[320] Complexity + Big O (Worksheet: Complexity Analysis)

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1

```
def search(L, target):
   for x in L:
      if x == target: #line A
         return True
   return False
```

assume this is asked unless otherwise stated

Let **f(N)** be the number of times line A executes, with `N=len(L)`. What is **f(N)** in each case?

Worst Case (target is at end of list): f(N) = ___________
Best Case (target is at beginning of list): f(N) = ___________
Average Case (target in middle of list): f(N) = ___________

A **step** is any unit of work with bounded execution time (it doesn't keep getting slower with growing input size).

We classify algorithm complexity by classifying the **order of growth** of a function f(N), where f gives the number of steps the algorithm must perform for a given input size.

Big O definition: if $f(N) \leq C * g(N)$ for large **N** values and some fixed constant **C**, then $f(N) \in O(g(N))$

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Worst Case (target is at end of list): f(N) = **$N \in O(N)$**

Best Case (target is at beginning of list): f(N) = ________

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Best Case (target is at beginning of list): f(N) = $\mathbf{1 \in O(1)}$

Average Case (target in middle of list): f(N) = ________

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Worst Case (target is at end of list): $f(N) = \mathbf{N} \in \mathbf{O(N)}$

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Average Case (target in middle of list): $f(N) = \frac{\mathbf{N}}{\mathbf{2}} \in \mathbf{O(N)}$

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We classify algorithm complexity by classifying the **order of growth** of a function f(N), where f gives the number of steps the algorithm must perform for a given input size.

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2

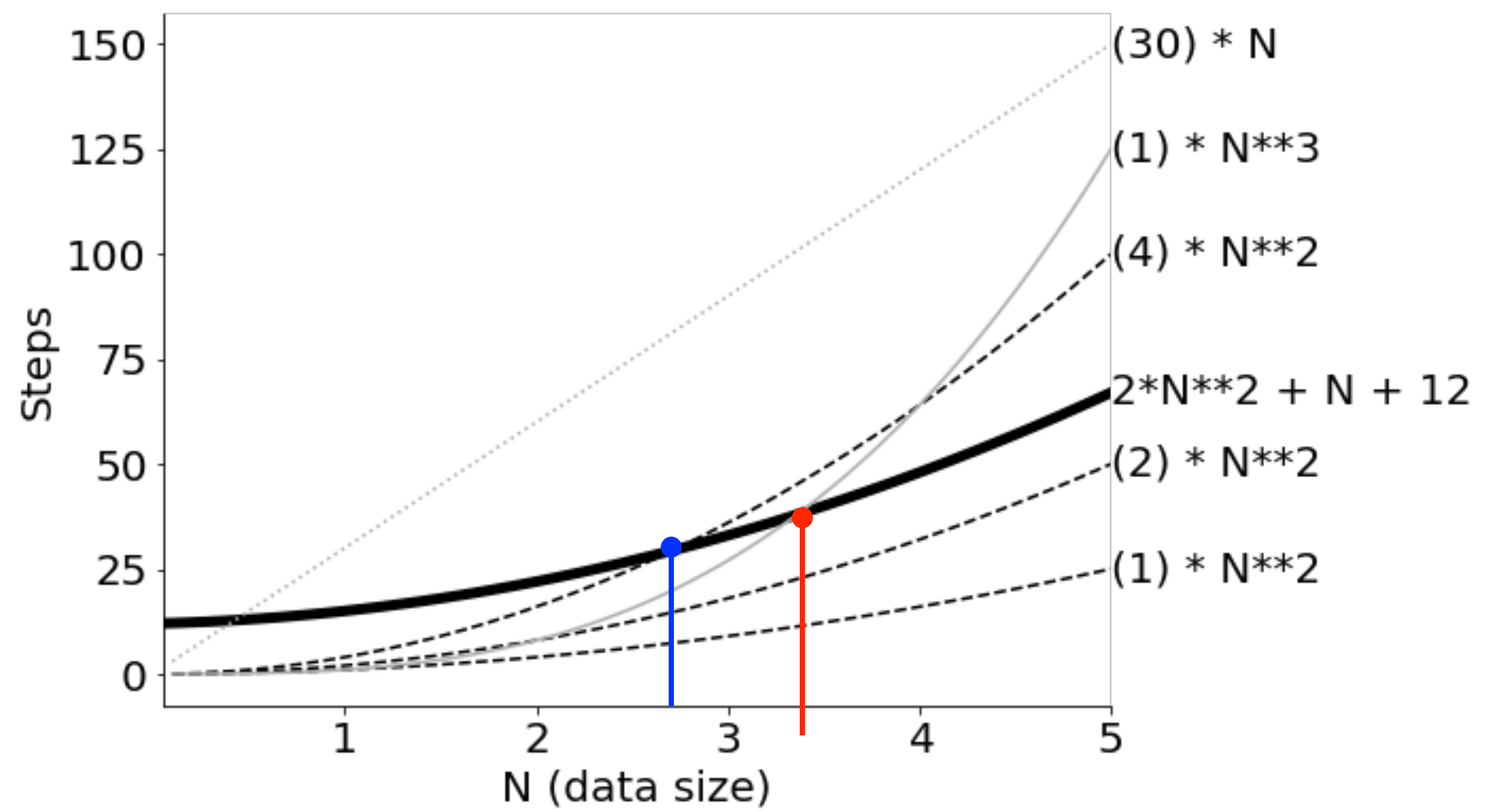
Let $f(N) = 2N^2 + N + 12$

If we want to show $f(N) \in O(N^3)$, what is a good lower bound on N? Let's have C=1.

To show $f(N) \in O(N^2)$, do we pick 1, 2, or 4 for the C? After picking C, what should we choose for N's lower bound?

What is more informative to show? $f(N) \in O(N^3)$ or $f(N) \in O(N^2)$?

Somebody claims $f(N) \in O(N)$, offering C=30 and N>0. Suggest an N value to counter their claim.



