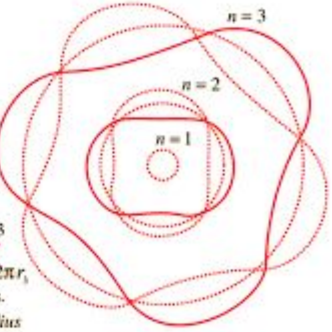
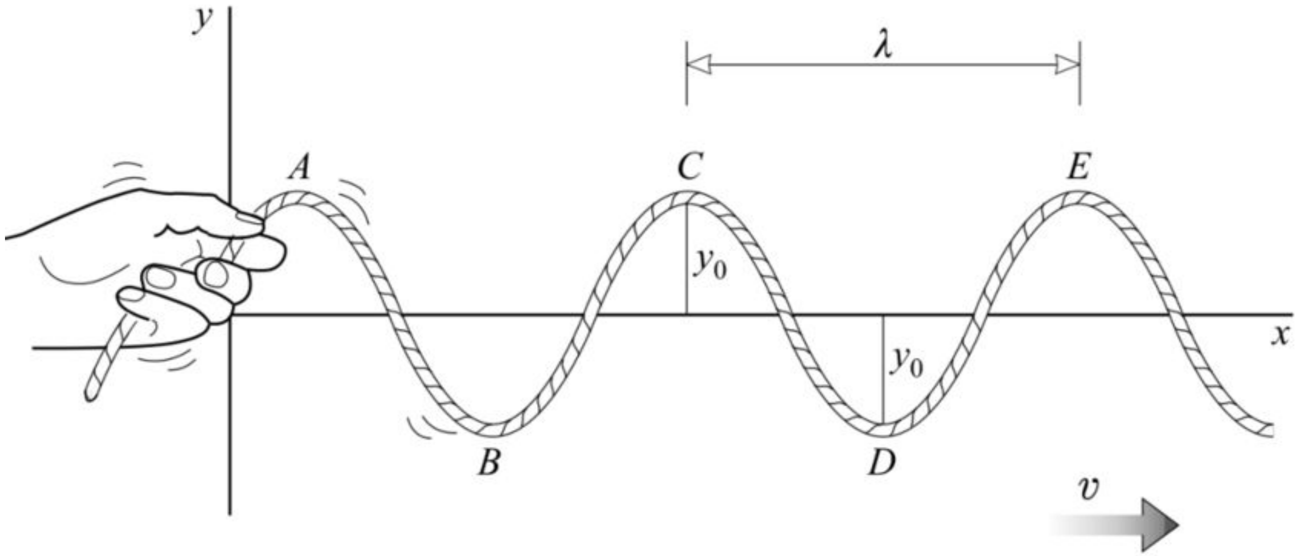
Simulating Waves Along a Circular Path

Jason Pruitt

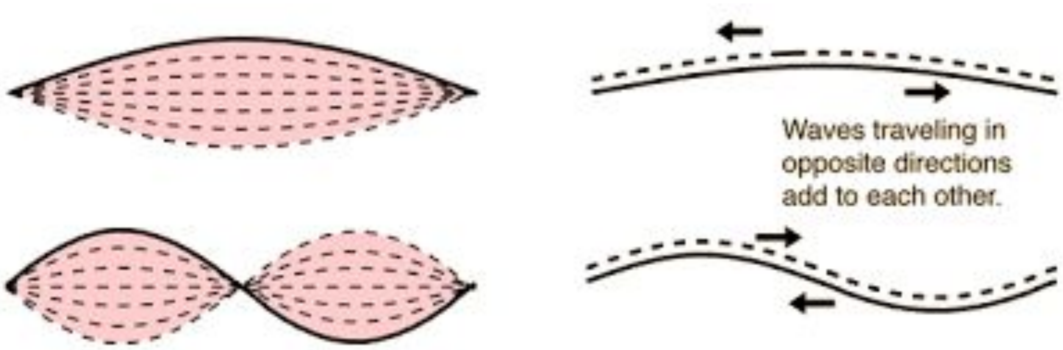


Waves on a String



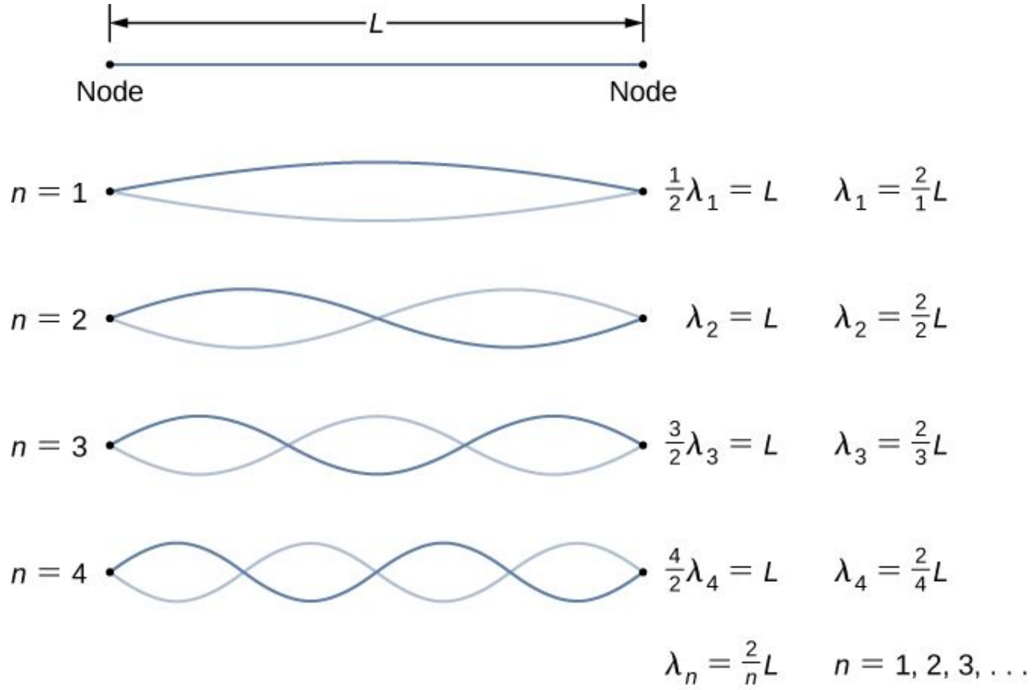
https://www.researchgate.net/figure/A-sinusoidal-wave-generated-in-a-stringsee-Wikipedia-18_fig1_262450664

