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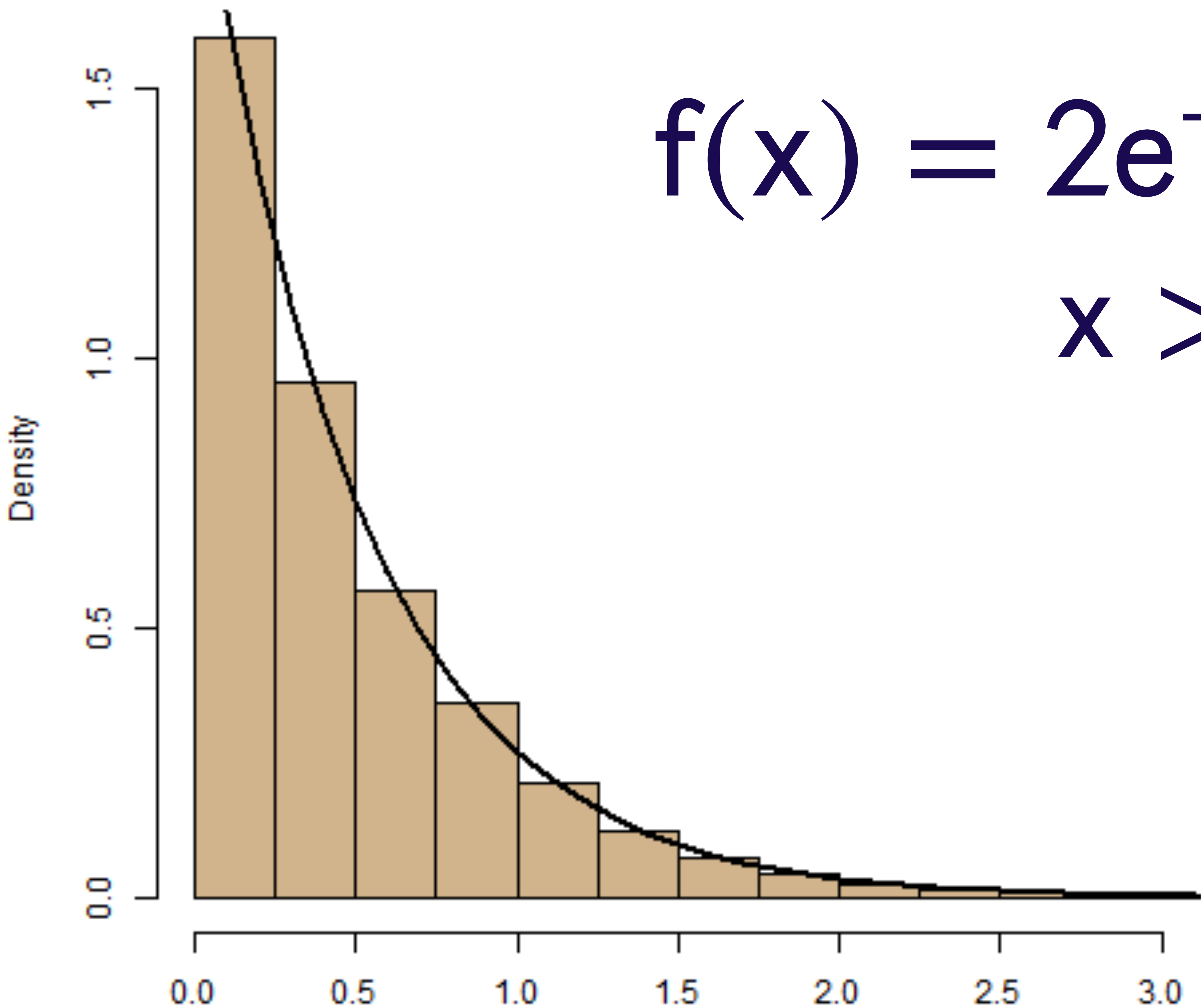

