[320] Complexity + Big O (Worksheet: Complexity Analysis)

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1

```
def search(L, target):
    for x in L:
        if x == target: #line A
            return True
    return False
```

assume this is asked unless otherwise stated

Let **f(N)** be the number of times line A executes, with `N=len(L)`. What is **f(N)** in each case?

Worst Case (target is at end of list): f(N) = ___________

Best Case (target is at beginning of list): f(N) = ___________

Average Case (target in middle of list): f(N) = ___________

A **step** is any unit of work with bounded execution time (it doesn't keep getting slower with growing input size).

We classify algorithm complexity by classifying the **order of growth** of a function f(N), where f gives the number of steps the algorithm must perform for a given input size.

Big O definition: if $\mathbf{f(N) \leq C * g(N)}$ for large **N** values and some fixed constant **C**, then $\mathbf{f(N) \in O(g(N))}$

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Average Case (target in middle of list): $f(N) = \mathbf{\frac{N}{2} \in O(N)}$

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2

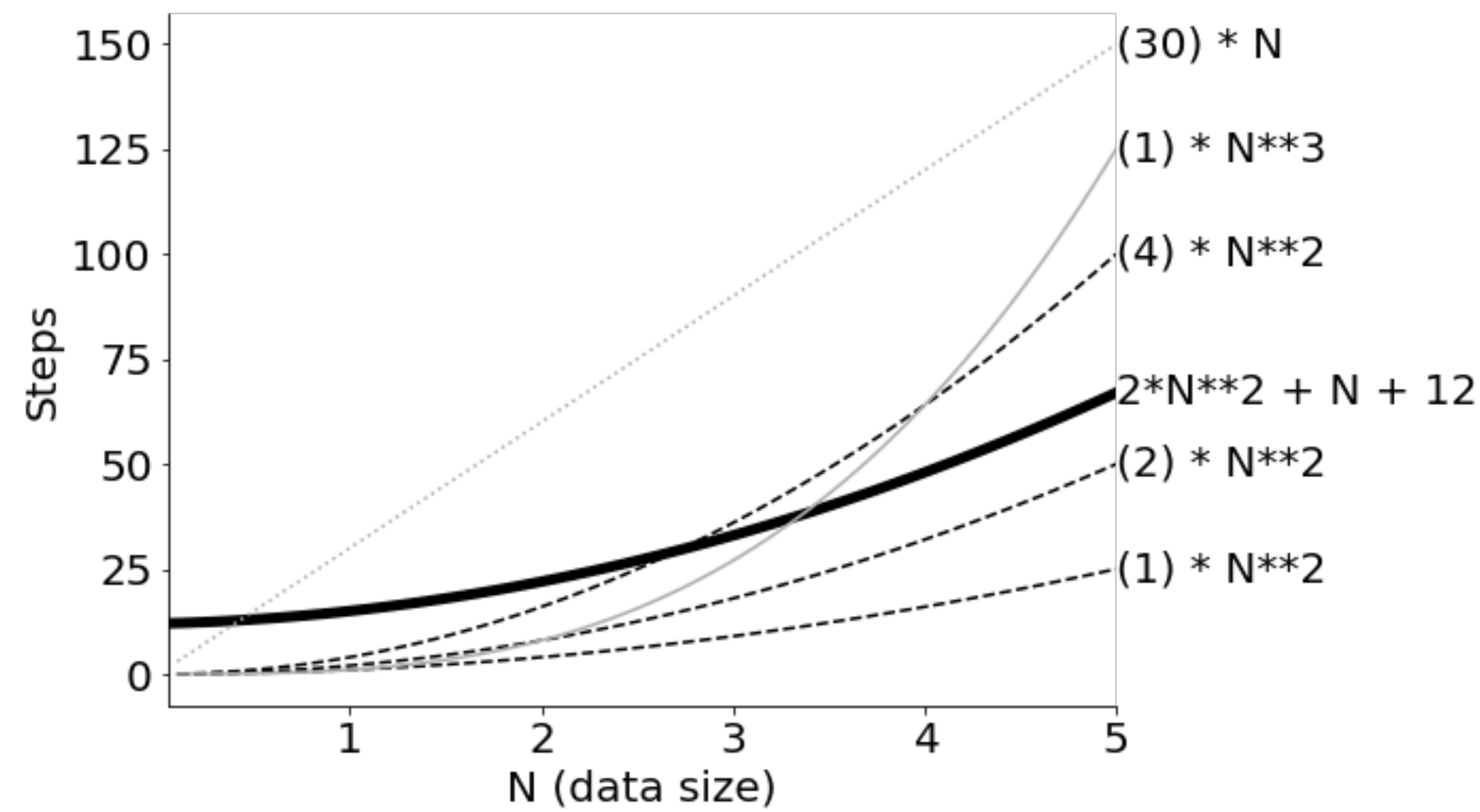
Let $f(N) = 2N^2 + N + 12$

If we want to show $f(N) \in O(N^3)$, what is a good lower bound on N? Let's have C=1.

