

Fin 585
Diether
Problem Set
The CAPM

Instructions

You can discuss these homework questions with other students in the class, but you must prepare your own answers, and submit your own write-up of the answers to Learning Suite. Your answers must be typed.

The main purpose of this assignment is to help you think about CAPM rather than evaluate your understanding of CAPM. The goal is to understand efficient markets much better after we discuss the assignment in class.

Questions

1. Suppose we are at time 1 and (i) each investor i is a risk averse expected utility maximizer; (ii) there is complete agreement among investors about the joint distribution of the payoffs on securities at time 2 (prices plus dividends or interest), and (iii) the joint distribution is multivariate normal. Thus, once prices of securities are set at time 1, the joint distribution of security returns is multivariate normal. In the scenario described above, unrestricted riskfree borrowing and lending and unrestricted short selling of positive variance securities leads to the conclusion that the value-weight market portfolio is mean-variance efficient. Outline the arguments. (Mathematical proofs are unnecessary. Verbal and/or geometric descriptions of the key arguments will suffice.)
2. Suppose we are in the world of the two-parameter portfolio model and optimal portfolios for investors are mean-variance efficient. Suppose Mr. X chooses the mean-variance efficient (MVE) portfolio e . Since e is on the MVE boundary, there is a linear relation between the expected return on any security j and its risk in portfolio e :

$$E(r_j) = E(r_{oe}) + \beta_{je} [E(r_e) - E(r_{oe})], \quad \text{where} \quad \beta_{je} = \frac{\text{cov}(r_j, r_e)}{\sigma^2(r_e)} \quad (1)$$

- a) "This is a variant of the familiar CAPM market equilibrium condition. It holds because the market must price securities to ensure that e is MVE." Assuming only what is stipulated in the statements before the quote, do you agree that (1) is a condition on equilibrium prices and expected returns?
- b) "Equation (1) means that the n available individual securities scatter randomly about the line in the $(E(r_j), \beta_{je})$ plane defined by this equation." Do you agree? (Words suffice. A derivation of (1) is unnecessary.)
- c) Are there securities with expected return equal to $E(r_{oe})$? If so, in what sense are

they riskless? Are they riskless in all MVE portfolios?

- d) Does the linear relation between $E(r_j)$ and β_{je} hold if j is another mean-variance-efficient portfolio?.
3. Suppose there is unrestricted riskfree borrowing and lending at the rate r_f , so all mean-variance efficient (MVE) portfolios combine riskfree lending or borrowing with a single tangency efficient portfolio T of all risky securities. Consider a portfolio e that is not in this MVE set, but is in the set of MVE portfolios that can be constructed using only risky securities. “Since e is in an MVE set, the risk-return relation (1) holds for all risky (positive variance) securities, as well as for the riskfree security F .” (Note, (1) refers to the equation labeled (1) in question #2). Are the two parts of the statement in quotes, after the first comma, correct?
4. Mr. Y thinks smoking is immoral. He instructs his financial advisor to construct MVE portfolios that do not include firms in the tobacco industry. Mr. Y then chooses a portfolio e from this restricted MVE set. Because this e excludes tobacco securities, it will not produce an expected return – risk relation like (1) that applies to all securities and portfolios. Indeed, his e will not produce an expected return – risk relation like (1) that applies to any securities or portfolios. Do you agree with the last two sentences? (Note, (1) refers to the equation labeled (1) in question #2).