2

Let $f(N) = 2N^2 + N + 12$

If we want to show $f(N) \in O(N^3)$, what is a good lower bound on N? Let's have C=1. **$N \geq 4$**

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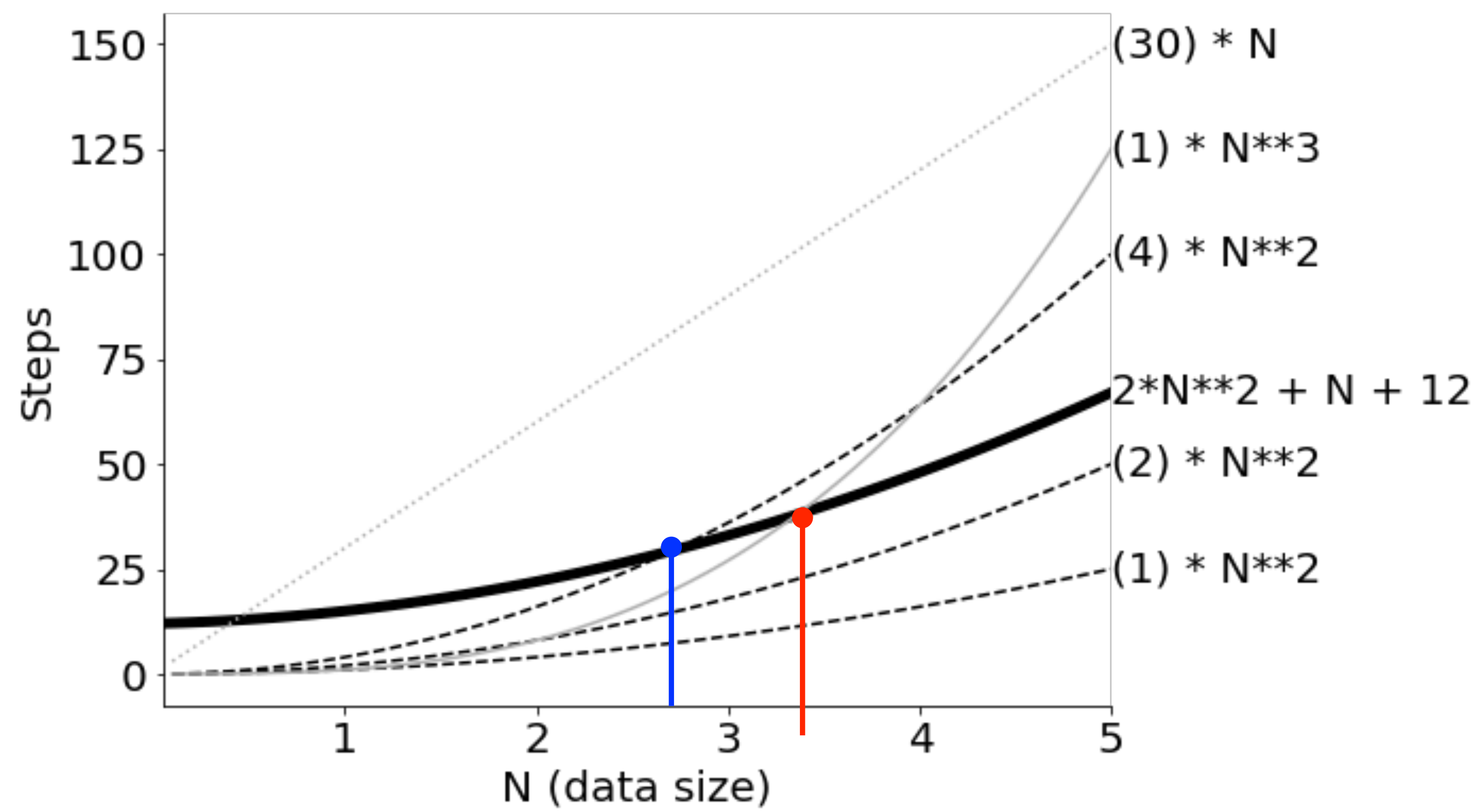

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$C = 4$

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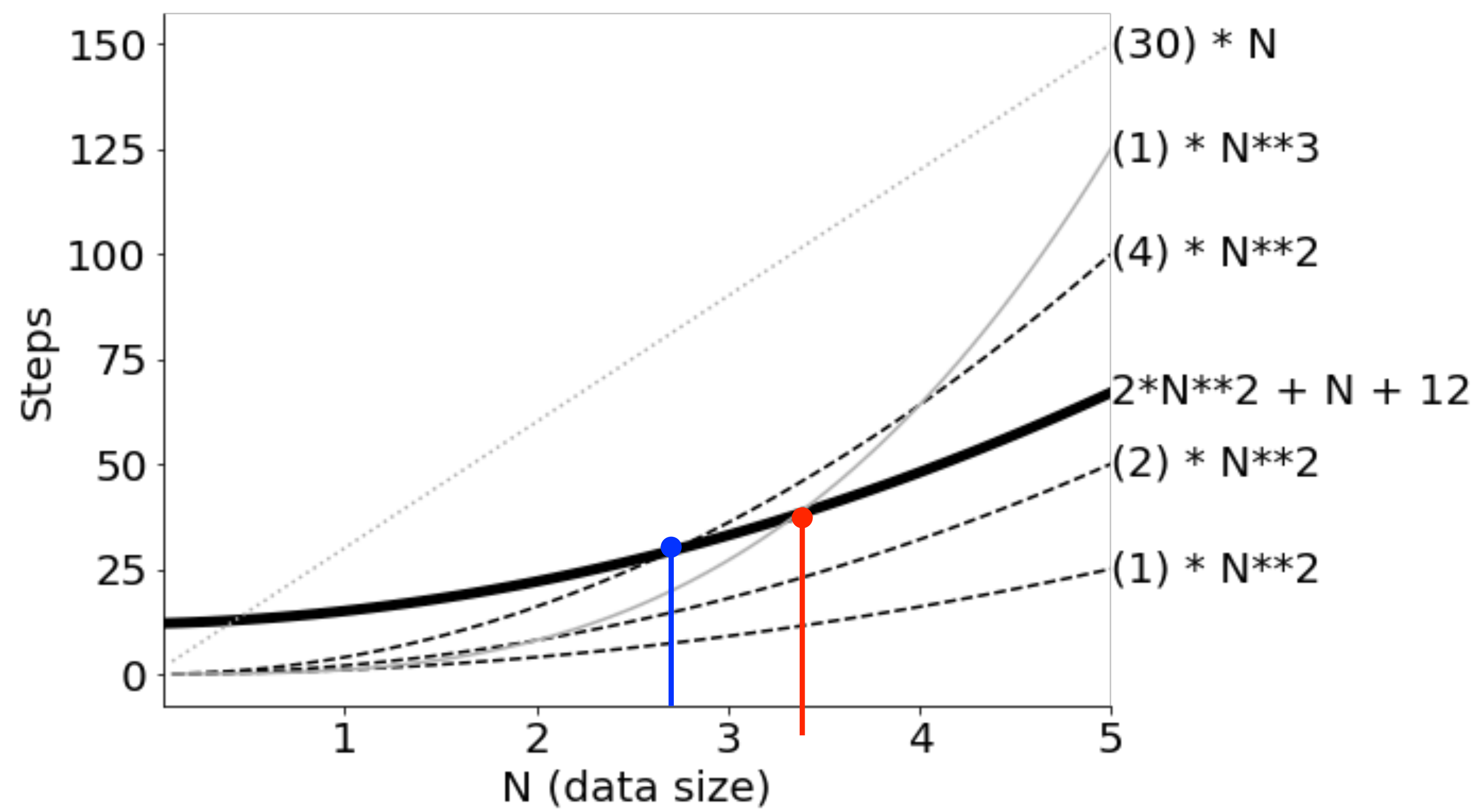

