# Near-Neighbor Search at Scale

Casey Stella

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- I'm here to talk about Near-Neighbor Searches
- In particular, about how they're hard to do efficiently at scale.

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- Many problems can be rephrased as a near neighbor search (or use it as a primary component)
  - Recommendation Systems
  - Contextual Marketing (i.e. ads)
  - Clustering data
  - Lots more

# Traditional Approach

- A naïve approach would be $O(n)$

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- It's also not clear that they work for non-$L_2$ metrics

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  - A lot of engineering work

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- Examples of these are HBase, Cassandra, MongoDB, MemcacheDB
- It would be very nice to be able to use this to find nearest neighbors, but how?

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- Often we are looking for data in a fixed radius.

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- We can use this to group our "near" vectors into buckets
- You can construct multiple hash functions (i.e. families) and compose to increase the accuracy at the expense of runtime complexity
- You can use multiple LSH functions and put the same input data into each bucket, thereby increasing accuracy at the expense of space complexity

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- Not all LSH functions have theoretical bounds about accuracy
  - Almost all research focuses on **nearest** neighbor searches
  - Practical alternative is to sample your data and measure

# Stable Distributions and the $L_k$ metric

- Based on the research of Piotr Indyk, et. al.[2]

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  - Know that the Normal distribution is 2-stable and the Cauchy distribution is 1-stable

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- Different choices of $a$ make different functions with the same characteristics.
- If you don't understand, that's ok..it's not terribly obvious.

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- Next up is min-hash
- `https://github.com/cestella/SpatialSearch`

# Conclusion

- Thanks for your attention
- Follow me on twitter @*casey_stella*
- Find me at
  - `http://caseystella.com`
  - `https://github.com/cestella`
- PS. If you like this...Explorys is hiring!