📘 Module 1 – Permutations, Combinations & Equal Probability

🎯 Learning Objectives

- Understand and calculate **probabilities in equally likely scenarios**
- Distinguish between **permutations** and **combinations**
- Apply **counting techniques** to compute event probabilities
- Use **“n choose k”** logic in real-world problems

🎲 Equally Likely Outcomes

If all **N** outcomes in a sample space **S** are equally likely:

P(A)=Number of outcomes in ANP(A) = \frac{\text{Number of outcomes in } A}{N}P(A)=NNumber of outcomes in A

🎲 Example 1: Rolling Two Six-Sided Dice

Sample space: 36 ordered pairs (i, j), where each i and j ∈ {1, 2, ..., 6}

Probability Examples:

- **P(First roll is a 1)**:
  Outcomes: (1,1), (1,2), … (1,6) → 6 outcomes
  P=636=16P = \frac{6}{36} = \frac{1}{6}P=366=61
- **P(Sum = 8)**:
  Outcomes: (2,6), (3,5), (4,4), (5,3), (6,2) → 5 outcomes
  P=536P = \frac{5}{36}P=365
- **P(Second roll is 2 more than first)**:
  Outcomes: (1,3), (2,4), (3,5), (4,6) → 4 outcomes
  P=436=19P = \frac{4}{36} = \frac{1}{9}P=364=91

🔢 Permutations

Definition: An ordered arrangement of **k** elements selected from a group of **n**.

P(n,k)=n!(n−k)!P(n, k) = \frac{n!}{(n - k)!}P(n,k)=(n−k)!n!

🔁 **Example:**

- Select a **President, VP, and Treasurer** from 60 members:

P(60,3)=60×59×58=60!57!=205,320P(60, 3) = 60 \times 59 \times 58 = \frac{60!}{57!} = 205,320P(60,3)=60×59×58=57!60!=205,320

- **Factorials**:
  - n!=n×(n−1)×⋯×1n! = n \times (n-1) \times \dots \times 1n!=n×(n−1)×⋯×1
  - 0!=10! = 10!=1 (by definition)

🔢 Combinations

Definition: An unordered group of **k** elements selected from **n** distinct elements.

C(n,k)=(nk)=n!k!(n−k)!C(n, k) = \binom{n}{k} = \frac{n!}{k!(n-k)!}C(n,k)=(kn)=k!(n−k)!n!

🔁 **Example:**

- Pick a **team of 3** from 60 (order does **not** matter):

C(60,3)=60!3!⋅57!=60 choose 3C(60, 3) = \frac{60!}{3! \cdot 57!} = 60 \text{ choose } 3C(60,3)=3!⋅57!60!=60 choose 3

> 🔁 Note: (nk)=(nn−k)\binom{n}{k} = \binom{n}{n-k}(kn)=(n−kn)

🧮 Example: Gender-Based Committee

- 60 total people (35 women, 25 men)

- Form a committee of 11
- Sample space: (6011)\binom{60}{11}(1160)

Q: What’s the probability of at least 5 women and 5 men?

Two valid cases:

- **5 men, 6 women**:
  (255) · (356)\binom{25}{5} \cdot \binom{35}{6}(525) · (635)
- **6 men, 5 women**:
  (256) · (355)\binom{25}{6} \cdot \binom{35}{5}(625) · (535)

P=(255)(356)+(256)(355)(6011)P = \frac{\binom{25}{5}\binom{35}{6} + \binom{25}{6}\binom{35}{5}}{\binom{60}{11}}P=(1160)(525)(635)+(625)(535)

🚌 Example: Buses with Defects

- 20 buses, 8 with visible cracks
- Sample size = 5 → (205)\binom{20}{5}(520)

Q1: Probability that exactly 4 have cracks?

- Choose 4 from the 8 cracked: (84)\binom{8}{4}(48)
- Choose 1 from the 12 non-cracked: (121)\binom{12}{1}(112)

P=(84) · (121)(205)P = \frac{\binom{8}{4} \cdot \binom{12}{1}}{\binom{20}{5}}P=(520)(48) · (112)

Q2: Probability that at least 4 have cracks?

- Exactly 4 cracks: (84) · (121)\binom{8}{4} \cdot \binom{12}{1}(48) · (112)
- Exactly 5 cracks: (85) · (120)\binom{8}{5} \cdot \binom{12}{0}(58) · (012)

P=(84)⋅(121)+(85)(205)P = \frac{\binom{8}{4} \cdot \binom{12}{1} + \binom{8}{5}}{\binom{20}{5}}P=(520)(48)⋅(112)+(58)

📌 Key Concepts & Formulas

Concept	Formula
Equally likely outcomes	P(A)=#A#SP(A) = \frac{\#A}{\#S}P(A)=#S#A
Permutations (order matters)	P(n,k)=n!(n−k)!P(n, k) = \frac{n!}{(n-k)!}P(n,k)=(n−k)!n!
Combinations (order doesn't matter)	C(n,k)=(nk)=n!k!(n−k)!C(n, k) = \binom{n}{k} = \frac{n!}{k!(n-k)!}C(n,k)=(kn)=k!(n−k)!n!
Relation: (nk)=(nn−k)\binom{n}{k} = \binom{n}{n-k}(kn)=(n−kn)	