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$C = 4$ and **$N \geq 3$**

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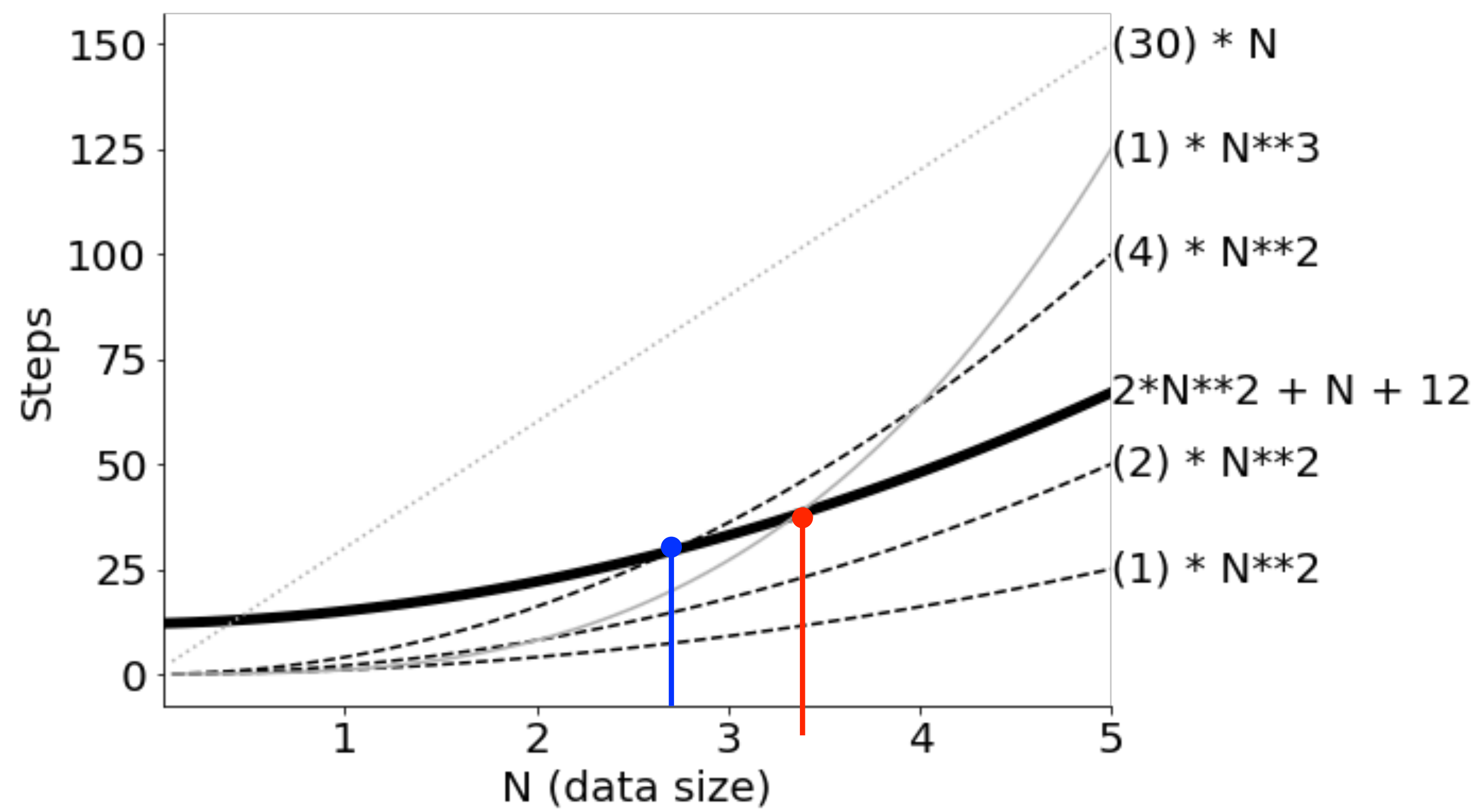