To show $f(N) \in O(N^2)$, do we pick 1, 2, or 4 for the C? After picking C, what should we choose for N's lower bound?

What is more informative to show? $f(N) \in O(N^3)$ or $f(N) \in O(N^2)$?

Somebody claims $f(N) \in O(N)$, offering C=30 and N>0. Suggest an N value to counter their claim.



2

Let $f(N) = 2N^2 + N + 12$

If we want to show $f(N) \in O(N^3)$, what is a good lower bound on N? Let's have C=1. $\mathbf{N \geq 4}$.

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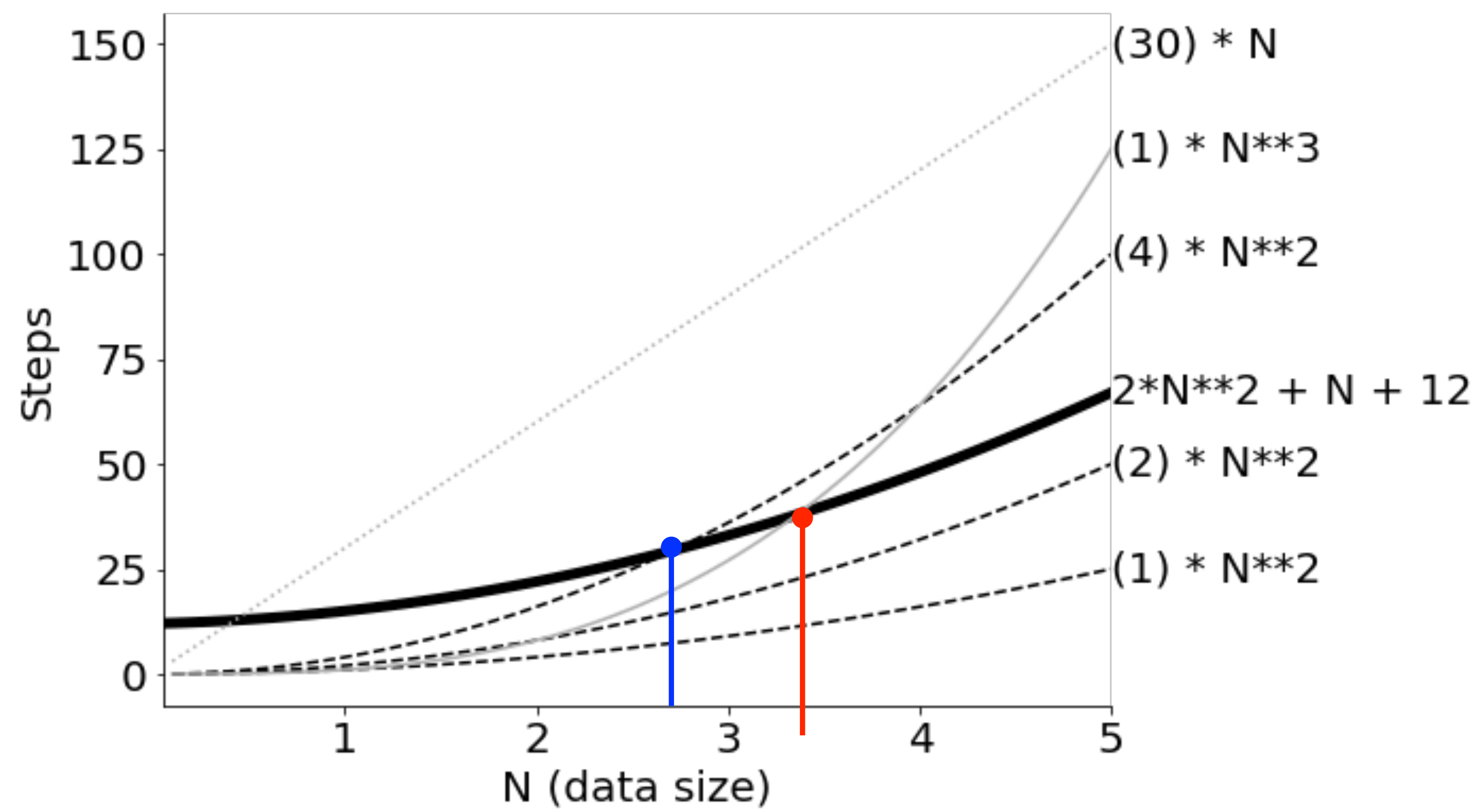

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If we want to show $f(N) \in O(N^3)$, what is a good lower bound on N? Let's have C=1. **$N \geq 4$.**

To show $f(N) \in O(N^2)$, do we pick 1, 2, or 4 for the C? After picking C, what should we choose for N's lower bound? **$C = 4$**

What is more informative to show? $f(N) \in O(N^3)$ or $f(N) \in O(N^2)$?

