Two-Layer Model of Earth using Central Pressure Method

Authors: J. Kamen, V. Karkour, D. Zurawski

Motivation

In this project, we use a function that creates a two-layer model of Earth using the "Central Pressure" method we utilized in class for a single homogenous layer model. We utilize formulas like the Core Mantle Boundary (CMB) and the Equations of State (EOS) to create a more realistic model of the Earth that is separated into two layers: the core (Fe) and the mantle (silicates). The arrays that the function creates are for pressure, radius, density, and mass. Further explanation of our code will be done in the methods section of our report. We analyze our model to determine if it is similar or equivalent to the Earth's data points as a reference for our data. With this project, we hope to better understand how adding another layer to a planet can increase its complexity as well as its mass and radius slope to see if it is merely a logarithmic line.

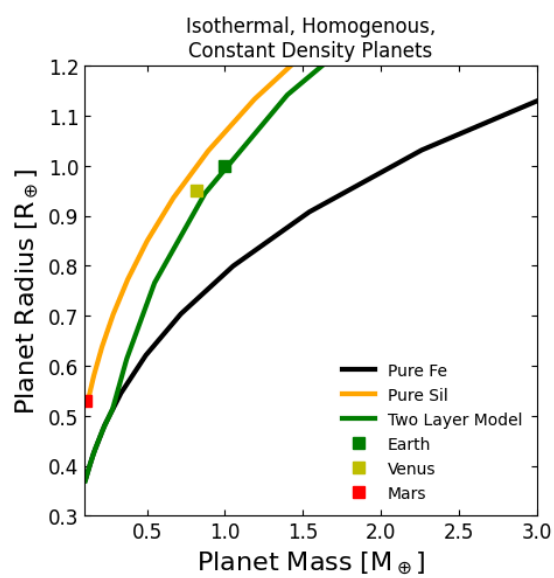
Methods

To begin, we create a copy of the class example code named "ModelingEarthUsingCentralPressure.ipynb". This code is very similar to our own, so we will explain this code in detail. In this code and our own, it is assumed that the planet's we will be creating are isothermal, meaning equal temperature, homogenous, meaning they have the same composition everywhere, and of constant density throughout the entire planet. The parameters of the function goes as follows: a pressure array from least pressure to greatest pressure, the surface pressure given in Pascals, the difference in radius for each step of the pressure, and the initial material given to describe the density. The code contains a function that takes a given density, pressure, and change of radius to output three important arrays used to create the planet: mass, radius, and density. Pressure is also an array included in this code, however, it is not outputted as a result until later. It initializes these four arrays within a for loop that iterates through the given pressure to create these arrays. For the radius array, it changes in an iteration of every 100 meters. The pressure array changes in accordance to the for loop by 1. The density array stays constant for each iteration, and the mass array changes for every 100 meters of the radius, in the isolated equation for mass: $dM = \frac{4}{3}\pi\rho dr$. When the code leaves this initialization of the function, it moves into a while loop that takes each value in the pressure array up until the surface pressure. It also uses the EOS to calculate the next value of gravity (as it must calculate everything below it as well), pressure at the latest given pressure, mass at the latest given pressure, and density at the latest given pressure. For Gravity, it uses the simple equation of gravity: $g = \frac{GM}{r^2}$. This pressure at the latest given iteration of pressure is described as a differential equation similar to the initialized mass: $dP = P_0 - \rho g dr$. The mass has a similar differential as its previous equation for initialization, so I will not describe it here, and the density is constant for the entirety of the example code's function. After all the calculations for the mass, pressure, and density are complete, it is then appended into the array to contain the calculation, which it then iterates to the next latest value of pressure, up until it reaches the surface pressure. Once the value is out of the while loop, it is then normalized compared to the Earth's radius, mass, and density, and then finally appended into the finalized array of the planet's radius, mass, and density. This is all that is done for the example code, and it works very effectively in creating each section of the planet. This function only creates a single layer planet that is isothermal, homogenous, and of constant density though. Our function works with the same assumptions and creates a two layer planet.

We will now begin explaining our code for the two layer planet model. It has a very similar function to the example code explained above, however to solve for a two layer model, we must find the point at which one density changes to another density. We do this by finding the radius of the core of the planet using the CMB equations. We can rearrange these equations to solve for the radius of the core. This equation is: $R_c = \left(\frac{\frac{3M_\oplus}{4\pi} - R_\oplus^3 \rho_m}{\rho_c - \rho_m}\right)^{(1/3)}$. With this value, we are able to switch the density in the while loop for whenever the radius of the array goes beyond the value we found in the CMB radius equation. So when it passes the radius of the core, it switches from the density of iron to the density of silicon. It continues utilizing the same equations as it switches between densities, and these are taken from the example code, and with this simple change, we are able to create a two layer model of a planet compared to Earth units.