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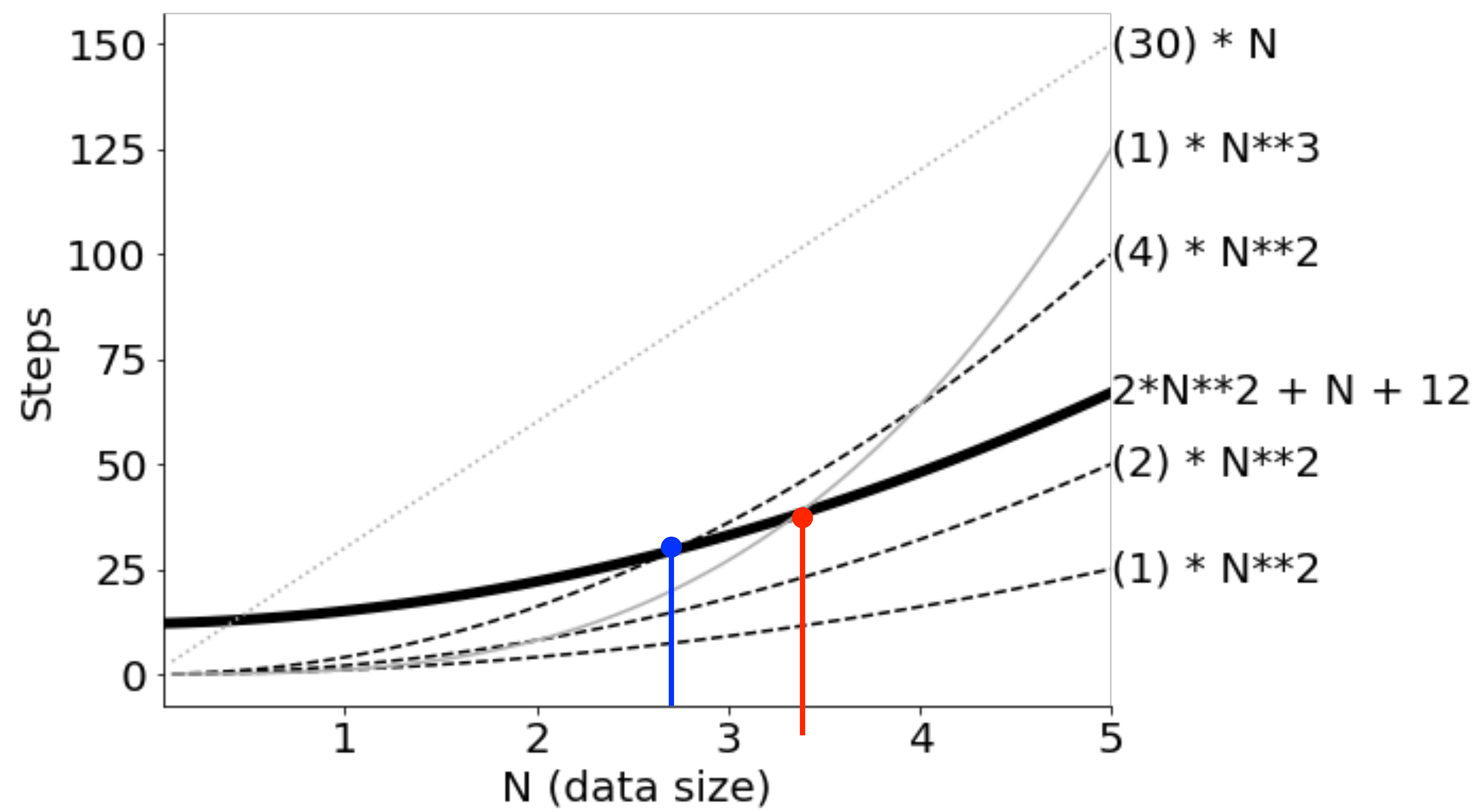

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$f(N) \in O(N^2)$ (tighter upper bound)

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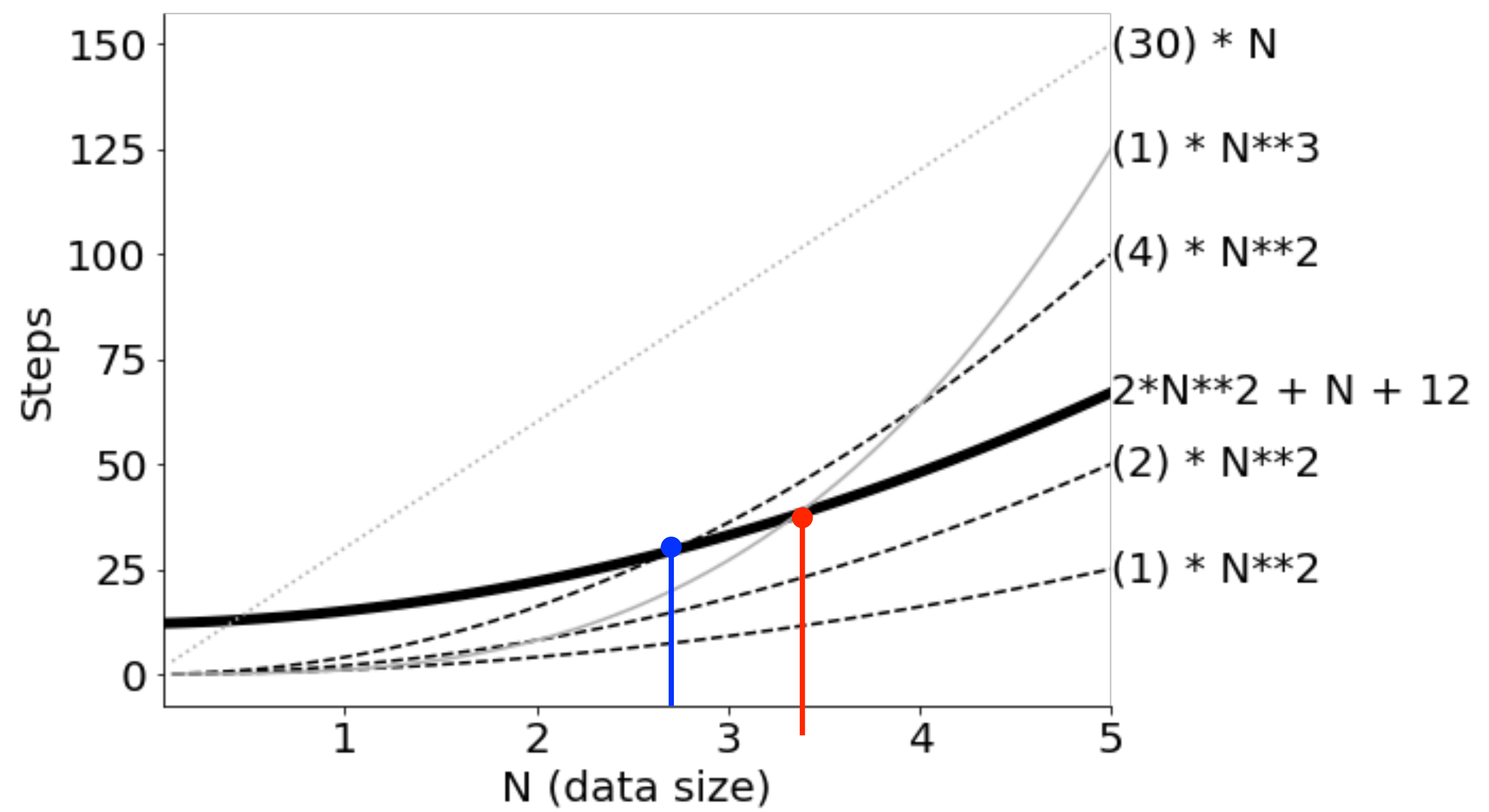