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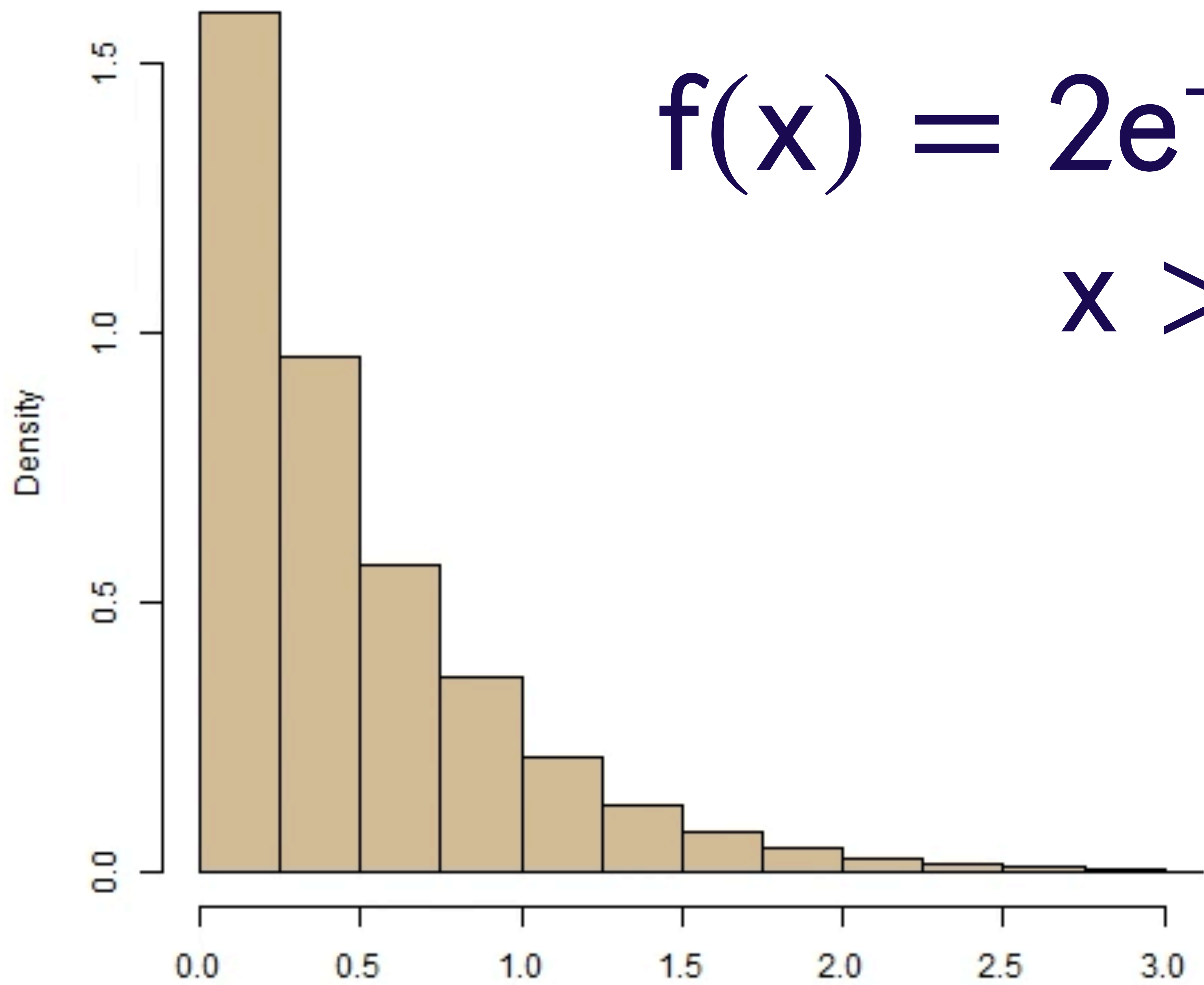

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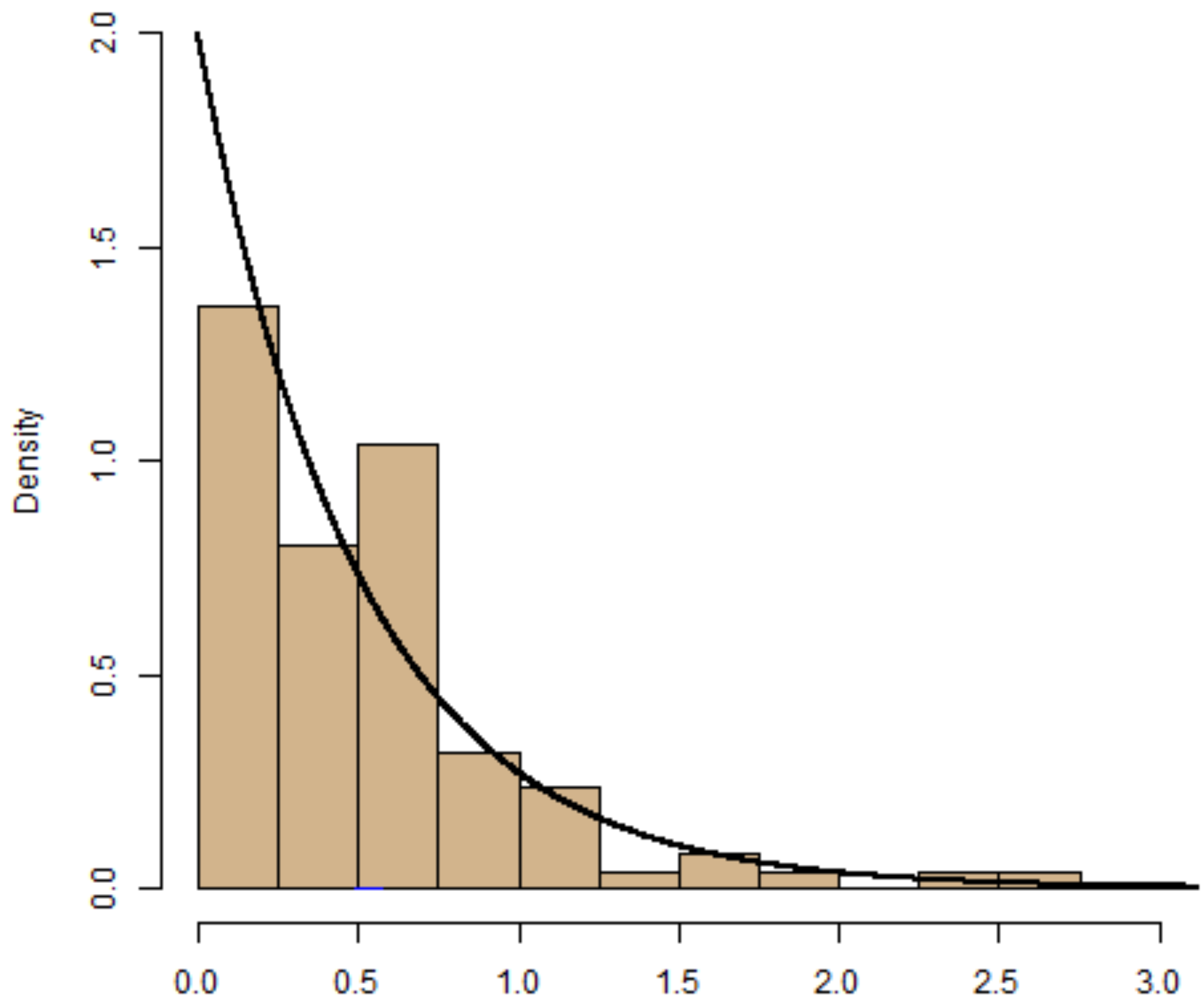

