Suppose that $X_1, X_2, \ldots, X_n$ is a random sample from the exponential distribution with rate $\lambda > 0$.

We know that

$$\sum_{i=1}^{n} X_i \sim \Gamma(n, \lambda)$$

and therefore that

$$2\lambda \sum_{i=1}^{n} X_i \sim \Gamma\left(n, \frac{1}{2}\right) = \Gamma\left(\frac{2n}{2}, \frac{1}{2}\right) = \chi^2(2n)$$

Suppose that $X_1, X_2, \ldots, X_n$ is a random sample from the exponential distribution with rate $\lambda > 0$.

We also know that

$$\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i \sim \Gamma(n, n\lambda)$$

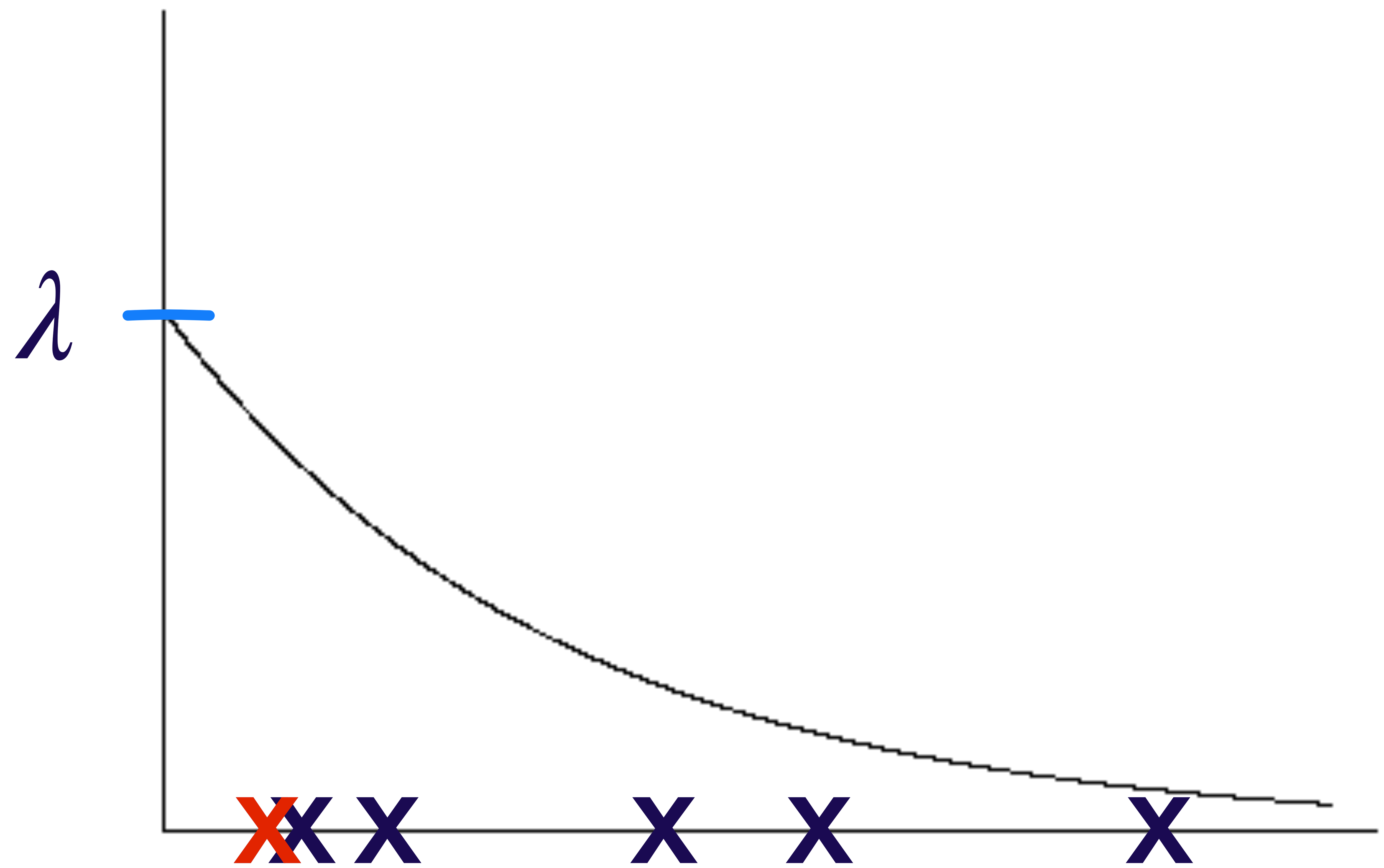
and therefore that

$$2n\lambda\overline{X} \sim \Gamma\left(n, \frac{1}{2}\right) = \Gamma\left(\frac{2n}{2}, \frac{1}{2}\right) = \chi^2(2n)$$

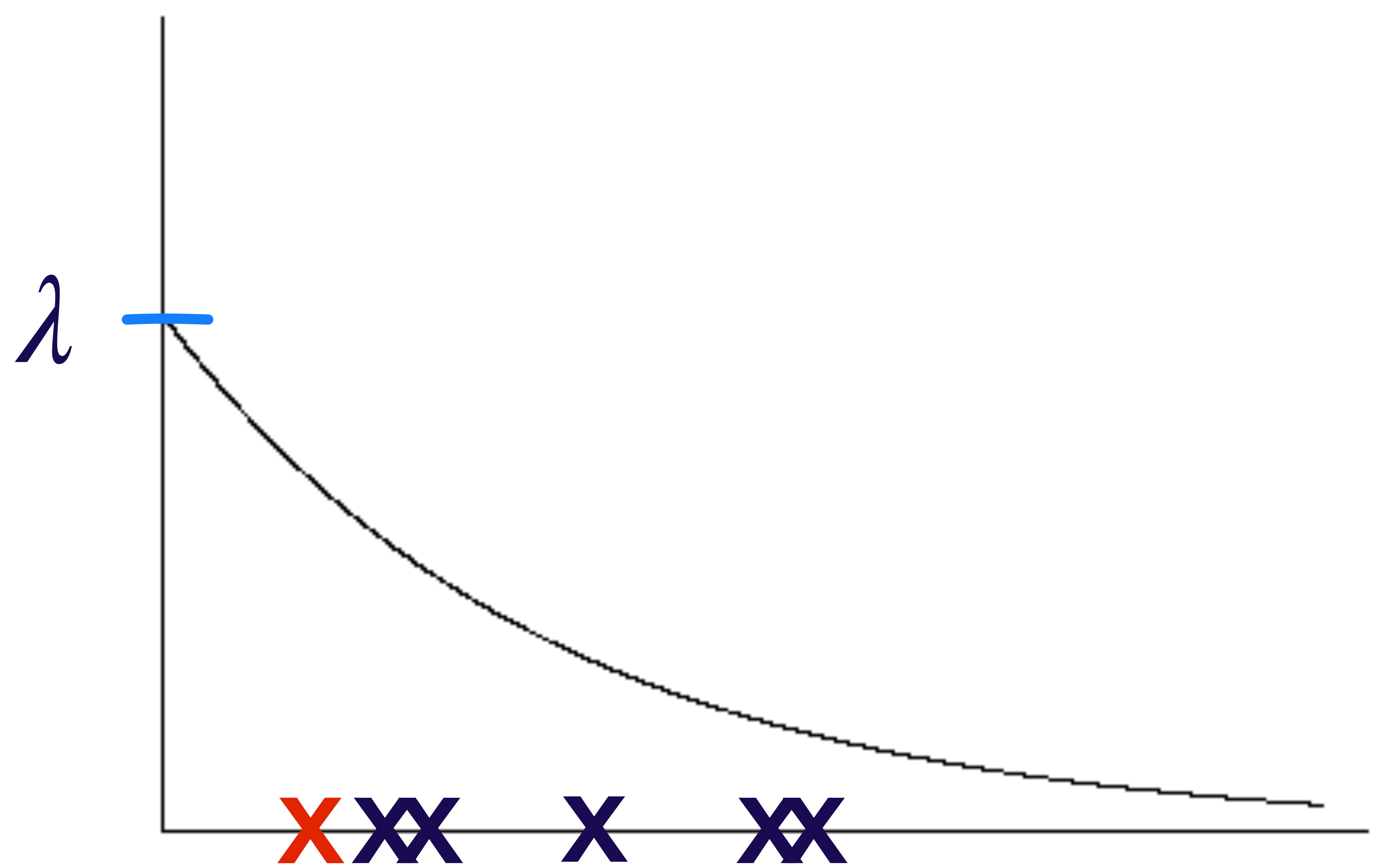
Suppose that $X_1, X_2, \ldots, X_n$ is a random sample from the exponential distribution with rate $\lambda > 0$.