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$\mathbf{f(N) \in O(N^2)}$ (tighter upper bound)

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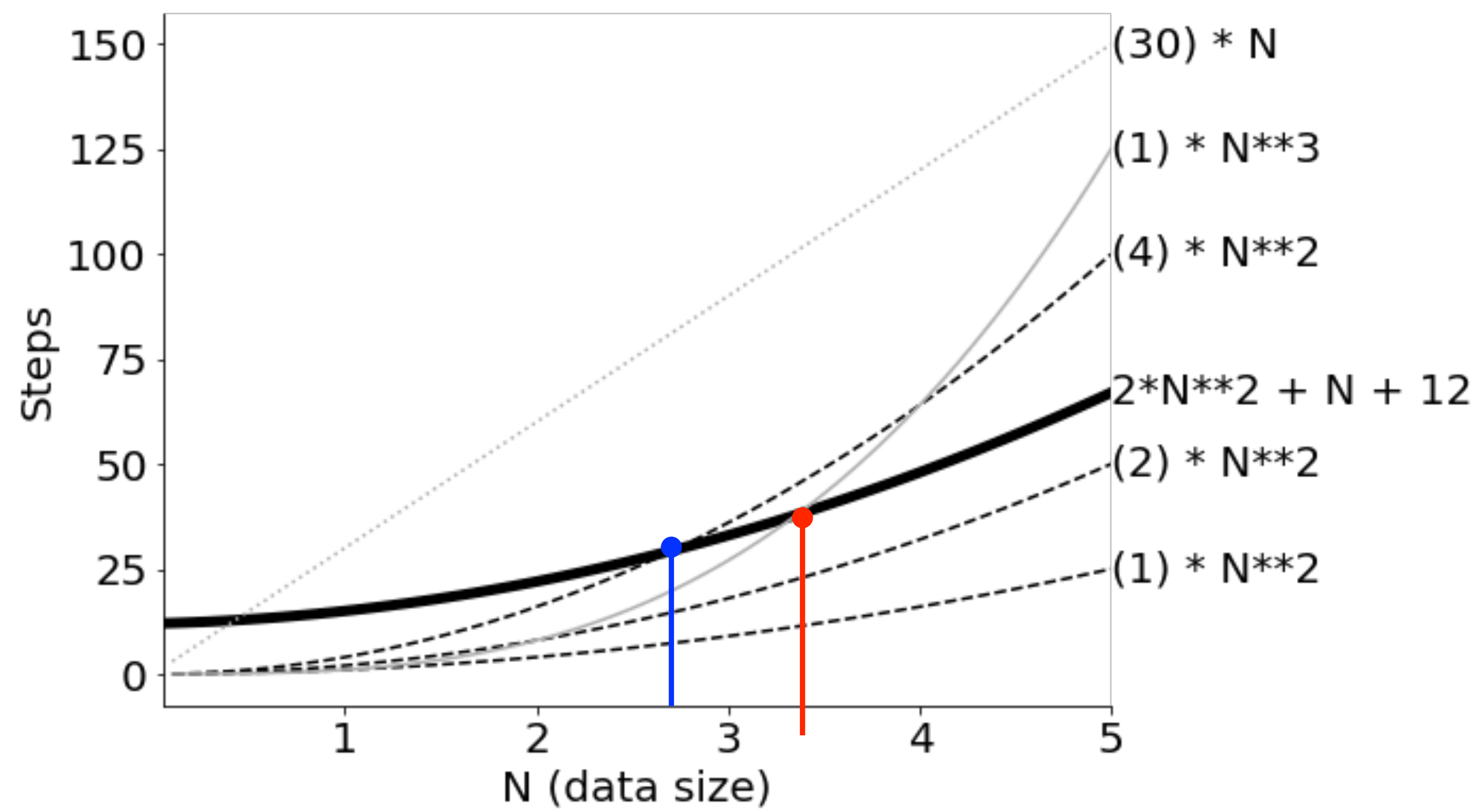
Assume $\mathbf{N = 20}$. and
$\mathbf{2N^2 + N + 12 \leq 30N}$.