Standing Waves



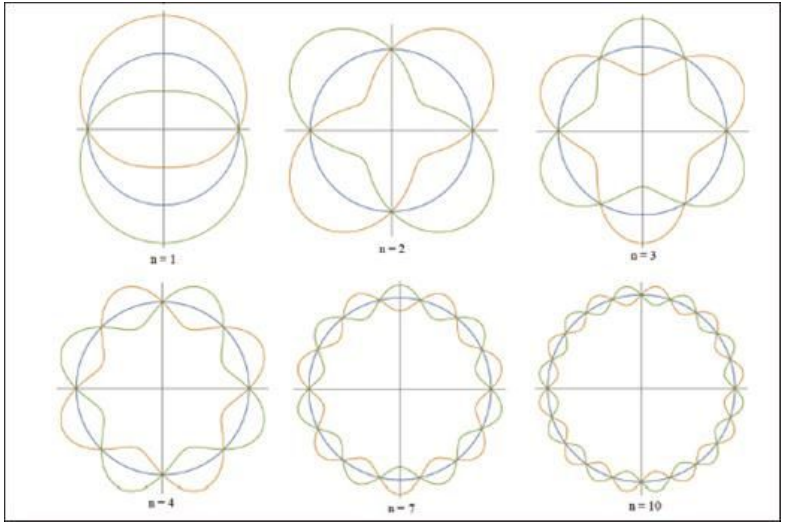
http://hyperphysics.phy-astr.gsu.edu/hbase/Waves/standw.html

Standing Waves



https://courses.lumenlearning.com/suny-osuniversityphysics/chapter/16-6-standing-waves-and-resonance/

Bohr Atomic Model



Methods

$$\frac{\partial \Psi}{\partial t} = v\frac{\partial \Psi}{\partial x},$$

Advection equation

$$\Psi_i^{n+1} = \Psi_i^n - \frac{c\Delta t}{\Delta x}(\Psi_{i+1}^n - \Psi_{i-1}^n),$$

Forward-Time Center-Space (FTCS)

$$\Psi_i^{n+1} = \frac{1}{2}(\Psi_{i+1}^n + \Psi_{i-1}^n) - \frac{c\Delta t}{\Delta x}(\Psi_{i+1}^n - \Psi_{i-1}^n),$$

Lax

$$\Psi_i^{n+1} = \Psi_i^n - \frac{1}{2}(\frac{c\Delta t}{\Delta x})(\Psi_{i+1}^n + \Psi_{i-1}^n) + \frac{1}{2}(\frac{c\Delta t}{\Delta x})^2(\Psi_{i+1}^n - \Psi_{i-1}^n - 2\Psi_i^n),$$