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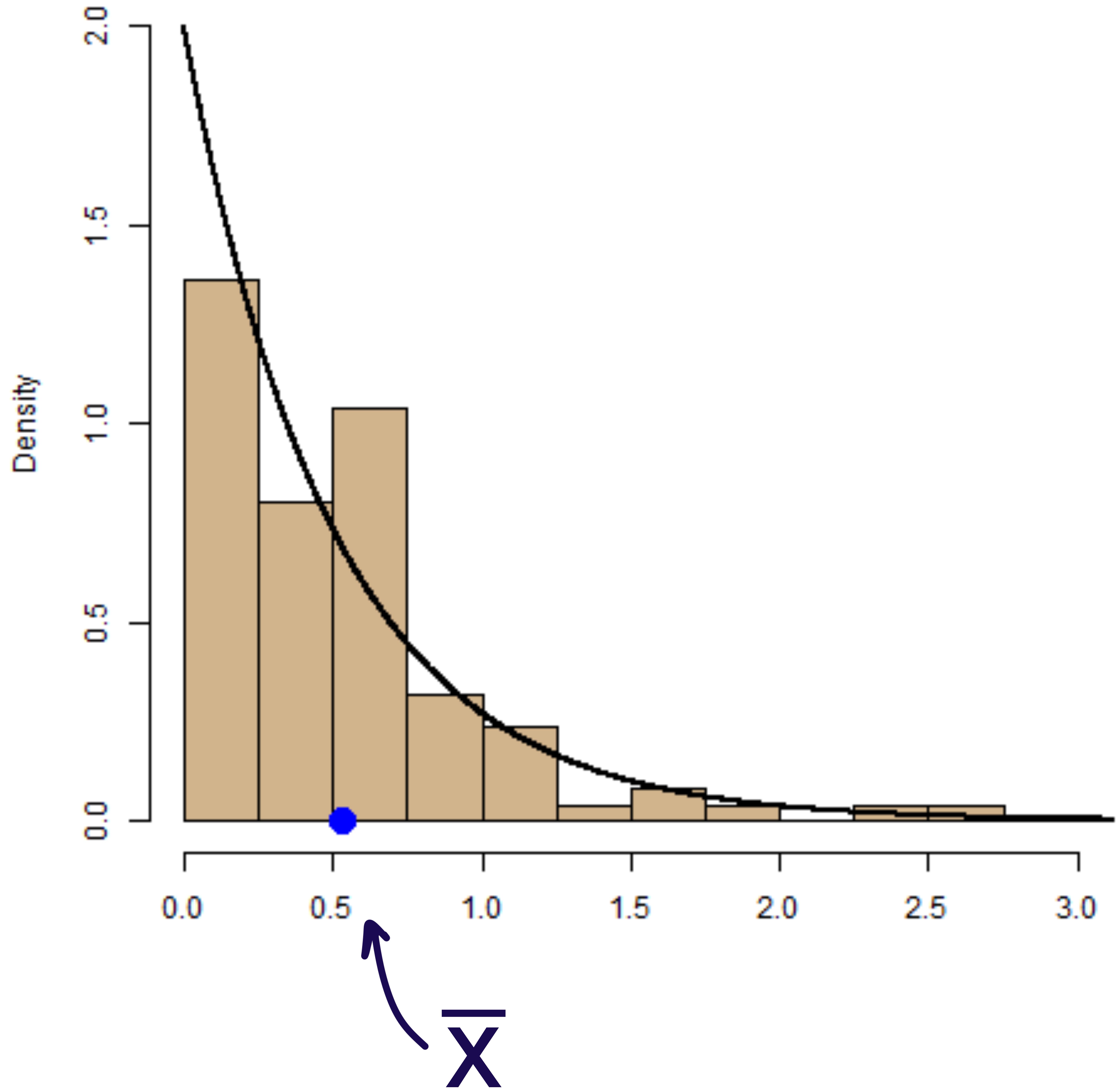

