Nuclear Magnetic Resonance

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Conceptual Questions

1. Protons have a magnetic moment μ due to their spin. This means that, in the presence of an external magnetic field, a proton will either be aligned or anti-aligned with the field. The energy difference between the aligned and anti-aligned states is

$$\Delta E = g\mu_N B_0, \tag{1}$$

where g is the proton g-factor, μ_N is the nuclear magneton, and B_0 is the strength of the external magnetic field. An incident photon that matches the transition energy, or "resonant" energy, can cause a proton to transition from aligned to anti-aligned and vice-versa.

2. For our lab, we sought to experimentally determine the g-factor for protons and Fluorine. Noticing from Equation 1 that we were able to relate field values to frequencies due to the relationship between photon energy and frequency (further explained in Result 3), we derived an expression for the resonance frequency as a function of magnetic field and fit a line to the data to extract the g-factor from the slope. However, since the apparatus did not allow for the Gaussmeter to reside next to the sample, we obtained a calibration curve to relate field values to current values.

3. The experiment was limited in that the allowed frequencies of the apparatus were between 16-19 MHz and so resonance at field values outside of that range were unobtainable. Since we only explored small field values we would not expect to see further energy level splitting described by the Paschen-Back effect.

4. This investigation allowed us to measure the g-factor best in that we were able to utilize the linear relationship between field and energy to obtain the g-factor from the slope. Since the energy of a photon is related to frequency and we were able to associate field values to current values, we were easily able to vary both parameters of frequency and field to achieve the resonance that marks each data point of the graphs displayed in Figures 1 and 2.

Results

1. We experimentally determined the proton g-factor to be 5.62±0.02, which is within 2σ or good agreement with the accepted value of 5.58. We also determined the Fluorine g-factor to be 5.29±.03, which is also within good agreement with the accepted value of 5.25.

2. The population ratio, described by

$$\frac{N_-}{N_+} = e^{\frac{g\mu_N B_0}{kT}}, \tag{2}$$

where N_- is the number of spin-down protons, N_+ is the number of spin-up protons, g is the proton g-factor, μ_N is the nuclear magneton, and B_0 is the strength of the external magnetic field, was found to be about 0.999998. This ratio encodes the very slight difference between the upward and downward transitions.

3. Given the relationship between photon energy and frequency described by

$$E_{photon} = hf, \tag{3}$$

where h is Planck's constant and f is the frequency of the photon, Equation 1 can be written as

$$hf = g\mu_N B_0. \tag{4}$$

Solving Equation 4 for frequency yields

$$f = \frac{g\mu_N B_0}{h}, \tag{5}$$

where f is the resonance frequency at the field B_0.

