Modelling optical waveguides Phys 423

In this lab we will be setting up computer models of optical waveguides to find the propagation constants and visualize the field distributions. We will use one-dimensional models (also known as slab waveguides), and to begin with we will focus on the TE case.

The first thing is to write down equations for the guidance conditions of waveguide modes and then solve these. Since the equations are transcendental, they need to be solved numerically. Once the propagation constants (meaning the z-components of the k vector) for the guided wave modes have been found we can then deduce the optical field distribution.

References

- Tamir, T. (Ed.) “Integrated Optics”. Springer Applied Physics Series (1975). This book is available for download through the library (https://lib.calpoly.edu/). I have copied out the first chapter and part of the second and put it with the supplementary material on Canvas. The first 10 pages of the second chapter contain information needed for our simulation.
- Hunsperger, R. G. “Integrated Optics: Theory and Technology” Springer (1991). Also available for download at the library. I have put Chapter 2 with the supplementary materials.

Solution of the transcendental equations

A side view of a slab guide is shown below

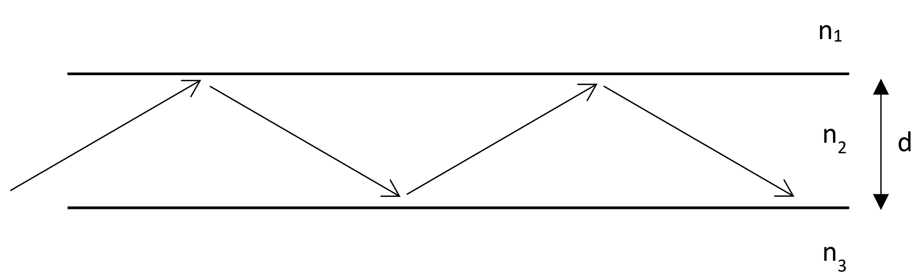

Figure 1: Geometry of the slab waveguide. The guide thickness is d and the refractive indexes of the guide and the cover/substrate are $n_2 > n_1, n_3$. The light propagating along the guide does so by bouncing at the interfaces at the top and bottom of the guide.

Within the “bouncing” interpretation the light bounces at the top and bottom of the guide and the guidance condition is given by

$$2kn_2dcos(\theta) + \phi_{2,3} + \phi_{2,1} = 2m\pi \tag{1}$$

Where k is the wavenumber and the phase shifts on reflection at the interfaces are given by the Fresnel formulas depending on whether or not we are using TE or TM modes. For the case of TE

$$\phi_{TE} = -2tan^{-1}\left(\frac{\sqrt{sin^2(\theta) - \left({}^{n_1}/{}_{n_2}\right)^2}}{\cos(\theta)}\right)$$

By solving the transcendental equation (1), we can find the bounce angle and thus the components of the wavevectors in the guide. Rather than expressing the propagation solution in terms of the angles of incidence it is better to express things directly in terms of the wavevector. If we imagine the waves propagating as shown in figure 1, we can write the wavevectors as shown in figure 2.

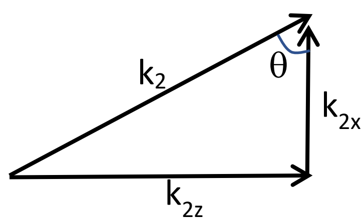

Figure 2: Wavevectors within the optical waveguide (where the refractive index is n_2). The guided wave propagates in the z direction and we can write $\vec{k_2} = \vec{k_{2z}} + \vec{k_{2x}}$. *Then* $k_2 = n_2k$ *and* $k_{2z} = n_{eff}k$ *where* n_{eff} *is the effective index and k is the free-space wavenumber. Note that we can write* $\cos(\theta) = {}^{k_{2x}}/{}_{k_2}$ *and* $\sin(\theta) = {}^{k_{2z}}/{}_{k_2}$

You should be able to rewrite the guidance condition in terms of the refractive indexes and the free space wavenumber, and put the formula into a form from which you can numerically solve for the effective index, where the effective index is defined in the caption to figure 2.

