Om lag polynomier

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Multiplikation

Multiplikation af et p'te grads lag-polynomium $\alpha(\mathrm{B})$ med et q'te grads lag-polynomium $\beta(\mathrm{B})$. Vi antager wlog at $p \geq q$.

$$\alpha(\mathrm{B}) = \alpha_0 + \alpha_1 \mathrm{B} + \cdots + \alpha_q \mathrm{B}^q + \alpha_{q+1}\mathrm{B}^{q+1} + \cdots + \alpha_p \mathrm{B}^p,$$
$$\beta(\mathrm{B}) = \beta_0 + \beta_1 \mathrm{B} + \cdots + \beta_q \mathrm{B}^q.$$

Så er $(p+q)$'te grads lag-polynomiet $\alpha(\mathrm{B})\beta(\mathrm{B})$ givet ved

$$\begin{aligned}
\alpha(\mathrm{B})\beta(\mathrm{B}) &= \sum_{i=0}^{q}\sum_{j=0}^{p} \alpha_j \beta_i \mathrm{B}^{i+j} \\
&= \sum_{k=0}^{p+q} \sum_{\{(i,j)\mid i+j=k, 0\leq i\leq q, 0\leq j\leq p\}} \alpha_j\beta_i \mathrm{B}^{i+j} \\
&= \sum_{k=0}^{p+q} \left(\sum_{j=\max(0,k-q)}^{\min(k,p)} \alpha_j \beta_{k-j} \right) \mathrm{B}^k.
\end{aligned}$$

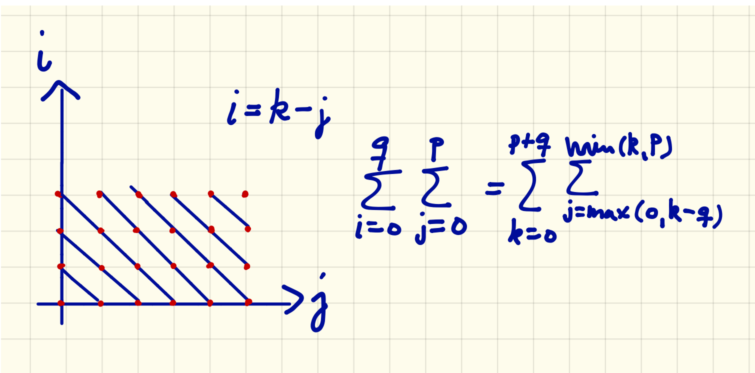

Figure 1: Sådan virker dobbeltsummationen for et eksempel med $p = 5$ og $q = 3$.

Inversion

Antag at $\alpha(\mathrm{B})$ er invertibel, og definér $\psi(\cdot)$ ved

$$\frac{1}{\phi_0 + \phi_1 \mathrm{B} + \cdots + \phi_p \mathrm{B}^p} x_t = (\psi_0 + \psi_1 \mathrm{B} + \psi_2 \mathrm{B}^2 + \cdots +) x_t.$$

Så er altså

$$\left(\sum_{j=0}^{p} \phi_j \mathrm{B}^j \right) \left(\sum_{i=0}^{\infty} \psi_i \mathrm{B}^i \right) = 1.$$

Løsningen for $\psi(\cdot)$ bliver

$$\begin{aligned} \psi_0 &= \frac{1}{\phi_0}, \\ \psi_k &= -\frac{1}{\phi_0} \sum_{j=1}^{\min(p,k)} \phi_j \psi_{k-j}, \text{ for } k \geq 1. \end{aligned}$$

`filter` funktionen i R

Kommandoen

```
y <- filter(x, c(1,psi),sides=1, method="convolution")
```

danner tidsrækken y_t ud fra x_t

$$\begin{aligned} y_t &= x_t + \psi_1 x_{t-1} + \psi_2 x_{t-2} + \cdots + \psi_k x_{t-k}, \text{ dvs.} \\ y_t &= \psi(\mathrm{B}) x_t = (1 + \psi_1 \mathrm{B} + \psi_2 \mathrm{B}^2 + \cdots + \psi_k \mathrm{B}^k) x_t \\ &= x_t + \sum_{j=1}^{k} \psi_j x_{t-j}, \end{aligned}$$

og hvor altså `psi` i koden ovenfor er vektoren $(\psi_1, \psi_2, \ldots, \psi_k)$.

Kommandoen

```
y <- filter(x, c(phi), method="recursive",init=startv),
```

hvor `phi` er vektoren $(\phi_1, \phi_2, \ldots, \phi_p)$ og `startv` er startvektoren $(y_0, y_{-1}, \ldots, y_{-p+1})$, danne tidsrækken y_t ud fra x_t

$$y_t = \sum_{j=1}^{p} \phi_j y_{t-j} + x_t.$$