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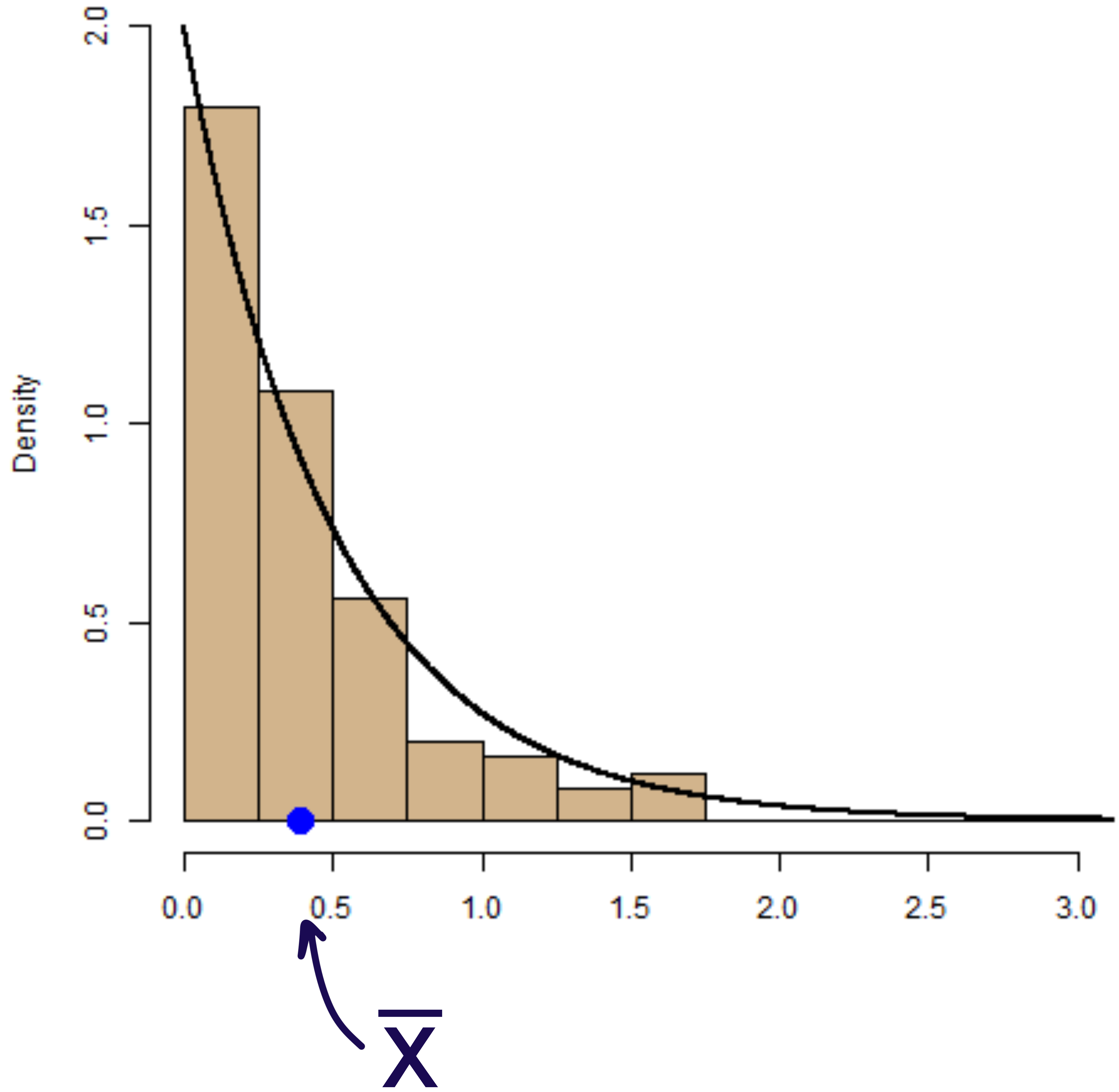

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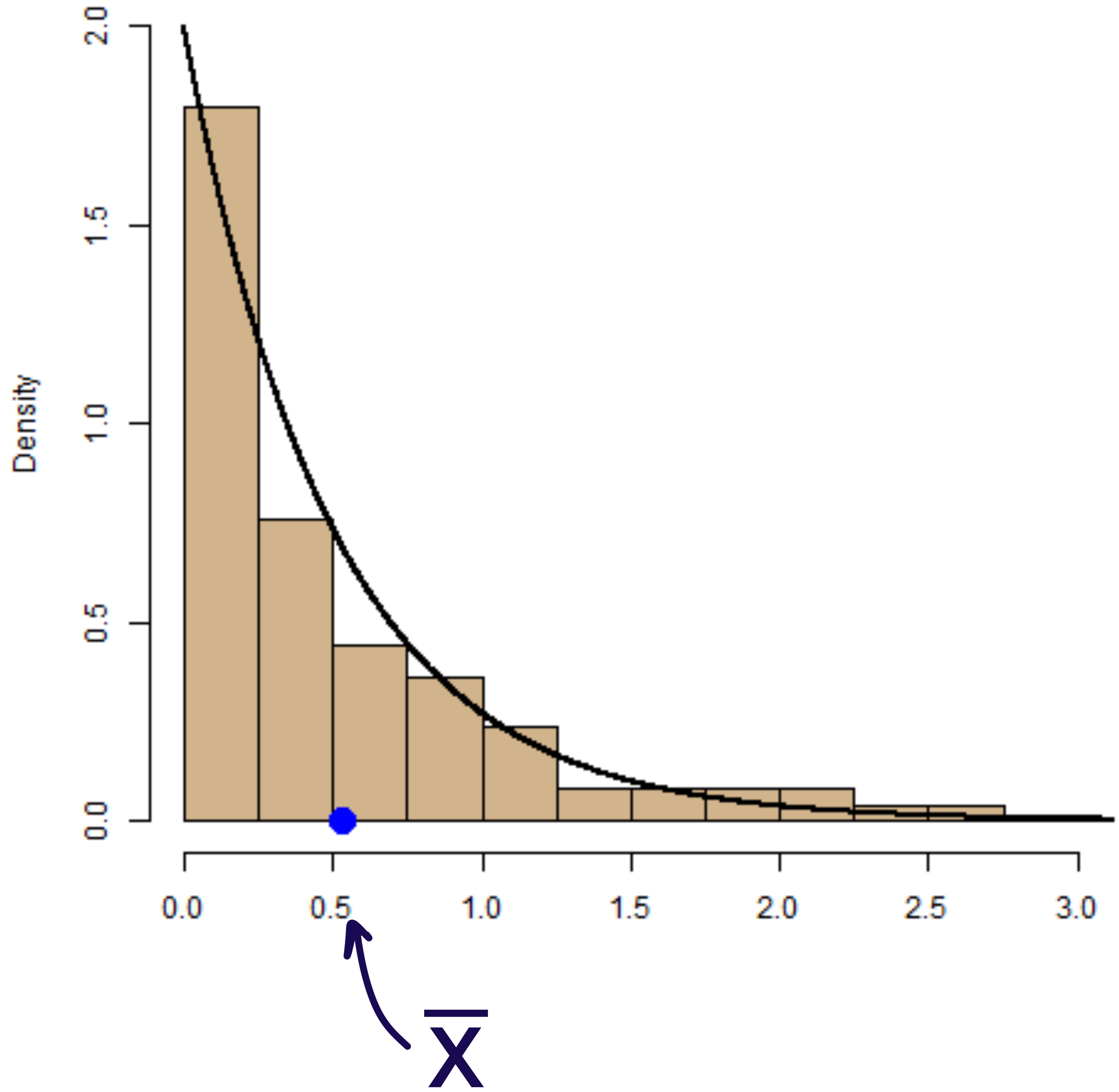

