STAT 443 – Winter 2015 – Assignment 2

due Thursday February 12 at the beginning of class

You may work in pairs if you choose; both names and ID numbers should appear on it, and both will receive the same mark. (No extra credit will be given for working alone.)

For any parts involving R, you should hand in the R code and output, as well as your interpretations of the output. You will NOT receive marks for uncommented R code or output.

1. Consider two independent, zero mean and stationary processes $\{X_t\}$ and $\{Z_t\}$, where $\gamma_X(h) = (0.8)^{|h|}$ and $\{Z_t\} \sim WN(0, 3^2)$. Define the new time series $\{Y_t\}$ to be $Y_t = X_t(Z_t + Z_{t-1})$.
   (a) Show that Y_t is an $MA(1)$ process. (Hint: we know a process is $MA(q)$ iff it is q-correlated)
   (b) Find the values of the parameters of Y_t, θ and σ^2. For which value(s) is Y_t is invertible?

2. Consider the time series $\{X_t\}$ defined by $X_t = \frac{5}{6}X_{t-1} - \frac{1}{6}X_{t-2} + Z_t - \frac{9}{20}Z_{t-1} + \frac{1}{20}Z_{t-2}$ where $Z_t \sim WN(0, \sigma^2)$
   (a) Show that $\{X_t\}$ is an $ARMA(2,2)$ process.
   (b) Is $\{X_t\}$ causal? Provide a detailed proof for your answer. If your answer is yes, provide the causal solution.
   (c) Is $\{X_t\}$ invertible? Provide a detailed proof for your answer.

3. Consider the $MA(1)$ process $X_t = Z_t + \theta Z_{t-1}$ where $\{Z_t\} \sim WN(0, \sigma^2)$. Suppose that the observation at time t is missing and we want to predict it based on the available observations. Let $\widehat{X}_t$ be the best linear predictor of X_t in terms of X_{t-1} and X_{t+1}, i.e. it minimizes the MSE.
   (a) Compute the predictor $\widehat{X}_t$.
   (b) Compute the mean square error (MSE) of the predictor $\widehat{X}_t$.

4. Let X_t be an $MA(3)$ process defined as $X_t = Z_t - 0.3Z_{t-1} + Z_{t-2} - 0.3Z_{t-3}$ where $Z_t \sim WN(0, 0.6^2)$.
   (a) Derive the best linear predictor of X_{t+1} in terms of the last 4 observations up to (and including) time t. You can use R or any other software to perform matrix inversion (if needed). In R, if A is a square matrix, `solve(A)` will give A^{-1}. Also, to multiply matrices A and B in R, use `A%*%B`.
   (b) Derive the best linear predictor of X_{t+h}, for $h \geq 4$. Explain logically why this makes sense for this process. Can you hypothesize the general result for an $MA(q)$ process?

5. Each of the three series posted - SeriesA, SeriesB, and SeriesC, all in SeriesforA2.txt - are examples of stationary time series. For each series, carry out the following steps:
   (a) Plot the series
   (b) Obtain the SACF and SPACF of the series, and use these to suggest some plausible $ARMA(p,q)$ models for the series. (Stick to low order models with $p + q \leq 4$ for which there is compelling evidence suggested by the plots.)
   (c) For each plausible model in (b), carry out an ARIMA fit using the following R code:
   ```
   seriesX.armapq <- arima(seriesX, order=c(p,0,q), method="ML")
   ```
   (d) Run diagnostics on each model from (c), including residual plot, SACF, SPACF, and QQ-plot as well as any other tests you deem relevant. Which models have residuals that look like white noise?
   (e) Among those models in (d) which pass, select the "best" one (if there is more than one) by choosing the one with the smallest `aic`. Which is the final model you choose for each series?