Instructions: Put your name, PID, section number, and TA's name on your blue book. No calculators or electronic devices are allowed. Turn off and put away your cell phone. You may use one page of handwritten notes, but no other resources. Make sure your solutions are clear and legible. **Show all of your work**. Credit will not be given for unreadable or unsupported answers. Please keep the questions in order in your blue book and clearly indicate which problem is on which page.

1. (10 points)

   (a) (4 pts) Find the equation of the line that passes through the points $(1,3)$ and $(-2,9)$. Express your answer in slope-intercept form ($y = mx + b$).

   - $\boxed{y = -2x + 5}$ Here the slope m is given by

     $$m = \frac{9-3}{-2-1} = \frac{6}{-3} = -2$$

     Then using the point slope form, we have

     $$\begin{aligned}(y-3) &= -2(x-1) \\ &= -2x + 2\end{aligned}$$

     so that

     $$y = -2x + 5$$

   (b) (4 pts) Find a real number t such that the line passing through the points (t,t) and $(2,4)$ is perpendicular to the line $y = -\frac{1}{5}x + 12$.

   - $\boxed{t = \frac{3}{2}}$ We want the slope

     $$m = \frac{t-4}{t-2}$$

     to be the slope that is perpendicular to the line $y = -\frac{1}{5}x + 12$. By taking negative reciprocals, we want

     $$\frac{t-4}{t-2} = -\frac{1}{-\frac{1}{5}} = 5$$

     This reduces to

     $$\begin{aligned}t - 4 &= 5(t-2) \\ &= 5t - 10\end{aligned}$$

     This reduces to

     $$6 = 4t$$

     or

     $$t = \frac{3}{2}$$

   (c) (2 pts) Find an equation of the line which passes through the point $(4,5)$ and is perpendicular to the line $y = 3$.

   - $\boxed{x = 4}$ The line $y = 3$ is a horizontal line. To get a vertical line, we have to set x equal to a number. To go through the point $(4,5)$ we need x to equal 4, so that $x = 4$.

2. (4 points)

(a) Find a formula which gives the distance between the points $(2t, -t)$ and $(0, 4)$ in terms of t.

- $\boxed{\sqrt{5t^2+8t+16}}$ Using the distance formula, we get

$$\begin{aligned} \text{dist} &= \sqrt{(4 - -t)^2 + (0-2t)^2} \\ &= \sqrt{(4+t)^2 + 4t^2} \text{(leaving it in this form is fine)} \\ &= \sqrt{16+8t+t^2+4t^2} \\ &= \sqrt{5t^2+8t+16} \end{aligned}$$

(b) Find the value of t which minimizes this distance (from part (a)).

- $\boxed{t = -\frac{4}{5}}$ The minimum of this quadratic expression happens at $t = -\frac{b}{2a}$. In this case $b = 8$ and $a = 5$ so

$$t = -\frac{8}{10} = -\frac{4}{5}$$

3. (4 points) Find the domain of the function

$$f(x) = \frac{x^4}{|4x-3|-5}.$$

Express your answer using interval notation.

- $\boxed{\left(-\infty, -\frac{1}{2}\right) \cup \left(-\frac{1}{2}, 2\right) \cup (2, \infty)}$ This makes sense as long as the denominator is not zero. In other words, we cannot have

$$|4x-3|-5=0$$

This is the same thing as

$$|4x-3|=5$$

So that means we cannot have

$$4x-3=5$$

and

$$4x-3=-5.$$

After solving for x, this means we can't have

$$x = 2 \text{ or } x = -\frac{1}{2}$$

In other words, our domain is

$$\left\{x : x \neq 2 \text{ or } x \neq -\frac{1}{2}\right\}.$$

Written in interval notation, this is

$$\left(-\infty, -\frac{1}{2}\right) \bigcup \left(-\frac{1}{2}, 2\right) \bigcup (2, \infty)$$

4. (8 points) Suppose you have a function $f(x)$ whose domain is the interval $[-1,3]$ and whose range is the interval $[1,5]$. Define a new function $g(x)$ by the following sequence of transformations:

i. Shift the graph of f vertically upwards by 4 units.
ii. Stretch the graph from (i) vertically by a factor of $\frac{1}{3}$.
iii. Stretch the graph from (ii) horizontally by a factor of $\frac{1}{3}$.

