Tests for the mean of a normal distribution.

Let $X_1, X_2, \ldots, X_n$ be a random sample from the normal distribution with mean μ and known variance σ^2.

Consider testing the simple versus simple hypotheses

$$H_0 : \mu = \mu_0 \qquad H_1 : \mu = \mu_1$$

where μ_0 and μ_1 are fixed and known.

$$H_0 : \mu = \mu_0$$
$$H_1 : \mu = \mu_1$$

Step One:

Choose an estimator for μ.

$$\hat{\mu} = \overline{X}$$

Step Two:

Give the "form" of the test.

Suppose that $\mu_0 < \mu_1$.

Reject H_0, in favor of H_1 if $\overline{X} > c$, where c is to be determined.

Developing a Test

$$H_0 : \mu = \mu_0$$
$$H_1 : \mu = \mu_1$$
$$\mu_0 < \mu_1$$

Step Three:

Find c.

$$\alpha = \mathrm{P}(\text{Type I Error})$$

$$= \mathrm{P}(\text{Reject } H_0 \text{ when true})$$

$$= \mathrm{P}(\overline{X} > c \text{ when } \mu = \mu_0)$$

$$= \mathrm{P}\left(\frac{\overline{X} - \mu_0}{\sigma/\sqrt{n}} > \frac{c - \mu_0}{\sigma/\sqrt{n}} \text{ when } \mu = \mu_0 \right)$$

Developing a Test

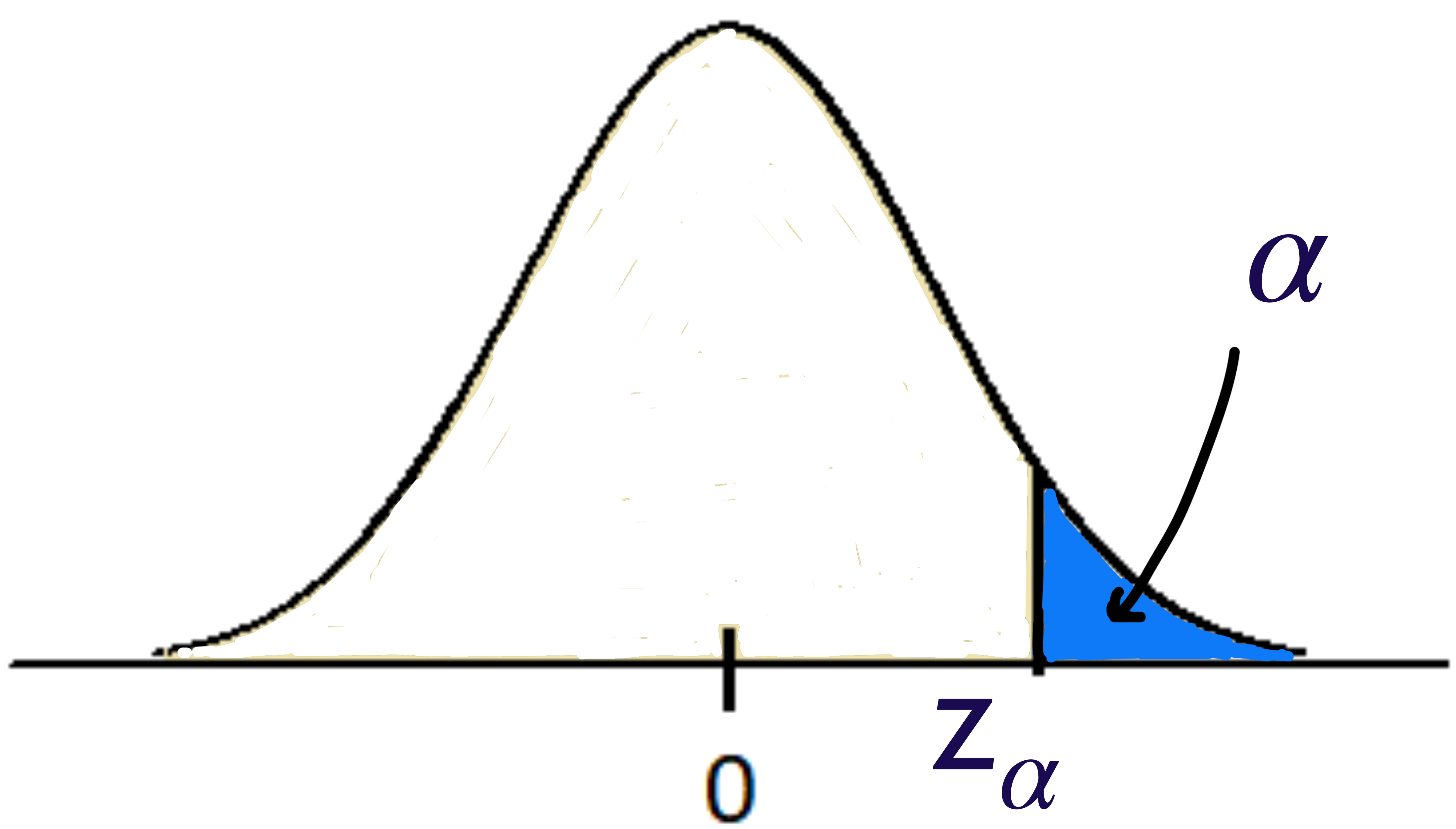
$$H_0 : \mu = \mu_0$$
$$H_1 : \mu = \mu_1$$
$$\mu_0 < \mu_1$$

Step Three:

Find c.

$$\alpha = P\left(Z > \frac{c - \mu_0}{\sigma/\sqrt{n}} \right)$$

where
$$Z \sim N(0,1)$$



Developing a Test

$$H_0 : \mu = \mu_0$$
$$H_1 : \mu = \mu_1$$
$$\mu_0 < \mu_1$$

Step Three:

$$\Rightarrow \quad \frac{c - \mu_0}{\sigma/\sqrt{n}} = z_\alpha$$

$$\Rightarrow \quad c = \mu_0 + z_\alpha \frac{\sigma}{\sqrt{n}}$$

$$H_0 : \mu = \mu_0$$
$$H_1 : \mu = \mu_1$$
$$\mu_0 < \mu_1$$

Developing a Test

Step Four:

Give a conclusion!

Reject H_0, in favor of H_1 if

$$\overline{X} > \mu_0 + z_\alpha \frac{\sigma}{\sqrt{n}}$$

$$H_0 : \mu = \mu_0$$
$$H_1 : \mu = \mu_1$$
$$\cancel{\mu_0 < \mu_1}$$
$$\mu_0 > \mu_1$$

Developing a Test

Step Four:

Give a conclusion!

Reject H_0, in favor of H_1 if

$$\overline{X} < \mu_0 + z_{1-\alpha}\frac{\sigma}{\sqrt{n}}$$

What ever happened to the Type II error? What is going to change if we look at composite hypotheses? What if we don’t know sigmasquared?

And what the heck is a P-value?

We’ll answer all these questions and more in the next module!