Week 3 Pre-Lab Reading

This weeks lab will make use of GNU Radio and GNU Radio Companion (GRC) for making signal processing flowgraphs. If you have not done so already, please install the software and ensure it is working properly before the lab. Installation instructions can be found at this link. We'll also be using an RTL-SDR to receive the FM signal. Instruction to install the necessary drivers can be found at this link, make sure you install the drivers for the correct device you'll be using in class. If you're unsure which device you'll be using, please contact your instructor.

1 Nyquist Sampling Theorem

When an analog signal is converted into digital format, the original signal is sampled (measured) at equally spaced time intervals, and the discrete sequence of samples is used to represent the original signal. The Nyquist theorem says that in order to accurately represent a signal, the sample rate must be at least twice the frequency of the highest frequency component of the signal. For a given sample rate f_s, the highest frequency signal that can be accurately measured is referred to as the Nyquist frequency f_N. For example, at a sample rate of $f_s = 16kHz$, the Nyquist frequency is $f_N = 8kHz$.

$$f_s \geq 2 * f_N \tag{1}$$

If the frequency of a signal is greater than the Nyquist frequency, an effect called aliasing will occur. Aliasing will cause false lower frequency components to appear in the sampled data.

The effect of aliasing can easily be shown using GNU radio. This simple flowgraph demonstrates the relationship between the frequency of a signal and the frequency of the sample rate.

- The "Signal Source" block produces a sine wave with a frequency adjustable by the first "QT GUI Range" block.

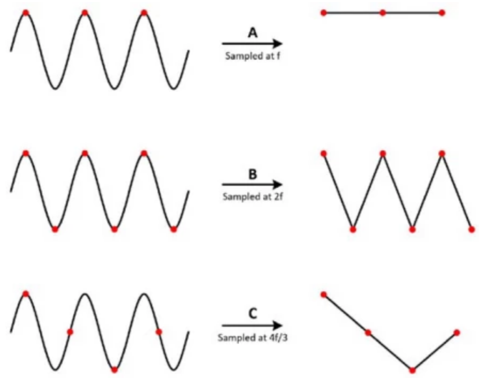


Figure 1: Undersampling a signal leads to an inaccurate reconstruction of the waveform and alias frequencies.

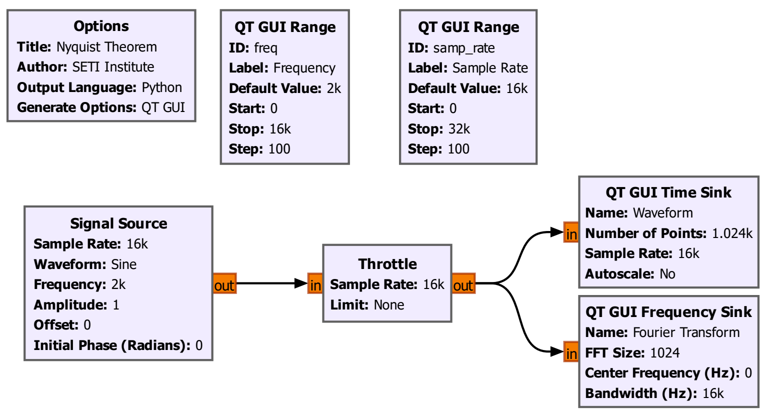


Figure 2: GNU Radio flowgraph demonstrating Nyquist sampling

- The "Throttle" block samples the signal at a frequency that is adjustable by the second "QT GUI Range" block.
- The "QT GUI Time Sink" block displays the sampled signal waveform.
- The "QT GUI Frequency Sink" block computes and displays the Fast Fourier Transform (FFT).

