetics computations.
- `fix kinetics/growth/monod` command: perform microbe growth and decay based on Monod kinetic.
- `fix kinetics/growth/energy` command: perform microbe growth and decay based on metabolic energy.
- `fix divide` command: perform microbe division.
- `fix eps_extract` command: perform EPS (Extracellular Polymeric Substances) production process on HETs (Heterotrophs).
- `fix death` command: perform microbe death process.
- `fix kinetics/ph` command: add the calculations of pH, iron strength and nutrient activity that effect the energy-based growth kinetics.
- `fix kinetics/thermo` command: add thermodynamic and liquid-gas transfer calculations that effect the energy-based growth kinetics.
- `fix kinetics/diffusion` command: solve mass balance of soluble substrates in biofilm and bulk liquid.
- `fix epsadh` command: bound microbes with EPS adhesive force
- `fix walladh` command: impose an adhesive force between wall and the microbes attaching to the wall.
- `fix shear` command: impose an additional shear force for each microbe in the group.
- `compute ntypes` command: define a computation that calculates total number of each species in the system.
- `compute biomass` command: define a computation that calculates total biomass of each species in the system.
- `compute diameter` command: define a computation that calculates floc equivalent

diameter.

- `compute dimension` command: define a computation that calculates fractal dimension.
- `compute diversity` command: define a computation that calculates diversity index of all species in the system
- `compute avg_height` command: define a computation that calculates biofilm average height.
- `compute roughness` command: define a computation that calculates biofilm roughness.
- `compute segregation` command: define a computation that calculates biofilm segregation index.
- `dump bio` command: dump microbe, biofilm, floc and kinetics information to one or more files every N timesteps.
- `dump grid` command: dump field information (pH, concentration, energy, etc) to one or more files every N timesteps in a format readable by the VTK visualisation toolkit.
- `read_bio_restart` command: read in a previously saved IbM simulation configuration from a restart file.

3.1.4 Run a simulation

Example:

```
...  
run 10000
```

A molecular dynamics simulation is run using the `run` command. Also see Appendix A for defining a multi-timescale run.

3.2 `atom_style` command

Syntax

```
atom_style bio
```

- *bio*: atom style for describing microbes

Description

Define a biological atom style used in IbM simulation. Classical LAMMPS provides different atom types that could be used by user. These are specified in the input script by the command: `atom_style`. Command must be used before a simulation is setup via `read_data_bio` command. A newer `atom_style`, named "bio", is added to describe microbe attributes. For the new atom style, the particles are spheres and each stores a per-particle diameter, mass, outer diameter, outer mass, etc.

3.3 read_data_bio command

Read in a data file containing information NUFEB needs to run a simulation, i.e, microbe, species, nutrient and computation domain. The file can be ASCII text or a gzipped text file (detected by a .gz suffix). The structure of the data file is important, though many settings and sections are optional or can come in any order. A typical example is the data file “atom.in ” in NUFEB/examples/biofilm-monod/ directory.

Format of the header of a data file

A data file has a header and a body. The header appears first. The first line of the header is always skipped; it typically contains a description of the file. Lines can have a trailing comment starting with ‘#’ that is ignored. If the line is blank (only whitespace after comment is deleted), it is skipped. If the line contains a header keyword, the corresponding value(s) is read from the line. If it doesn’t contain a header keyword, the line begins the body of the file.

Header lines can come in any order. The value(s) are read from the beginning of the line. Thus the keyword atoms should be in a line like “44 atoms”; the keyword ylo yhi should be in a line like “0.0 1.0e-04 ylo yhi”. The following list is the headers that are required for running a NUFEB simulation.

- atoms = # of atoms (microbes) in system
- atom types = # of types (microbial species) in system
- nutrients = # of nutrients in system
- xlo xhi = simulation box boundaries in x dimension (horizontal)
- ylo yhi = simulation box boundaries in y dimension (horizontal)
- zlo zhi = simulation box boundaries in z dimension (vertical)

Example:

```
IbM Simulation

44  atoms
5   atom types
5   nutrients

0.0 1.0e-04 xlo xhi
0.0 4.0e-05 ylo yhi
0.0 1.0e-04 zlo zhi

...
```

Format of the body of a data file

The body of the file contains zero or more sections. The first line of a section has only a keyword. The next line is skipped. The remaining lines of the section contain values. The number of lines depends on the section keyword as described below. Zero or more blank lines can be used between sections. Sections can appear in any order, with a few exceptions as noted in the following sections.

These are the section keywords for the body of the file.

- Atoms, Type Name, Growth Rate, Consumption Rate, Yield, Maintenance, Decay, Electron Donor, Dissipation.
- Nutrients, Diffusion Coeffs, Mass Transfer Coeffs.
- Ks, Catabolism Coeffs, Anabolism Coeffs, Decay Coeffs, Nutrient Energy, Type Energy, Charge Number.

3.3.1 Atoms section

- one line per atom
- line syntax: atom-ID type-ID inner-diameter density x y z outer-diameter
- unit: diameter (m), density (kg m^{-3})

Example:

Atoms

```
1 1 1.0e-6 150 0.5e-5 0.5e-5 1e-6 1.2e-6
2 2 1.0e-6 150 1.5e-5 0.5e-5 1e-6 1.0e-6
```

Define initial atoms (microbes) and types (microbial species) in the system. The Atoms section must appear before all other sections in the data file. The atoms can be listed in any order. For atom_style bio, the particles are spheres. atom-ID is used to identify the atom throughout the simulation and in dump files. Normally, it is a unique value from 1 to N atoms for each atom. The type-ID is a 2nd identifier attached to an atom. Normally, it is a number from 1 to N, identifying which species the microbe belongs to. The inner-diameter and outer-diameter specify the inner and outer sizes of a finite-size spherical microbe. Organism such as Heterotroph (HET) excretes Extracellular Polymeric Substances (EPS) which is initially accumulated as a extra shell beyond the particle. The outer-diameter of these species is the initial size of EPS shell which must be greater or equal than the inner-diameter. For the species that do not produce EPS, their outer-diameter should be equal to the inner-diameter. The density is used in conjunction with the particle volume to set the mass of each particle as $\text{mass} = \text{density} \times \text{volume}$. x,y,z specify the (x,y,z) coordinates of atoms. These must be inside the simulation box.

3.3.2 Nutrients section

- one line per nutrient
- line syntax: nutrient-ID nutrient-name nutrient-status=g/l xbc ybc zbc S_{domain}
 S_{bc}
- units: fix kinetics/growth/energy = mol L^{-1} ;
fix kinetics/growth/monod = kg m^{-3}

Example:

Nutrients

```
1 o2 l pp pp nd 0.002 0.002
2 nh4 l nn nn nn 0 1e-3
3 go2 g nn nn nn 1e-3 1e-3
```

Define nutrients and their initial settings in the system. The Nutrients section must appear before all other sections except Atoms section in the data file. nutrient-ID is used to identify the nutrient throughout the simulation. The nutrient-ID is a 2nd identifier attached to a nutrient. Normally, it is a number from 1 to N. nutrient-name assigns a string to each nutrient which is used to define nutrient attribute parameters in other sections. nutrient-status can be either l (liquid) or g (gas). It is suggested that the nutrient-name of any gas nutrient should start with prefix 'g', e.g, go2 or gco2.