Lax-Wendroff

Methods

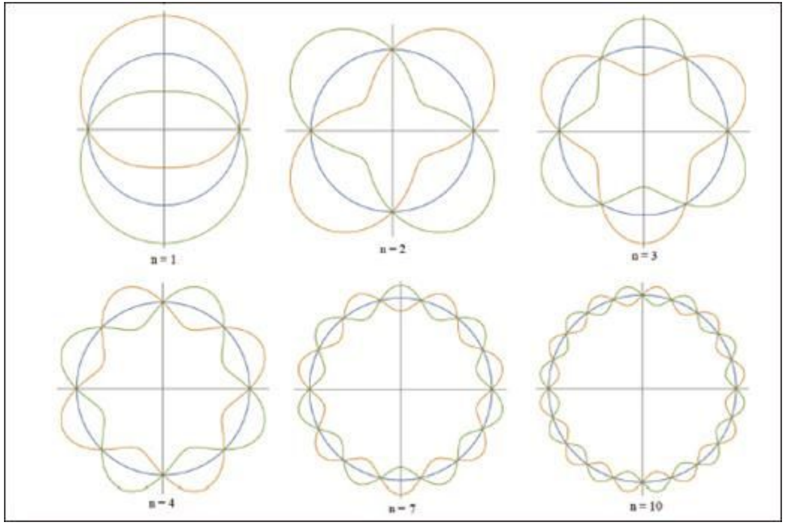
$$\Psi_t(x,t) = Asin(k(x - vt)).$$

$$+$$

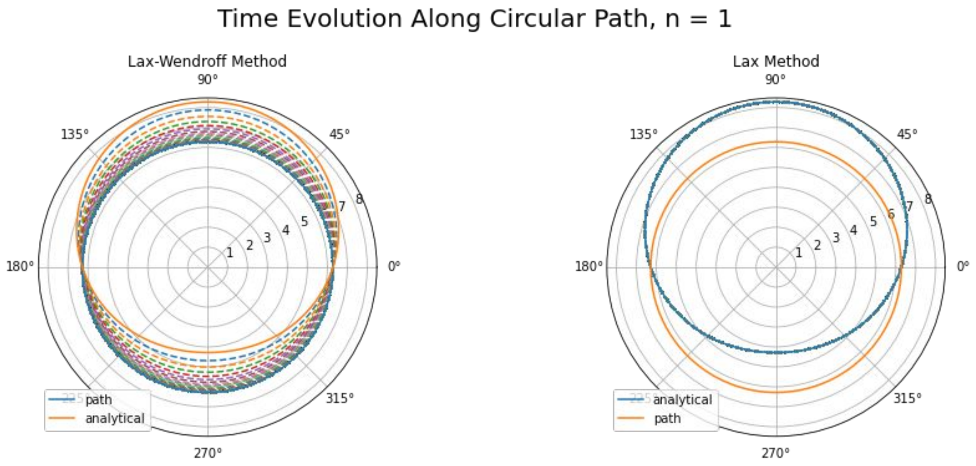
$$\Psi_t(x,t) = Asin(k(x + vt)),$$

$$= 2Asin(kx)cos(kvt)$$

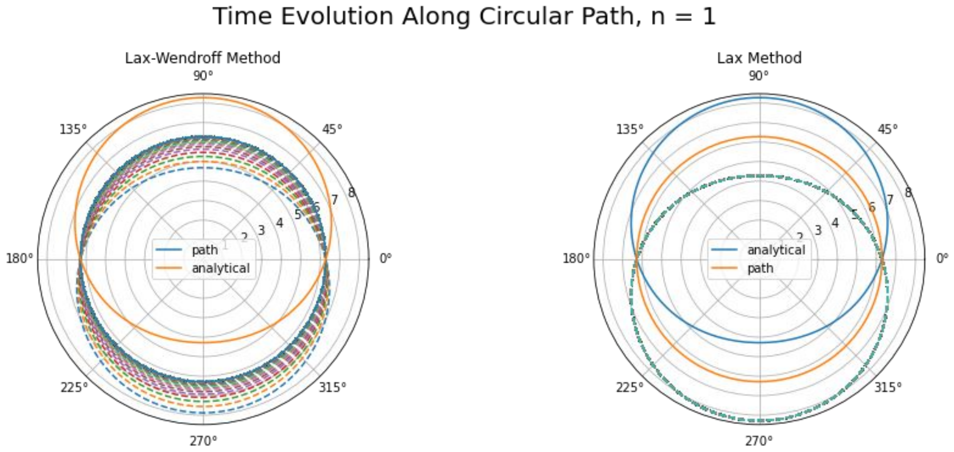
Results



n = 1
n = 2
n = 3
n = 4
n = 7
n = 10

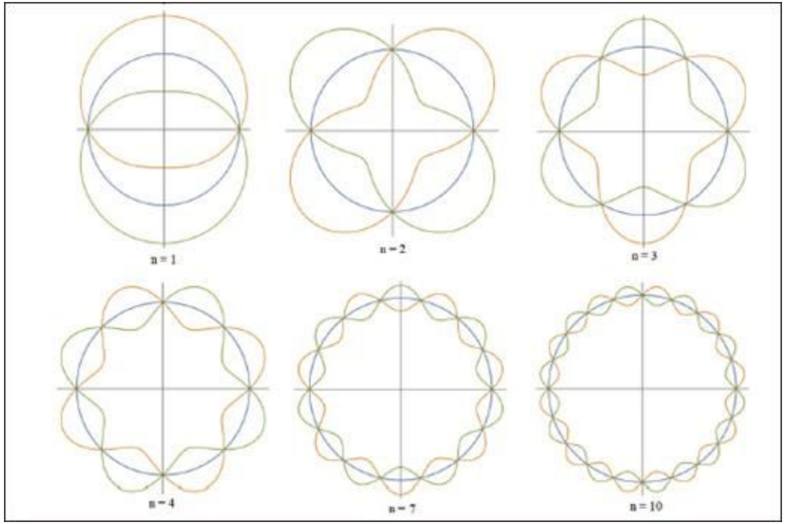


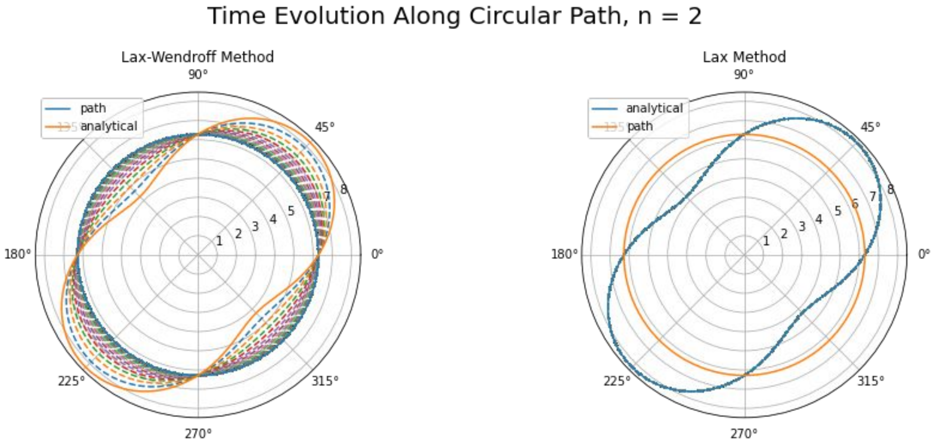
Time Evolution Along Circular Path, n = 1
Lax-Wendroff Method
path
analytical
Lax Method
analytical
path
90°
45°
0°
315°
270°
180°
135°

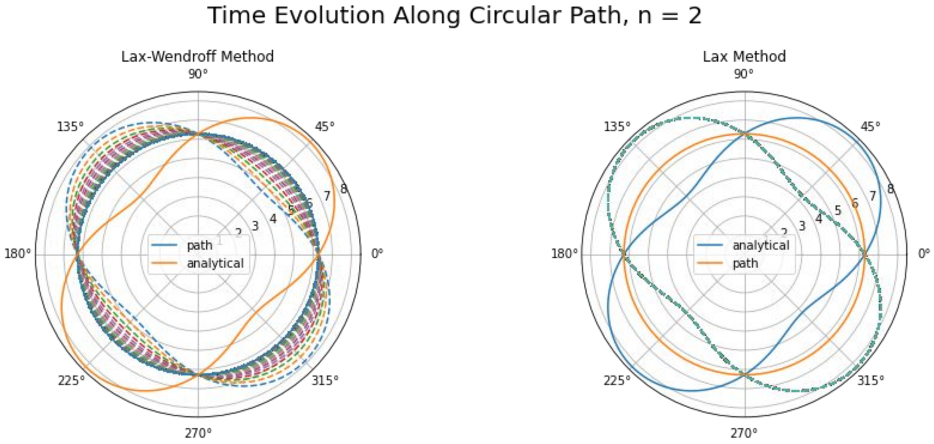


Time Evolution Along Circular Path, n = 1
Lax-Wendroff Method
path
analytical
Lax Method
analytical
path
90°
45°
0°
315°
270°
225°
180°
135°

