

Example:

A random sample of 500 people in a certain **county** which is about to have a national election were asked whether they preferred “Candidate A” or “Candidate B”.

From this sample, 320 people responded that they preferred Candidate A.

A random sample of 400 people in a **second county** which is about to have a national election were asked whether they preferred “Candidate A” or “Candidate B”.

Example:

From this second county sample, 268 people responded that they preferred Candidate A.

$$\hat{p}_1 = \frac{320}{500} = 0.64$$

$$\hat{p}_2 = \frac{268}{400} = 0.67$$

Test

$$H_0 : p_1 = p_2 \quad H_1 : p_1 \neq p_2$$

Change to:

$$H_0 : p_1 - p_2 = 0$$

$$H_1 : p_1 - p_2 \neq 0$$

Estimate $p_1 - p_2$ with $\hat{p}_1 - \hat{p}_2$.

For large enough samples,

$$\hat{p}_1 \overset{\text{approx}}{\sim} N \left(p_1, \frac{p_1(1 - p_1)}{n_1} \right)$$

and

$$\hat{p}_2 \overset{\text{approx}}{\sim} N \left(p_2, \frac{p_2(1 - p_2)}{n_1} \right)$$

$$\hat{p}_1 - \hat{p}_2 \sim N(?, ?)$$

$$E[\hat{p}_1 - \hat{p}_2] = E[\hat{p}_1] - E[\hat{p}_2] = p_1 - p_2$$

$$\text{Var}[\hat{p}_1 - \hat{p}_2] \stackrel{\text{indep}}{=} \text{Var}[\hat{p}_1] + \text{Var}[\hat{p}_2]$$

$$= \frac{p_1(1 - p_1)}{n_1} + \frac{p_2(1 - p_2)}{n_2}$$

$$\frac{\hat{p}_1 - \hat{p}_2 - (p_1 - p_2)}{\sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}} \sim N(0, 1) \quad = 0 \text{ under } H_0$$

$$= ? \text{ under } H_0$$

Use estimators for p_1 and p_2 **assuming they are the same.**

- Call the common value p .
- Estimate by putting both groups together.

In the example with

$$\hat{p}_1 = \frac{320}{500} = 0.64 \qquad \hat{p}_2 = \frac{268}{400} = 0.67$$

we have

$$\hat{p} = \frac{320 + 268}{500 + 400} = \frac{588}{900} = \frac{49}{75}$$

$$\approx 0.6533$$

Use

approx

$$Z := \frac{\hat{p}_1 - \hat{p}_2 - (p_1 - p_2)}{\sqrt{\frac{\hat{p}(1 - \hat{p})}{n_1} + \frac{\hat{p}(1 - \hat{p})}{n_2}}} \sim N(0, 1)$$

$$= \frac{\hat{p}_1 - \hat{p}_2 - (p_1 - p_2)}{\sqrt{\hat{p}(1 - \hat{p}) \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}}$$

Two-tailed test with z-critical values...

Example:

320 people out of a random sample of size 500 from County 1 prefer Candidate A.

268 people out of a random sample of size 400 from County prefer Candidate A.