Using the slider at the top of the window, we can change either the sample rate or the signal frequency. In figure 3, a 2 kHz signal is being sampled at 16 kHz, and the resulting waveform and Fourier transform are displayed. The waveform is clearly a sine wave, and a peak can be seen in the Fourier transform display at the correct incoming frequency of 2 kHz. In figure 4 however, the signal frequency is increased to 10 kHz while the sample rate is kept fixed at 16 kHz. Instead of a peak at 10 kHz, we see an alias frequency of 6 kHz in the Fourier transform display, as well as significant degradation of the waveform.

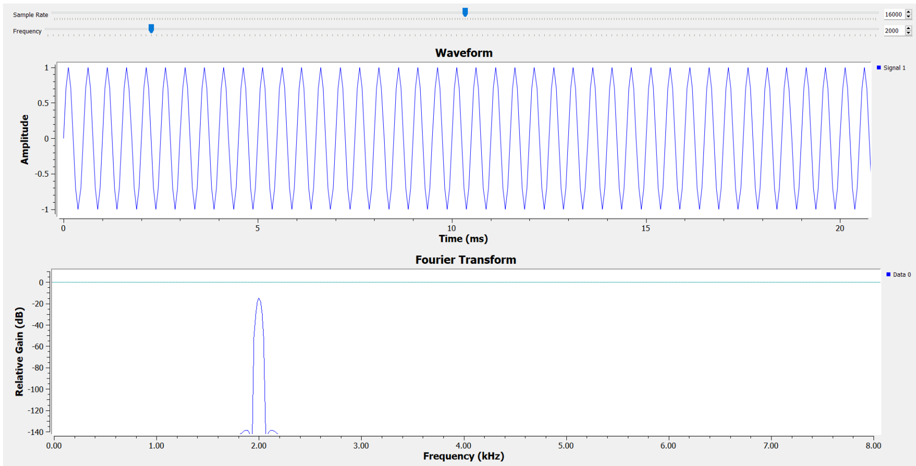


Figure 3: A 2 kHz sine wave sampled at 16 kHz showing an accurate waveform and correct frequency.

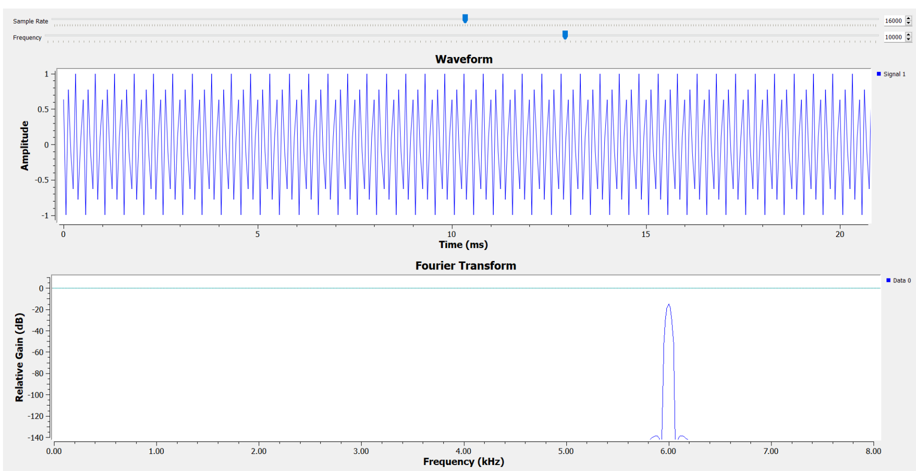


Figure 4: A 10 kHz sine wave sampled at 16 kHz showing a degraded waveform and alias frequency at 6 kHz.

