Clase pasada: → Metodo de Euler

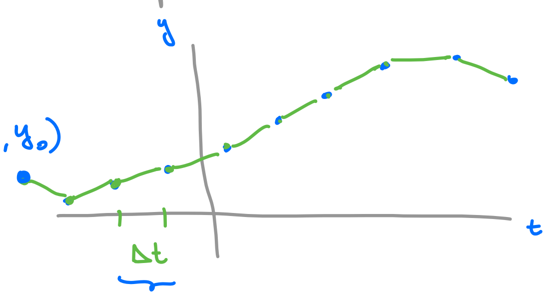
solucion $y' = F(y,t)$ donde $y = y(t)$.

Sucesiones recurrentes
$$\begin{cases} t_{i+1} = t_i + \underbrace{\Delta t}_{\to \text{ Fijo}} \\ y_{i+1} = y_i + \Delta t \cdot \underbrace{F(y_i, t_i)}_{\to \text{ ec. dif.}} \end{cases}$$

$$\{(t_i, y_i)\}_{i=1}^{n}$$

Graficamente

Dato inicial → (t_0, y_0)



. Tamaño de paso
· discretizacion

Ver colab ejemplos

1) $$\frac{d\theta}{dt} = F(\theta, \omega, t)$$
$$\frac{d\omega}{dt} = G(\theta, \omega, t)$$

pseudo-codigo

previo $\begin{cases} \theta = [\,] \,, & \Delta t \\ \omega = [\,] \,, & \theta_0, \omega_0, t_0 \end{cases}$

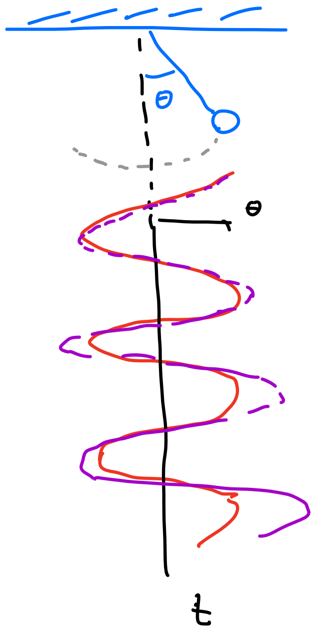
1) def sistema (θ, ω, t)
$\frac{d\theta}{dt} = \ldots$
$\frac{d\omega}{dt} = \ldots$
return $\frac{d\theta}{dt}, \frac{d\omega}{dt}$

→ t = np.arange$(t_{min}, t_{max}, \Delta t)$

2) for t_i in t :
$\frac{d\theta}{dt}, \frac{d\omega}{dt}$ = system $(\theta_i, \omega_i, t_i)$

Euler → $\theta_{i+1} = \theta_i + \Delta t \frac{d\theta}{dt}$

Euler → $\omega_{i+1} = \omega_i + \Delta t \frac{d\omega}{dt}$



Metodo de Adams

¿como mejorar el metodo de Euler?

Serie Taylor

$$y(t_i+\Delta t) = \underbrace{\underbrace{y(t_i) + \Delta t\, y'(t_i)}_{\text{Euler}} + \frac{\Delta t^2}{2}\, y''(t_i) + \dots \frac{\Delta t^n}{n}\, y^{(n)}(t_i)}_{\text{Adams}} + \dots$$

↓

problema: $y''(t_i)$

↓

si Δt pequeño, entonces

$$y''(t_i) \approx \frac{y'(t_i) - y'(t_{i-1})}{\underbrace{t_i - t_{i-1}}_{\Delta t}}$$

reemplazando en Taylor y truncando hasta la derivada de segundo orden tenemos

$$y(t_{i+1}) \approx y(t_i) + \Delta t\, y'(t_i) + \frac{\Delta t^{\cancel{2}}}{2}\left(\frac{y'(t_i) - y'(t_{i-1})}{\cancel{\Delta t}}\right)$$

Como $y'(t_i) = F(y_i, t_i)$ tenemos que

$$y(t_{i+1}) \approx y(t_i) + \Delta t\, F(y_i, t_i) + \frac{\Delta t}{2} F(y_i, t_i) - \frac{\Delta t}{2} F(y_{i-1}, t_{i-1})$$

usando $y(t_{i+1}) = y_i$ tenemos

$$y_{i+1} \approx y_i + \frac{3}{2}\Delta t\, F(y_i, t_i) - \frac{\Delta t}{2} F(y_{i-1}, t_{i-1})$$

Método de Adams →

$$\boxed{y_{i+1} \approx y_i + \frac{\Delta t}{2}\left(3\underbrace{F(y_i, t_i)}_{\text{paso anterio}} - \underbrace{F(y_{i-1}, t_{i-1})}_{\text{paso mas anterior}}\right)}$$

depende de dos pasos.

Nota: se necesitan (t_0, y_0) → lo da el problema ✓

(t_1, y_1) → lo calcula con euler ✓