$$\hat{p}_1 = \frac{320}{500} = 0.64$$

$$\hat{p}_2 = \frac{268}{400} = 0.67$$

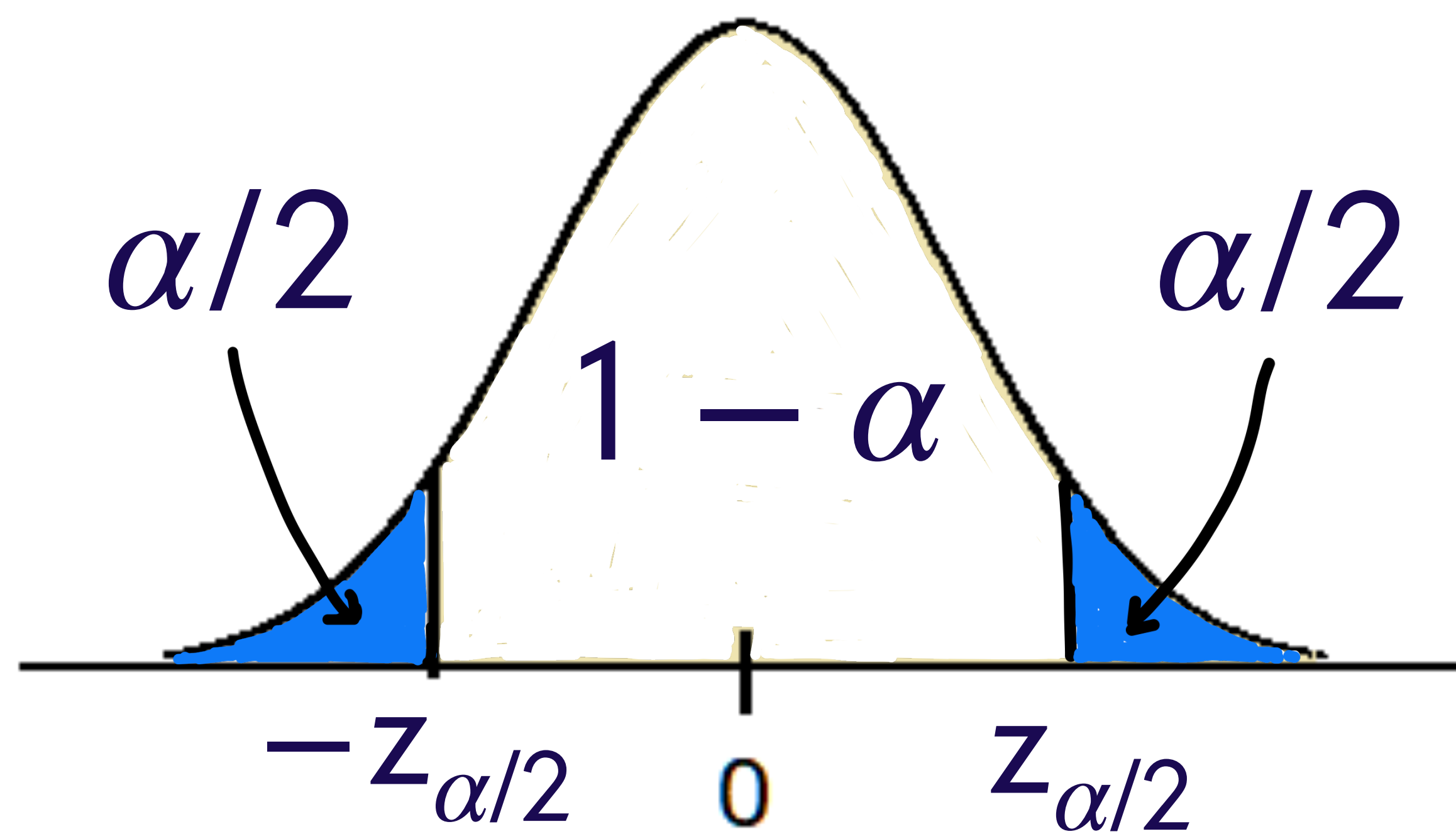
Use $\alpha = 0.05$.

Example:

$$\hat{p} = \frac{320 + 268}{500 + 400} = \frac{588}{900} = \frac{49}{75}$$

$$Z = \frac{0.64 - 0.67 - 0}{\sqrt{0.6533(1 - 0.6533) \left(\frac{1}{500} + \frac{1}{400} \right)}}$$

$$\approx -0.9397$$



$$z_{0.025} = 1.96$$

`qnorm(1 - 0.05/2)`

$Z = -0.9397$ does not fall in the rejection region!

Example:

The data suggests that the true proportion of people who like Candidate A is not different for the 2 counties.

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prop.test(x = c(320, 268), n = c(500, 400))
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```
2-sample test for equality of proportions with
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data:  c(320, 268) out of c(500, 400)
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```
chi-squared = 0.75556, df = 1, p-value = 0.3847
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alternative hypothesis: two.sided
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95 percent confidence interval:
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```
0.094648  0.034648
```

```
sample estimates:
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```
prop 1 prop 2
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```
0.64    0.67
```