Suppose that $X_1, X_2, \ldots, X_n$ is a random sample from the exponential distribution with rate $\lambda > 0$.

Construct a 95% confidence interval for λ.

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Step One: Choose a statistic.

Choose one whose distribution you know and is one that depends on the unknown parameter.

$$\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$$

- $$\sum_{i=1}^{n} X_i \sim \Gamma(n, \lambda)$$
- $$X \sim \Gamma(\alpha, \beta) \quad c > 0$$
  $$\Rightarrow \quad cX \sim \Gamma(\alpha, \beta/c)$$
- **So,**
  $$\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i \sim \Gamma(n, n\lambda)$$

Step Two: Find a function of the statistic and the parameter you are trying to estimate whose distribution is known and parameter free.

$$\overline{X} \sim \Gamma(n, n\lambda)$$

$$\Downarrow$$

$$\lambda\overline{X} \sim \Gamma(n, n)$$

$$\lambda \overline{X} \sim \Gamma(n, n)$$

Question: Can we find critical values for the $\Gamma(n, n)$ distribution?

Answer: We can use R.

qgamma(0.975,n,n) and qgamma(0.025,n,n)

Historically, people used tables.

Transform gamma into chi-squared.

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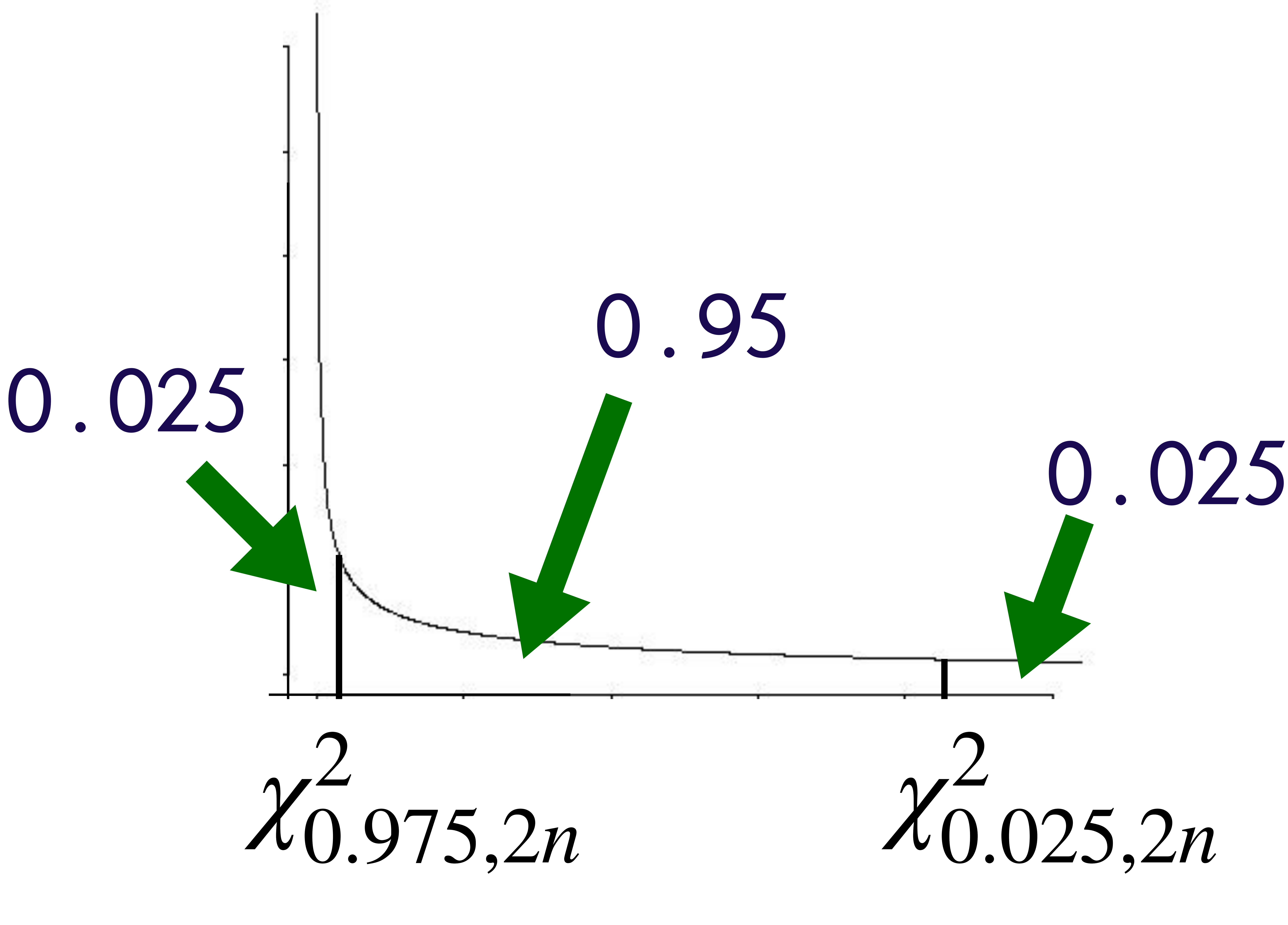
$$\overline{X} \sim \Gamma(n, n\lambda) \quad \Rightarrow \quad \lambda\overline{X} \sim \Gamma(n, n)$$

$$\Rightarrow \quad 2n\lambda\overline{X} \sim \Gamma\left(n, \frac{1}{2}\right)$$

$$= \Gamma\left(\frac{2n}{2}, \frac{1}{2}\right) = \chi^2(2n)$$

Step Three: **Find appropriate critical values.**



Step Four: Put your statistic from Step Two between the critical values and solve for the unknown parameter "in the middle".

$$2n\lambda\overline{X} \sim \chi^2(2n)$$

$$\chi^2_{0.975,2n} < 2n\lambda\overline{X} < \chi^2_{0.025,2n}$$

$$\frac{\chi^2_{0.975,2n}}{2n\overline{X}} < \lambda < \frac{\chi^2_{0.025,2n}}{2n\overline{X}}$$