Lab 2 - Fourier Optics

Jason Pruitt

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1 Fresnel-Kirchoff Integral

Since the contribution to the electric field at point P is given by

$$dE_P = (\frac{E_A da}{r})e^{i(\omega t - kr)}, \tag{1}$$

and given approximation

$$r = r_0(1 - 2\frac{(xX + yY)}{r_0^2})^{1/2}, \tag{2}$$

which can be reduced by binomial expansion to

$$r = r_0(1 - \frac{(xX + yY)}{r_0^2}), \tag{3}$$

The electric field at point P is then obtained by integrating over the aperture,

$$E_P = (\frac{e^{i(\omega t - kr_0)}}{Z}) \iint E_A(x, y)e^{ik(xX+yY)/r_0} dxdy. \tag{4}$$

Furthermore, since the relative amplitude function can be defined by

$$A_P = ZE_P e^{i(\omega t - kr_0)} \tag{5}$$

it is then equivalent to write the expression as

$$A_P = \iint E_A(x, y)e^{ik(xX+yY)/r_0} dxdy. \tag{6}$$

Given angular spatial frequency relations

$$k_x \equiv \frac{kX}{r_0} \text{ and } k_y \equiv \frac{kY}{r_0}, \tag{7}$$

Equation 6 can then be reformed as

$$A_P = \iint E_A(x, y) e^{ik(xk_x + yk_y)/r_0} dk_x dk_y. \tag{8}$$

With this, we see that E_P and A_P are related by the two dimensional Fourier transform pair,

$$f(x, y) = \frac{1}{(2\pi)^2} \iint_{-\inf}^{+\inf} g(k_x, k_y) e^{-ik(xk_x + yk_y)/r_0} dk_x dk_y,$$

(9)

and

$$g(k_x, k_y) = \iint_{-\inf}^{+\inf} f(x, y) e^{ik(xk_x + yk_y)/r_0} dx dy. \tag{10}$$

[1]

2 Figures

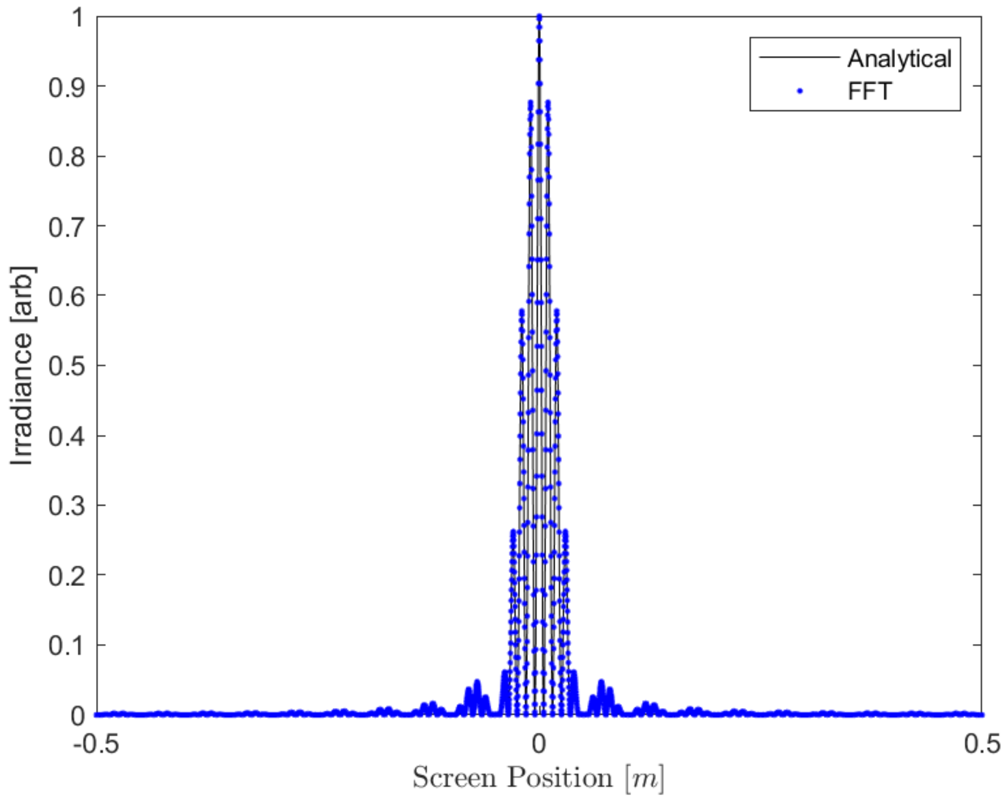

Figure 1: Irradiance pattern for 1-D Double Slit obtained from Fourier transform with analytical expression superimposed