To begin with, we will look at the symmetric guide where the refractive index is the same above and below the guide, so n_1=n_3, and we will see the effect of increasing the thickness of the guide. The parameters to be used are wavelength = 633 nm, n_1=n_3=1.4, n_2=1.5, and vary the thickness from zero to 2 microns. You should be able to generate a graph like figure 3 which is a plot of the effective index vs thickness for thicknesses of zero up to 2 μm in steps of 0.01 μm. As you can see this type of calculation allows you to predict things like the maximum thickness for which there is only single mode propagation. Thinking in terms of “bounce angle” what angle does the light in this mode bounce off each interface? What is the speed of the modes as they propagate along the waveguide? Does the mode propagate faster or slower as we increase the mode number?

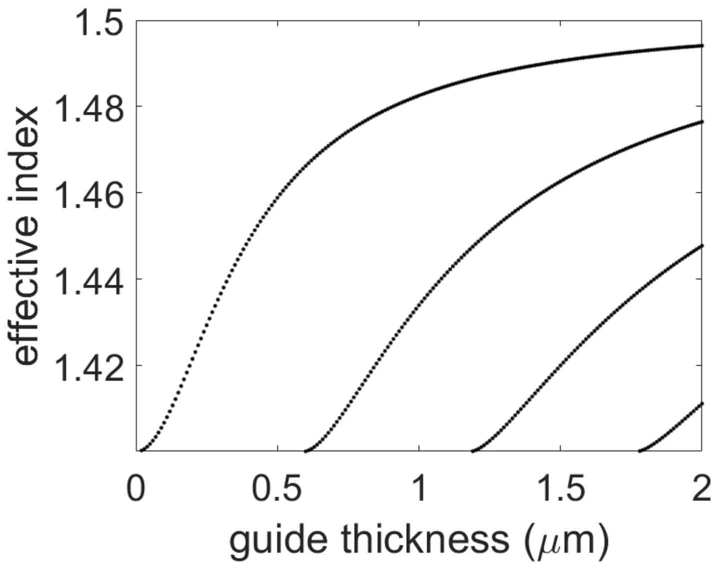

Figure 3: Effective index as a function of guide thickness. Wavelength = 633 nm, n_1=n_3=1.4, n_2=1.5.

There is a piece of code below that will get you started. All it does is plot a function as n_{eff} is varied between n_1 and n_2. As you can see (figure 4) the function passes through zero and the value of n_{eff} at that point is the solution. You will have to do a search for the zero point.

```
% a program that plots the function of neff
% TE mode equation
% The effective index of the guided wave is
% when the function = 0
% subtract integral multiples of pi to get
% higher modes

%jps 2021

lam=0.633e-6; %wavelength, meters
k=2*pi/lam;
n1=1.4; %cover and substrate
n2=1.5; %guide

d=1.7e-6; %guide thickness (m)

neff=n1:0.001:n2;

func=d*sqrt(k*k*(n2*n2-neff.*neff))-...
2*atan(sqrt(neff.*neff-n1*n1)./sqrt(n2*n2-neff.*neff))-pi;

plot(neff, func,'*');

xlabel('neff','FontSize',16);
ylabel('func','FontSize',16);
grid on
```

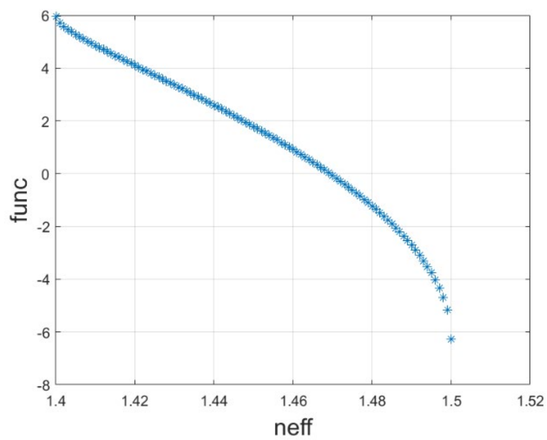

Figure 4: This function is generated with the above program. The correct value of the effective index is when func=0. The program has a guide of 1.7 microns and (note the negative pi) is plotting the function for the 2nd mode. The zero is close to neff = 1.47, and you can see in figure 3 that this is the effective index for the 2nd mode at that thickness.