2 Fourier Transformations

The Fourier theorem says that any periodic and continuous signal can be decomposed into a sum of sinusoidal components of different frequencies and amplitudes. The Fourier transform (FT) is a mathematical tool that allows you to find the individual sine and cosine terms that, when added together, will reproduce the original signal. The Fourier transform $F(k)$ of an arbitrary periodic function $f(x)$ is given by equation 2, while the inverse Fourier transform is given by equation 3.

$$F(k) = \int_{-\infty}^{\infty} f(x) e^{-2\pi ikx} dx \tag{2}$$

$$f(x) = \int_{-\infty}^{\infty} F(k) e^{2\pi ikx} dk \tag{3}$$

A Fourier transform works on continuous mathematical expressions, however real data is not continuous, but discrete. Therefore when working with real scientific data, a discrete Fourier transform (DFT) is used. A DFT works much the same way as a traditional FT, with the main difference being that it converts a discrete sequence of samples of a signal into discrete samples of the DFT. The DFT transforms a sequence of N complex numbers (the values of the signal samples $\{x_n\} := x_0, x_1, ..., x_{N-1}$) into another sequence of complex numbers ($\{X_k\} := X_0, X_1, ..., X_{N-1}$) given by equation 4, while the inverse DFT is given by equation 5.

$$X_k = \sum_{n=0}^{N-1} x_n e^{-2\pi ikn/N} \tag{4}$$

$$x_n = 1/N \sum_{k=0}^{N-1} X_k e^{2\pi ikn/N} \tag{5}$$

Sometimes described as "the most important numerical algorithm of our lifetime", the Fast Fourier Transform (FFT) is a very efficient algorithm for computing the DFT. FFTs are widely used in many disciplines, including digital signal processing for radio astronomy.

3 Filters

Various kinds of filters are often used in signal processing to separate, modify, or enhance specific frequencies ranges of a signal. Filters can be used to remove noise or other forms of interference, and to extract information from a signal. Here we'll cover the most commonly used types of filters and show how they are used in signal processing applications. First let's define some frequently used terms. A diagram of these terms overlaid on a Band Pass filter can be see in figure 6.

- Attenuate - to decrease the amplitude of a signal.

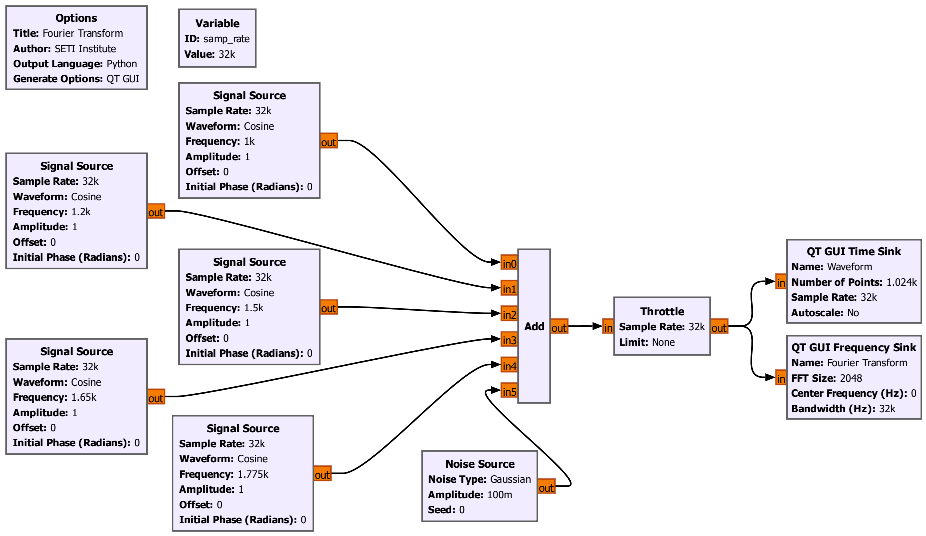


Figure 5: GRC flowgraph showing how a fast Fourier transform can decompose a signal into the frequencies it's made of.

- Cutoff frequency - the frequency at which a filter's response begins to significantly attenuate or change.
- Passband - the range of frequencies a filter allows to pass through with minimal attenuation.
- Stopband - the range of frequencies a filter will attenuate.
- Transition band - the frequency range between the passband and stopband where the filter transitions from allowing signals to attenuating them, or vice versa.
- Center frequency - the midpoint of the passband or stopband.