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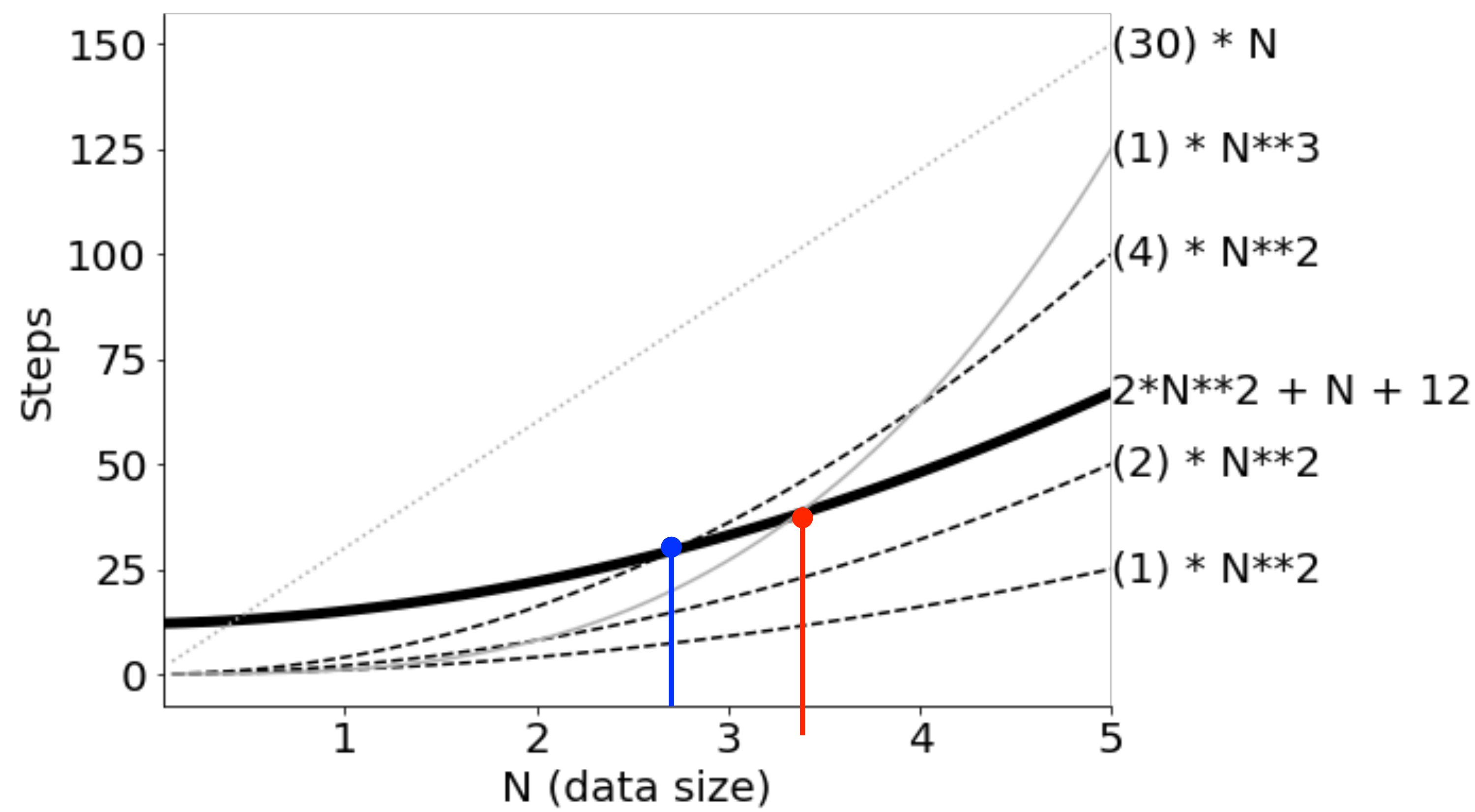
$\mathbf{f(N) \in O(N^2)}$ (tighter upper bound)

Somebody claims $f(N) \in O(N)$, offering C=30 and N>0. Suggest an N value to counter their claim.

Assume $\mathbf{N = 20.}$ and
$\mathbf{2N^2 + N + 12 \leq 30N.}$
However, $\mathbf{800 + 20 + 12 \not\leq 600.}$



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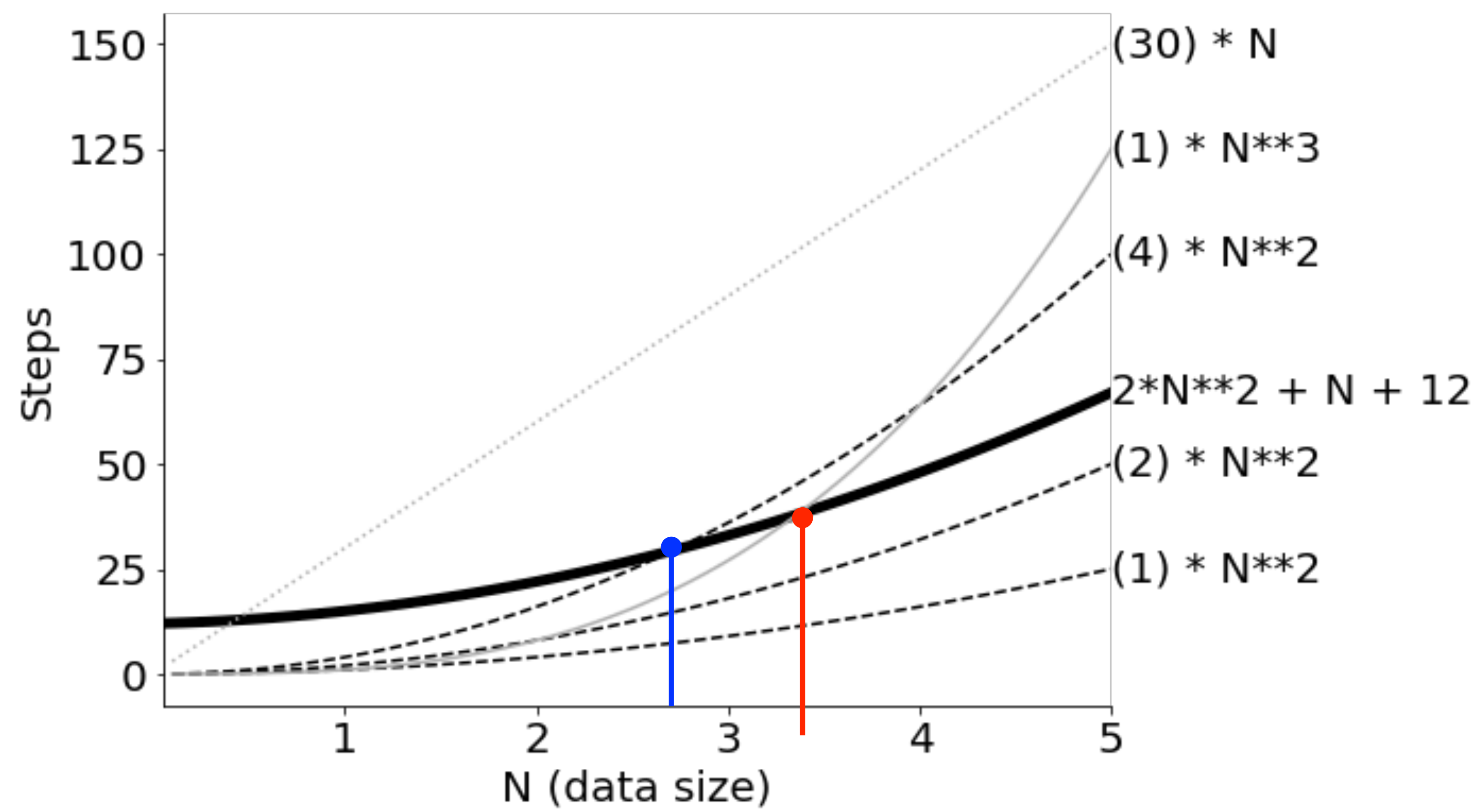
Somebody claims $f(N) \in O(N)$, offering C=30 and N>0. Suggest an N value to counter their claim.