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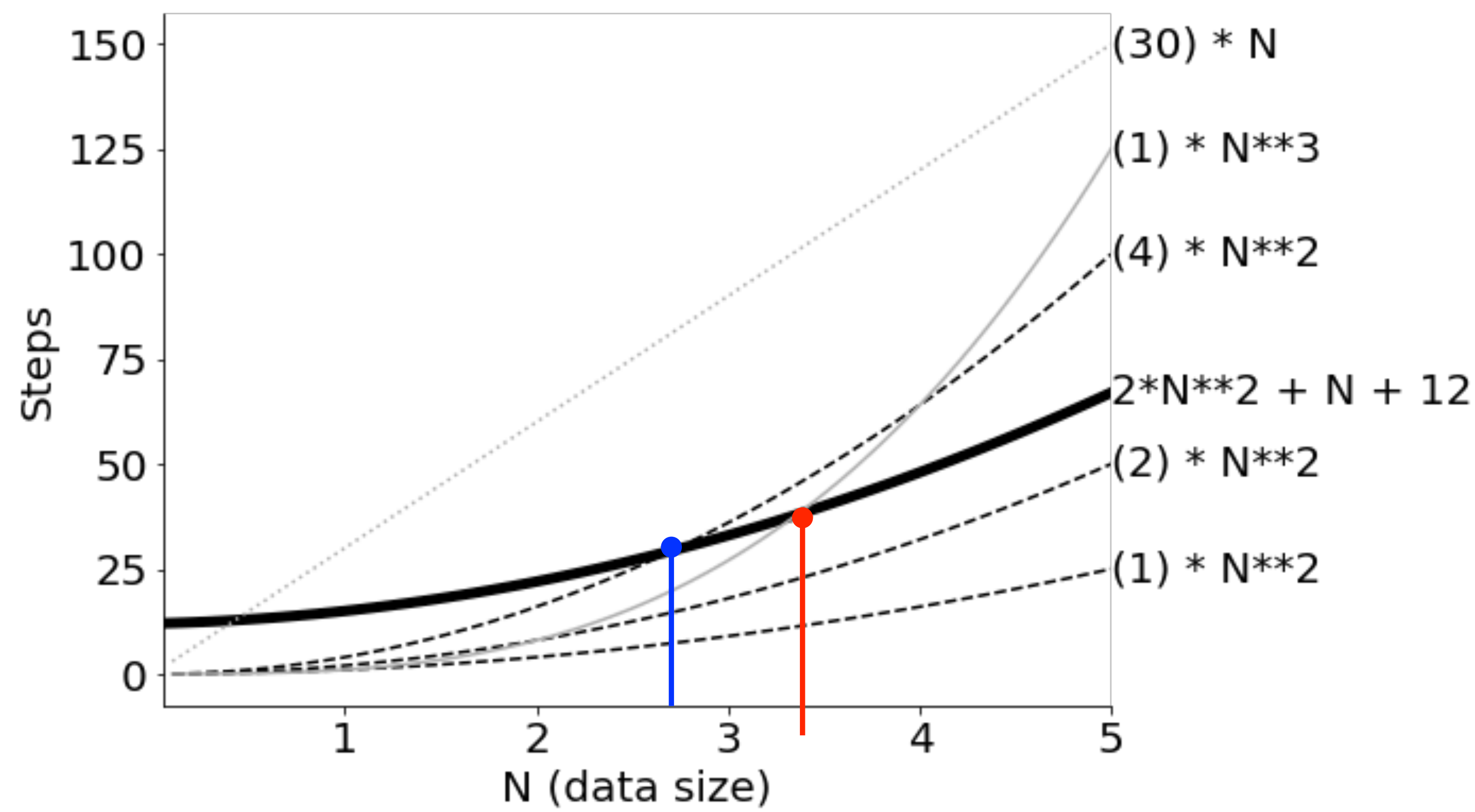