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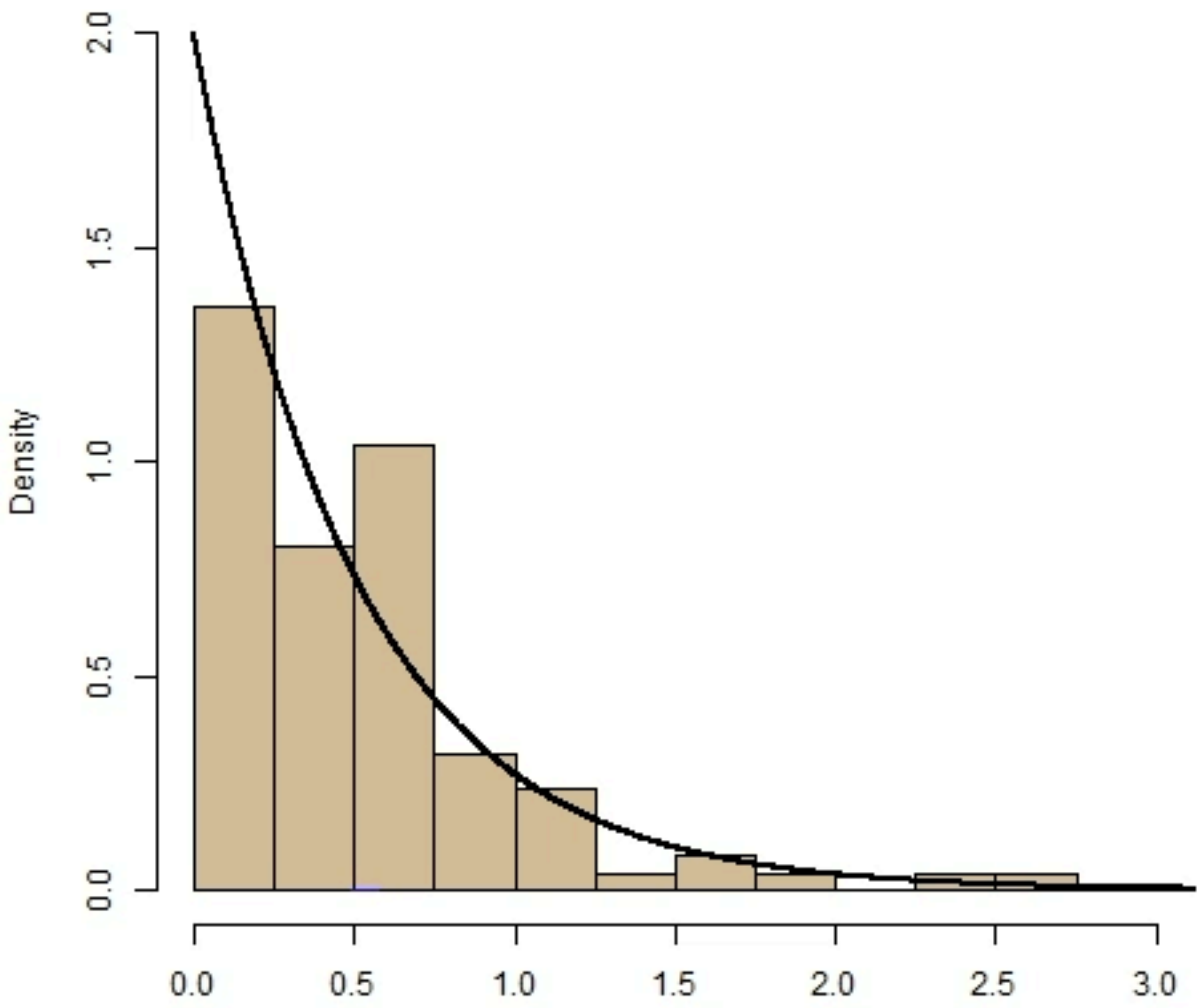