How to find the zero?

It's perhaps easiest to do a binary split. Thus, start at either end of the range which runs from n_1 to n_2. Call these values "left" and "right". Find the midpoint between them, which is (left+right)/2. Then, if the product of (left x midpoint) is less than zero we know that the zero lies between left and midpoint – otherwise it lies between midpoint and right. If it is the former case then make right=midpoint and start again. If it is the latter, make left=midpoint. You can keep going until the change from iteration to iteration is below some tolerance, say 0.0001. Or you could do 10 iterations which, for this problem, should have about the same effect.

Asymmetric waveguides

Once you have written the program for the symmetric TE case it will hopefully not be too difficult to adapt it to the asymmetric case.

For the asymmetric guide replace the cover index with that of air, make the guide index 1.5095 and the substrate 1.4711. These are more realistic parameters than we used above, and would be typical of glass waveguides. Keep the wavelength at 633 nm. You now have different phase shifts at the top and bottom and you will have to incorporate that into your program. Once you have the program running you will notice that below a certain thickness the waveguide will not support any modes. This is in distinction to the symmetric guide where this cutoff of the zeroth order mode does not occur (see figure 3). FYI, the cutoff thickness for the m=0 mode with the above parameters is about 0.5 microns.

What about the optical fields?

Once we have the propagation constant we should then be able to write down the electric field distribution in and near the guide.

Consider again the TE mode with the wave propagating in the z direction and the guide confined between two planes in the x direction and unconfined in the y direction (see figure 5)

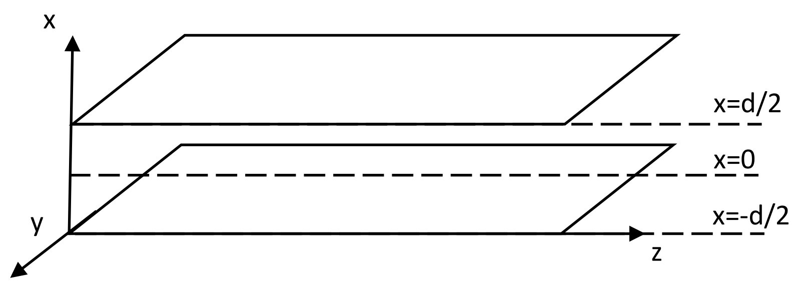

Figure 5: Revisiting the geometry of the slab guide.

We know that the propagation vector component in the x direction is real within the guide and imaginary outside (remember the evanescent field?) so we could write the field within the various regions shown in figure 5 as

$$E_y(x) = A_1 exp(-\alpha_{1x}x) \qquad x > {}^{d}/_{2}$$

$$E_y(x) = A_2 \cos(k_{2x}x + \phi) \qquad |x| < d\ /\ 2$$

$$E_y(x) = A_3 exp(\alpha_{3x}x) \qquad x < -d\ /\ 2$$

If we could get the As and the ϕ, we could plot these things. We know that the field is continuous across the boundaries but there are 4 unknowns (actually three since the overall amplitude is arbitrary) and two boundaries. However, we can get another constraint……

Starting with Faradays Law, we can write

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

We have TE waves so there is only oscillation in the y-direction and no variation in the y or z directions. Because these are harmonic waves the differentiation with time brings out iω and we get

$$\frac{\partial E_y}{\partial x} = i\omega B_z$$

Because the tangential component of the magnetic field is continuous, this means that across any interface we can say that the spatial derivative of the electric field is also continuous.

In summary, for TE waves, across a boundary

$$E_y \ is \ constant$$

$$\frac{dE_y}{dx} \ is \ constant$$

By using these boundary conditions, we can then solve for the constants in the field equations above. Using the continuity at the x=d/2 plane we can get

$$\tan\left(\frac{k_{2x}d}{2} + \phi\right) = \frac{\alpha_{1x}}{k_{2x}}$$

which will give ϕ. Then we can, say, set A_1=1 and solve to get

$$A_2 = \frac{exp\left(-\frac{\alpha_{1x}d}{2}\right)}{\cos\left(\frac{k_{2x}d}{2} + \phi\right)}$$

And

$$A_3 = A_2 \frac{\cos\left(-\frac{k_{2x}d}{2} + \phi\right)}{exp\left(-\frac{\alpha_{3x}d}{2}\right)}$$

Using this approach, we can stitch together the fields inside and outside the waveguide. An example is shown in figure 6. The brightness of the light would be obtained by taking the square of the amplitude so the observed brightness would feature four bright regions. You can see some experimental results for the output of a slab guide in figure 2.4 of the Hunsperger book, while figure 2.2 has a nice illustration of the various waveguide modes that can be excited.