3.1 Band Pass Filters

Band Pass filters are designed to allow a specific range of frequencies to pass while attenuating frequencies outside this range. This is useful for isolating signals within a specific frequency range of interest and reducing the noise coming from other frequencies. In communication systems, Band Pass filters are used for demodulation, where they extract the baseband signal from the modulated carrier wave.

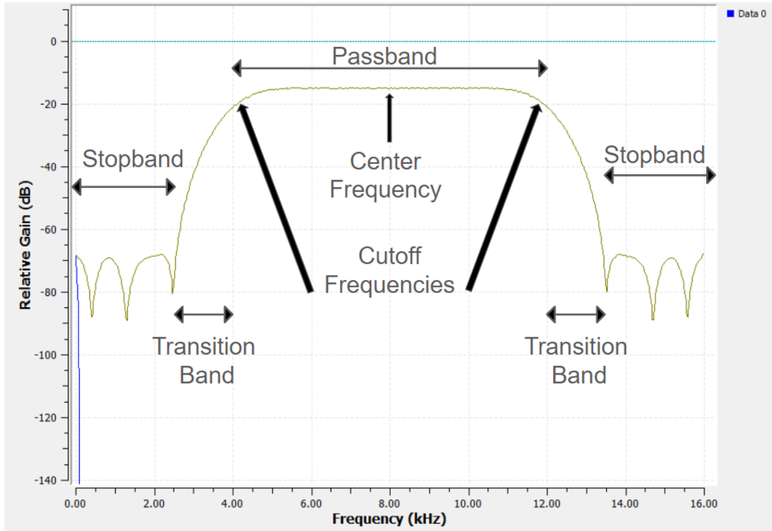


Figure 6: A Band Pass filter with labeled terms.

3.2 Low Pass Filters

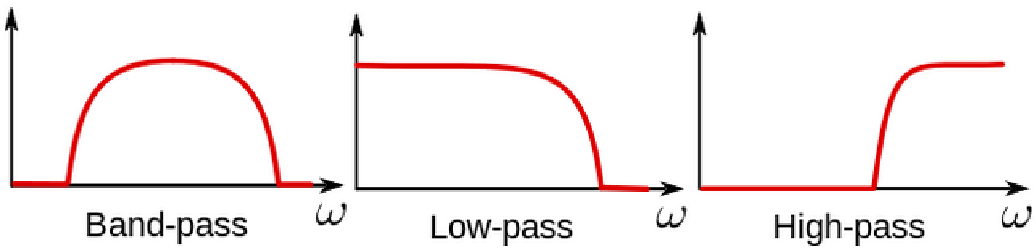
As the name implies, Low Pass filters pass lower frequency components of a signal while attenuating higher frequencies. Low Pass filters play a crucial role in signal processing by removing high-frequency components, reducing noise and interference, improving signal quality, and facilitating accurate signal analysis and interpretation. In digital communication applications, Low Pass filters are commonly used for baseband signal processing, channel equalization, and modulation and demodulation.

3.3 High Pass Filters

High Pass filters pass higher frequency components of a signal while attenuating lower frequencies. High Pass filters are used in signal preprocessing stages to remove low-frequency noise and interference from communication signals. This helps in improving the signal-to-noise ratio (SNR) and enhancing the overall quality of transmitted and received signals. They can also be employed for transient detection in radio astronomy, where they highlight rapid changes or events in signal intensity that occur at higher frequencies compared to the background noise and steady signals.

4 FM Transmitters (Frequency Modulation)

Frequency modulation (FM) is a popular way to broadcast audio signals over relatively short distances. FM radio begins with the generation of a high-frequency carrier wave.



This carrier wave typically falls around 88 to 108 MHz for FM radio broadcasts. The audio signal, which carries the sound information to be transmitted, is then combined with the carrier wave to modulate the frequency of the carrier wave. In FM modulation, the frequency of the carrier wave varies in response to the amplitude of the audio signal. The amount by which the carrier frequency varies or deviates from its center frequency (the carrier's unmodulated frequency) is known as the frequency deviation. This deviation is directly proportional to the amplitude of the audio signal.