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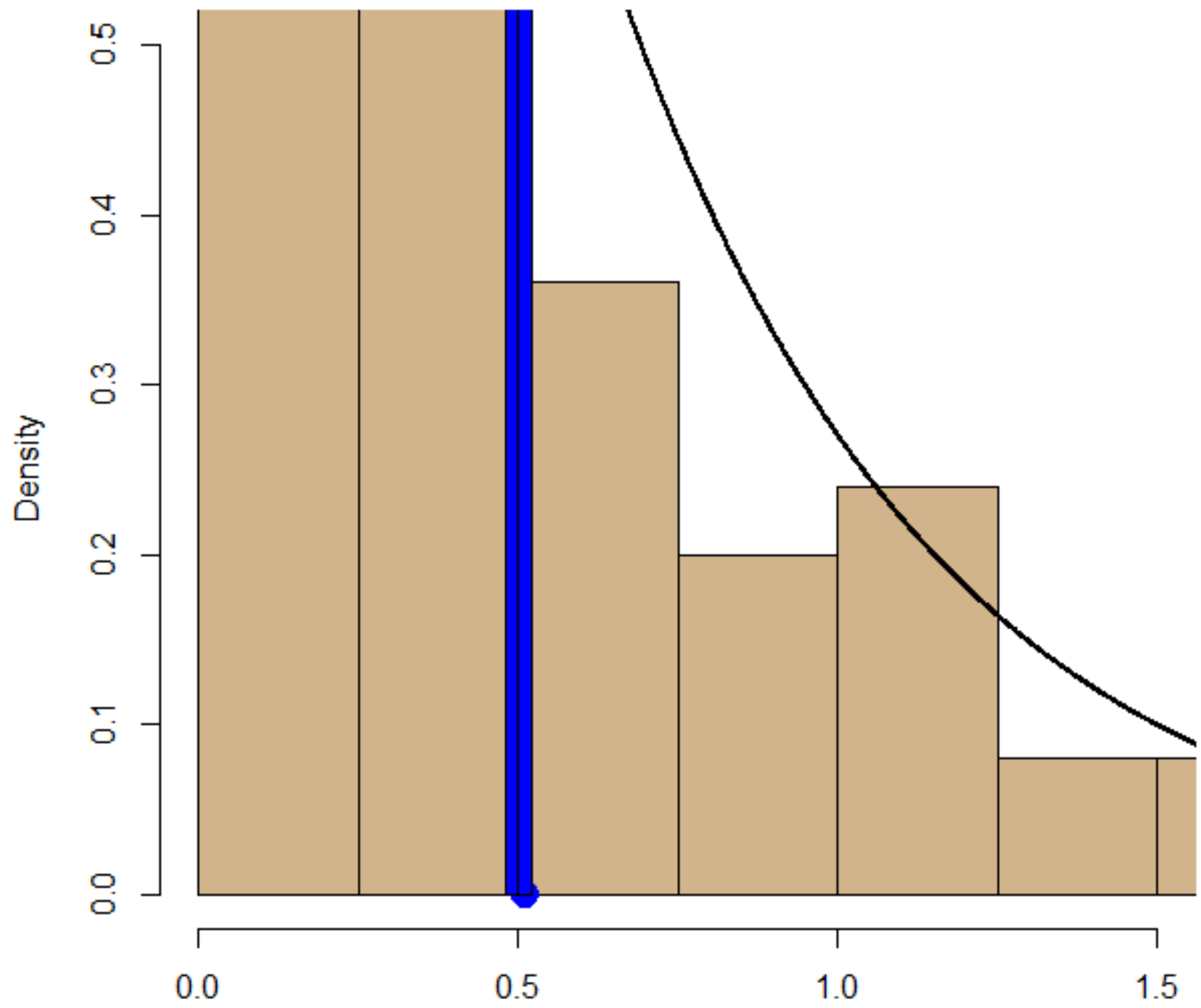

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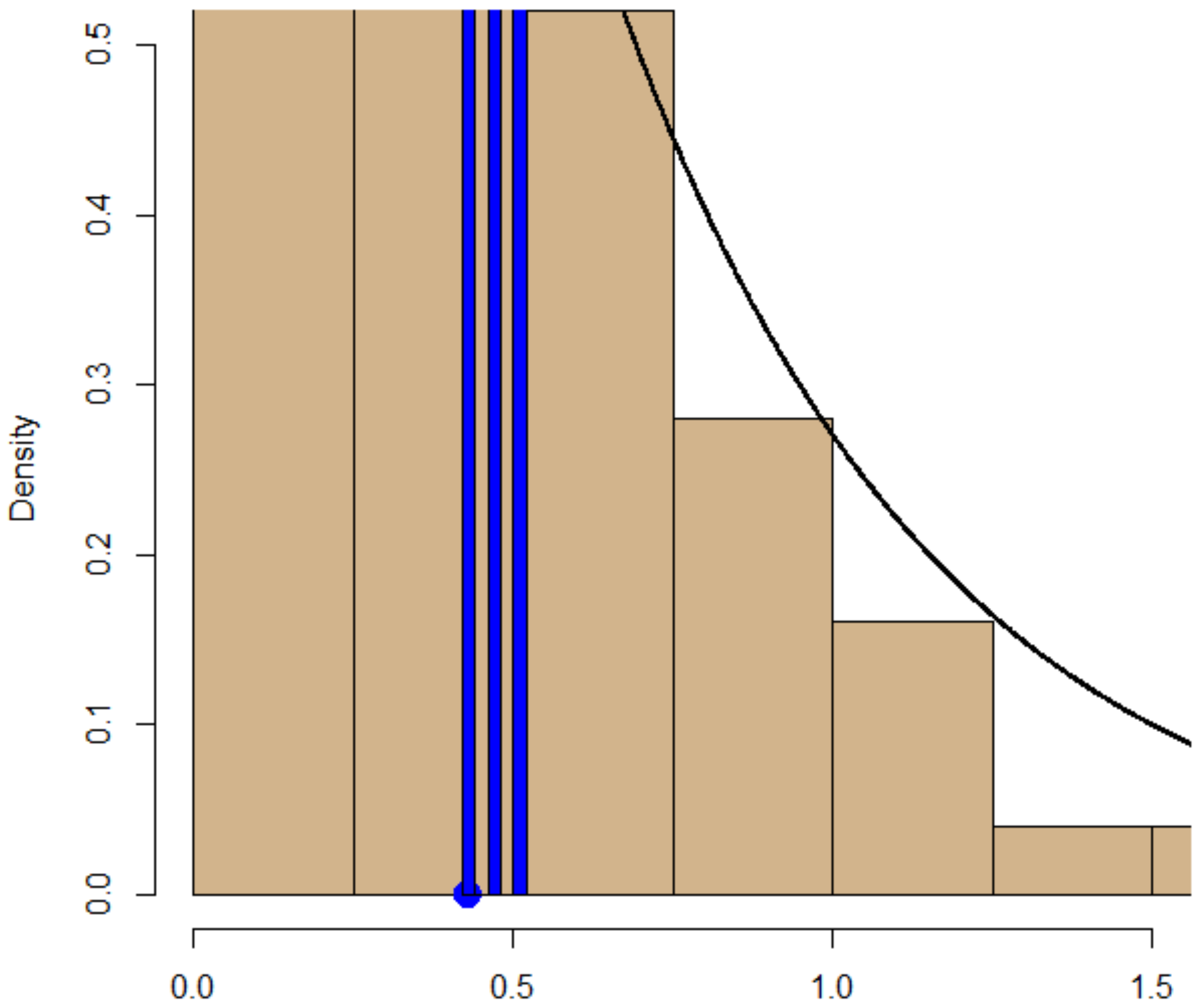

