Photon statistics Physics 423

The purpose of this lab is to investigate the distribution of photon arrivals due to a constant irradiance and a quasi-monochromatic (pseudo thermal) light source.

References

The following documents, available on Canvas, give the necessary background for photon statistics. They should be read in the following order.

- Poisson Distribution Derivation.
- "Photon Counting Statistics – An undergraduate experiment." Koczyk, P. *et al*. Am. J. Physics. **64**, 240-245, (1996).
- Photon counting. Cal Poly Q-Lab manual (PhotonCountW2017).

The idea behind the lab is that we are going to use the random number generator on your computer to give us numbers drawn from a uniform distribution and then use those numbers to simulate the arrival of photons. (Uniform distribution means that numbers in the interval 0 to 1 are equally likely).

The results of the simulations should be able to reproduce the data shown in figures 3 and 4 of the paper by Koczyk et al.

Matlab uses the function rand to return a uniform random number and if I type rand/return on the command line I get 0.8147. If I type rand/return again I get 0.9058. You should try this. Now shut down Matlab and restart it. If I type rand/return again I get 0.8147. Is that a coincidence? No, random numbers made by computers are not truly random in the same way that random numbers made by observing nuclear decays are random. The numbers are generated by a deterministic process that, given the same starting point, leads to the same sequence. Although the numbers are not random, they exhibit statistical behavior that is essentially indistinguishable from that of random numbers. To ensure we have new sequences every time we run the random number generator we use a different starting condition. This is called "seeding" the random number generator, and a common choice is to seed with the clock on your computer. The command for this is rng('shuffle') and it should be added at the start of your programs. That way, you do not end up using the same seed again and again.

The starting point for this lab is really the binomial distribution, and how it leads to the Poisson distribution. If you have taken Physics 340 you will have come across the Poisson distribution, and I have included a derivation of the distribution with the materials for this lab.

Part 1: The constant intensity case

Imagine generating a sequence of uniformly distributed random numbers, say N of them. This represents N trials. The chance of n photon arrivals would then be obtained by examining the numbers in your sequence and if any particular number is less than n/N we could count that as the presence of a photon. Clearly, n is the average number of photons you would expect in this experiment.

To see if we get the Poisson distribution we can thus run the simulation for a certain number of trials, and count the number of photons. This process is then repeated a great number of times so that you can figure the probability of obtaining 0 photons, 1 photon, 2 photons etc. This data can be compared to the functional form of the Poisson distribution.

Part 2: The pseudo-thermal case

Now the light irradiance is constantly changing. At any instant the value of the irradiance will give you the mean value to use with the random number generator to figure the number of photons that arrive. The situation is sketched in the Qlab manual (PhotonCountWinter2017). What we need is some way of generating the distribution of irradiance that would be found with this type of source. That distribution for the irradiance can be calculated and it is given by

$$P(I) = \frac{1}{I_{av}} \exp(-\frac{I}{I_{av}})$$

Where I stands for irradiance. This is what is called a negative exponential distribution.

The irradiance of the light is proportional to the photon count rate λ, so we can write

$$P(\lambda) = \frac{1}{\lambda_{av}} \exp(-\frac{\lambda}{\lambda_{av}})$$

This is the probability that at a randomly chosen point in the light field the photon count rate is λ.

What we must do is generate the probability distribution for the light irradiance, take values from that distribution, and use these values as the input to the binomial distribution. But how do you find the values for the probability of the light having a certain irradiance? That is, how can we draw probabilities from the above distribution? It can be shown that random deviates from the negative exponential distribution can be obtained as

$$-\lambda_{av} log_e(x) \qquad (1)$$

Where x is the uniform random deviate. To get to equation (1) you would need to know about the transformation of random variables. That takes us a bit far off course, but if you are interested the following document explains it all very nicely
https://www.cl.cam.ac.uk/teaching/2003/Probability/prob11.pdf

Your report

Your report will demonstrate how the Poisson and Bose-Einstein distributions arise.

You would need to submit the following:

- Code that plots the Poisson distribution, given the mean value you selected. The code would also compute the theoretical Poisson distribution for that mean value. This code should be thoroughly annotated so that the reader can clearly see what each part of your program is up to.

- Plots of the data you get, presented in the format shown in Figure 3 of the Kocyk paper. You should have a least 4 different mean values. You also need to supply an informative caption, telling the reader what they are looking at.
- Code that plots the Bose-Einstein distribution, given the mean value you selected. The code would also compute the theoretical distribution for that mean value. This code should be thoroughly annotated so that the reader can clearly see what each part of your program is up to.
- Plots of the data you get, presented in the format shown in Figure 4 of the Kocyk paper. You should have a least 4 different mean values. You also need to supply an informative caption, telling the reader what they are looking at.