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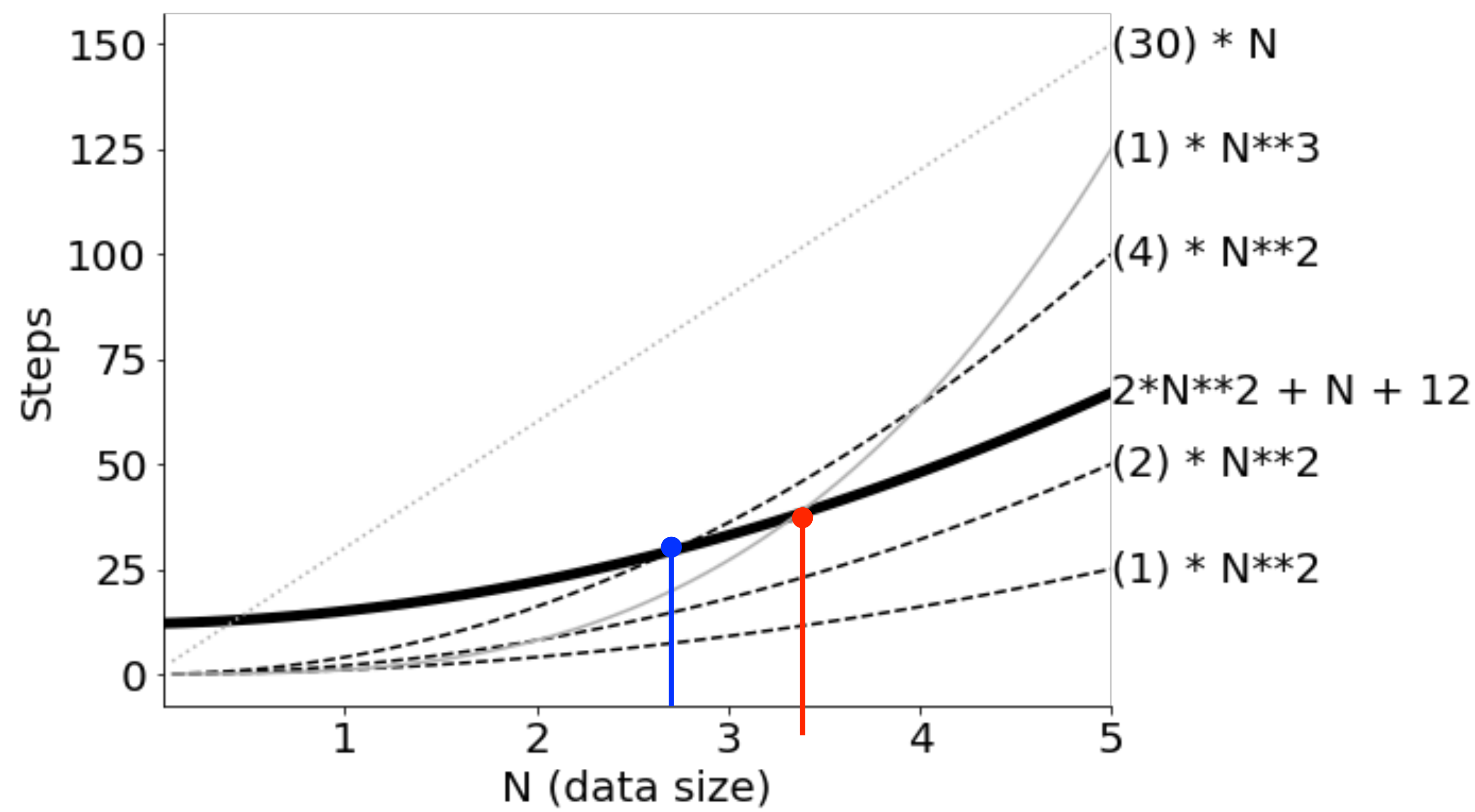

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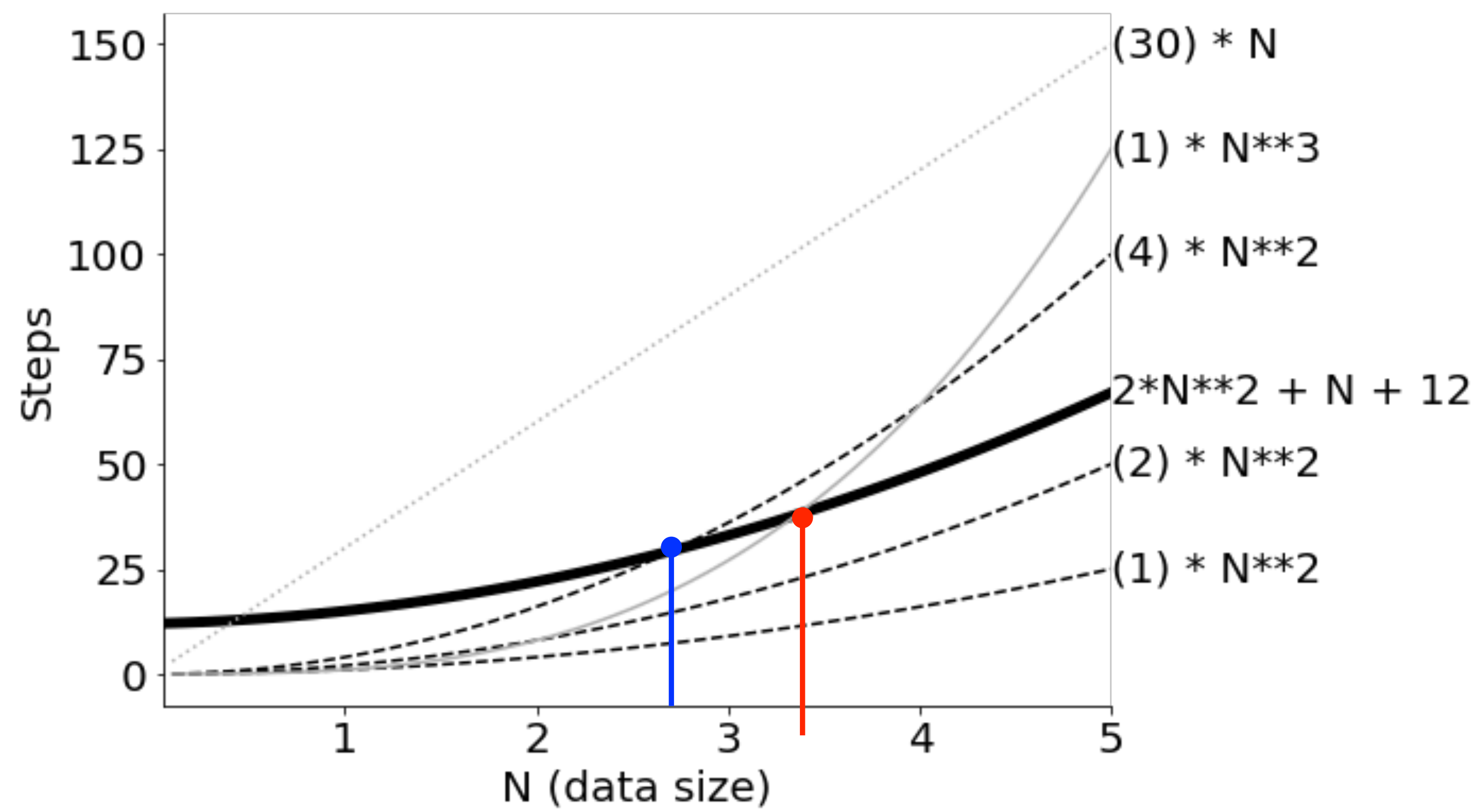