Three types of boundary conditions can be defined on each domain surface: n = Neumann, d = Dirichlet and p = Periodic. For example, $zbc = nd$ applies Neumann BC on zlo surface and Dirichlet on zhi surface, while $xbc = pp$ defines periodic BC on xlo and xhi surfaces. S_{domain} defines the initial concentration within the simulation domain. S_{bc} define the inlet concentrations on six boundary surfaces. The unit of concentration depends on the growth command used in the simulation. S is considered as in mol L^{-1} if fix kinetics/growth/energy command is defined, while S is in $kg\ m^{-3}$ if fix kinetics/growth/monod is defined.

3.3.3 Type Name section

- one line per type
- line syntax: type-ID type-name

Example:

Type Name

```
1 het
2 aob
```

Assign a string to each species type defined in Atoms section. The type name must be in accordance with the type-ID. Such name is used to define species attribute parameters in other sections.

3.3.4 Growth Rate section

- one line per type
- line syntax: type-name value
- units: s^{-1}

Example:

Growth Rate

```
het 0.0000695
aob 0.0000088
```

Define maximum specific growth rate of each species. The Growth Rate section must be defined if `fix kinetics/growth/monod` command is used in the simulation.

3.3.5 Yield section

- one line per type
- line syntax: type-name value
- units: `fix kinetics/growth/energy` = mol mol⁻¹;
`fix kinetics/growth/monod` = kg kg⁻¹

Example:

Yield

```
het 0.61
aob 0.33
```

Define biomass yield of each species. The Yield section must be defined if `fix kinetics/growth/energy` or `fix kinetics/growth/monod` command is used in the simulation.

3.3.6 Consumption Rate section

- one line per type
- line syntax: type-name value
- units: mol mol⁻¹ s⁻¹

Example:

Consumption Rate

```
aob 0.000283611
nob 0.000702222
```

Define maximum specific nutrient consumption rate of each species. The Consumption Rate section must be defined if `fix kinetics/growth/energy` command is used in the simulation.

3.3.7 Maintenance section

- one line per type
- line syntax: type-name value
- units: `fix kinetics/growth/energy` = mol mol⁻¹ s⁻¹;
`fix kinetics/growth/monod` = s⁻¹

Example:

Maintenance

aob 0.000123376

nob 0.000134422

Define maintenance rate of each species. The Maintenance section must be defined if `fix kinetics/growth/energy` or `fix kinetics/growth/monod` command is used in the simulation.

3.3.8 Decay Rate section

- one line per type
- line syntax: type-name value
- units: `fix kinetics/growth/energy` = mol mol⁻¹ s⁻¹;
`fix kinetics/growth/monod` = s⁻¹

Example:

Decay Rate

aob 0.000003694

nob 0.00000127314

Define decay rate of each species. The Decay Rate section must be defined if `fix kinetics/growth/energy` or `fix kinetics/growth/monod` command is used in the simulation.

3.3.9 Electron Donor section

- one line per type
- line syntax: type-name value

Example:

Electron Donor

aob 0.9

nob 2.9

Define electron donor of each species. The Electron Donor section must be defined if `fix kinetics/thermo` command is used with the argument `f_yield = unfix`.

3.3.10 Dissipation section

- one line per type
- line syntax: `type-name value`
- units: kJ mol^{-1}

Example:

```
Dissipation

aob 3500
nob 3500
```

Define dissipation constant of each species. The Dissipation section must be defined if `fix kinetics/thermo` command is used with the argument `f_yield = unfix`.

3.3.11 Diffusion Coeffs section

- one line per nutrient
- line syntax: `nutrient-name value`
- units: $\text{m}^2 \text{s}^{-1}$

Example:

```
Diffusion Coeffs

o2 2e-9
go2 0
```

Define diffusion coefficient of each nutrient. The Diffusion Coeffs section must be defined if `fix kinetics/diffusion` command is used in the simulation.

3.3.12 Mass Transfer Coefficient section

- one line per nutrient
- line syntax: `nutrient-name value`
- units: s^{-1}

Example:

```
KLa

o2 0.0056
go2 0.0056
```

Define mass transfer coefficient (KLa) of each nutrient. The Mass Transfer Coeffs section must be defined if `fix kinetics/thermo` command is used with the argument `f_reaction=close`.

3.3.13 Ks section

- one line per type
- line syntax: `type-name value-1 value-2 ... value-Nnutrient`
- units: `fix kinetics/growth/energy` = mol L⁻¹;
`fix kinetics/growth/monod` = kg m⁻³

Example:

```
Nutrients

1 nh3 1 0.002 0.002 0.002 0.002 0.002 0.002 0.002
2 o2 1 2.8e-4 2.8e-4 2.8e-4 2.8e-4 2.8e-4 2.8e-4 2.8e-4

Ks

aob 1.71e-04 1.88e-05
```

Define half-velocity constants of each species. The Ks section must be defined if `fix kinetics/growth/energy` or `fix kinetics/growth/monod` command is used in the simulation. The order of the Ks values of each species must be consistent with the nutrient IDs defined in the Nutrients section. In the above example, $K_{s_{nh3}}=1.71e-04$, and $K_{s_{o2}}=1.88e-05$. $K_{s_{nutrient}}=0$ indicates that the nutrient will not be taken into account when solving Monod equation.

3.3.14 Catabolism Coeffs section

- one line per type
- line syntax: `type-name value-1 value-2 ... value-Nnutrient`

Example:

```
Nutrients

1 nh3 1 0.002 0.002 0.002 0.002 0.002 0.002 0.002
2 no2 1 1e-3 1e-3 1e-3 1e-3 1e-3 1e-3 1e-3

Catabolism Coeffs

aob -1 1
```

Define catabolism coefficients of each species. The section must be defined if `fix kinetics/growth/energy` or `fix kinetics/thermo` command is used in the simulation. The order of the coefficients of each species must be consistent with the nutrient IDs defined in the Nutrients section.

3.3.15 Anabolism Coeffs section

- one line per type
- line syntax: type-name value-1 value-2 ... value-Nnutrient

Example:

```
Nutrients

1 nh3 1 0.002 0.002 0.002 0.002 0.002 0.002 0.002
2 no2 1 1e-3 1e-3 1e-3 1e-3 1e-3 1e-3 1e-3

Aatabolism Coeffs

aob -0.9 0.7
```

Define anabolism coefficients of each species. The section must be defined if `fix kinetics/growth/energy` or `fix kinetics/thermo` command is used for the simulation. The order of the coefficients of each species must be consistent with the nutrient IDs defined in the `Nutrients` section.

3.3.16 Decay Coeffs section

- one line per species
- line syntax: species-name value-1 value-2 ... value-Nnutrient

Example:

```
Nutrients

1 nh3 1 0.002 0.002 0.002 0.002 0.002 0.002 0.002
2 no2 1 1e-3 1e-3 1e-3 1e-3 1e-3 1e-3 1e-3

Decay Coeffs

aob -0.9 0.7
```

Define decay coefficients of each species. The section must be defined if `fix kinetics/growth/energy` or `fix kinetics/growth/monod` command is used in the simulation. The order of the coefficients of each species must be consistent with the nutrient IDs defined in the `Nutrients` section.

3.3.17 Nutrient Energy section

- one line per nutrient
- line syntax: nutrient-name not-hydrated-form fully-protonated0form 1st-deprotonated-form 2nd-deprotonated-form 3rd-deprotonated-form form-flag

Example:

Nutrient Energy

```
nh3 inf -79.37 -26.57 inf inf 3
o2  inf 16.4   inf   inf inf 2
```

Define Gibbs energy value for each defined nutrient. The section must be defined if `fix kinetics/growth/energy` command is used in the simulation.

