Excercise Questions 1

1. Evaluate the following limits:

a) $\lim_{x\to 0}\frac{\cos x-1}{x}$ ans: 0

b) $\lim_{x\to \frac{\pi}{4}}\frac{\sin x-\cos x}{\cos 2x}$ ans:$-\frac{\sqrt{2}}{2}$

c) $\lim_{x\to 0} x^4\cos(\frac{2}{x})$ ans:0

d) $\lim_{x\to 64}\frac{\sqrt[3]{x}-4}{\sqrt{x}-8}$ ans: $\frac{1}{3}$

e) $\lim_{x\to \frac{\pi}{2}}\frac{\cos x}{\frac{\pi}{2}-x}$ ans: 1

f) $\lim_{x\to \frac{\pi}{3}}\frac{1-2\cos x}{\pi-3x}$ ans: $-\frac{\sqrt{3}}{3}$

g) $\lim_{x\to 0}\frac{\arcsin x}{x}$ ans:1

h) $\lim_{x\to 0}\frac{\cos x-\cos\sqrt{x}}{\sin^2 x}$ ans:$-\frac{1}{4}$

i) $\lim_{x\to 0}\frac{3^x-3^{-x}}{3^x+3^{-x}}$ ans:0

j) $\lim_{x\to+\infty} x\left(\sqrt{9x^2+1}-3x\right)$ ans:$\frac{1}{6}$

k) $\lim_{n\to+\infty}\left(1+\frac{x}{n}\right)^n$ ans:e^x

l) $\lim_{x\to+\infty}\left(\frac{x+3}{x-1}\right)^{x+3}$ ans:e^4

m) $\lim_{x\to+\infty} e^{-x}\cos x$ ans:0

n) $\lim_{x\to+\infty} e^{\sin x-x}$ ans:0

o) $\lim_{x\to+\infty}\sqrt{\frac{2x+\sqrt{3x}}{5x}}$ ans:$e\sqrt{\frac{2}{5}}$

p) $\lim_{x\to+\infty} x\sin\frac{5}{2x}$ ans:$\frac{5}{2}$

q) $\lim_{x\to+\infty}\left(1+\frac{r}{ax+b}\right)^{px+q}$ ans:$e^{\frac{rp}{q}}$

r) $\lim_{x\to 0}\frac{a^x-b^x}{x}$ ans:$\frac{1}{\ln(\frac{a}{b})}$

s) $\lim_{x\to-\infty}\left(x+\sqrt{x^2-a^2x}\right)$ ans:$-\frac{a^2}{2}$

2. Find the values of a, b and c so that the following function is continuous

$$f(x) = \begin{cases} 6 - 3bx & , x \leq -2 \\ cx^2 - ax + 4 & -2 < x \leq -1 \\ 6 - bx & -1 < x \leq 1 \\ ax^2 + c & 1 < x \end{cases}$$

3. Check the continuity of the following functions:

a) $f(x) = \begin{cases} x \sin \frac{1}{x} & , x \neq 0 \\ 0 & , x = 0 \end{cases}$

b) $f(x) = \begin{cases} \sin x & , 0 \leq x \leq \frac{\pi}{2} \\ \cos x & , \frac{\pi}{2} < x \leq \pi \end{cases}$

c) $f(x) = \begin{cases} 1 - x^2 & , -1 < x < 0 \\ 0 & , x = 0 \\ 1 - x & , 0 < x < 1 \end{cases}$