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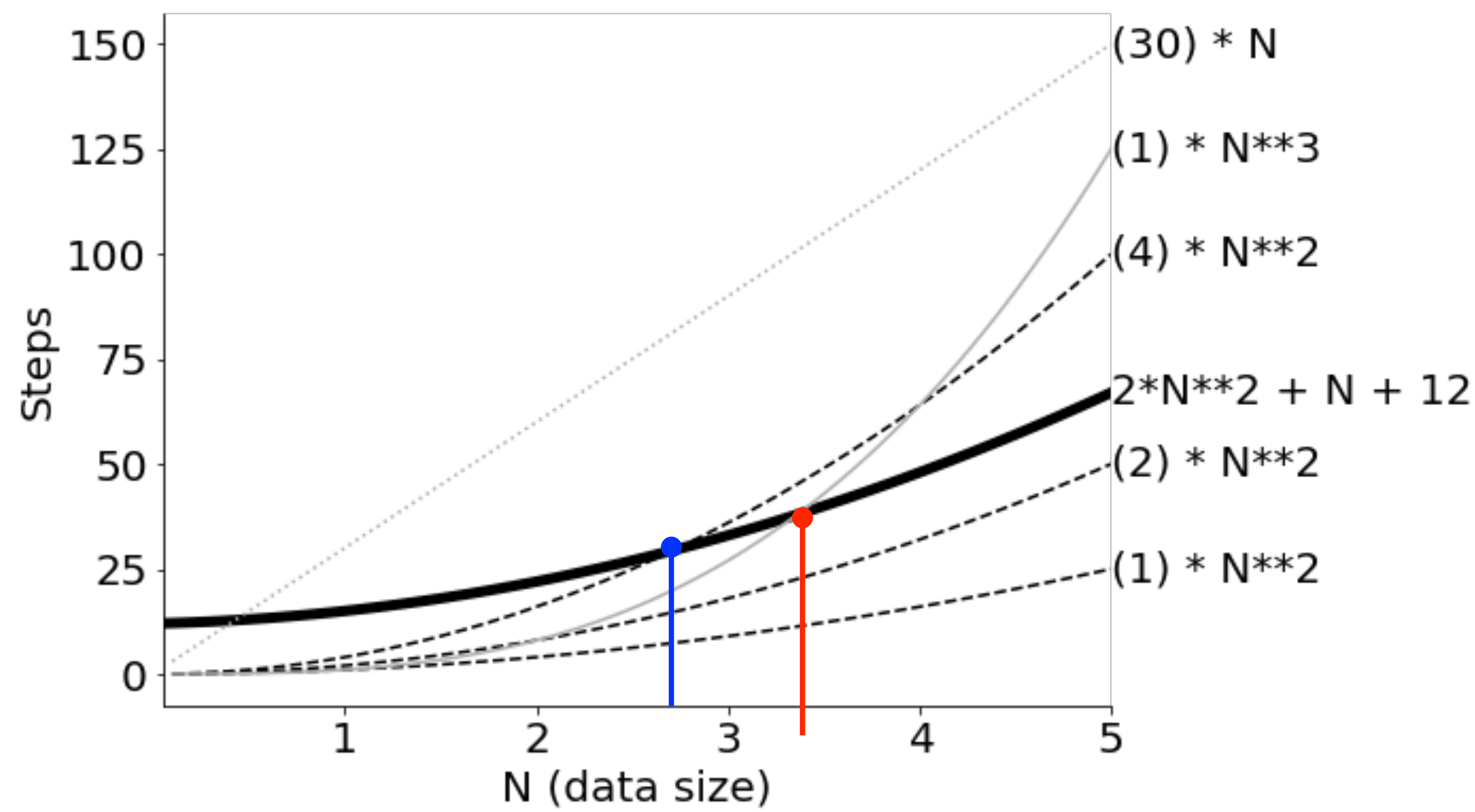
$f(N) \in O(N^3)$ or $f(N) \in O(N^2)$?
$\mathbf{f(N) \in O(N^2)}$ (tighter upper bound)