3.3.18 Type Energy section

- one line per type
- line syntax: type-name not-hydrated-form fully-protonated-form 1st-deprotonated-form 2nd-deprotonated-form 3rd-deprotonated-form form-flag

Example:

Type Energy

```
aob inf -67 inf inf inf 2
```

Define Gibbs energy value for each defined type. The section must be defined if `fix kinetics/growth/energy` command is used in the simulation.

3.3.19 Charge Number section

- one line per nutrient
- line syntax: nutrient-name not-hydrated-form fully-protonated-form 1st-deprotonated-form 2nd-deprotonated-form 3rd-deprotonated-form

Example:

Charge Number

```
nh3 na 1 0 na na
no2 na 0 -1 na na
```

Define the charge number of ion for each nutrient. The section must be defined if `fix kinetics/ph` command is used in the simulation.

3.4 fix kinetics command

Syntax

```
fix ID group-ID kinetics Nevery nx ny nz v_diffT v_layer  
keyword value
```

- *ID* : user-assigned name for the fix
- *group-ID* : ID of the group of microbes to apply the fix to
- *kinetics* : style name of this fix command
- *Nevery* : call kinetics-related functions every this many timesteps
- *nx, ny, nz* : number of grid elements, x, y, z planes
- *v_diffT* : diffusion timestep (s)
- *v_layer* : thickness of boundary layer (m)
- optional keyword = *temp* or *rth* or *gvol* or *rg* or *ph* or *demflag* or *niter*

temp value = temperature (default: 298.15K)

rth value = universal gas constant for thermo ($8.3144e^{-3}$ kJ mol⁻¹ K⁻¹)

demflag value = 0 or 1 = flag of DEM run (0)

niter value = maximum # of iterations in kinetics integration (-1)

Required data sections

Atoms, Nutrients, Nutrient Energy* sections

* If fix kinetics/growth/energy command is used.

Examples

```
variable diffT equal 1e-4  
variable layer equal 2e-4  
  
fix k1 all kinetics 100 60 12 30 v_diffT v_bl  
fix k1 all kinetics 100 30 6 20 v_diffT v_bl niter 1000
```

Description

Set parameters and perform kinetics integration process. The procedure is shown in Algorithm 1 and 2 depending on the use of monod or energe growth model. The integration invokes several sub-fix commands and executes them in sequence.

nx, ny, nz set the number of Cartesian grids in *x, y*, and *z* directions of the simulation box, respectively. NUFEB uses cubic Cartesian coordinate system. Thus, it is important to make sure the equal length of grid edges when assign the values. For example, for the simulation domain with $size = 1e^{-4} \times 5e^{-5} \times 1e^{-4}$, $nx = 20, ny = 10, nz = 20$ is a valid partition, which generates 4000 3D cubic grids. The edge length of each grid is $5e^{-6}(m)$.

v_layer defines thickness of boundary layer between the maximum height of biofilm and bulk. This boundary layer forms part of the pure liquid region within the domain, and in this region only diffusion governs the local concentration. Beyond the boundary layer region the liquid is assumed to be well-mixed, and so the solute concentrations are

Algorithm 1 (kinetics integration procedure (Monod based growth))

```
1: while nutrient concentration does not reach steady state OR # of iteration < niter
   do
2:   perform reaction update based on nutrient consumption
     (fix kinetics/growth/monod)
3:   update nutrient concentration field (fix kinetics/diffusion)
4: end while

5: perform microbe growth ( fix kinetics/growth/monod)
6: update bulk concentration(fix kinetics/diffusion)
7: update boundary layer position
```

Algorithm 2 (kinetics integration procedure (energy based growth))

```
1: while nutrient concentration does not reach steady state OR # of iteration < niter
   do
2:   update pH field (fix kinetics/ph)
3:   perform reaction update based on gas-liquid exchange (fix kinetics/thermo)
4:   perform reaction update based on nutrient consumption
     (fix kinetics/growth/energy)
5:   update nutrient concentration field (fix kinetics/diffusion)
6: end while

7: perform buffer pH(fix kinetics/ph)
8: update bulk concentration(fix kinetics/diffusion)
9: update boundary layer position
```

kept equal to their concentration in an attached bulk compartment. Note that if v_{layer} is less than zero or greater than z_{hi} , it is assumed that the boundary layer region is always from biofilm to z_{hi} .

temp and *rth* are the optional parameters for the calculation of Gibb free energy.

demflag is used to manually switch on or off kinetics integration process. The process will be suspended during mechanical integration if *demflag* = 1. You can find more details on how to use this setting in Appendix A.

niter sets the maximum number of iteration for solving nutrient mass balance.

3.5 fix kinetics/growth/monod command

Syntax

```
fix ID group-ID kinetics/growth/monod keyword value
```

- *ID* : user-assigned name for the fix
- *group-ID* : ID of the group of microbes to apply the fix to
- *kinetics/growth/monod* : style name of this fix command
- optional keyword = *epsdens* or *etahet*

epsdens value = EPS density (default: 30 kg m⁻³)

etahet value = reduction factor in HET anoxic growth (default: 0)

Required data sections

Atoms, Nutrients, Growth Rate, Yield, Maintenance, Ks, Decay Rate sections

Examples

```
fix kgm all kinetics/growth/monod
fix kgm HET kinetics/growth/monod epsdens 35
fix kgm AOB kinetics/growth/monod epsdens 30 etahet 0.6
```

Description

Perform microbe growth based on Monod kinetics. Four commonly found microbial functional groups and two inert states are implemented: heterotrophs (HET), ammonia oxidizing bacteria (AOB), nitrogen oxidizing bacteria (NOB), anammox (ANA), extracellular polymeric substances (EPS) and inerts (DEAD). The process of growth and decay is described by the following ordinary differential equation:

$$\frac{dm_i}{dt} = r_i m_i$$

where m_i is the mass of the particulate microbe of the i th microbe and r_i is the specific growth rate. r is determined by Monod kinetic equation driven by the local concentration of nutrients (substrate, nh4, no2, o2). The decay is assumed to be first order. The stoichiometric matrix for the particulate and soluble components can be found in table1 and table2 in appendix.

For each species defined in system, the corresponding growth model is recognised by the first three characters of its type name given in Type Name section. For example, species with type name 'het', 'het-1' or 'heterotrophic' will be considered as in heterotrophs functional group. For the nutrients involved in the growth model, they need to be defined in a given manner: sub (substrate), o2 (oxygen), nh4 (ammonium), no2 (nitrite), and no3 (nitrate). It is possible to define multiple species within the same functional group. For example, having two heterotrophic species 'hetr' and 'hety' with different growth rates and yield coefficients in data file would be:

```

....
Nutrients

    1 sub 1 1e-4 1e-4 1e-4 1e-4 1e-4 1e-4 1e-4
    2 o2 1 5e-5 5e-5 5e-5 5e-5 5e-5 5e-5 5e-5

Type Name

    1 hetr
    2 hety

Growth Rate

    hety 0.000022778
    hetr 0.000045556

Yield

    hety 0.046
    hetr 0.023
....

```

If HET and EPS are defined in the system, *epsdens* is the optional parameters to set the density of HET shell. *etahet* is the reduction factor of HET anoxic growth. Nutrients no2 and no3 have to be defined if *etahet* is greater than 0.

