Closed-form DSM method for the free vibration of orthotropic rectangular thin plates

Four SOV-type eigenvalue equations

$$\alpha_1=\chi\sqrt{\sqrt{\left(\frac{D_{12}I_2}{D_{11}I_1}-2\frac{D_{66}I_3}{D_{11}I_1}\right)^2-\frac{D_{22}I_4}{D_{11}I_1}+b^4\Omega_x^4}+\frac{D_{12}I_2}{D_{11}I_1}-2\frac{D_{66}I_3}{D_{11}I_1}}$$

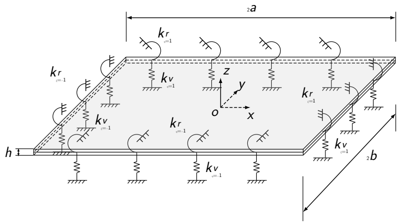
$$\beta_1=\chi\sqrt{\sqrt{\left(\frac{D_{12}I_2}{D_{11}I_1}-2\frac{D_{66}I_3}{D_{11}I_1}\right)^2-\frac{D_{22}I_4}{D_{11}I_1}+b^4\Omega_x^4}-\frac{D_{12}I_2}{D_{11}I_1}+2\frac{D_{66}I_3}{D_{11}I_1}}$$

$$\alpha_2=\frac{1}{\chi}\sqrt{\sqrt{\left(\frac{D_{12}J_2}{D_{22}J_1}-2\frac{D_{66}J_3}{D_{22}J_1}\right)^2-\frac{D_{11}J_4}{D_{22}J_1}+\frac{a^4D_{11}}{D_{22}}\Omega_y^4}+\frac{D_{12}J_2}{D_{22}J_1}-2\frac{D_{66}J_3}{D_{22}J_1}}$$

$$\beta_2=\frac{1}{\chi}\sqrt{\sqrt{\left(\frac{D_{12}J_2}{D_{22}J_1}-2\frac{D_{66}J_3}{D_{22}J_1}\right)^2-\frac{D_{11}J_4}{D_{22}J_1}+\frac{a^4D_{11}}{D_{22}}\Omega_y^4}-\frac{D_{12}J_2}{D_{22}J_1}+2\frac{D_{66}J_3}{D_{22}J_1}}$$

+

Two eigenvalue equations from boundary condition



The orthotropic rectangular plate with all edges elastically restrained.

$$\det(\mathbf{R}_x)=0$$

$$\det(\mathbf{R}_y)=0$$

SOV-DSM formulation and solution

Traditional SOV methods

- Six unknown variables: α_1, β_1, Ω_x, α_2, β_2, Ω_y.
- Four nonlinear and two highly nonlinear transcendental equations.
- Expensive computational cost and convergence difficulties.

Construct SOV-DS matrices

$$\mathbf{K}_x=\mathbf{R}_x\mathbf{Q}_x^{-1}$$
$$\mathbf{K}_y=\mathbf{R}_y\mathbf{Q}_y^{-1}$$

- Two unknown variables: Ω_x,Ω_y.
- Two highly complex SOV-DS matrices: $\mathbf{K}_x(\Omega_x)$ and $\mathbf{K}_y(\Omega_y)$.

Enhanced Wittrick–Williams algorithm to solve SOV-DS matrices:

$$J(p)=J(\bar{p}_1)-J_k(\bar{p}_1)+J_k(p,\omega^*)$$

The problem of solving $J_0(p_1)$ is transformed into solving $J(\bar{p}_1)$.

If $\bar{p}_1$ = S–S boundary condition, closed-form expression to find $J(\bar{p}_1)$:

$$b\Omega_{x,n_x}^4=\left[\left(\frac{n_x\pi}{2\chi}\right)^2-\frac{D_{12}S_2}{D_{11}S_1}+2\frac{D_{66}S_3}{D_{11}S_1}\right]^2-\left(\frac{D_{12}S_2}{D_{11}S_1}-2\frac{D_{66}S_3}{D_{11}S_1}\right)^2+\frac{D_{22}S_4}{D_{11}S_1}$$

$$a\Omega_{y,n_y}^4=\frac{D_{22}}{D_{11}}\left\{\left[\left(\frac{n_y\pi\chi}{2}\right)^2-\frac{D_{12}T_2}{D_{22}T_1}+2\frac{D_{66}T_3}{D_{22}T_1}\right]^2-\left(\frac{D_{12}T_2}{D_{22}T_1}-2\frac{D_{66}T_3}{D_{22}T_1}\right)^2+\frac{D_{11}T_4}{D_{22}T_1}\right\}$$

Other specific boundary condition:

- $\bar{p}_1=S\text{–}G$.
- $\bar{p}_1=G\text{–}G$.
- $J(\bar{p}_1)$ available.

Mode shape computation for arbitrary boundary conditions

First-order Taylor series:

$$\mathbf{R}_{x,k}(\omega_k)\mathbf{A}_k=\mathbf{R}_{x,a}\mathbf{A}_k+(\omega_k-\omega_a)\mathbf{R}'_{x,a}\mathbf{A}_k+O\left((\omega_k-\omega_a)^2\right)=0$$

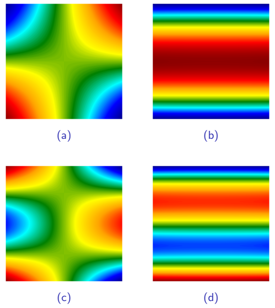
Inverse iteration procedure:

$$\bar{\mathbf{A}}^{(i+1)}=\mathbf{R}_{x,a}^{-1}\mathbf{R}'_{x,a}\mathbf{A}^{(i)}$$

Mode shape:

$$\phi(\xi)=A_1\sin(\alpha_1\xi)+A_2\cos(\alpha_1\xi)+A_3\sinh(\beta_1\xi)+A_4\cosh(\beta_1\xi)$$
$$\psi(\eta)=B_1\sin(\alpha_2\eta)+B_2\cos(\alpha_2\eta)+B_3\sinh(\beta_2\eta)+B_4\cosh(\beta_2\eta)$$

The square orthotropic plate with FFFF boundary conditions:



(a) the first mode; (b) the second mode; (c) the third mode; (d) the fourth mode.

Advantages:

- General solution for arbitrary boundary conditions.
- Closed-form DSM.
- Explicit closed-form expression for the J_0 term.
- Avoids solving highly nonlinear equations.