The modulated FM signal is transmitted through antennas and propagates outward. FM radio receivers, such as we are using today, receive these signals and uses an analog to digital converter (ADC) to represent the signal in digital format. The receiver demodulates the FM signal, extracting the audio signal while filtering out the carrier wave and any noise or interference.

FM radio offers several advantages over other modulation techniques:

- FM provides better audio fidelity compared to AM (Amplitude Modulation) radio, with less susceptibility to noise and interference.
- FM radio can cover a wider frequency range, allowing for more channels and higher bandwidth for broadcasting music and speech.
- FM can transmit stereo audio signals, enabling the broadcast of stereo music to listeners.

Let's denote the carrier wave as $C(t)$, which has a frequency f_c and angular frequency $\omega_c = 2\pi f_c$. The equation of the carrier wave can be written as:

$$C(t) = A_c \cdot cos(\omega_c t + \phi_c) \tag{6}$$

Here A_c is the amplitude of the carrier wave, ω_c is the angular frequency of the carrier wave, ϕ_c is the phase of the carrier wave, and t is time.

We'll denote the modulating signal (often called the baseband signal) as $m(t)$, which represents the audio signal to be transmitted. Similar to the carrier wave equation, we can write the equation of the modulating signal as:

$$m(t) = A_m \cdot cos(\omega_m t + \phi_m) \tag{7}$$

Here A_m is the amplitude of the modulating signal, ω_m is the angular frequency of the modulating signal, ϕ_m is the phase of the modulating signal, and t is time.

Frequency modulation involves varying the frequency of the carrier wave based on the instantaneous amplitude of the modulating signal. The frequency modulated signal $FM(t)$ can be written as:

$$FM(t) = A_c \cdot cos(\omega_c t + 2\pi k_f \int_0^t m(\tau)d\tau + \phi_c) \tag{8}$$

Here k_f is called the modulation index, representing how much the frequency of the carrier wave changes per unit amplitude change of the modulating signal. The τ term is an arbitrary dummy variable that makes the integration easier.

The frequency deviation Δf is the maximum frequency deviation from the carrier frequency f_c, and is given by $\Delta f = k_t \cdot A_m$.

5 FM Receivers (Frequency Demodulation)

Frequency demodulation is the process of extracting the modulating signal $m(t)$ from the transmitted signal $FM(t)$. The received signal which we'll call $r(t)$, differs from the transmitted signal only in the amplitude and the addition of noise from transmission and propagation, can be written as:

$$r(t) = A_r \cdot cos(\omega_c t + 2\pi k_f \int_0^t m(\tau)d\tau) + n(t) \tag{9}$$

Here A_r is the received signal amplitude, ω_c is the angular frequency of the carrier wave, $\theta(t)$ is the phase angle introduced by the modulating signal $m(t)$ as well any phase shifts due to the channel or other factors, and $n(t)$ is the noise present in the received signal. For the sake of simplicity let's assume that no noise was introduced into the signal during transmission and propagation, so $n(t) = 0$ and can be ignored.

The phase $\theta(t)$ of the FM signal is directly proportional to the integral of the modulating signal, given by:

$$\theta(t) = \omega_c t + 2\pi k_f \int_0^t m(\tau)d\tau \tag{10}$$

The instantaneous phase change can be obtained by differentiating the phase with respect to time:

$$\Delta\theta(t) = \frac{d\theta(t)}{dt} = \omega_c + 2\pi k_f m(t) \tag{11}$$

Finally, the original modulating signal $m(t)$ can be obtained using a demodulator that measures the phase change over small intervals:

$$m(t) = \frac{1}{2\pi k_f}\Delta\theta(t) \tag{12}$$