Assume $\mathbf{N = 20}$. and
$\mathbf{2N^2 + N + 12 \leq 30N}$.
However, $\mathbf{800 + 20 + 12 \nleq 600}$.
Therefore, the suggest value of
$\mathbf{N = 20}$.



3

```
nums = [...]

first100sum = 0

for x in nums[:100]:
    first100sum += x
print(first100sum)
```

If we increase the size of nums from 20 items to 100 items, the code will probably take ________ times longer to run.

If we increase the size of nums from 100 to 1000, will the code take longer? Yes / No

The complexity of the code is O(______), with N=len(nums).

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nums = [...]

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for x in nums[:100]:
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If we increase the size of nums from 20 items to 100 items, the code will probably take **5** times longer to run.

If we increase the size of nums from 100 to 1000, will the code take longer? Yes / No

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If we increase the size of nums from 100 to 1000, will the code take longer? Yes / No

No

The complexity of the code is O(______), with N=len(nums).

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No

The complexity of the code is O(**1**), with N=len(nums).

4

Each of the following list operations are either O(1) or O(N), where N is len(L). Circle those you think are O(N).

```
L.insert(0, x)    L.pop(0)     x = L[0]       x = max(L)     x = len(L)

L.append(x)       L.pop(-1)    L2.extend(L)   x = sum(L)     found = X in L
```

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5

```
L = [...]
for x in L:
    avg = sum(L) / len(L)
    if x > 2*avg:
        print("outlier", x)
```

What is the big O complexity?

Is there a way to optimize the code?

5

```
L = [...]
for x in L: N + 1 steps
    avg = sum(L) / len(L)
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$$\mathbf{O((N+1)N) = O(N^2 + N)}$$

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What is the big O complexity?

$$\mathbf{O((N+1)N) = O(N^2 + N) = O(N^2)}$$

Is there a way to optimize the code?

Calculate avg outside the loop.

6

```
A = [...]
B = [...]

for x in A:
    for y in B:
        print(x*y)
```

how would you define the variable(s) to describe the size of the input data?

The complexity of code is

O(______________)

6

```
A = [...]
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for x in A:
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$len(A) = M$

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how would you define the variable(s) to describe the size of the input data?

$len(A) = M$ and $len(B) = N$

The complexity of code is

O(_______________)

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$len(A) = M$

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$\mathbf{M+1}$ steps

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$len(A) = M$ and $len(B) = N$

The complexity of code is

$O((\mathbf{M+1})(\mathbf{N+1})) = O(\mathbf{MN+M+N+1}) = O(\mathbf{MN})$

7

```
# assume L is already sorted, N=len(L)
def binary_search(L, target):
    left_idx = 0 # inclusive
    right_idx = len(L) # exclusive
    while right_idx - left_idx > 1:
        mid_idx = (right_idx + left_idx) // 2
        mid = L[mid_idx]
        if target >= mid:
            left_idx = mid_idx
        else:
            right_idx = mid_idx

    return right_idx > left_idx and L[left_idx] == target
```

how many times does this step run when N = 1? N = 2? N = 4? N = 8?

If **f(N)** is the number of times this step runs, then f(N) =

The complexity of binary search is O(________)

Assume target = 20

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1	3	5	8	10	20	73	80

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# assume L is already sorted, N=len(L)
def binary_search(L, target):
    left_idx = 0 # inclusive
    right_idx = len(L) # exclusive
    while right_idx - left_idx > 1:
        mid_idx = (right_idx + left_idx) // 2
        mid = L[mid_idx]
        if target >= mid:
            left_idx = mid_idx
        else:
            right_idx = mid_idx

    return right_idx > left_idx and L[left_idx] == target
```

how many times does this step run when N = 1? N = 2? N = 4? N = 8?

If **f(N)** is the number of times this step runs, then f(N) =

The complexity of binary search is O(________)

Assume target = 20

Idx	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
	1	3	5	8	10	20	73	80	

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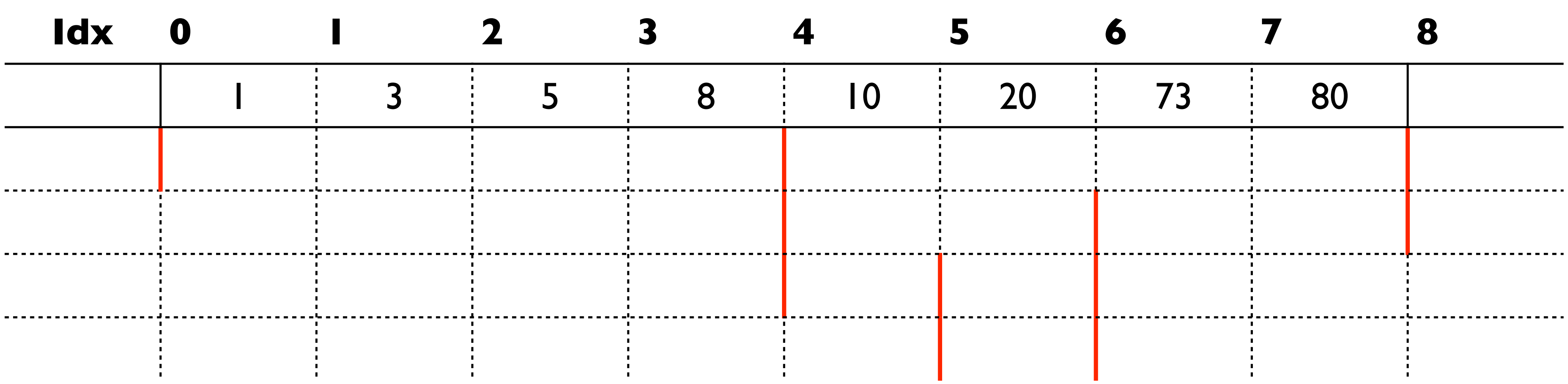
The complexity of binary search is O(_________)

Assume target = 20



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Assume target = 20

Idx	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
	1	3	5	8	10	20	73	80	
cut									
cut									
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how many times does this step run when N = 1? N = 2? N = 4? N = 8?

If **f(N)** is the number of times this step runs, then f(N) = **$\log_2 N$**

The complexity of binary search is O(_________)

Assume target = 20

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how many times does this step run when N = 1? N = 2? N = 4? N = 8?

If **f(N)** is the number of times this step runs, then f(N) = **$\log_2 N$**

The complexity of binary search is O(**logN**)

Assume target = 20

Idx	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
	1	3	5	8	10	20	73	80	
cut									
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8