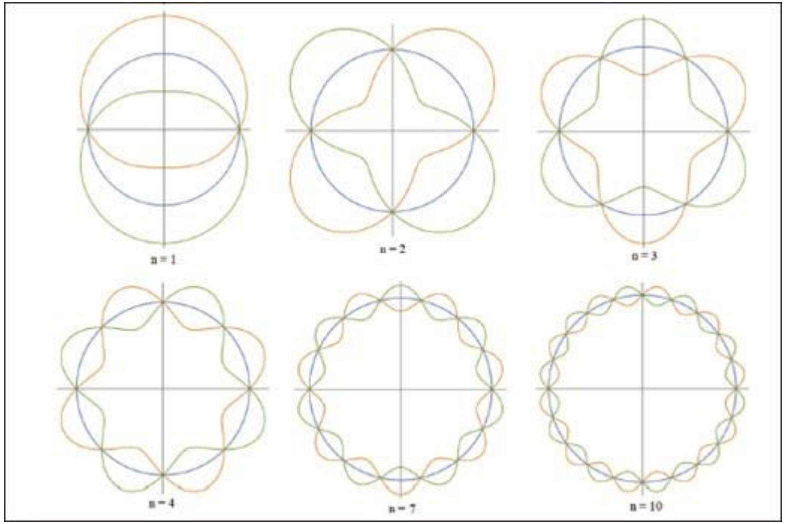
Results

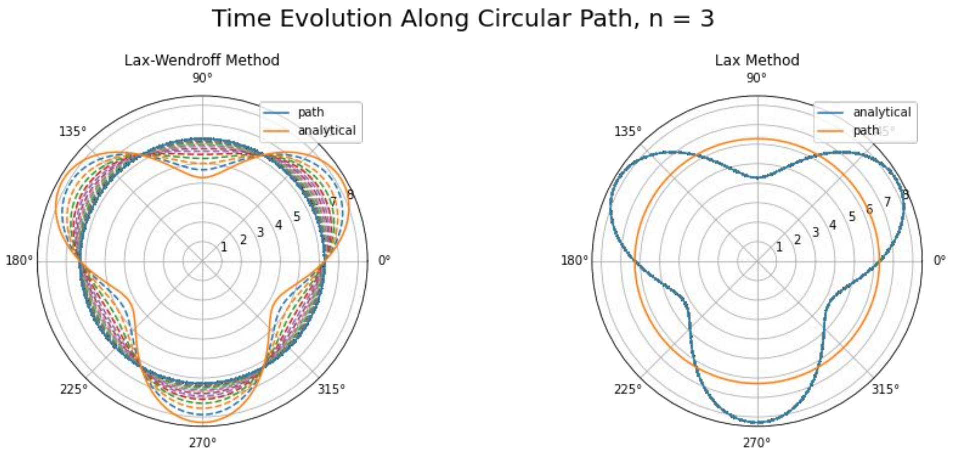

n = 1
n = 2
n = 3
n = 4
n = 7
n = 10

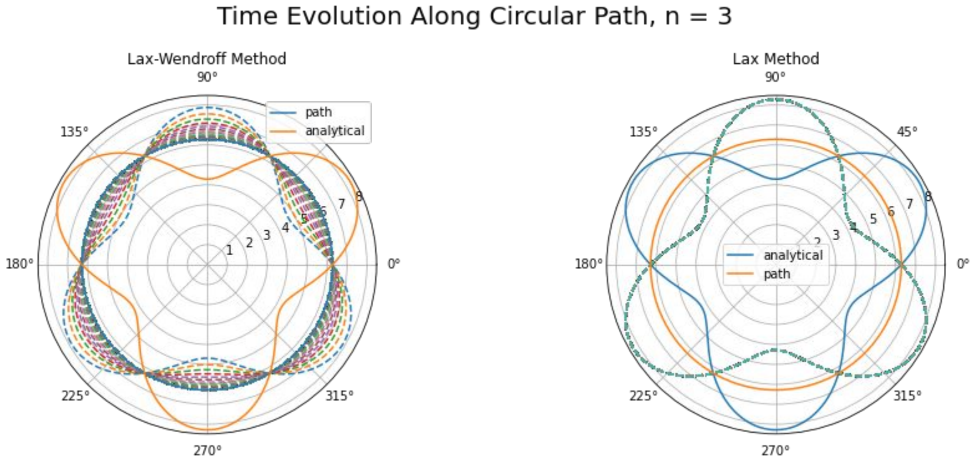

Time Evolution Along Circular Path, n = 2
Lax-Wendroff Method
path
analytical
Lax Method
analytical
path


Time Evolution Along Circular Path, n = 2
Lax-Wendroff Method
path
analytical
Lax Method
analytical
path

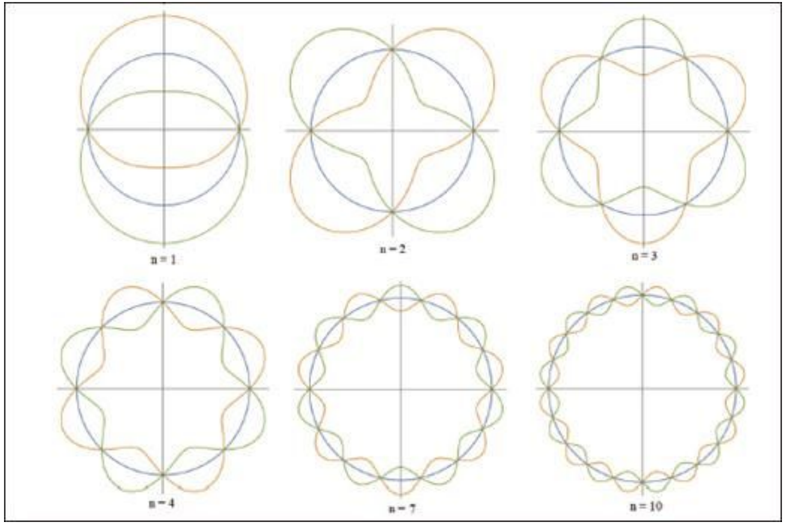
Results

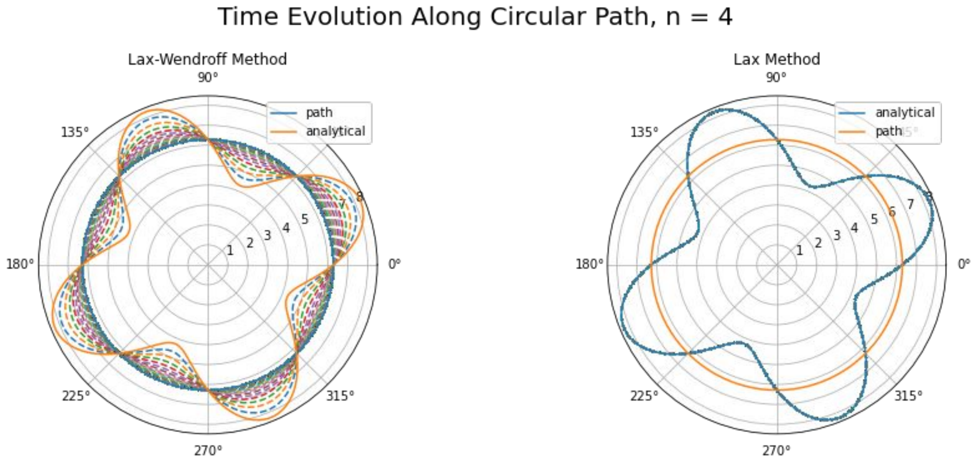

n = 1
n = 2
n = 3
n = 4
n = 7
n = 10

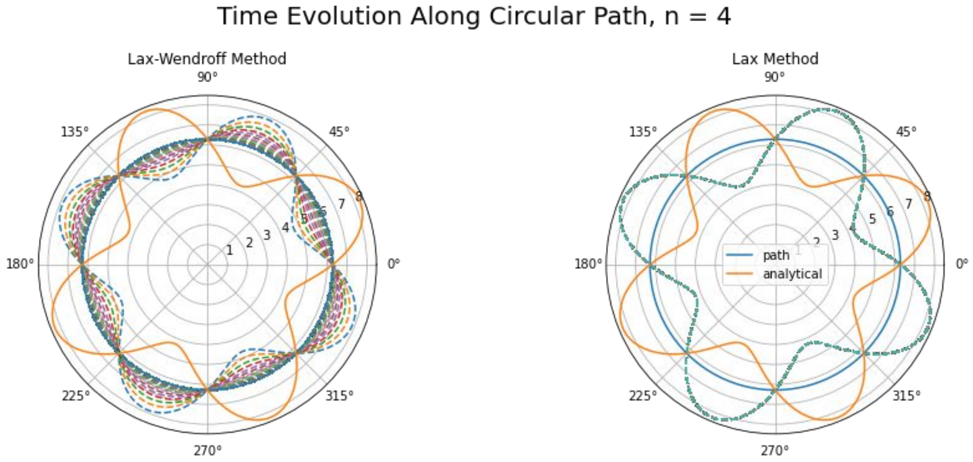

Time Evolution Along Circular Path, n = 3
Lax-Wendroff Method
path
analytical
Lax Method
analytical
path


