CRWN 88
Homework 1

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FSA We describe a general procedure for making the prediction under the following assumptions. First, given step, the participant is equally likely to move to any of the squares adjacent to to his current square. Next, a participant may move into square they have already visited. Finally, if the participant finds himself in a square that is removed, we consider that this is the "final" square they visit (the rules were unclear for cases dealt with in these last two assumptions).

We can model the experiment using a series of Markov chains, one for each stage of the experiment. We call a stage of the experiment a series of moves the participant makes on the grid, followed by the removal of a square. Define each of the n square in the grid to be one of n states in the Markov chain (We label the states/squares $1, 2, ..n$). For a given stage i, for each of the n states, we encode the probabilities of transitioning from that state to each of n states in the $n \times n$ transition matrix T_i, where the (j, k)-entry is 0 if the squares k and j are not adjacent and $\frac{1}{a}$ otherwise, where a is the number of squares adjacent to square k (as the participant moves with equal probability to any of the squares adjacent to his current square).

$$T_i = \begin{bmatrix} P(1|1) & P(2|1) & \dots \\ P(1|2) & P(2|2) & \dots \\ \vdots & \vdots & \ddots \end{bmatrix}$$

Define $S_i = \begin{bmatrix} P(1)_i & P(2)_i & \dots \end{bmatrix}$ as the vector whose entries are the probabilities of each possible state at stage i. To compute the probability of each state at the end of the i-th stage, we use the following formula:

$$S_i = S_{i-1} \cdot T^k$$

Where k is the number of moves the participant makes on the grid for that stage. However we must still account for the removal of a square k from the grid at the end of each stage. We do this as follows:

(i) Choose k to be the state/square corresponding to the *smallest*, or tied for smallest, entry in S_i, (i.e. with the lowest probability)

(ii) Represent the stage $i+1$ with a new Markov chain which does not include square/state k. Compute the transition probabilities as before (accounting for the fact that $k$is no longer one of the possible squares/states

(iii) Remove the entry in S_i corresponding to the state/square k and record that entry as the "final" probability of state k.

Now to compute the square we will predict the participant will finish in, begin by selecting a starting square, and set the entries of S_0 to $\frac{1}{a}$, where a is the number of squares adjacent to the starting square, if they correspond to a square adjacent to the starting one, and to 0 if not. We then repeatedly apply the process for modeling each stage described above, until all but one square/state has been removed. We can then predict that the participant will finish the experiment in that square.

This square is merely the most likely that participants will finish the experiment in the predicted square, and factors such as grid size or the allowed number of moves for each stage may influence the final probability. For instance, the last two stages of the experiment done in class were fully deterministic due the number of moves allowed.

(2) It seems unlikely that machine learning was used to make the prediction, given that the possibility of making a fairly good prediction using a much method similar to the Markov chain technique described in (1). Furthermore, it seems unclear how data sourced from our educational emails could be used to predict moves in this experiment. The method behind this trick may have been described that way to avoid giving away too much of the real method. If participants have some idea of how the 'real' method works, they might go out of their way to make moves that the method deems unlikely and thus make the prediction less accurate - for example, with the method described (1), squares which are adjacent to few other squares will gradually be assigned lower probabilities.

(3) The first application that comes to mind for the method would be the design and analysis of public spaces, in situations where it is reasonable to approximate the movement of people by random walks on a grid, say in malls, pedestrain downtown areas or or convention centers. For example, we might examine how changing the layout of an area - simulated by removing squares from the grid -alters traffic, or look at the places experiencing the most traffic to decide where to open a business.

Notes: In (1), I did not show the proof for the procedure because it would have been lengthy to reproduce. Also, I did not add the computations I did to verify that the procedure worked for the in-class experiment because the question only asked to explain the procedure that could have been used.