3.6 fix kinetics/growth/energy command

Syntax

```
fix ID group-ID kinetics/growth/energy v_EPSdens
```

- *ID* : user-assigned name for the fix
- *group-ID* : ID of the group of microbes to apply the fix to
- *kinetics/growth/energy* : style name of this fix command
- *v_EPSdens* : EPS density (kg m⁻³)

Required data sections

Atoms, Nutrients, Consumption Rate, Yield, Maintenance, Ks, Catabolism Coeffs, Anabolism Coeffs, Decay Rate sections.

Examples

```
variable EPSdens equal 30  
  
fix kge all kinetics/growth/energy v_EPSdens
```

Description

Perform microbe growth and decay based on metabolic energy. The growth of each bacteria is described by calculating the amount of energy available for its metabolism in each grid, see (Gogulancea V *et al.* 2019) for more details¹

$$\begin{aligned}\mu &= Y^{max} \cdot (q^{met} - m^{req}) && \text{if } q^{met} > m^{req} \\ \mu &= 0 && \text{if } q^{met} = m^{req} \\ \mu &= -k_d \frac{q^{met} - m^{req}}{m^{req}} && \text{if } q^{met} < m^{req}\end{aligned}$$

The maximum growth yield Y^{max} can be either dynamic which is calculated using Energy Dissipation Method implemented in `fix kinetics/thermo` command, or static which is defined in Yield section. m^{req} is the maintenance rate defined in Maintenance section. q^{met} is the metabolic rate which depends on the availability of limiting nutrients (in particular, their dissociation forms). k_d is the decay rate defined in Decay section. Then, microbial growth is considered only if the cell is harvesting more energy than the necessary to maintain. Otherwise, microbe will maintain or decay linearly with the lack of energy with a constant. The function also calculates consumption rate R for each microbe based on the nutrient concentration of the grid where the microbe reside. The results will be used for solving diffusion-reaction equation in `kinetics/diffusion` function.

¹Gogulancea V, Gonzalez-Cabaleiro R, Li B, et al. Individual based model links thermodynamics, chemical speciation and environmental conditions to microbial growth. *Frontiers in Microbiology* 2019, 10, 1871.

3.7 fix kinetics/ph command

Syntax

```
fix ID group-ID kinetics/ph
```

- *ID* : user-assigned name for the fix
- *group-ID* : ID of the group of microbes to apply the fix to
- *kinetics/ph* : style name of this fix command
- *fix* or *dynamic*: fix pH or dynamic pH (default: *fix*)
- optional keyword = *ph* or *buffer*

ph value = influence ph (default: 7.5)

buffer value = *phlo phhi* = ph buffer (default: 6.5 9)

Required data sections

Nutrients, Charge Number sections.

Examples

```
fix kge all kinetics/ph fix
fix kge all kinetics/ph dynamics ph 7.5 buffer 6 8
```

Description

The fix enables pH dynamics and updates the concentrations of nutrient dissociation forms in the liquid solution. A Newton-Raphson implicit method is implemented to find the root of the following equation:

$$[H+] - [OH^-] = \sum_j^{i=1} z[S^{z-}] \quad (1)$$

where j is the total number of all the dissociation forms contributing to the pH, z is the number of charge of each form defined in Charge Number section, C^{z-} is the concentration of each form calculated based on Gibbs free energy and temperature.

Keyword pH defines initial pH value. *dynamics* enables pH dynamics, while the keyword *fix* disables pH dynamics calculation and a constant pH is applied.

The optional keyword *buffer* enables buffering for pH regulation. *phlo* and *phhi* defines pH range. Two additional chemical species (Na and CL) must be defined in the Nutrients section if this keyword is used.

3.8 fix kinetics/thermo command

Syntax

```
fix ID group-ID kinetics/thermo keyword value
```

- *ID* : user-assigned name for the fix
- *group-ID* : ID of the group of microbes to apply the fix to
- *kinetics/thermo* : style name of this fix command
- optional keyword = *reactor* or *yield* or *pressure*

yield value = *fix* or *dynamic* = flag of dynamic yield (default: *fix*)

reactor value = *close* or *open* = flag of reactor state (default: *open*)

gvol value = gas volume ($8e^{-14}$ L)

rg value = universal gas constant for gas transfer ($0.08205746 \text{ atm L mol}^{-1} \text{ K}^{-1}$)

Required data sections

Atoms, Nutrients, Catabolism Coeffs, Anabolism Coeffs, Nutrient Activity Coeffs, Type Activity Coeffs, Dissipation*, Mass Transfer Coeffs**, Electron Donor* sections.

* If *yield-flag* = *dynamic*.

** If *reactor-flag* = *close*.

Examples

```
fix kge all kinetics/thermo
fix kge all kinetics/thermo yield dynamic reactor close
```

Description

Calculate Gibbs free energy and liquid-gas transfer that affect the energy-based growth kinetics. The Gibbs free energy of catabolism ΔG_{cat} and anabolism ΔG_{ana} is given by:

$$\Delta G = \Delta G_0 + R_{th}T(M^T \ln(S_{liq}))$$

where R_{th} and T are the universal gas constant and temperature defined in `fix kinetics` command respectively. M is the catabolism and anabolism coefficients derived from Catabolism Coeffs and Anabolism Coeffs sections, and S_{liq} is the concentrations of nutrient dissociation forms calculated in `fix kinetics/ph` command.

If *yield* = *dynamic*, a dynamic yield value will be calculated at each grid. The growth yield is calculated by assuming that the Gibbs energy values of the anabolic and catabolic equations plus an energy dissipation term give a fair approximation:

$$Y = -\frac{\Delta G_{cat}}{\Delta G_{ana} + \Delta G_{dis}} + eD$$

where ΔG_{dis} is the dissipation constant defined in Dissipation section and eD is electron donor defined in Electron Donor section.

Some of the soluble components might transfer in an important proportion to the gas phase. The equilibrium between the liquid and the gas, disturbed by the acid-base reactions and the bacterial activity occurring in the liquid, could move towards production and/or consumption of mass that would affect the reaction term when solving diffusion-reaction. Flag *reactor = close* triggers the gas field to be considered in the simulation domain. To solve gas-liquid transfer of any chemical component, both of its liquid and gas status must be given in Nutrients section. For example, if liquid dioxide is named as 'co2', then its corresponding gas status 'gco2' must be defined in order to solve for their gas-liquid transfer.

The reaction rate from gas to liquid $R_{G \rightarrow L}$ of a given nutrient can be expressed by the following equation :

$$R_{G \rightarrow L} = K_L a \cdot (S_{gas} - \frac{S_{liq}}{K_H})$$

here the rate is determined by its mass transfer coefficient $K_L a$ defined in Mass Transfer Coeffs section, the gas concentration S_{gas} in the head space, the concentration of dissociation form S_{liq} , and Henry's constant K_H . The transference from liquid to gas is formalised as follow:

$$R_{L \rightarrow G} = \frac{-R_{G \rightarrow L} V_{gas}}{R_g T}$$

where V_{gas} and R_g are the gas volume of the head space (gvol), and ideal gas constant (rg) defined in the command respectively.

