Notes: Introduction to Random Variables and Discrete Distributions

- A random variable represents a numerical outcome of a probabilistic experiment.
- Example: In a fair coin flip, define $X = 1$ if heads, $X = 0$ if tails. X is a random variable.
- Convention: Use capital letters (e.g., X) for random variables.
- The probability mass function (PMF), denoted $f(x)$, gives $P(X = x)$.
  - Example: $f(1) = 1/2$, $f(0) = 1/2$, $f(7) = 0$
  - The PMF shows how probabilities are distributed across possible values.

Bernoulli Distribution:

- Models a binary outcome (success/failure) with success probability p.
- Notation: $X \sim \text{Bern}(p)$
- PMF: $f(x) = p^x * (1-p)^{(1-x)}$ for $x = 0$ or 1
- Can also use indicator functions to express PMFs.

Indicator Functions:

- Defined as $I_A(x) = 1$ if x is in A, 0 otherwise.
- Compact way to define PMFs and conditions.
- Alternative notations exist but the concept is consistent.

Geometric Distribution:

- Models number of trials until the first success (version 1):
  - Values: 1, 2, 3, ...
  - PMF: $f(x) = (1-p)^{(x-1)} * p$
- Alternate definition (version 0): Number of failures before the first success
  - Values: 0, 1, 2, ...

  - PMF: $f(x) = (1-p)^x * p$

- Notation: $X \sim \text{Geom}(p)$, sometimes Geom_0 or Geom_1 to indicate version.

Key Takeaways:

- PMFs describe how probability is assigned to discrete random variables.

- Bernoulli and geometric distributions are foundational in probability theory.

- Indicator functions simplify notation and allow precise condition control in expressions.