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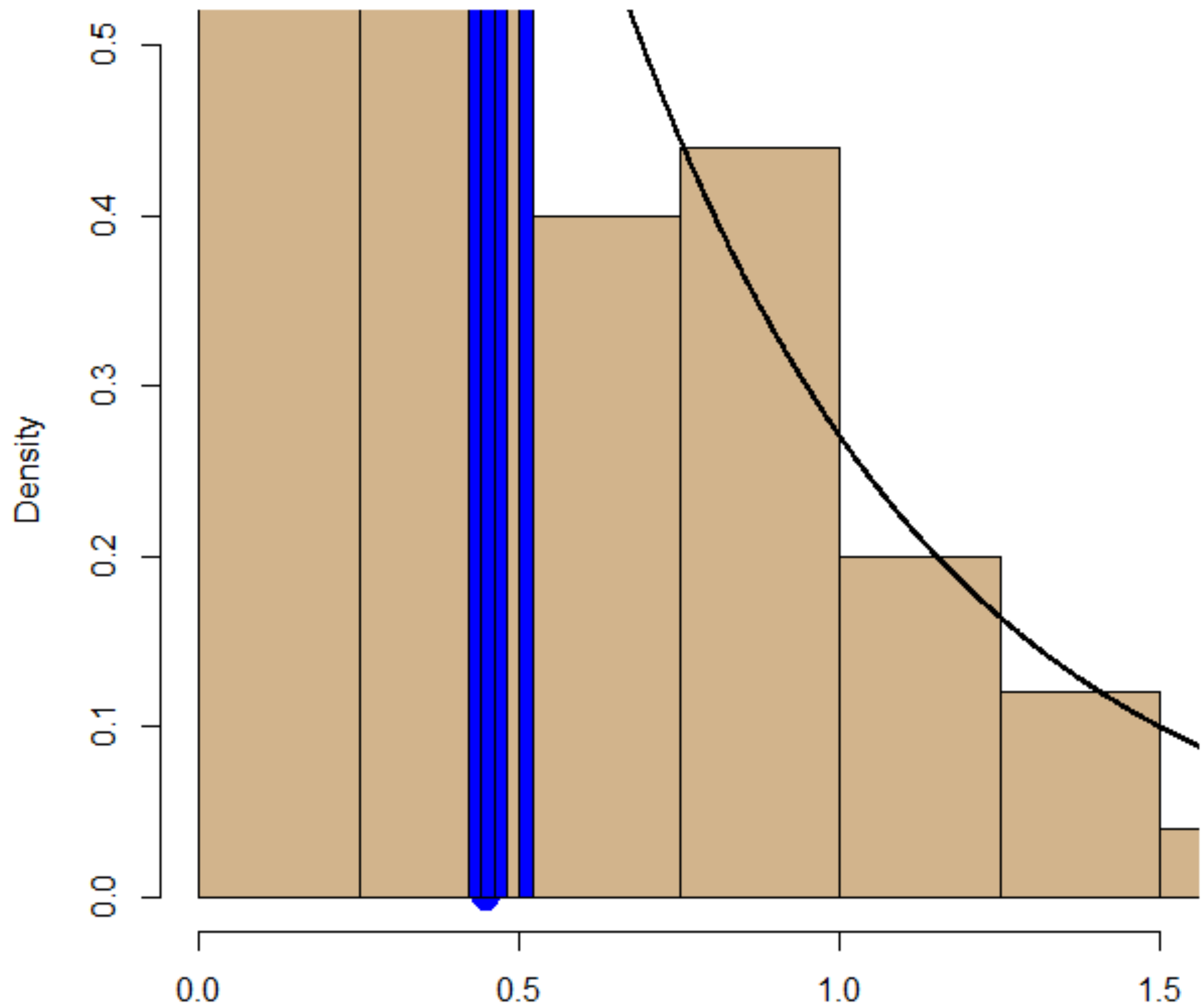