3.9 fix kinetics/diffusion command

Syntax

```
fix ID group-ID kinetics/diffusion v_tol keyword value
```

- *ID* : user-assigned name for the fix
- *group-ID* : ID of the group of microbes to apply the fix to
- *kinetics/diffusion* : style name of this fix command
- *v_tol* : absolute tolerance for convergence
- *xbc, ybc, zbc*: boundary condition mode flag on x, y and z planes (*xbc = ybc = zbc = nn or nd or pp or dn or dd*)
- optional keyword = *bulk* or *srate* or *closedflag*

bulk values = q rvol af

q = volumetric flow rate (m³ s⁻¹)

rvol = volume of biofilm reactor (m³)

af = total biofilm area in the reactor (m²)

srate value = effects of shear rate on the diffusion

closedflag value = 0 or 1 = flag of concentration in closed system (default: 0)

dcflag value = 0 or 1 or 2 = flag of effective diffusion mode (default: 0)

dcratio value = effective diffusion factor for *dcflag* = 2 (default: 0.8)

Required data sections

Nutrients, Diffusion Coeffs sections.

Examples

```
variable tol equal 1e-6
...
fix kd all kinetics/diffusion v_tol pp pp nd
fix kd all kinetics/diffusion v_tol pp pp nd bulk 2.31e-7 1.25e-3 0.1
fix kd all kinetics/diffusion v_tol pp pp nn closedflag 1
fix kd all kinetics/diffusion v_tol pp pp nd dcflag 2 dcratio 0.6
```

Description

Solve mass balance of soluble substrates in biofilm and bulk liquid. Nutrient concentration distribution in the rectangular simulation domain is calculated by solving the advection-diffusion-reaction equation for each soluble component defined in Nutrients section. For each nutrient, the mass balance equation is given by,

$$\frac{\partial S}{\partial t} = \nabla \cdot (D_e \nabla S) - \vec{U} \cdot \nabla S + R$$

where *R* is the reaction rate calculated in *kinetics/growth/monod* or *kinetics/growth/energy* and/or *fix kinetics/thermo* commands, and *D* is the effective diffusion coefficient defined in Diffusion Coeffs section. The equation is discretised on a Marker-And-Cell

(MAC) uniform grids defined by nx , ny and nz in `kinetics` command. The temporal and spatial derivatives of the transport equation are discretised by Forward Euler and Central Finite Differences, respectively. The equation is solved for the steady state solution and the convergence criteria is defined by v_tol .

The optional keyword *bulk* enables the update of mass balances of solutes in the bulk liquid for dynamic conditions:

$$\frac{dS^{(b)}}{dt} = \frac{Q}{V}(S^{(in)} - S^{(out)}) + \frac{A_f}{VL_YL_Z} \int_0^{L_X} \int_0^{L_Y} \int_0^{L_Z} R(x, y, z) dz dy dx$$

where $R(x, y, z)$ is the reaction rate at each grid voxel, Q is the volumetric flow rate (q), V is the volume of biofilm reactor (*rvol*), A_f is the total biofilm area in the reactor (*af*) and L is the size of simulation domain.

If none of the surfaces uses Dirichlet boundary condition, i.e, no nutrient supplement from environment, then we assume the system is a closed system. For such system, the optional keyword *closedflag* = 0 indicates that nutrient concentration is updated after each diffusion process based on biological timestep:

$$\frac{dS(x, y, z)}{dt} = \frac{Res_{x,y,z}}{diffT}$$

where Res is residual value at last diffusion iteration, $diffT$ is the diffusion timestep. If the optional keyword *closedflag* = 1, we consider that the gradient will be negligible so the concentration will only vary due to the reaction rate:

$$\frac{dS(x, y, z)}{dt} = \frac{1}{nx \cdot ny \cdot nz} \int_0^{nx} \int_0^{ny} \int_0^{nz} R(x, y, z) dz dy dx$$

where $S(x, y, z)$ is the nutrient concentration in grid (x, y, z) , nx , ny and nz are the numbers of grid in x , y and z axis, respectively.

The optional keyword *dcflag* affects effective diffusion factor in biofilm region. By default, the diffusion coefficient of solute in biofilm region is the same as in liquid region. If *dcflag* = 1, a dynamic factor is considered:

$$d_f(x, y, z) = 1 - \frac{0.43 \cdot X(x, y, z)^{0.92}}{11.19 + 0.27 \cdot X(x, y, z)^{0.99}}$$

where $X(x, y, z)$ is the biomass concentration in grid (x, y, z) . If *dcflag* = 2, the factor is considered as a constant ratio which can be defined from the keyword *dcratio*.

3.10 fix divide command

Syntax

```
fix ID group-ID divide Nevery v_EPSdens v_divMass seed
```

- *ID* : user-assigned name for the fix
- *group-ID* : ID of the group of microbes to apply the fix to
- *divide* : style name of this fix command
- *Nevery* : call the function every this many timesteps
- *v_EPSdens* : density of EPS (kg m^{-3})
- *v_divDia* : threshold diameter value for microbe division (m)
- *seed* : random seed for cell orientation
- optional keyword = *demflag*

demflag value = 0 or 1 = flag of DEM run (default: 0)

Required data sections

Atoms section

Examples

```
variable EPSdens equal 30  
variable divDia equal 1.3e-6  
  
fix d1 all divide 500 v_EPSdens v_divDia 111  
fix d1 HET divide 1000 v_EPSdens v_divDia 1453
```

Description

Perform division for the microbes in a group. Division occurs if the diameter of a microbe reaches the user-specified threshold value *v_divDia*; the cell then divides into two daughter cells. The total mass of the two daughter cells is always conserved from the parent cell. One daughter cell is (uniformly) randomly assigned 40% - 60% of the parent cell's mass, and the other gets the rest. Also, one daughter cell takes the location of the parent cell while the centre of the other daughter cell is (uniformly) randomly chosen at a distance *d* (distance between the centres of the two agents) corresponding to the sum of the diameter of the daughters.

v_EPSdens is required for HET division as they are surrounded by EPS shell. *v_divDia* defines the threshold diameter value that a microbe starts dividing.

3.11 fix eps_extract command

Syntax

```
fix ID group-ID eps_extract Nevery v_EPSratio v_EPSdens seed
```

- *ID* : user-assigned name for the fix
- *group-ID* : ID of the group of microbes to apply the fix to
- *eps_extract* : style name of this fix command
- *Nevery* : call the function every this many timesteps
- *v_EPSratio* : extraction threshold ratio between outer-radius and inner-radius of HET
- *v_EPSdens* : density of EPS
- *seed* : random seed for cell orientation
- optional keyword = *demflag*

demflag value = 0 or 1 = flag of DEM run (default 0)

Required data sections

Atoms section

Example

```
variable v_EPSratio equal 1.25
variable v_EPSdens equal 30

fix d1 HET eps_extract 500 v_EPSratio v_EPSdens 123
```

Description

Perform EPS production process in the simulation. To use the command, a species named “eps” must be defined in Atoms section.

The Monod-based growth model allows active heterotrophs (HET) to secrete EPS into their neighbouring environment. The EPS play an important role in microbial aggregation by offering a protective medium. We assume that EPS are secreted by heterotrophs only. Initially, EPS is accumulated as an extra shell around a HET particle. When the relative thickness of the EPS shell of the HET particle exceeds a certain threshold value, around half (uniformly random ratio between 0.4-0.6) of the EPS mass excretes as a separate EPS particle and is (uniformly) randomly placed next to the HET.

3.12 fix death command

Syntax

```
fix ID group-ID death Nevery v_deadDia
```

- *ID* : user-assigned name for the fix
- *group-ID* : ID of the group of microbes to apply the fix to
- *death* : style name of this fix command
- *Nevery* : call the function every this many timesteps
- *v_deadDia* : threshold diameter value for microbe death

Required data sections

Atoms section

Example

```
variable v_deadDia equal 0.8e-6  
  
fix d1 HET death 500 v_deadDia
```

Description

Perform microbe death process in the simulation. To use the command, a species named “dead” must be defined in Atoms section.