Time Evolution Along Circular Path, n = 3
Lax-Wendroff Method
path
analytical
Lax Method
analytical
path

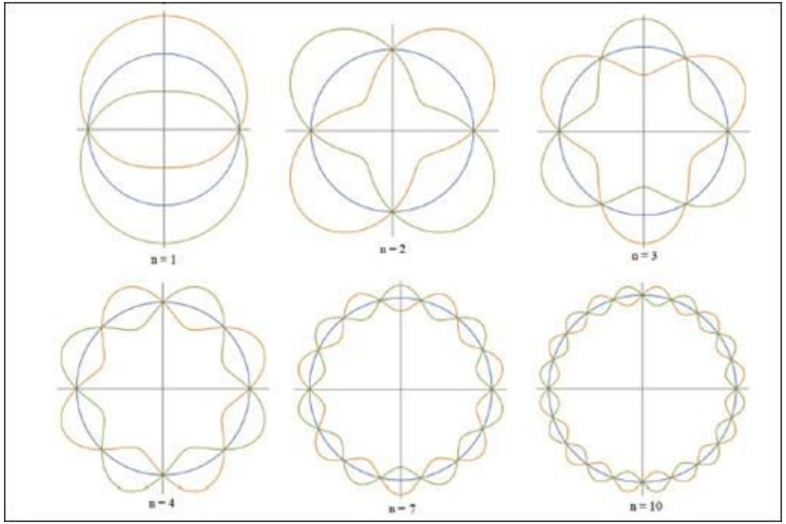
Results

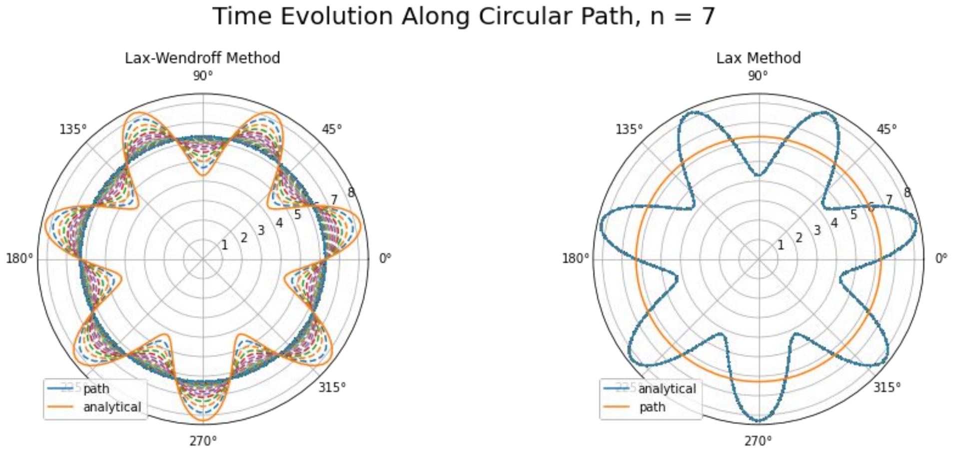

n = 1
n = 2
n = 3
n = 4
n = 7
n = 10

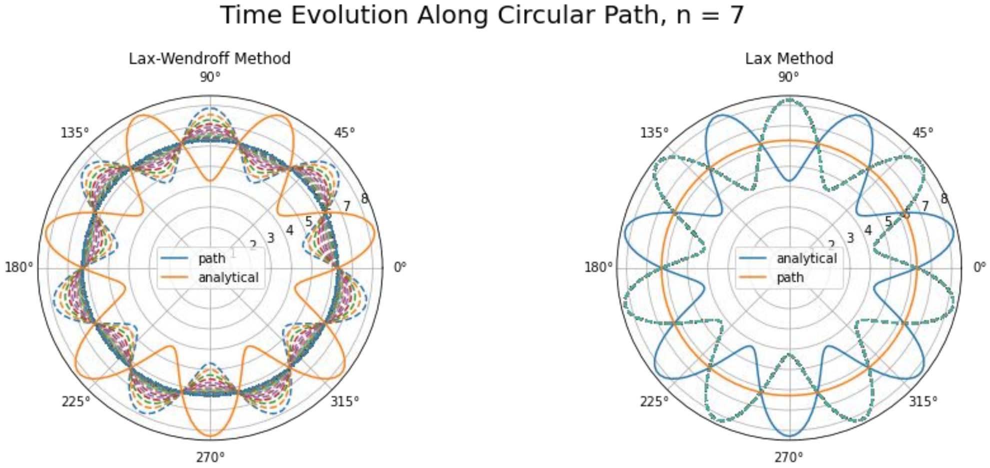

Time Evolution Along Circular Path, n = 4
Lax-Wendroff Method
Lax Method
path
analytical
analytical
path


Time Evolution Along Circular Path, n = 4
Lax-Wendroff Method
Lax Method
path
analytical