What is the distribution of the minimum?

Suppose that $X_1, X_2, \ldots, X_n$ is a random sample from the exponential distribution with rate $\lambda > 0$.



Suppose that $X_1, X_2, \ldots, X_n$ is a random sample from the exponential distribution with rate $\lambda>0$.



Let $Y_n = \min(X_1, X_2, \ldots, X_n)$.

The cdf for each X_i is

$$F(x) = P(X_i \le x) = 1 - e^{-\lambda x}$$

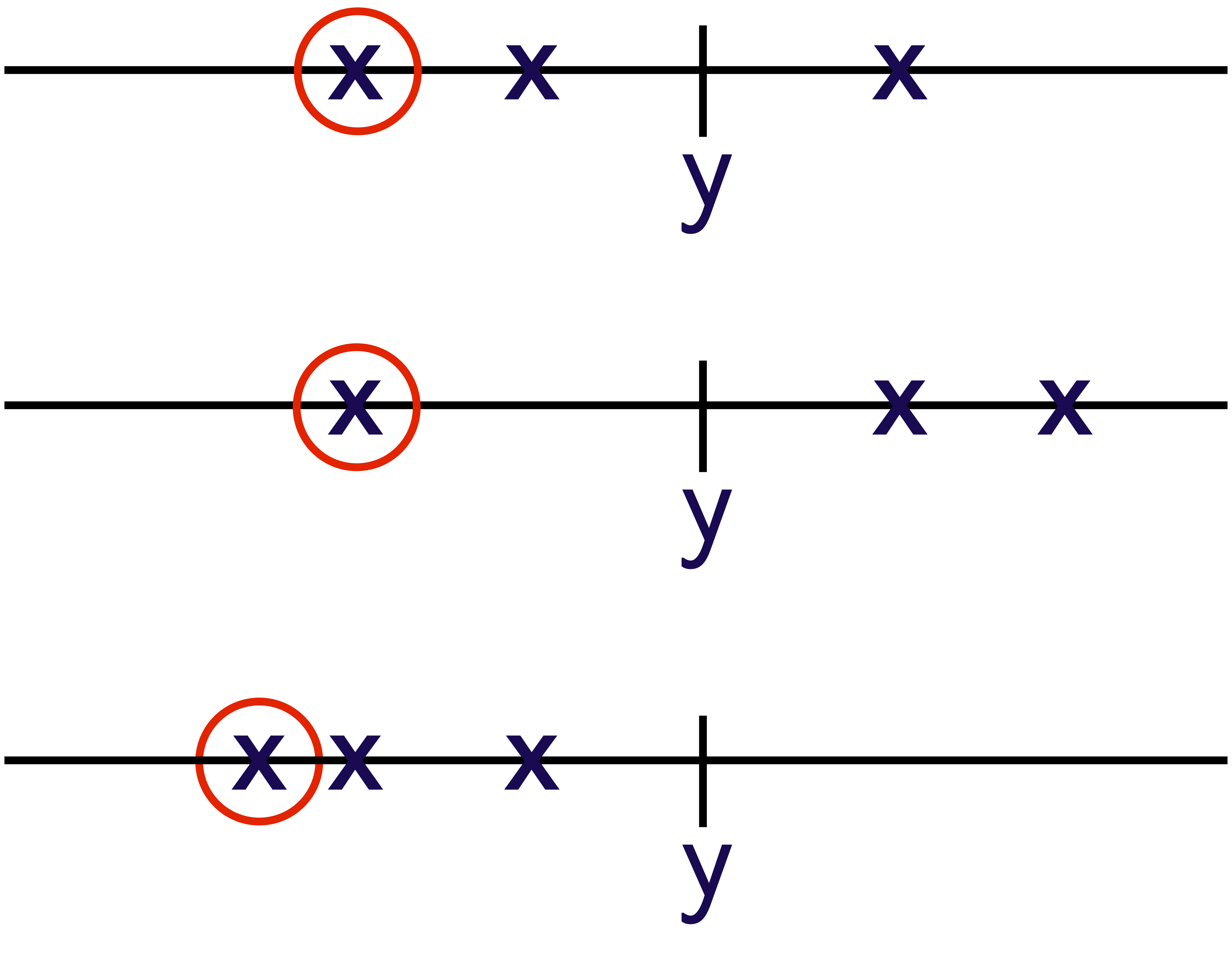
The cdf for Y_n is

$$F_{Y_n}(y) = P(Y_n \le y)$$

$$= P(\min(X_1, X_2, \ldots, X_n) \le y)$$

$$F_{Y_n}(y) = P(Y_n \leq y)$$

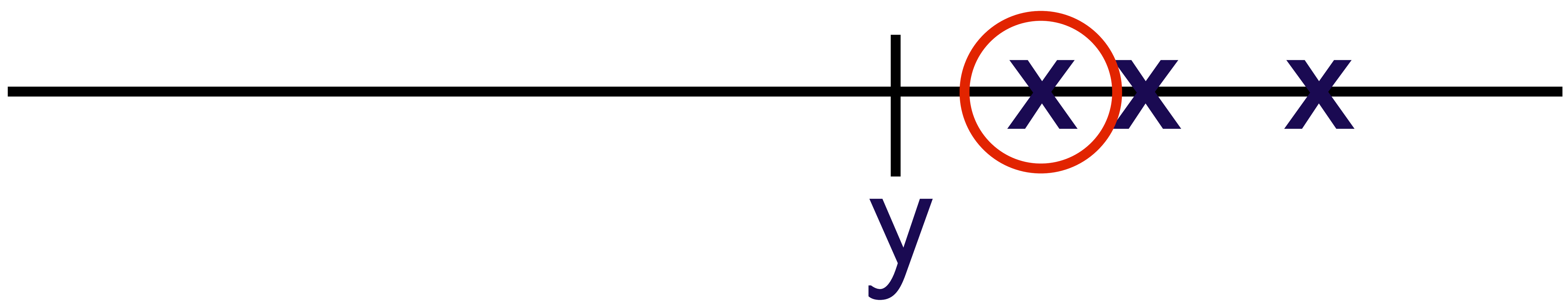
$$= P(\min(X_1, X_2, \ldots, X_n) \leq y)$$



$$F_{Y_n}(y) = P(Y_n \le y)$$

$$= P(\min(X_1, X_2, \ldots, X_n) \le y)$$

$$= 1 - P(\min(X_1, X_2, \ldots, X_n) > y)$$



$$= 1 - P(X_1 > y, X_2 > y, \ldots, X_n > y)$$

$$\overset{\text{indep}}{=} 1 - P(X_1 > y) \cdot P(X_2 > y) \cdots P(X_n > y)$$

$$\overset{\text{ident}}{=} 1 - [P(X_1 > y)]^n = 1 - [1 - F(y)]^n$$

$$
\begin{aligned}
F_{Y_n}(y) &= P(Y_n \leq y) \\
&= 1 - [1 - F(y)]^n \\
&= 1 - [1 - (1 - e^{-\lambda y})]^n \\
&= 1 - [e^{-\lambda y}]^n \\
&= 1 - e^{-n\lambda y}
\end{aligned}
$$

$$
f_{Y_n}(y) = \frac{d}{dy} F_{Y_n}(y) = n\lambda e^{-n\lambda y}
$$

The minimum of n iid exponential with rate λ is exponential with rate nλ !