The size of a microbe decreases when nutrients are limited or the energy available is not sufficient to meet maintenance requirements. Microbes which shrink below a user-specified minimum diameter *v_deadDia* are considered as dead. Their type then changes to the dead type. Dead cells do not perform biological activities, but their biomass will linearly convert to substrate to be consumed by other microbes.

3.13 fix epsadh command

Syntax

```
fix ID group-ID epsadh Nevery v_ke f_adhmodel
```

- *ID* : user-assigned name for the fix
- *group-ID* : ID of the group of microbes to apply the fix to
- *epsadh* : style name of this fix command
- *Nevery* : call the function every this many timesteps
- *v_ke* : spring stiffness
- *f_adhmodel* : adhesive force model flag (*f_adhmodel* = 1 or 2)

Required data sections

Atoms section

Example

```
variable ke equal 5e+10  
  
fix d1 HET epsadh 1 v_ke 1
```

Description

The excreted EPS mass from the HET particles can be employed as a parameter of adhesion force models between the particles. The EPS link between the particles are treated as much more stiffer springs, but only employing the attractive forces. Total effective EPS mass M_{ij}^{eps} is calculated between the microbes, and a spring stiffness k_e is defined per unit mass. The forces calculated according to the effective spring stiffness ($M_{ij}^{eps} k_e$) multiplied by the separation distance between two particles (model 1), or inverse of the separation distance (model 2).

The EPS-mediated binding forces are calculated as:

$$\vec{F}_{eps,ij} = M_{ij}^{eps} k_e (d_{ij} - d_{0ij}) \cdot \frac{\vec{d}_{ij}}{d_{ij}}$$
$$\vec{F}_{a,i} = \sum_{j=1}^N \vec{F}_{eps,ij}$$

where d_{0ij} is the sum of the radii of two interacting particles and d_{ij} is the distance between centres of two particles.

3.14 fix walladh command

Syntax

```
fix ID group-ID walladh v_kanc f_wallstyle lo hi
```

- *ID* : user-assigned name for the fix
- *group-ID* : ID of the group of microbes to apply the fix to
- *walladh* : style name of this fix command
- *v_kanc* : adhesive strength
- *f_wallstyle* : specify a pair of walls in a dimension (*f_wallstyle* = *xplane* or *yplane* or *zplane*)
- *lo*, *hi* : position of lower and upper plane

Required data sections

Atoms section

Examples

```
variable kanc equal 50  
  
fix xw all walladh v_kanc xplane 0.0 1.0e-04  
fix yw all walladh v_kanc yplane 0.0 5.0e-05
```

Description

Impose an adhesive force between the wall (the boundary of simulation domain) and the microbes attaching to the wall. The force is calculated as the product of adhesive strength and overlap distance.

3.15 fix shear command

Syntax

```
fix ID group-ID shear Nevery v_viscosity v_shearRate v_height  
f_direction start end
```

- *ID* : user-assigned name for the fix
- *group-ID* : ID of the group of microbes to apply the fix to
- *shear* : style name of this fix command
- *Nevery* : call the function every this many timesteps
- *v_viscosity*: dynamic viscosity of fluid
- *v_shear-rate*: rate of change of velocity
- *v_height* : distance to the stationary point from bottom wall
- *f_direction* : direction of the force (*direction* = *zx* or *zy*)
- *start, end* : time range for applying the force

Required data sections

Atoms section

Examples

```
variable viscosity equal 0.5  
variable shearRate equal 0.6  
variable height equal 5e-5  
fix s1 all shear 10 v_viscosity v_shearRate v_height zx 5 500
```

Description

Impose an additional shear force each microbe in the group. The shear force is calculated according to the drag force created on a sphere in Stokes flow, and it is given by:

$$\vec{F}_{f,i} = 6\pi\mu r_i \vec{v}_r$$

where μ is dynamic viscosity of fluid, r_i is radius of particle and \vec{v}_r is local fluid velocity relative to the particle. The parameter *height* is a user-defined value where the directions of flow above and below the stationary point are in opposition.

3.16 compute ntypes command

Syntax

```
compute ID group-ID ntype
```

- *ID* : user-assigned name for the computation
- *group-ID* : ID of the group of atoms to perform the computation on
- *ntype* : style name of this compute command

Examples

```
compute myNtypes all ntypes
```

Description

Define a computation that calculates total number of microbes of each species in the system. Result values are saved in a global vector that can be output via `dump bio` or `thermo_style` command.

3.17 compute biomass command

Syntax

```
compute ID group-ID biomass
```

- *ID* : user-assigned name for the computation
- *group-ID* : ID of the group of atoms to perform the computation on
- *biomass* : style name of this compute command

Examples

```
compute myMass all biomass
```

Description

Define a computation that calculates total biomass of each species in the system. Result values are saved in a global vector that can be output via `dump bio` or `thermo_style` command.

3.18 compute diameter command

Syntax

```
compute ID group-ID diameter
```

- *ID* : user-assigned name for the computation
- *group-ID* : ID of the group of atoms to perform the computation on
- *diameter* : style name of this compute command

Examples

```
compute myDia all diameter
```

Description

Define a computation that calculates floc equivalent diameter. The specified group must be “all”. The equivalent diameter at time t $d_{t,eqv}$ is computed by the formula:

$$d_{t,eqv} = \sum_{k=1}^n \sqrt[3]{\frac{6V_{kt}}{\pi}}$$

where V_{kt} is volume of each individual spherical particle k at time t . Result value is saved in a global scalar that can be output via `dump bio` or `thermo_style` command.

3.19 compute dimension command

Syntax

```
compute ID group-ID dimension
```

- *ID* : user-assigned name for the computation
- *group-ID* : ID of the group of atoms to perform the computation on
- *dimension* : style name of this compute command

Examples

```
compute myDimen all dimension
```

Description

Define a computation that calculates fractal dimension. The specified group must be “all”. The fractal dimension F_{Dt} is computed by the formula:

$$F_{Dt} = \frac{\log(R_a/R_m)}{\log(n)}$$

where n is total number of particles in the system, $R_a = \sqrt{\frac{\sum_{k=1}^n m_k d_k^2}{\sum_{k=1}^n m_k}}$ and $R_m = \frac{\sum_{k=1}^n r_k}{n}$, d_k , r_k and m_k are the particle diameter, radius and mass respectively. Result value is saved in a global scalar that can be output via `dump bio` or `thermo_style` command.

3.20 compute diversity command

Syntax

```
compute ID group-ID diversity
```

- *ID* : user-assigned name for the computation
- *group-ID* : ID of the group of atoms to perform the computation on
- *diversity* : style name of this compute command

Examples

```
compute myDiver all diversity
```

Description

Define a computation that calculates diversity index in a system. The specified group must be “all”. The diversity index D_t at time t is computed by the formula:

$$D_t = 1 - \frac{\sum n(n-1)}{N(N-1)}$$

Where n is the total number of organism of a particular specie and N is the total number of microbes of all species. Result value is saved in a global scalar that can be output via `dump bio` or `thermo_style` command.