```
s1 = tuple("...") # could be any string
s2 = tuple("...")
```

```
# version A
import itertools

matches = False
for p in itertools.permutations(s1):
    if p == s2:
        matches = True
```

```
# version B
s1 = sorted(s1)
s2 = sorted(s2)
matches = (s1 == s2)

Examples, merge sort, quick sort
```

assumed sorted is O(N log N)

what is the complexity of version A?

what is the complexity of version B?

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s1 = tuple("...") # could be any string
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$len(\mathrm{s1}) = N$
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For Example, s1 = (A, B, C), then permutations of s1 are
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For Example, s1 = (A, B, C), then permutations of s1 are
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ABC   BCA
ACB   CAB
BAC   CBA

N choices	N-1 choices	N-2 choices	...	2 choices	1 choice

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$N!$ steps

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Examples, merge sort, quick sort
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Examples, merge sort, quick sort

assumed sorted is O(N log N)

what is the complexity of version A? O(**N * N!**)

what is the complexity of version B?

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what is the complexity of version A? $O(\mathbf{N} * \mathbf{N}!)$

what is the complexity of version B? $O(\mathbf{N}\ \log\mathbf{N} + \mathbf{N}\ \log\mathbf{N})$

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what is the complexity of version B? $O(\mathbf{N}\ \log\mathbf{N} + \mathbf{N}\ \log\mathbf{N}) = O(\mathbf{2N}\ \log\mathbf{N})$

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what is the complexity of version B? $O(\mathbf{N}\ \log\mathbf{N} + \mathbf{N}\ \log\mathbf{N}) = O(\mathbf{2N}\ \log\mathbf{N}) = O(\mathbf{N}\ \log\mathbf{N})$

9

```
def selection_sort(L):
    for i in range(len(L)):
        idx_min = i
        for j in range(i, len(L)):
            if L[j] < L[idx_min]:
                idx_min = j
        # swap values at i and idx_min
        L[idx_min], L[i] = L[i], L[idx_min]

nums = [2, 4, 3, 1]
selection_sort(nums)
print(nums)
```

if this runs **f(N)** times, where **N=len(L)**,

then **f(N)** =

The complexity of selection sort is

O(______________)

9

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The complexity of selection sort is

O(_______________)

i	# of items for the inner for loop
0	N
1	N-1
2	N-2
...	...
N-1	1
N	0

9

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if this runs **f(N)** times, where **N=len(L)**,

then **f(N)** = $\mathbf{N + (N-1) + (N-2) + \ldots + 2 + 1 + 0}$

The complexity of selection sort is

O(_____________)

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if this runs **f(N)** times, where **N=len(L)**,

then **f(N)** = $\mathbf{N + (N-1) + (N-2) + \ldots + 2 + 1 + 0}$

$= \mathbf{\frac{N(N+1)}{2}}$

The complexity of selection sort is

O(____________)

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if this runs f(N) times, where N=len(L),

then f(N) $= \mathbf{N + (N-1) + (N-2) + \ldots + 2 + 1 + 0}$

$= \frac{\mathbf{N(N+1)}}{\mathbf{2}} = \frac{\mathbf{N^2+N}}{\mathbf{2}}$

The complexity of selection sort is

O(____________)

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if this runs **f(N)** times, where **N=len(L)**,

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$$= \frac{\mathbf{N(N+1)}}{\mathbf{2}} = \frac{\mathbf{N^2+N}}{\mathbf{2}} = \frac{\mathbf{N^2}}{\mathbf{2}} + \frac{\mathbf{N}}{\mathbf{2}}$$

The complexity of selection sort is

O(______________)

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```

if this runs f(N) times, where N=len(L),

then $f(N) = N + (N-1) + (N-2) + \ldots + 2 + 1 + 0$

$$= \frac{N(N+1)}{2} = \frac{N^2+N}{2} = \frac{N^2}{2} + \frac{N}{2}$$

The complexity of selection sort is

$$O(\frac{N^2}{2} + \frac{N}{2}) = O(\frac{N^2}{2})$$

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if this runs **f(N)** times, where **N=len(L)**,

then **f(N)** $= \mathbf{N + (N-1) + (N-2) + \ldots + 2 + 1 + 0}$

$$= \frac{\mathbf{N(N+1)}}{\mathbf{2}} = \frac{\mathbf{N^2 + N}}{\mathbf{2}} = \frac{\mathbf{N^2}}{\mathbf{2}} + \frac{\mathbf{N}}{\mathbf{2}}$$

The complexity of selection sort is

$$O\left(\frac{\mathbf{N^2}}{\mathbf{2}} + \frac{\mathbf{N}}{\mathbf{2}}\right) = O\left(\frac{\mathbf{N^2}}{\mathbf{2}}\right) = O(\mathbf{N^2})$$

i	**# of items for the inner for loop**
0	N
1	N-1
2	N-2
...	...
N-1	1
N	0