Somebody claims $f(N) \in O(N)$, offering C=30 and N>0. Suggest an N value to counter their claim.

Assume $\mathbf{N = 20}$. and
$\mathbf{2N^2 + N + 12 \leq 30N}$.



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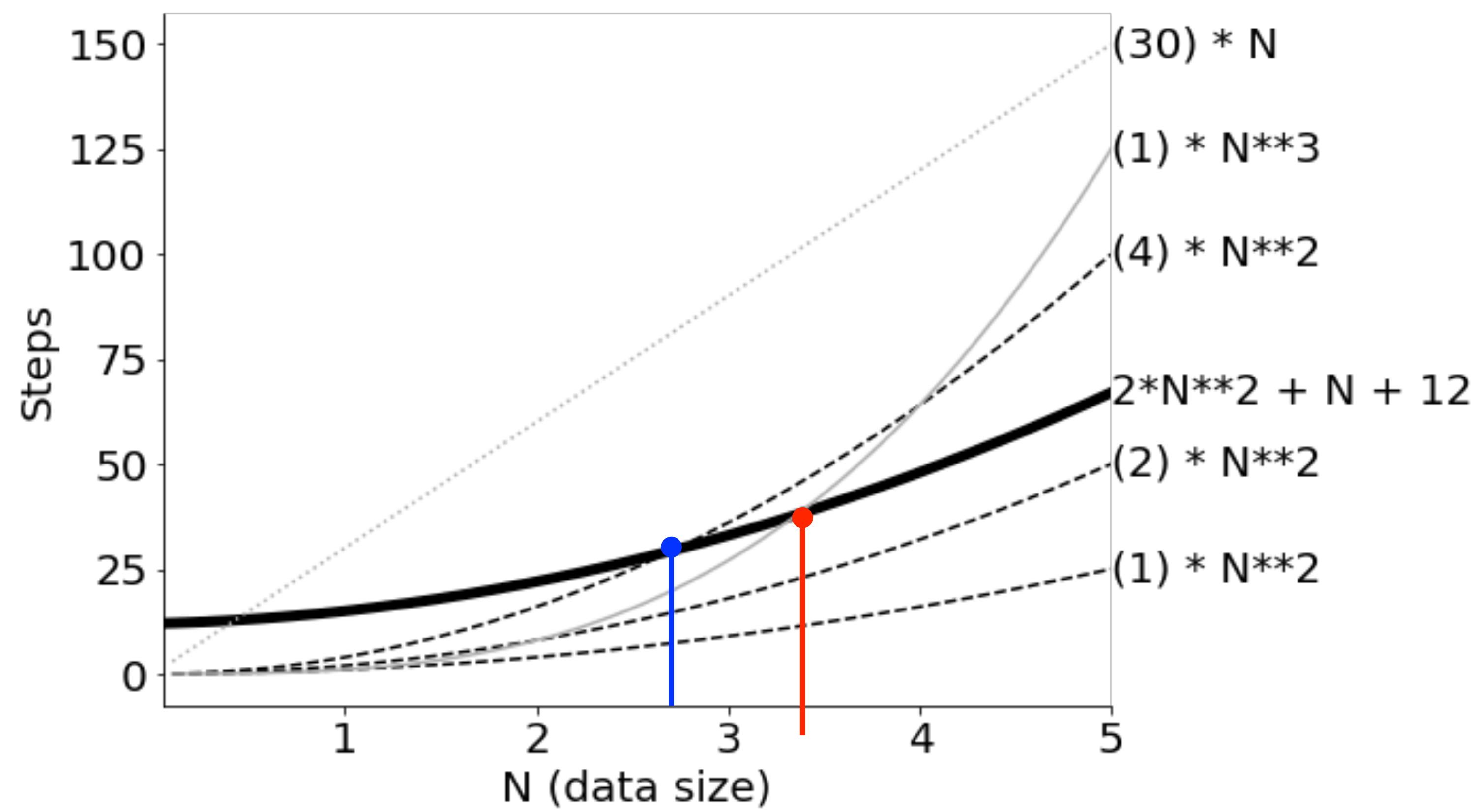

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Somebody claims $f(N) \in O(N)$, offering C=30 and N>0. Suggest an N value to counter their claim.