3.21 compute avg_height command

Syntax

```
compute ID group-ID avg_height nx ny
```

- *ID* : user-assigned name for the computation
- *group-ID* : ID of the group of atoms to perform the computation on
- *avg_height* : style name of this compute command
- optional keyword = *nx ny*

nx value *ny* value = number of grid elements in x, y planes

Examples

```
compute myHeight all avg_height  
compute myHeight all avg_height nx 50 ny 50
```

Description

Define a computation that calculates biofilm average height. The specified group must be “all”. The formula is given by:

$$\bar{h} = \frac{1}{L_x L_y} \int \int h(x, y) dx dy$$

where $h(x, y)$ is biofilm height (measured in z direction) at position (x, y) on the substratum. By default, the simulation domain is discretised automatically where the size of each grid element is the maximum cut-off distance for all atom pairs used in mechanical iteration. Result value is saved in a global scalar that can be output via `dump bio` or `thermo_style` command.

3.22 compute roughness command

Syntax

```
compute ID group-ID roughness nx ny
```

- *ID* : user-assigned name for the computation
- *group-ID* : ID of the group of atoms to perform the computation on
- *roughness* : style name of this compute command
- optional keyword = *nx ny*

nx value *ny* value = number of grid elements in x, y planes

Examples

```
compute myRough all roughness
compute myRough all roughness nx 50 ny 50
```

Description

Define a computation that calculates biofilm roughness. The specified group must be “all”. The formula is given by:

$$roughness = \frac{1}{L_x L_y} \int \int (h(x, y) - \bar{h})^2 dx dy$$

where $h(x, y)$ is biofilm height (measured in z direction) at position (x, y) on the substratum. \bar{h} is biofilm average height. By default, the simulation domain is discretized automatically where the size of each grid element is the maximum cut-off distance for all atom pairs used in mechanical iteration. Result value is saved in a global scalar that can be output via `dump bio` or `thermo_style` command.

3.23 compute segregation command

Syntax

```
compute mySeg all segregation v_cutoff
```

- *ID* : user-assigned name for the computation
- *group-ID* : ID of the group of atoms to perform the computation on
- *segregation* : style name of this compute command
- *v_cutoff* : cutoff distance for neighbour list calculation (m)

Examples

```
variable cutoff equal 1e-6  
  
compute mySeg all segregation v_cutoff
```

Description

Define a computation that calculates biofilm segregation index. The specified group must be “all”. The formula is given by:

$$\sigma_t = \frac{1}{M} \sum_{i=1}^M \left(\frac{1}{N} \sum_{j=1}^N \rho(c_i, c_j) \right)$$

$$\text{where } \rho(c_i, c_j) = \begin{cases} 0, & c_j \text{ is not the same type as } c_i \\ 1, & c_j \text{ is the same type as } c_i \end{cases}$$

j is neighbour of atom *i*, variable *v_cutoff* defines neighbour list of each atom. Result value is saved in a global scalar that can be output via `dump bio` or `thermo_style` command.

3.24 dump bio command

Syntax

```
dump ID group-ID bio Nevery file args
```

- *ID* : user-assigned name for the dump
- *group-ID* : ID of the group of atoms to be dumped
- *bio* : style name of this dump command
- *Nevery* : dump every this many timesteps
- *file* : name of file to write dump info to
- *args* : list of arguments for dump info

```
possible dump info = biomass, ntypes, diameter, dimension,  
                    diversity, avg_height, roughness,  
                    segregation, avg_con, avg_ph, bulk
```

```
biomass = total biomass of each species  
ntypes = total number of microbes of each species  
diameter = floc equivalent diameter  
dimension = fractal dimension  
diversity = diversity index  
avg_height = biofilm average height  
roughness = biofilm roughness  
segregation = segregation index  
avg_con = average nutrient concentration  
bulk = nutrient concentration in bulk liquid  
avg_ph = average pH
```

Examples

```
compute myMass all biomass  
compute myRough all roughness  
compute myHeight all avg_height  
  
dump d0 all bio 3600 biomass roughness avg_height
```

Dump IbM attributes to one or more files every *Nevery* timesteps. The related computes of specified attributes must be defined in prior. The dumped attribute information are mostly related to biofilm and floc. To dump atom information (e.g, mass, diameter, position), use `dump custom` command, or `dump vtk` command. The `dump bio` command creates a new directory named `/Result` in the current example directory to save the output data.

3.25 dump grid command

Syntax

```
dump ID group-ID grid Nevery file args
```

- *ID* : user-assigned name for the dump
- *group-ID* : ID of the group of atoms to be dumped
- *grid* : style name of this dump command
- *Nevery* : dump every this many timesteps
- *file* : name of file to write dump info to
- *args* : list of arguments for dump info

```
possible dump info = con, upt, gas, act, yie, cat, ana
```

```
con = nutrient concentration  
upt = nutrient update rate  
gas = gas field  
act = nutrient activity  
yie = growth yield  
cat = Gibbs free energy of anabolism  
ana = Gibbs free energy of catabolism
```

Examples

```
dump dv all grid 10000 grid_%.vti con  
dump dv all grid 500 grid_%.vti yie cat
```

Dump a snapshot of field information to one or more files every *Nevery* timesteps in a format readable by the VTK visualisation toolkit, e.g. ParaView.

Similarly with `dump custom/vtk` command. The VTK format uses a single snapshot of the system per file, thus a wildcard “*” must be included in the filename, as discussed below. Otherwise the dump files will get overwritten with the new snapshot each time.

Dump filenames can contain two wildcard characters. If a “*” character appears in the filename, then one file per snapshot is written and the “*” character is replaced with the timestep value. For example, `grid*.vti` becomes `grid0.vti`, `grid10000.vti`, `grid20000.vti`, etc.

If a “%” character appears in the filename, then each of *P* processors writes a portion of the dump file, and the “%” character is replaced with the processor ID from 0 to *P*-1 preceded by an underscore character. For example, `grid%.vti` becomes `grid0.vti`, `grid1.vti`, ... `gridP-1.vti`, etc. This creates smaller files and can be a fast mode of output on parallel machines that support parallel I/O for output.

3.26 dump bio/hdf5 command

Syntax

```
dump ID group-ID bio/hdf5 Nevery file args
```

- *ID* : user-assigned name for the dump
- *group-ID* : ID of the group of atoms to be dumped
- *bio/hdf5* : style name of this dump command
- *Nevery* : dump every this many timesteps
- *file* : name of file to write dump info to
- *args* : list of arguments for dump info

```
possible dump info = id, type, x, y, z, vx, vy, vz, fx, fy, fz,  
                    radius, con, upt, gas, act, yie, cat, ana
```

```
id = atom id  
type = atom type  
x, y, z = atom position  
vx, vy, vz = atom velocity  
fx, fy, fz = atom force  
radius = atom radius  
con = nutrient concentration  
upt = nutrient update rate  
gas = gas field  
act = nutrient activity  
yie = growth yield  
cat = Gibbs free energy of anabolism  
ana = Gibbs free energy of catabolism
```

Examples

```
dump dh0 all bio/hdf5 3600 dump_*.h5 id type radius x y z con cat ana hyd
```

Dump filenames can contain a wildcard character. If a “*” character appears in the filename, then one file per snapshot is written and the “*” character is replaced with the timestep value. For example, grid*.vti becomes grid0.vti, grid10000.vti, grid20000.vti, etc.