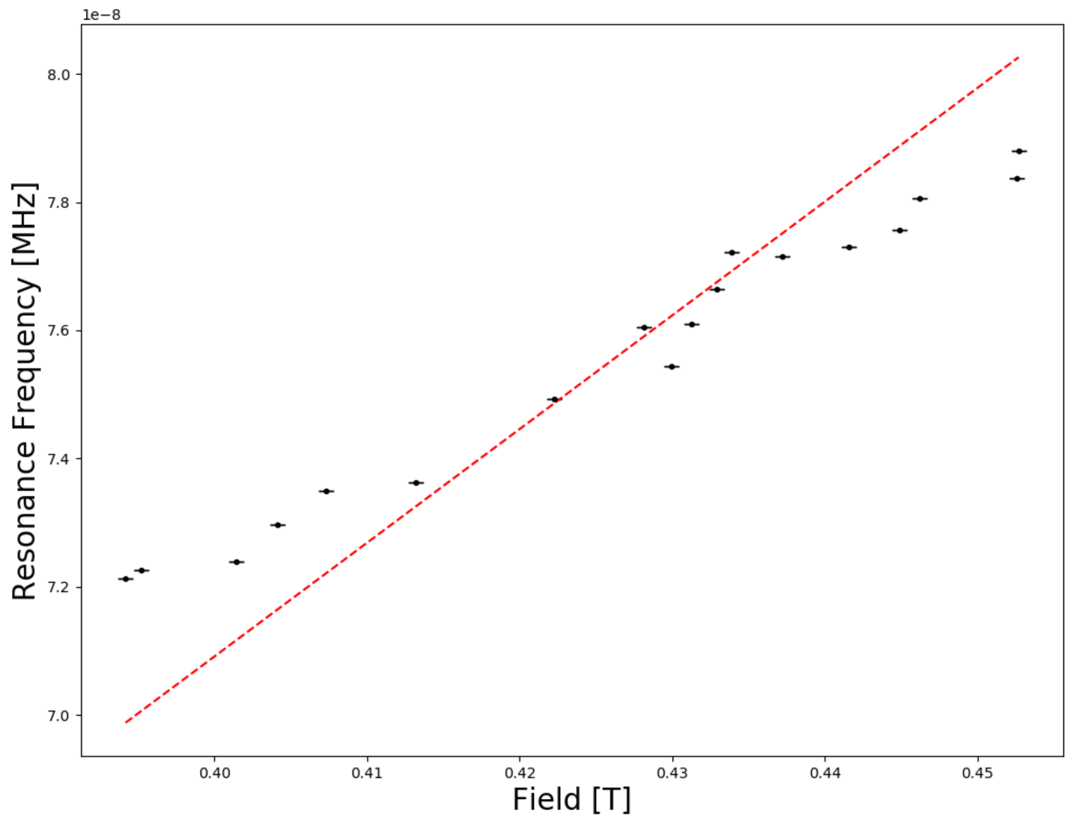

Figure 1: Resonance Frequencies vs Fields for Water, where the g-factor is the slope over the nuclear magneton, μ_N.

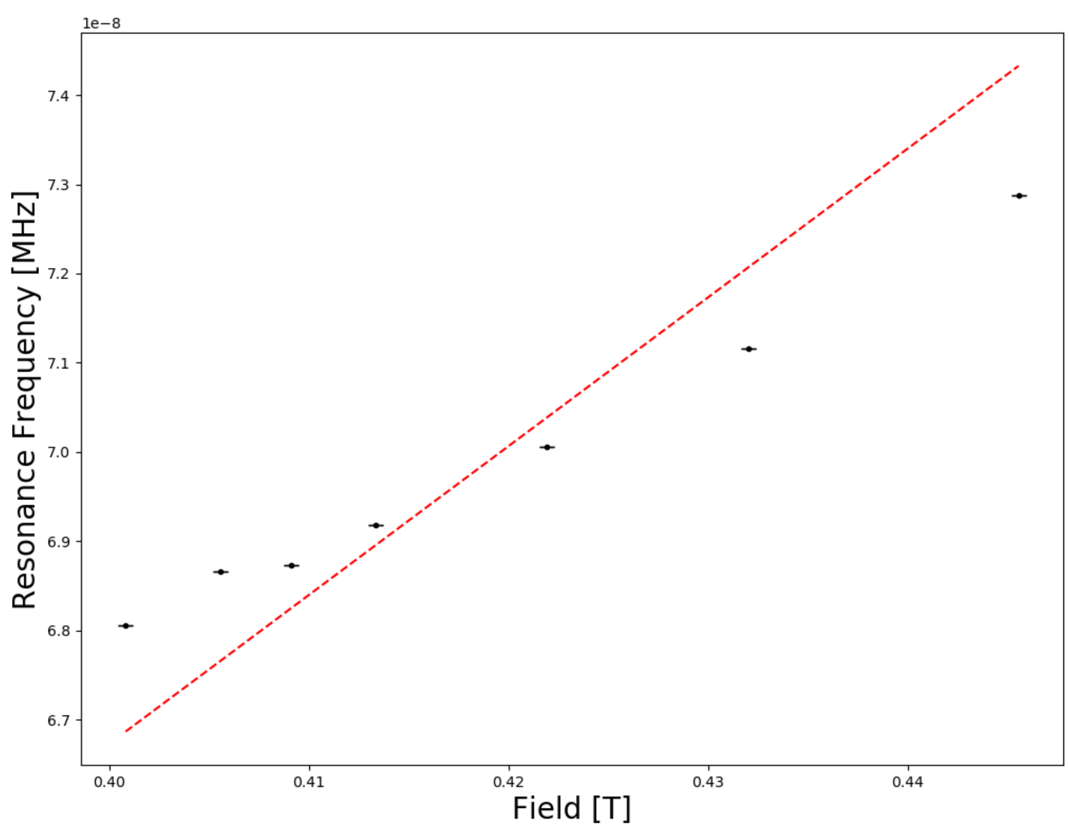

Figure 2: Resonance Frequencies vs Fields for Fluorine, where the g-factor is the slope over the nuclear magneton, μ_N.

4. To verify the equivalence between the ratio of the resonance frequencies and the ratio of the g-factors of water and Fluorine, we found the associated field for water at 17.5 MHz. Without changing the magnetic field, the Fluorine sample was inserted and the resonance frequency for the unchanged field was found. To show that this relationship always holds, we compared the ratio of the y values from the fits to the slopes of the fits since the y values were resonance frequencies and the slopes were proportional to the g-factor over the nuclear magneton.