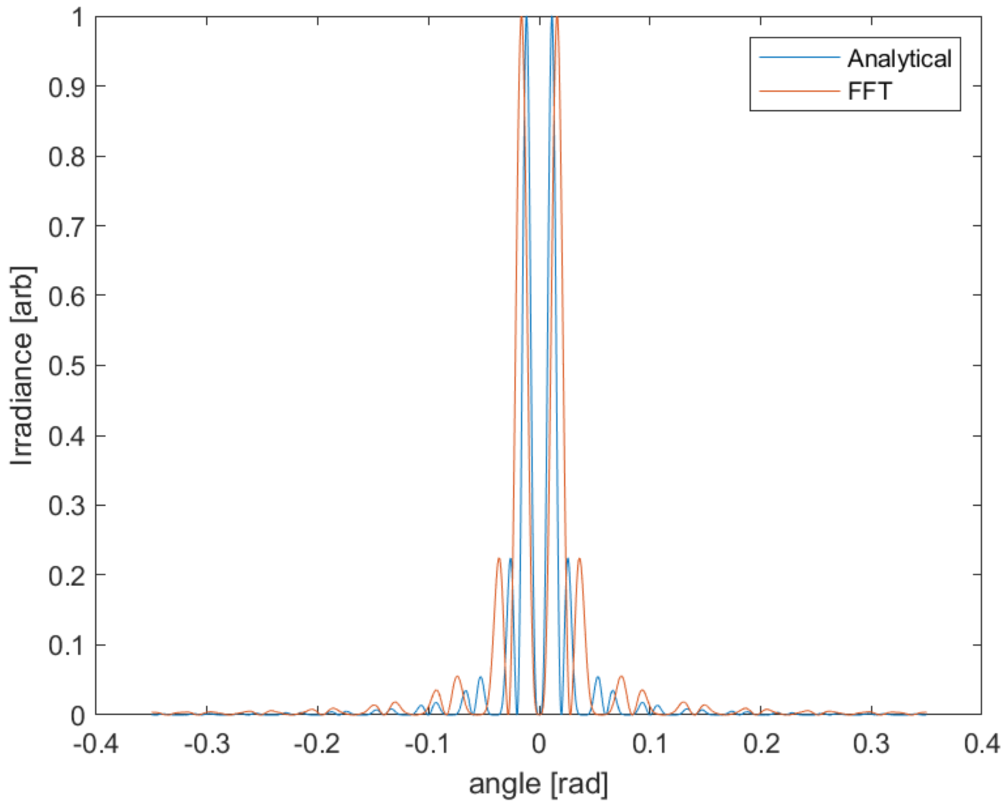

Figure 2: Irradiance pattern for 1-D Single Slit with phase element obtained from Fourier transform with analytical expression superimposed