The TM case

In the TM case the boundary conditions are going to change, but it should not be too hard to adapt your mode-finding program. We could also find the fields but that would also involve different boundary conditions at the edges of the waveguide.

Your report

You should hand in for your lab:

1) Program to generate n_{eff} vs d graphs for TE modes for a waveguide of index 1.5095, substrate index 1.4711 and the cover index 1.0. Free space light wavelength is 633 nm. You should include

as a pdf a plot showing the effective index vs guide thickness for guide thickness ranging from 0 to 10 microns (it will look similar to figure 3, above). You should provide a thorough caption for the graph so the reader knows what they are looking at.

2) Plots of the electric field of the m=0 and m=1 modes for a 2 micron wide guide with the above parameters. Should look similar to figure 6, and should be informatively captioned.
3) Graphs of effective index vs thickness for the TM modes of the same waveguide with the thickness varying from 0 to 10 microns. You do not need to submit the program. The graphs will look very similar to those for the TE, but will differ in detail because of the different boundary conditions.
4) EXTRA CREDIT! Create an image of the output of the guide for both modes in part 2 (it should look similar to figure 2.4 of Hunsperger).
5) EXTRA EXTRA CREDIT! Make an animation of the oscillations of the electric field showing how the fields oscillate with time. Submit the result as a GIF or MPEG or AVI. But don't make it too big.

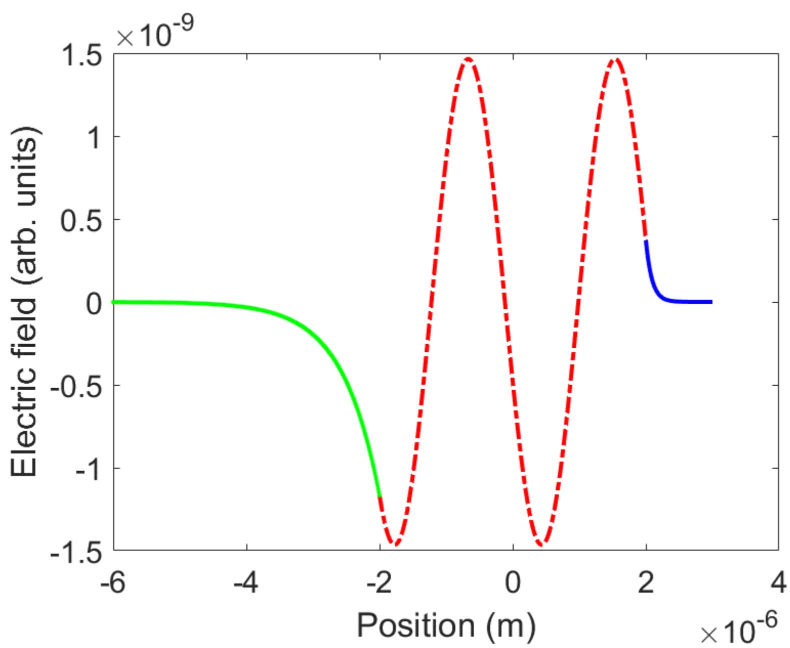

Figure 6. The electric field distribution for the m=3 mode in a 4-micron thick waveguide. Waveguide index is 1.5095, substrate index is 1.4711 and the cover index is 1.0. Free space light wavelength is 633 nm. The center of the guide (x=0, see figure 5) is at position zero and the field within the guide is shown as chained red. The evanescent fields in the air and the substrate are shown in blue and green, respectively. Note that the penetration into the substrate is much greater than into the air because the substrate is closer in index to that of the guide.