Instructions: Put your name, PID, section number, and TA's name on your blue book. No calculators or electronic devices are allowed. Turn off and put away your cell phone. You may use one page of handwritten notes, but no other resources. Make sure your solutions are clear and legible. **Show all of your work**. Credit will not be given for unreadable or unsupported answers. Please keep the questions in order in your blue book and clearly indicate which problem is on which page.

1. (9 points) Let the rational function $r(x)$ be defined by

$$r(x) = \frac{2x^2 + 3x + 6}{x^2 - 4x - 5}$$

   (a) (3 pts) Find the domain of $r(x)$.

   $\boxed{\{x : x \neq 5, -1\}}$ We cannot have the denominator equal zero. In other words, we can't have

$$x^2 - 4x - 5 = 0$$

   But

$$x^2 - 4x - 5 = (x - 5)(x + 1)$$

   So $x \neq 5$ and $x \neq -1$.

   (b) (2 pts) Find the horizontal asymptote of $r(x)$ if it exists. If not, state that it does not exist.

   $\boxed{y = 2}$ Taking the leading degree terms on both the numerator and denominator, for x large, we get

$$r(x) \approx \frac{2x^2}{x^2} = 2$$

   (c) (4 pts) By polynomial long division (or any other method you want), write $r(x)$ in the form

$$A + \frac{Bx + C}{x^2 - 4x - 5}$$

   where A, B, and C are real numbers.

   $\boxed{r(x) = 2 + \frac{11x+16}{x^2-4x-5}}$ We have

$$\begin{array}{r} 2 \\ x^2 - 4x - 5 \overline{\smash{\big)}\, 2x^2 + 3x + 6} \\ -\left(2x^2 - 8x + 10\right) \\ \hline 11x + 16 \end{array}$$

2. (6 points) Solve the equation for x:

$$\log_4(x + 1) + \log_4(x - 2) = 1$$

   (i.e. find all real x that satisfy the equation)

   $\boxed{x = 3}$ We have

$$\log_4(x + 1) + \log_4(x - 2) = 1$$

   Using a log rule, we get

$$\log_4((x + 1)(x - 2)) = 1$$

which means

$$(x+1)(x-2) = 4^1 = 4$$

So

$$x^2 - x - 2 - 4 = 0$$

Or

$$x^2 - x - 6 = 0$$

Since

$$x^2 - x - 6 = (x-3)(x+2)$$

The solutions to this are $x = -2$ and $x = 3$. You cannot put $x = -2$ into the original equation so the only solution is $x = 3$.

3. (7 points) Suppose u and v are two numbers such that

$$\log_3(u) = 6.3 \qquad \text{and} \qquad \log_3(v) = 2.5$$

Find:

(a) (2 pts) $\log_3\left(\frac{u^2}{v}\right)$

$\boxed{10.1}$ We have

$$\begin{aligned} \log_3\left(\frac{u^2}{v}\right) &= \log_3\left(u^2\right) - \log_3(v) \\ &= 2\log_3(u) - \log_3(v) \\ &= 2(6.3) - 2.5 \\ &= 12.6 - 2.5 \\ &= 10.1 \end{aligned}$$

(b) (3 pts) $\log_9\left(u^2\right)$

$\boxed{6.3}$ Using the change of base formula we have

$$\log_9\left(u^2\right) = \frac{\log_3\left(u^2\right)}{\log_3(9)}$$

But $\log_3(9) = 2$, so

$$\begin{aligned} \log_9\left(u^2\right) &= \frac{\log_3\left(u^2\right)}{\log_3(9)} \\ &= \frac{2\log_3(u)}{2} \\ &= \log_3(u) \\ &= 6.3 \end{aligned}$$

(c) (2 pts) $\log_3\left(v\sqrt[3]{u}\right)$

$\boxed{4.6}$ We have

$$\begin{aligned}\log_3\left(v\sqrt[3]{u}\right) &= \log_3(v) + \log_3\left(u^{1/3}\right)\\ &= \log_3(v) + \frac{1}{3}\log_3(u)\\ &= 2.5 + \frac{6.3}{3}\\ &= 2.5 + 2.1\\ &= 4.6\end{aligned}$$

4. (5 points) Suppose a bacteria culture initially has a population of 100 cells and also suppose that it doubles in population every 2 hours. How much time will pass before the population reaches 1400 cells?

   (Your answer may or may not be an integer and your answer may include logs or exponents- That is expected because you do not have a calculator)

   $\boxed{2\log_2(14)}$ We have

$$A(t) = 100 \cdot 2^{t/2}$$

   We want the time such that $A(t) = 1400$. Thus we get

$$1400 = 100 \cdot 2^{t/2}$$

   so that

$$14 = 2^{t/2}$$

   Taking $\log_2$ of both sides, we get

$$\log_2(14) = t/2$$

   or

$$t = 2\log_2(14)$$

5. (7 points) Solve the system of equations:

$$\begin{aligned}x + 2e^y &= 5\\ 2x + e^y &= 4\end{aligned}$$

   (i.e. find all pairs (x, y) that satisfy both equations).

   $\boxed{x = 1, y = \ln(2)}$ We have by the first equation

$$x = 5 - 2e^y.$$

   Plugging this into the second equation, we get

$$2(5 - 2e^y) + e^y = 4$$

   or

$$-3e^y = -6$$

   So that

$$e^y = 2$$

   Plugging this back into $x = 5 - 2e^y$ we get

$$x = 5 - 2(2) = 1$$

   Thus $x = 1$ and $y = \ln(2)$

6. (6 points) Suppose for a snack you have some strawberries and cherries. Each strawberry has 4 Calories and each cherry has 5 Calories. Suppose the snack has a total of 116 Calories and that you have 25 pieces of fruit. How many fruits of each kind do you have?

   $\boxed{\text{9 strawberries and 16 cherries}}$ Let s be the number of strawberries and let c be the number of cherries. Then we have the equations

$$\begin{aligned} s + c &= 25 \\ 4s + 5c &= 116 \end{aligned}$$

   Adding -4 times the first equation to the second, we get

$$\begin{aligned} s + c &= 25 \\ c &= 16 \end{aligned}$$

   Plugging $c = 16$ into the first, we get $s = 9$