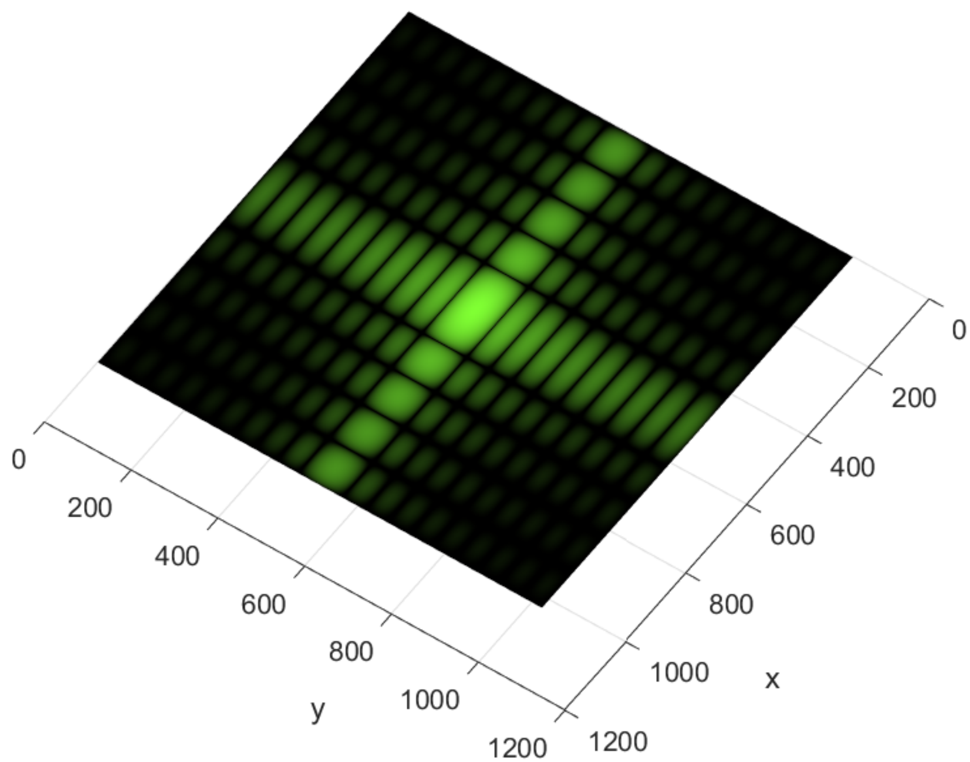

Figure 3: Irradiance pattern for 2-D Double Slit obtained from Fourier transform plotted on a logarithmic scale

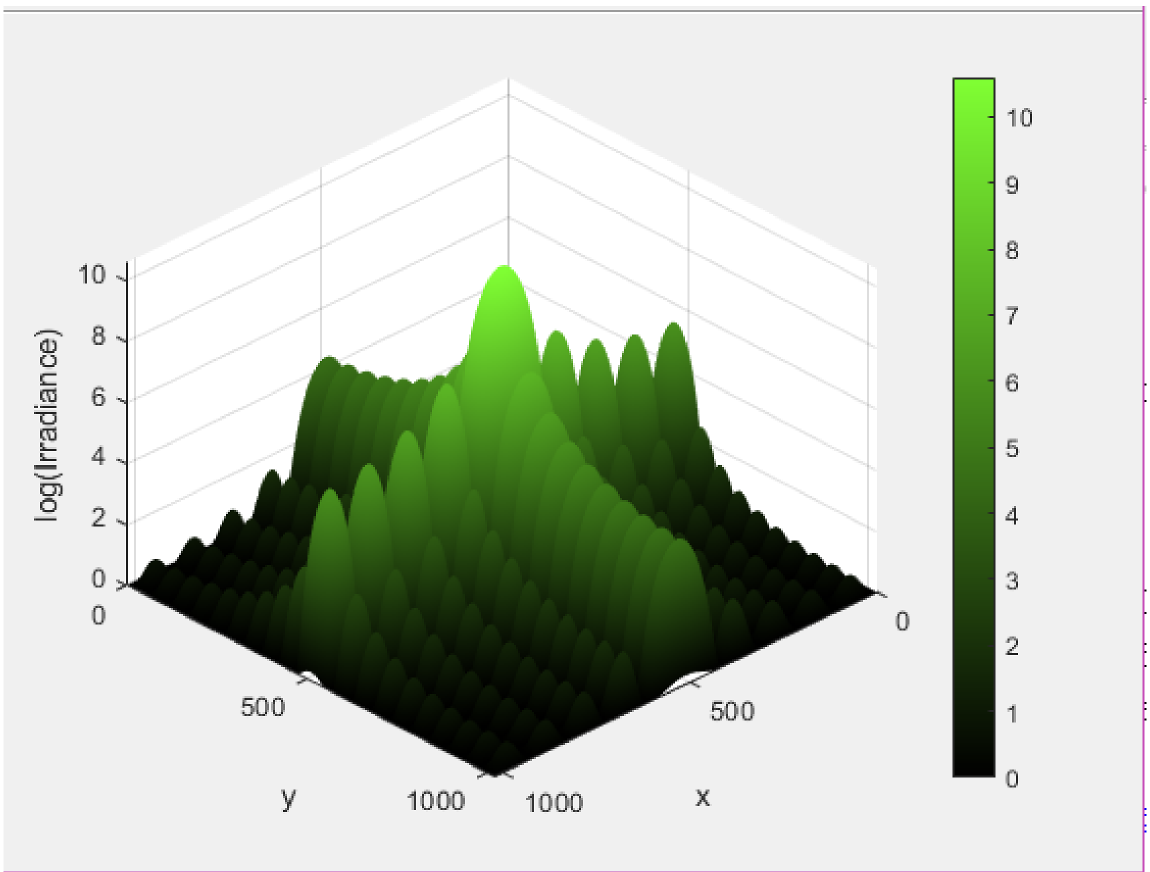

Figure 4: Side view of previous figure with colorbar for scale

References

[1] Pedrotti. *Introduction to Optics, Third Edition.*