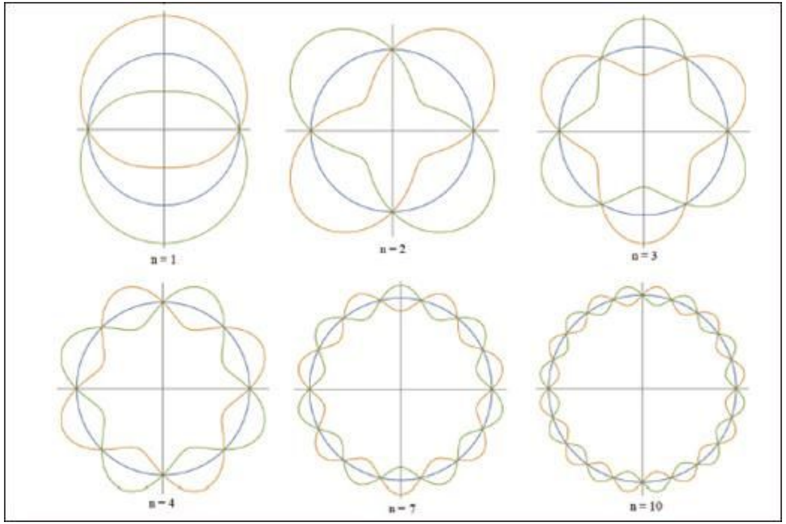
Results


n = 1
n = 2
n = 3
n = 4
n = 7
n = 10

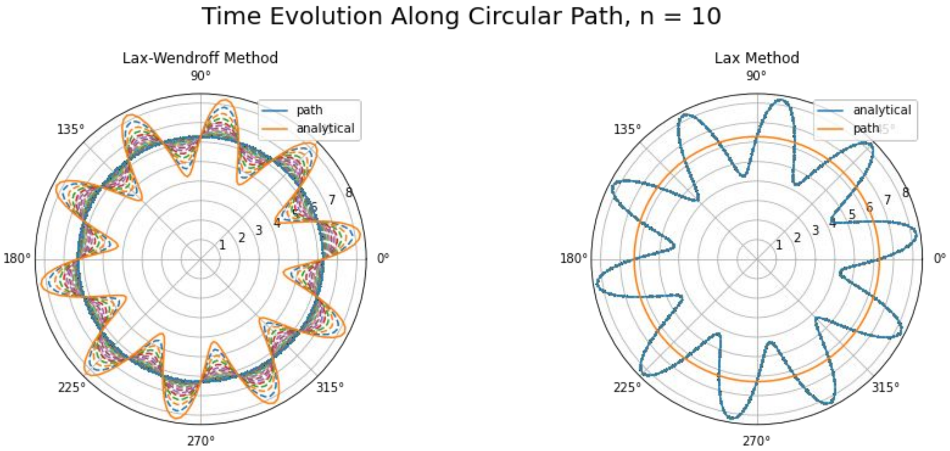

Time Evolution Along Circular Path, n = 7
Lax-Wendroff Method
path
analytical
Lax Method
analytical
path


Time Evolution Along Circular Path, n = 7
Lax-Wendroff Method
path
analytical
Lax Method
analytical
path

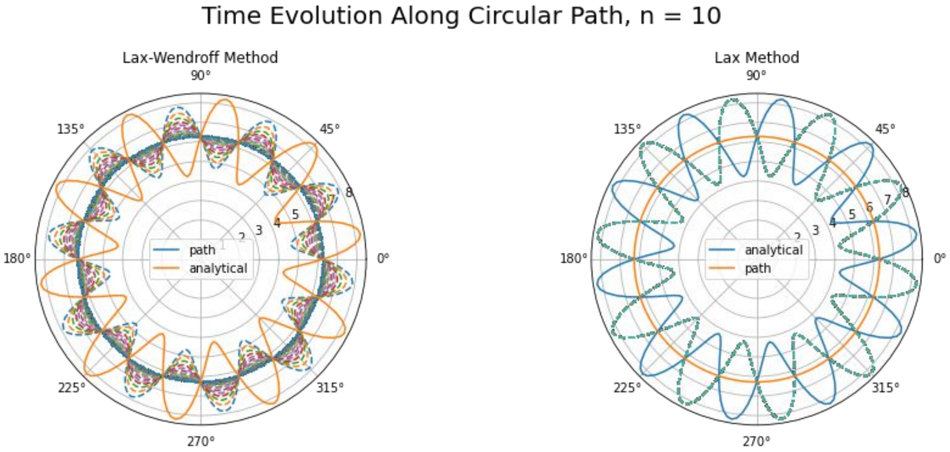
Results



n = 1
n = 2
n = 3
n = 4
n = 7
n = 10

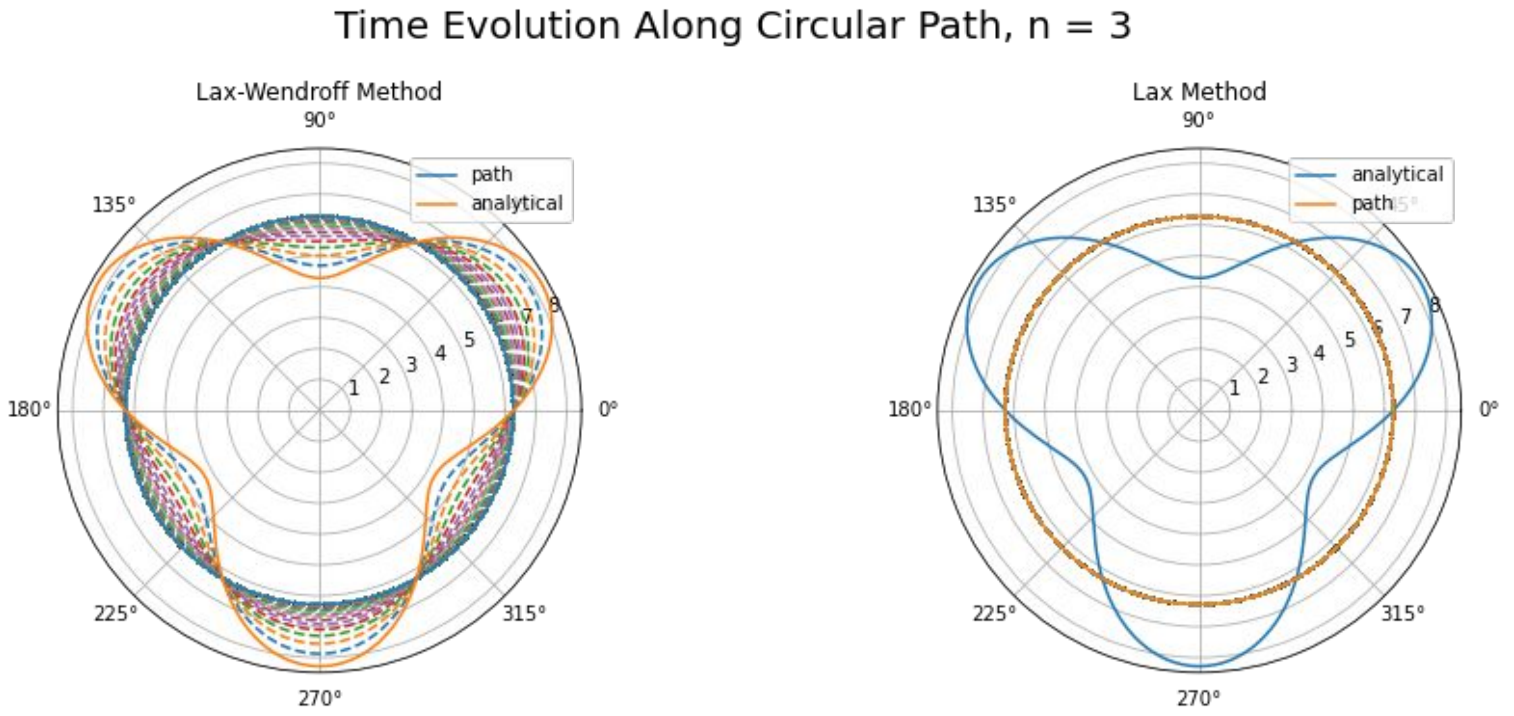


Time Evolution Along Circular Path, n = 10
Lax-Wendroff Method
path
analytical
Lax Method
analytical
path
90°
135°
180°
225°
270°
315°
0°