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A Define a multi-timescale run

The time scale of biological processes (≈ 1 hour) is in general much slower than the mechanical ($\approx 10^{-8}$ hours) and chemical processes ($\approx 10^{-6}$ hours). Thus, the later two can be treated as instantaneous processes without considering biological activities. To solve such system, these processes are usually operated sequentially. The coupling between the multiple timescales relies on the pseudo steady-state approximation and the frozen state: when a steady state solute concentration is reached at each biological timestep, the concentration is assumed to remain unchanged (frozen state) until the next biological step.

In NUFEB, a general way to model the above system is to use multi-timescale simulation scheme. A main loop with larger time-scale is defined to perform biological activities. At each timestep of the main loop, two sub-loops are defined to solve chemical and mechanical processes until they reach a pseudo-steady state or certain maximum steps. Examples can be found in NUFEB/example/competition-energy directory. The inputscript is something like:

```
...
pair_style gran/hooke/history 1.e-4 NULL 1.e-5 NULL 0.0 1
...
fix k1 all kinetics 1 100 5 100 v_diffT v_layer demflag 0 niter 10000
fix d1 all divide 1 v_EPSdens v_divDia 31231 demflag 0
...
#####Define Bio-loop and DEM-subloop#####

run 4320 pre no post no every 1 &
"fix_modify k1 demflag 1" &
"fix_modify d1 demflag 1" &
"timestep 1e-3" &
"run 10000 pre no post no" &
"timestep 600" &
"fix_modify k1 demflag 0" &
"fix_modify d1 demflag 0" &
```

The main (biological) loop has 4320 steps. The *every* keyword is used to break a NUFEB run into a series of shorter runs where the sub-loops are inserted. The *pre* and *post* keywords are used to streamline the setup, clean-up, and associated output to the screen that happens before and after each shorter run. In this case, they should be specified as “no” so the initial setup and full timing summary are skipped.

At each shorter run (biological step), the simulation will first proceed chemical and biological processes (e.g, fix kinetics, and fix divide). Then these processes will be switched off by setting *demflag* = 1 via *fix modify* command. This will make sure that the chemical and biological processes will not proceed during mechanical sub-loop. Then we change to the mechanical timestep (1e-3 s) and perform a 10000 steps sub-simulation for the mechanical interaction (e.g, impose frictional force by gran/hooke/history). After the mechanical loop, we reset the timestep to biological timescale and also switch on chemical and biological processes ready for the next step by setting *demflag* = 0.

Table 1: Stoichiometric matrix for particulate and soluble components implemented in Monod-based growth model. Here Y_i is the yield for microbial functional group i (i = HET, AOB, NOB, ANAMMOX, EPS and DEAD), μ_i and b_i are the maximum specific growth and decay rates, respectively, η_i is the reduction factor in anoxic conditions, S_j is the concentration of nutrient j (j = Substrate, O_2 , NH_4 , NO_2 , NO_3), $K_{i,j}$ is the affinity constant between nutrient j and functional group i , bm_i is the maintenance rate for i , and X is the biomass density

Process	Soluble					Particulate			Kinetic Expression
	S_{sub}	S_{O_2}	S_{NH_4}	S_{NO_2}	S_{NO_3}	X_H	X_A	X_N	
HET	Aerobic growth	$-\frac{1}{Y_H}$	$-\frac{1-Y_H-Y_E}{Y_H}$			1			$\mu_H \frac{S_{sub}}{K_{H,O_2}+S_{sub}} \frac{S_{O_2}}{K_{H,O_2}+S_{O_2}} X_H$
	Anoxic growth on NO_2	$-\frac{1}{Y_H}$			$-\frac{1-Y_H-Y_E}{2.86Y_H}$	1			$\eta_H \mu_H \frac{S_{sub}}{K_{H,sub}+S_{sub}} \frac{S_{NO_2}}{K_{H,NO_2}+S_{NO_2}} \frac{K_{H,O_2}}{K_{H,O_2}+S_{O_2}} X_H$
	Anoxic growth on NO_3	$-\frac{1}{Y_H}$		$-\frac{1-Y_H-Y_E}{1.17Y_H}$	-1	1			$\eta_H \mu_H \frac{S_{sub}}{K_{H,sub}+S_{sub}} \frac{S_{NO_3}}{K_{H,NO_3}+S_{NO_3}} \frac{K_{H,O_2}}{K_{H,O_2}+S_{O_2}} X_H$
	Aerobic Maintenance		-1			-1			$bm_H \frac{S_{O_2}}{K_{H,O_2}+S_{O_2}} X_H$
	Anoxic Maintenance on NO_2			$-\frac{1}{1.17}$		-1			$\eta_H bm_H \frac{S_{NO_2}}{K_{H,NO_2}+S_{NO_2}} \frac{K_{H,O_2}}{K_{H,O_2}+S_{O_2}} X_H$
	Anoxic maintenance on NO_3				$-\frac{1}{2.86}$	-1			$\eta_H bm_H \frac{S_{NO_3}}{K_{H,NO_3}+S_{NO_3}} \frac{K_{H,O_2}}{K_{H,O_2}+S_{O_2}} X_H$
	Decay					-1			$b_H X_H$
AOB	Aerobic growth	$-\frac{3.42-Y_A}{Y_A}$	$-\frac{1}{Y_A}$	$\frac{1}{Y_A}$			1		$\mu_A \frac{S_{NH_4}}{K_{A,NH_4}+S_{NH_4}} \frac{S_{O_2}}{K_{A,O_2}+S_{O_2}} X_A$
	Maintenance		-1				-1		$bm_A \frac{S_{O_2}}{K_{A,O_2}+S_{O_2}} X_A$
	Decay						-1		$b_A X_A$
NOB	Aerobic growth	$-\frac{1.15-Y_N}{Y_N}$		$-\frac{1}{Y_N}$	$\frac{1}{Y_N}$			1	$\mu_N \frac{S_{NO_2}}{K_{N,NO_2}+S_{NO_2}} \frac{S_{O_2}}{K_{N,O_2}+S_{O_2}} X_N$
	Maintenance		-1					-1	$bm_N \frac{S_{O_2}}{K_{N,O_2}+S_{O_2}} X_N$
	Decay							-1	$b_N X_N$

Table 2: Continue Table 1. Stoichiometric matrix for particulate and soluble components implemented in Monod-based growth model.

	Process	Soluble					Particulate			Kinetic Expression
		S_{sub}	S_{O_2}	S_{NH_4}	S_{NO_2}	S_{NO_3}	X_{AN}	X_{E}	X_{D}	
ANAMMOX	Anoxic growth			$-\frac{1}{Y_{\text{AN}}}$	$-(\frac{1}{Y_{\text{AN}}} + \frac{1}{1.14})$	$\frac{1}{1.14}$	1			$\mu_{\text{AN}} \frac{S_{\text{NH}_4}}{K_{\text{AN,NH}_4} + S_{\text{NH}_4}} \frac{S_{\text{NO}_2}}{K_{\text{AN,NO}_2} + S_{\text{NO}_2}} \frac{K_{\text{O}_2}}{K_{\text{AN,O}_2} + S_{\text{O}_2}} X_{\text{AN}}$
	Maintenance		-1				-1			$bm_{\text{AN}} \frac{S_{\text{O}_2}}{K_{\text{AN,O}_2} + S_{\text{O}_2}} X_{\text{AN}}$
	Decay						-1			$b_{\text{AN}} X_{\text{AN}}$
EPS	Decay	1						-1		$b_{\text{E}} X_{\text{E}}$
DEAD	Decay	1							-1	$b_{\text{D}} X_{\text{D}}$