Assume $\mathbf{N = 20.}$ and
$\mathbf{2N^2 + N + 12 \leq 30N}$.
However, $\mathbf{800 + 20 + 12 \nleq 600}$.

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If we want to show $f(N) \in O(N^3)$, what is a good lower bound on N? Let's have C=1. $\mathbf{N \geq 4}$

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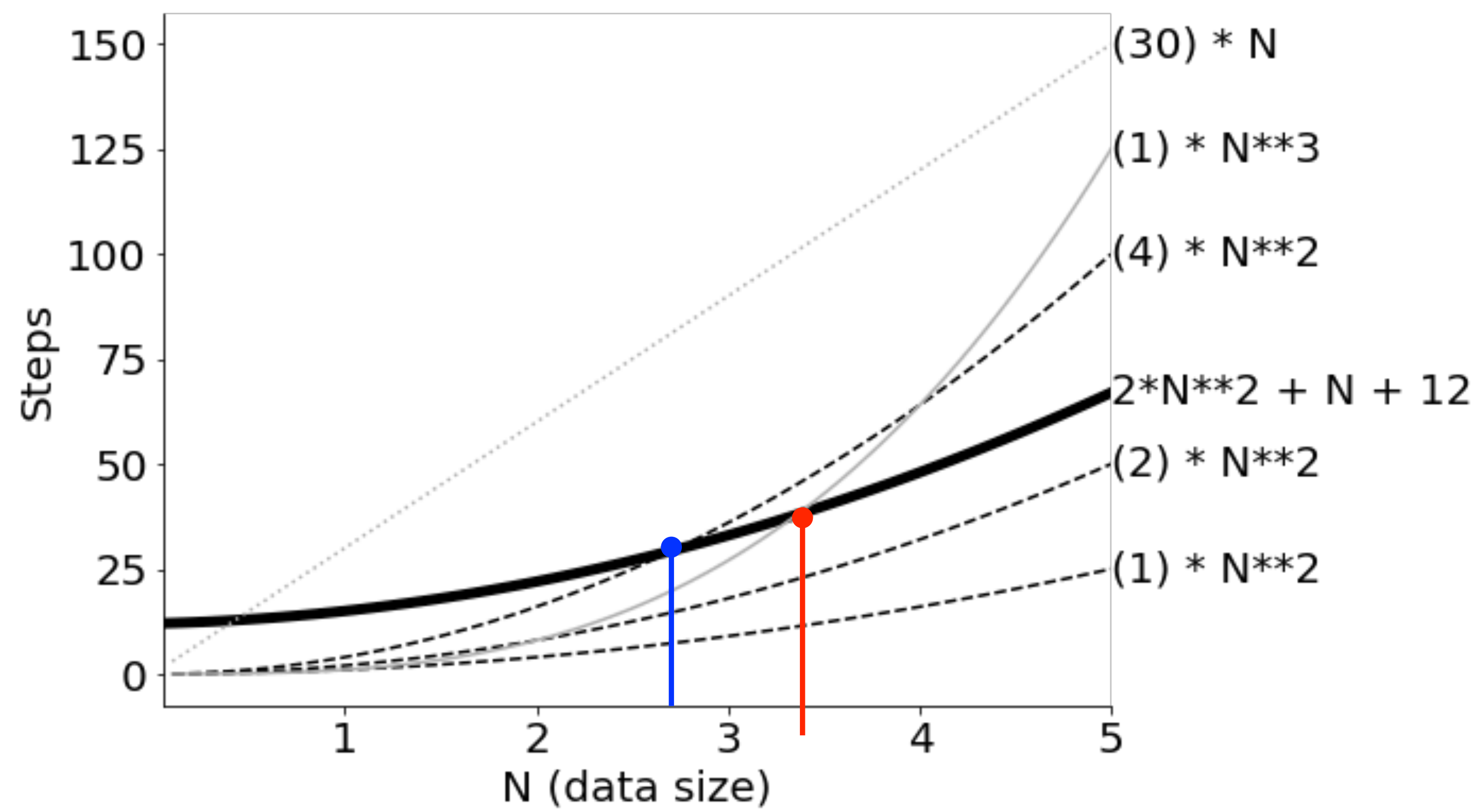
Somebody claims $f(N) \in O(N)$, offering C=30 and N>0. Suggest an N value to counter their claim.

Assume $\mathbf{N = 20}$. and
$\mathbf{2N^2 + N + 12 \leq 30N}$.
However, $\mathbf{800 + 20 + 12 \nleq 600}$.
Therefore, the suggest value of
$\mathbf{N = 20}$.