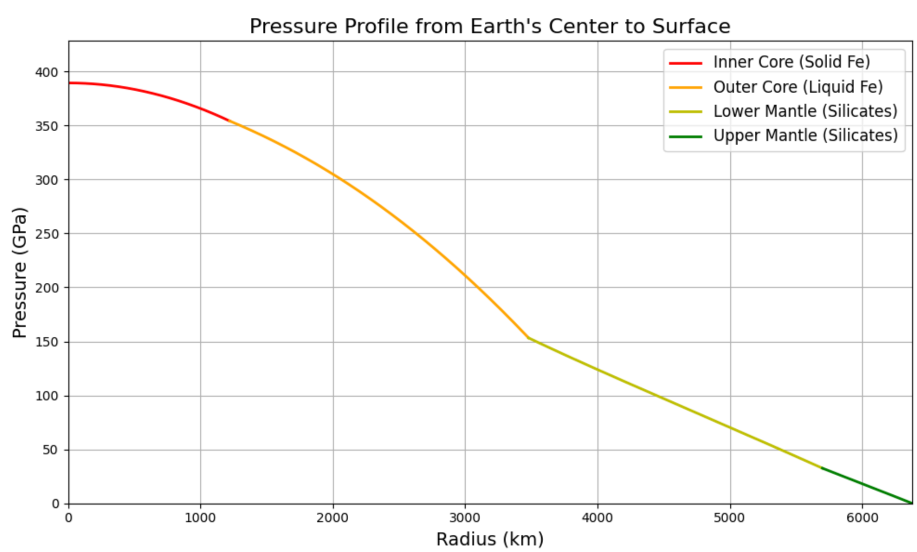
Results

We use our method of creating a two layer model planet using pressures from 10^2 to 10^3 in steps of 15. This pressure list is then multiplied by 10^9 in accordance with the example code's creation of a planet. The resulting figure is shown on the right. The figure has the x-axis as the planet's mass and the y-axis as the planet's radius. The legend at the bottom right differentiates each line and data point on the plot. For the pure iron line, we can see that the radius constantly decreases for each value of mass that is added on, showing a decreasing slope as the function continues. In the pure silicon line, we see that the radius does



decrease as mass increases, however, it decreases at a much slower rate compared to the pure iron line. Comparing both lines, we can suspect that as the density of a planet increases, the mass to radius relationship decreases. We understand this because of the difference in densities of iron and silicon, 11500 and 4500 kg/m^3 respectively. From our figure as well, we can see that our two layer model of a planet follows a similarly to the pure iron line from 0 to ~$0.3M_{\oplus}$. After this point, it jumps up dramatically, attempting to match the pure silicon line. It cannot do so however, due to its addition of iron in the core at the 0-$0.3M_{\oplus}$ range. Looking at the two layer model line, we are can see that the model fits and passes through the Earth data point of $1M_{\oplus}$and $1R_{\oplus}$. This means that our two layer model follows a similar trend to Earth's composition, which mainly consists of a mantle and core. Included in this figure are other rocky planets in our solar system. Analyzing Venus' composition from the mass-radius relationship, we can see that Venus follows a similar composition to our two layer model as well as Earth. In Mars' composition, we see that it follows the trend of the pure silicon line, however we understand that Mars' composition is not purely silicon. This could hint at the single layer model being too simplistic in terms of estimations of other planets due to the model being reliant on Earth-like data and normalizing data to Earth units.

Another figure we created with our planetary data is a pressure profile that transitions between regions of the core and mantle. For this, we created a two-layer model for an Earth-like planet and then plotted where the estimated densities and



core radii of each region would be for the given model. This can be seen in the plot above. In this plot, we can see that as the core transitions into the mantle, the pressure goes from a decreasing quadratic to a negative linear slope. This allows us to understand that not only does density change the mass-radius relationship in a planet, in doing so, it also changes the pressure of a planet, as we can see from the transition between iron and silicon. This also allows us to believe that in a pure iron and pure silicon model, one would have a quadratic decreasing slope for the entirety of the pressure profile while the other will merely have a negative linear slope. This analysis can be incorrect however, as our model may be too simplistic for the pressure profile to take into account, since we utilize many assumptions about our model.

Conclusion

From this project we have created a two layer model inspired from the Central Pressure method, understanding how changing the amount of layers in a planet can affect its mass and radius, and how different densities of materials can change the mass-radius relationship at planetary scales. This project has proven to us that while a mass-radius relationship can be complicated, it is not a logarithmic slope that can be defined so simply. With this, our motivation was successful. This project was very enjoyable, and much more tame compared to the other projects. We hope in the future that as this class continues beyond the scope of our semester, it can allow for smaller, less intense projects such as this.