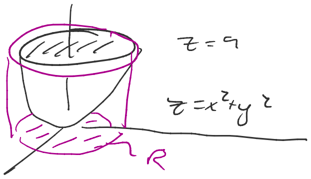
Triple Integrals in Cylindrical

Friday, December 8, 2023 4:24 PM



$z = 9$

$z = x^2+y^2$

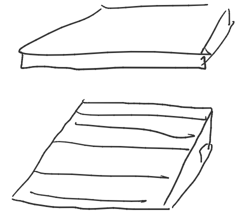
total Mass = ?

const mass density

δ

$dm = \delta\, dV$

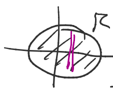
$\delta = \frac{dm}{dV}$



$\iint \delta\, dA$

$\iiint_D \delta\, dV = \iiint_D dm$

$z = 0 \Rightarrow$



$x^2+y^2 = 3$

$y = \pm\sqrt{3-x^2}$

$\rightarrow 0 \le r \le 3$ $0 \le \theta \le 2\pi$ vs $-3 \le x \le 3$ $-\sqrt{3-x^2} \le y \le \sqrt{3-x^2}$

$dA = r\,dr\,d\theta$ ⇐ don't forget

$$\delta \iint_R \int_{x^2+y^2}^{z=9} dz\, dA \rightarrow \delta \iint \left[\int_{x^2+y^2}^{9} dz\right] dA$$

$$\delta \iint (9 - x^2+y^2)\, dA$$

$$\delta \int_0^{2\pi}\int_0^{3} (9 - r^2)\, r\,dr\,d\theta$$

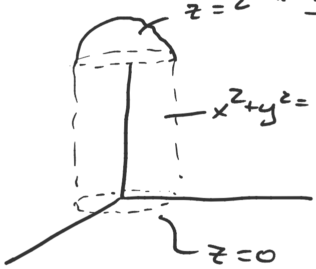
$$\delta \int_0^{2\pi}\int_0^{3} (9r - r^3)\, dr\,d\theta$$

$$\delta\, 2\pi \left[\frac{9}{2}r^2 - \frac{1}{4}r^4\right]_0^3$$

$$= \frac{81\pi\delta}{2}$$

ex] D is bounded b/t

$z = 2 - x^2 - y^2 \rightarrow \left\{ \begin{array}{l} x^2+y^2 = 1 \text{ — cylinder} \\ z \ge 2 - x^2 - y^2 \\ xy \text{ plane} \end{array} \right\}$



M_{tot} if $\delta = \sqrt{x^2+y^2} \longrightarrow r$

cylindrical

$dV = dz\, r\, dr\, d\theta$

spherical

$\{\rho, \theta, \phi\}$

$$\int_0^{2\pi}\int_0^{1}\int_0^{2-r^2} r\, dz\, r\,dr\,d\theta$$

$$\int_0^{2\pi}\int_0^{1}\int_0^{2-r^2} r^2\, dz\,dr\,d\theta$$

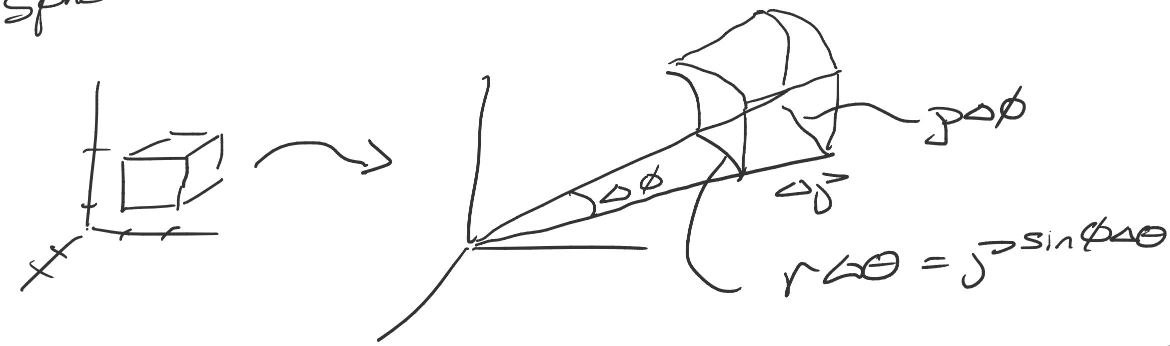
$$\int_0^{2\pi}\int_0^{1} r^2(2-r^2)\, dr\,d\theta$$

$$2\pi \int^{1} 2r^2 - r^4\, dr = \frac{14}{15}\pi$$

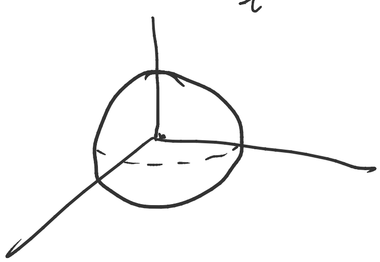
$$2\pi \int_0^1 2r^2 - r^4\,dr = \frac{14}{15}\pi$$

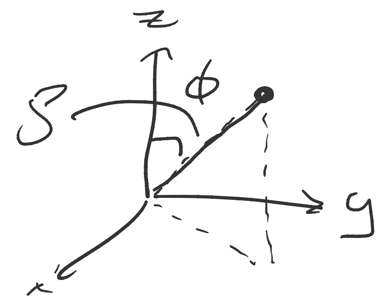
$$dV = \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$

spherical



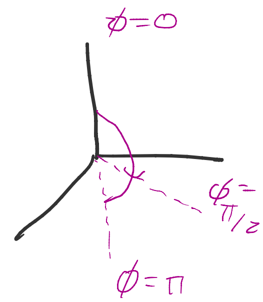
ex] Vol of sphere radius a





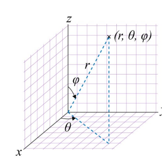
$$V = \iiint_D dV$$

$$\int_0^{2\pi}\int_0^{\pi}\int_0^{a} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$



$$\int_0^{2\pi}\int_0^{\pi} \frac{1}{3} a^3 \sin\phi \, d\theta$$

$\phi =$



$$\frac{2\pi}{3} a^3 \int_0^{\pi} \sin\phi$$

$$\left[-\cos\phi\right]_0^{\pi}$$

$$\frac{2\pi a^3}{3}\left(-(-1) - -1\right)$$

$$\boxed{\text{Vol sphere} = \frac{4}{3}\pi a^3}$$