3

```
nums = [...]

first100sum = 0

for x in nums[:100]:
    first100sum += x
print(first100sum)
```

If we increase the size of nums from 20 items to 100 items, the code will probably take ________ times longer to run.

If we increase the size of nums from 100 to 1000, will the code take longer? Yes / No

The complexity of the code is O(_____), with N=len(nums).

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If we increase the size of nums from 100 to 1000, will the code take longer? Yes / No

No

The complexity of the code is O(______), with N=len(nums).

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If we increase the size of nums from 100 to 1000, will the code take longer? Yes / No

No

The complexity of the code is O(**1**), with N=len(nums).

4

Each of the following list operations are either O(1) or O(N), where N is len(L). Circle those you think are O(N).

```
L.insert(0, x)    L.pop(0)     x = L[0]        x = max(L)     x = len(L)

L.append(x)       L.pop(-1)   L2.extend(L)    x = sum(L)     found = X in L
```

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5

```
L = [...]
for x in L:
    avg = sum(L) / len(L)
    if x > 2*avg:
        print("outlier", x)
```

What is the big O complexity?

Is there a way to optimize the code?

5

```
L = [...]
for x in L: N + 1 steps
    avg = sum(L) / len(L)
    if x > 2*avg:
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L = [...]
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$\mathbf{N+1}$ steps (for x in L)

$\mathbf{N}$ steps (avg = sum(L) / len(L))

What is the big O complexity?

$$\mathbf{O((N+1)N) = O(N^2+N)}$$

Is there a way to optimize the code?

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L = [...]
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What is the big O complexity?

$$\mathbf{O((N+1)N) = O(N^2 + N) = O(N^2)}$$

Is there a way to optimize the code?

Calculate avg outside the loop.

6

```
A = [...]
B = [...]

for x in A:
    for y in B:
        print(x*y)
```

how would you define the variable(s) to describe the size of the input data?

The complexity of code is

O(________________)

6

```
A = [...]
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for x in A:
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$len(A) = M$

how would you define the variable(s) to describe the size of the input data?

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how would you define the variable(s) to describe the size of the input data?

$len(A) = M$ and $len(B) = N$

The complexity of code is

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$\mathbf{M+1}$ steps

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for x in A: $\mathbf{M+1}$ steps

for y in B: $\mathbf{N+1}$ steps

how would you define the variable(s) to describe the size of the input data?

$len(A) = M$ and $len(B) = N$

The complexity of code is

$O((\mathbf{M+1})(\mathbf{N+1})) = O(\mathbf{MN+M+N+1}) = O(\mathbf{MN})$

8

```
s1 = tuple("...") # could be any string
s2 = tuple(“...")
```

```
# version A
import itertools

matches = False
for p in itertools.permutations(s1):
    if p == s2:
        matches = True
```

```
# version B
s1 = sorted(s1)
s2 = sorted(s2)
matches = (s1 == s2)
```

assumed sorted is O(N log N)

what is the complexity of version A? O(_______________)

what is the complexity of version B? O(_______________)

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For Example, s1 = (A, B, C), then permutations of s1 are
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ACB CAB
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N! steps

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# version B
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assumed sorted is O(N log N)

what is the complexity of version A? O(____________)

what is the complexity of version B? O(____________)

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assumed sorted is O(N log N)

what is the complexity of version A? O(**N * N!**)

what is the complexity of version B? O(______________)

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$len(s1) = N$
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matches = False
for p in itertools.permutations(s1):
    if p == s2:
        matches = True
```

`for p in itertools.permutations(s1):` **N**! steps

`if p == s2:` **N** steps

```
# version B
s1 = sorted(s1)
s2 = sorted(s2)
matches = (s1 == s2)
```

`s1 = sorted(s1)` **N** log**N**

`s2 = sorted(s2)` **N** log**N**

Example, merge sort, quick sort

assumed sorted is O(N log N)

what is the complexity of version A? O(**N** * **N**!)

what is the complexity of version B? O(_______________)

8

```
s1 = tuple("...") # could be any string
s2 = tuple("...")
```

$len(s1) = N$
$len(s2) = N$

```
For Example, s1 = (A, B, C), then permutations of s1 are
```

ABC BCA
ACB CAB
BAC CBA

```
# version A
import itertools

matches = False
for p in itertools.permutations(s1): N! steps
    if p == s2: N steps
        matches = True
```

```
# version B
s1 = sorted(s1)      N logN
s2 = sorted(s2)      N logN
matches = (s1 == s2)

Example, merge sort, quick sort
```

assumed sorted is O(N log N)

what is the complexity of version A? $O(\mathbf{N} * \mathbf{N}!)$

what is the complexity of version B? $O(\mathbf{2N}\ \log\mathbf{N}) = O(\mathbf{N}\ \log\mathbf{N})$