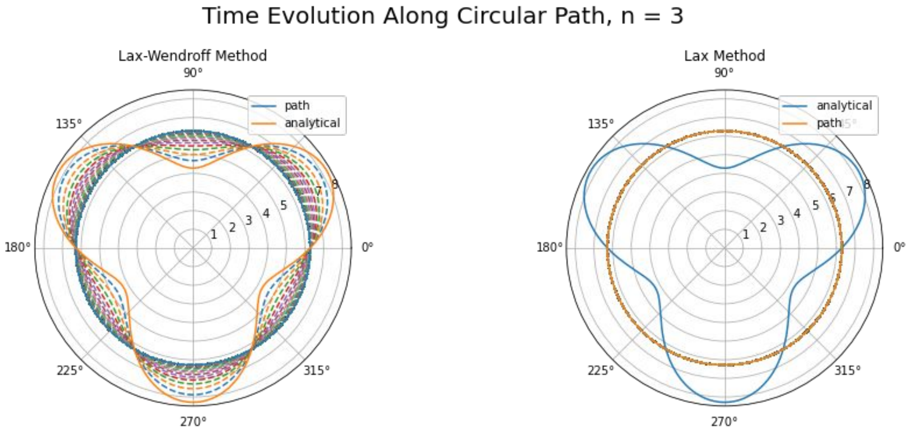


Time Evolution Along Circular Path, n = 10
Lax-Wendroff Method
path
analytical
Lax Method
analytical
path
90°
45°
135°
180°
225°
270°
315°
0°

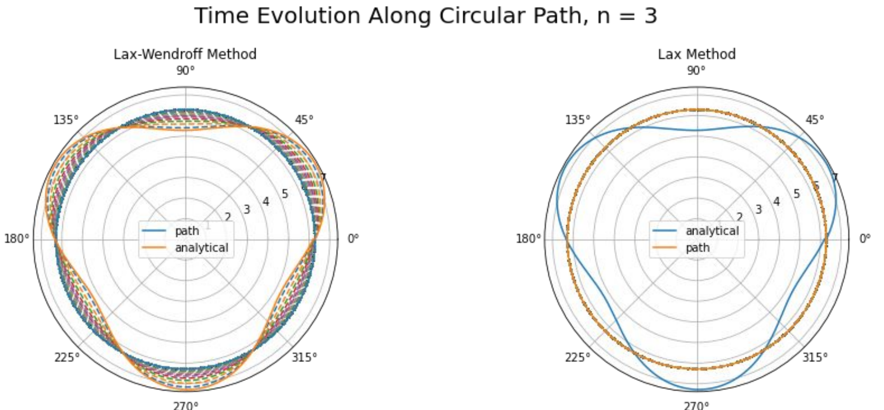
Results - Lax Addendum



Results - Different Initial Condition

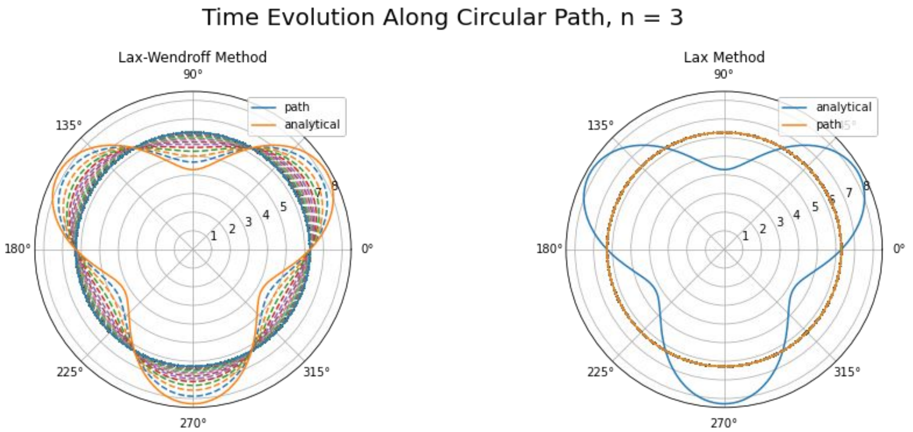


$$\Psi_{right} + \Psi_{left} = Asin(k(x - vt)) + Asin(k(x + vt))$$

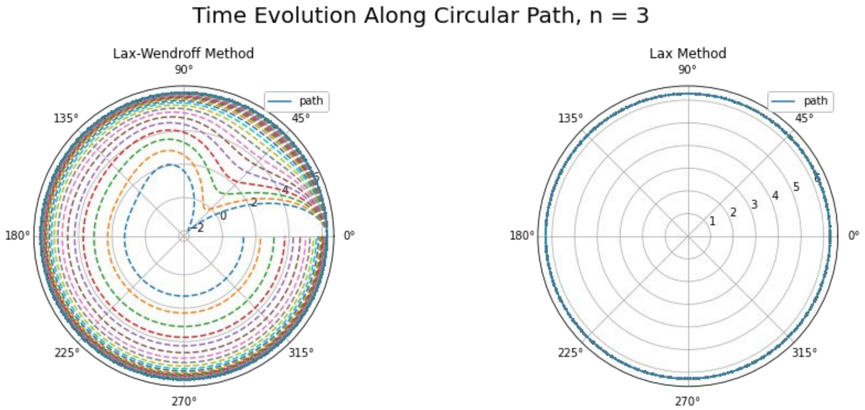


$$\Psi(x, 0) = sin(\frac{2n\pi x}{R}),$$

Results - Different Initial Condition

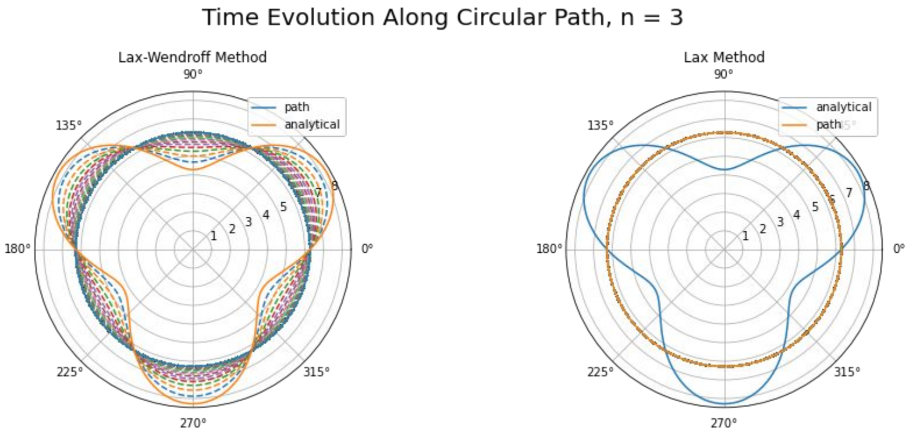


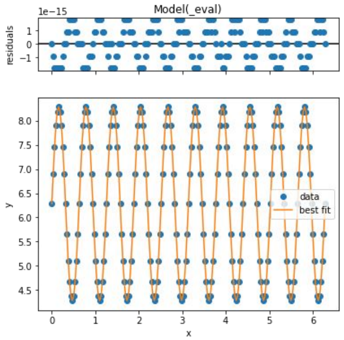
$$\Psi_{right} + \Psi_{left} = Asin(k(x - vt)) + Asin(k(x + vt))$$