(a) (4 pts) Find a formula for g in terms of f.

- $\boxed{g(x) = \frac{1}{3}f(3x) + \frac{4}{3}}$ Following the steps, we get $f(x) \longrightarrow f(x)+4 \longrightarrow \frac{1}{3}(f(x)+4) \longrightarrow \frac{1}{3}(f(3x)+4)$.
  After simplifying, this is
  $$\frac{1}{3}f(3x) + \frac{4}{3}$$

(b) (2 pts) What is the domain of g?

- $\boxed{\left[-\frac{1}{3}, 1\right]}$ We start with domain $[-1,3]$. Only step iii changes the domain. It shrinks it to $\left[-\frac{1}{3}, 1\right]$

(c) (2 pts) What is the range of g?

- $\boxed{\left[\frac{5}{2}, \frac{9}{2}\right]}$ Step i changes the range from $[1,5]$ to $[5,9]$
  Step ii changes it from $[5,9]$ to $\left[\frac{5}{3}, \frac{9}{3}\right]$.

5. (7 points) Define the function $f(x) = \frac{3}{x-5}$.

(a) Find a formula for the inverse function f^{-1}.

- $\boxed{f^{-1}(y) = \frac{3+5y}{y}}$ To find the inverse, we set $y = f(x)$ and solve for x. That is:
  $$y = \frac{3}{x-5}$$
  which is
  $$yx - 5y = 3$$
  which is
  $$yx = 3 + 5y$$
  Dividing by y we get
  $$x = \frac{3+5y}{y}$$
  In other words
  $$f^{-1}(y) = \frac{3+5y}{y}$$

(b) State the domain and the range of f.

- $\boxed{\text{Domain is } \{x : x \neq 5\} \text{ and Range is } \{y : y \neq 0\}}$ The second one is true because the range of f is the same thing as the domain of f^{-1}

(c) State the domain and the range of f^{-1}.

- $\boxed{\text{Domain is } \{y : y \neq 0\} \text{ and Range is } \{x : x \neq 5\}}$

6. (4 points) Suppose $f(x)$ is an odd function and $g(x)$ is an even function which both have domain $(-\infty, \infty)$. (*You do not need to justify your answer on these problems*)

   (a) Is $(g \circ f)(x)$ an even function, an odd function, or not necessarily either?

   - $\boxed{\text{Even}}$ To see this, we have

   $$(g \circ f)(-x) = g(f(-x)) = g(-f(x)) = g(f(x)) = (g \circ f)(x)$$

   The 2nd equality is true because f is odd and the 3rd equality is true because g is even. The above says

   $$(g \circ f)(-x) = (g \circ f)(x)$$

   which is another way of saying $g \circ f$ is even.

   (b) Is $(f \circ f)(x)$ an even function, an odd function, or not necessarily either?

   - $\boxed{\text{Odd}}$ To see this, we have

   $$(f \circ f)(-x) = f(f(-x)) = f(-f(x)) = -f(f(x)) = -(f \circ f)(x)$$

   In other words

   $$(f \circ f)(-x) = -(f \circ f)(x)$$

   which is another way of saying $f \circ f$ is odd.

7. (3 points) Find a value of b such that -1 is a zero of the polynomial

$$p(x) = 3 + 5x + 6x^2 + bx^3 + 2x^{91}.$$

- $\boxed{b = 2}$ We want $0 = p(-1)$, but

$$\begin{aligned} p(-1) &= 3 + 5(-1) + 6(-1)^2 + b(-1)^3 + 2(-1)^{91} \\ &= 3 - 5 + 6 - b - 2 \end{aligned}$$

So we want

$$\begin{aligned} 0 &= 3 - 5 + 6 - b - 2 \\ &= 2 - b \end{aligned}$$

Or $b = 2$