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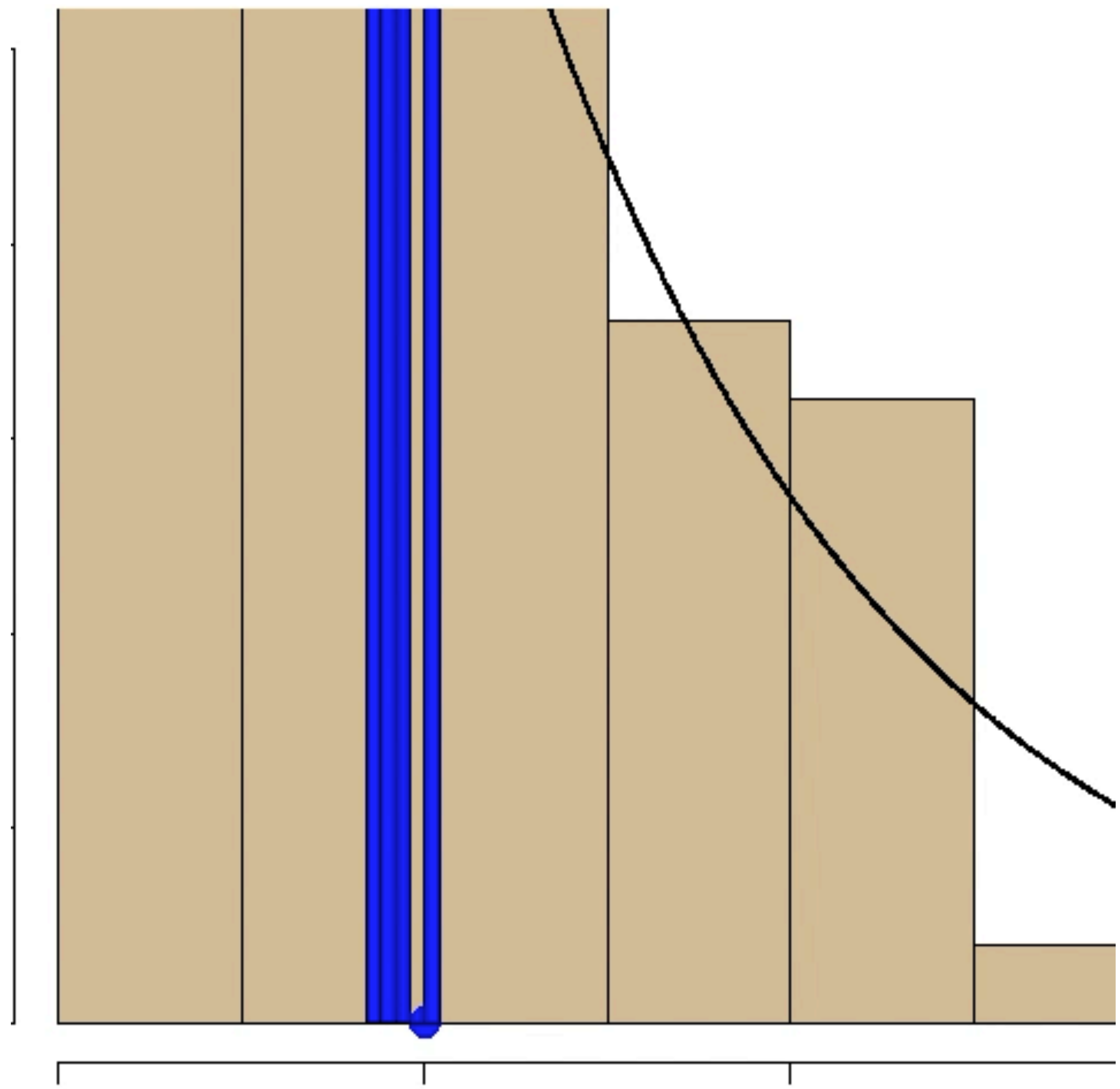

Convergence in Distribution

Let $X_1, X_2, X_3, \ldots$ be a sequence of random variables and let X be another random variable.

If $$\lim_{n\to\infty} P(X_n \le x) = P(X \le x)$$

i.e. $$\lim_{n\to\infty} F_n(x) = F(x)$$

Then the sequence $\{X_n\}$ **converges in distribution** to X.

Notation: $X_n \xrightarrow{d} X$

The Central Limit Theorem

Let $X_1, X_2, X_3, \ldots$ be a sequence of random variables from any distribution with mean μ and variance $\sigma^2 < \infty$.

Let

$$\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i \qquad (\overline{X} = \overline{X}_n)$$

Then

$$\frac{\overline{X} - \mu}{\sigma/\sqrt{n}} \xrightarrow{d} N(0, 1)$$

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We write

$$\overline{X} \overset{\text{asymp}}{\sim} N\left(\mu, \frac{\sigma^2}{n}\right)$$

In general, $X_n \sim N(a_n, b_n)$

means $$\frac{X_n - a_n}{\sqrt{b_n}} \xrightarrow{d} N(0, 1)$$

Note that $\dfrac{\overline{X} - \mu}{\sigma/\sqrt{n}} \xrightarrow{d} N(0, 1)$

$$\Rightarrow \quad \frac{\frac{1}{n}\sum_{i=1}^{n} X_i - \mu}{\sigma/\sqrt{n}} \xrightarrow{d} N(0, 1)$$

$$\Rightarrow \quad \frac{\sum_{i=1}^{n} X_i - n\mu}{\sigma\sqrt{n}} \xrightarrow{d} N(0, 1)$$

$$\Rightarrow \quad \sum_{i=1}^{n} X_i \overset{asymp}{\sim} N(n\mu, \sigma^2 n)$$

The Point?

Let $X_1, X_2, X_3, \ldots$ be a sequence of random variables from any distribution with mean μ and variance $\sigma^2 < \infty$.

An approximate, large sample, test of

$$H_0 : \mu \leq \mu_0 \qquad H_1 : \mu > \mu_0$$

is to reject H_0, in favor of H_1 if

$$\overline{X} > \mu_0 + z_\alpha \frac{\sigma}{\sqrt{n}}$$