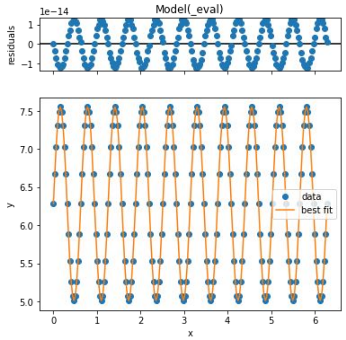


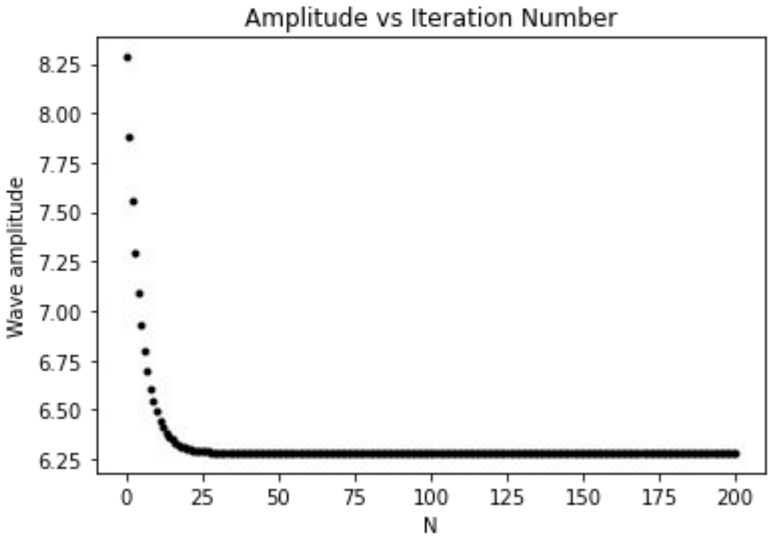
$$\Psi(x, 0) = cos(\frac{\pi x}{\sigma})e^{\frac{-x^2}{2\sigma^2}},$$

Results - Amplitude decay



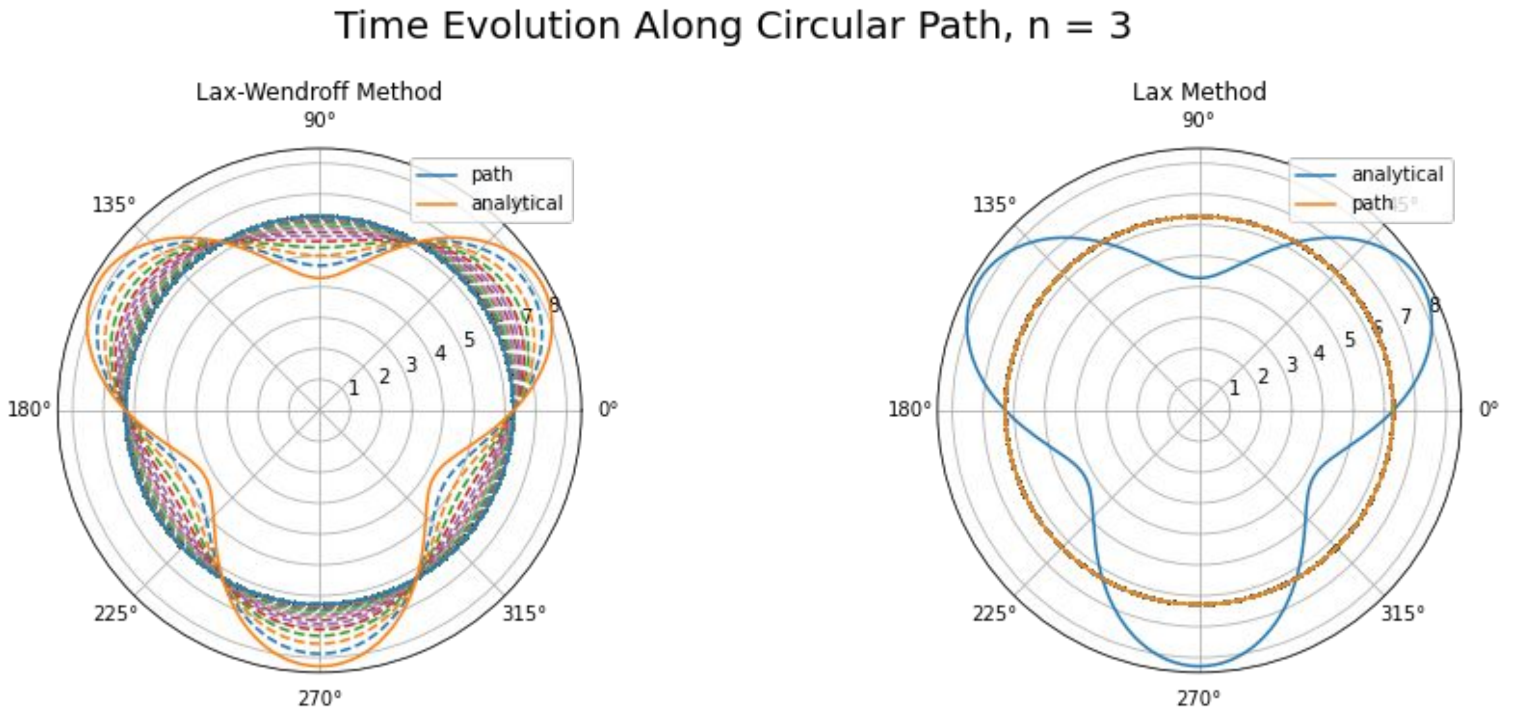






$$\neq 2Asin(kx)cos(kvt)$$

To Conclude


Time Evolution Along Circular Path, n = 3
Lax-Wendroff Method
path
analytical
90°
135°
180°
225°
270°
315°
0°
1
2
3
4
5
7
8
Lax Method
analytical
path
90°
135°
180°
225°
270°
315°
0°
1
2
3
4
5
7
8