Module 1 Overview

- Introduces key foundational concepts: **population vs. sample**, **probability**, and **sample** space
- Emphasizes that probability is the foundation of statistics and data science

Ⅲ What is Statistics?

- **Definition**: The science of using data to gain knowledge
- Involves ethical collection and analysis (based on test assumptions, not morality)
- **Population**: Full set of individuals/objects of interest
- Sample: Subset drawn from the population (often due to size or inaccessibility)

Statistics vs. Probability

- **Probability**: Assume population characteristics → infer about samples
- **Statistics**: Use sample characteristics → infer about populations

Sample Spaces and Events

- Probability provides a framework to handle randomness and uncertainty
- Examples:
 - Coin flips
 - o Purchase behavior
 - Stock price predictions

Key Terms:

- **Experiment**: Any process that generates observations
- Sample Space (S): Set of all possible outcomes
- **Event**: A single or collection of outcomes
- Cardinality: Number of outcomes (denoted |S|)

Sample Space Examples

- 1. Single coin flip:
 - S = {0, 1} where 0 = head, 1 = tail
 - |S| = 2
- 2. Two coin flips:
 - o S = {00, 01, 10, 11}
 - |S| = 4
- 3. Flip until first tail:
 - S = {T, HT, HHT, HHHT, ...}
 - \circ |S| = ∞ (infinite outcomes)
- 4. Car inspection for 3 defects (engine, seatbelt, paint):
 - Each outcome: 3-digit binary string (e.g., 100 = engine only)
 - |S| = 8

Set Notation and Events

- Union (A ∪ B): Outcomes in A or B (or both)
- Intersection (A ∩ B): Outcomes in both A and B
- Complement (Ac): Outcomes not in A
- Mutually Exclusive: $A \cap B = \emptyset$ (no overlap)

Car Defects Example – Event Definitions

- **Event A**: Engine problem → outcomes with 1 in first digit
 - A = {100, 110, 101, 111}
- Event B: Exactly one defect
 - o B = {100, 010, 001}
- Event C: Exactly two defects
 - o C = {110, 101, 011}

Combining Events:

- $A \cap B = \{100\}$
- $A^c = \{000, 010, 001\}$
- $A^c \cup B = \{000, 010, 001, 100\}$
- **B** ∩ **C** = ∅ (no overlap)

Venn Diagrams

• Used for visualizing unions, intersections, and complements

- Each circle represents an event (e.g., A, B, C)
- Every outcome from S appears exactly once in the diagram

Example Placement:

- $A \cap B = \{100\}$
- $A \cap C = \{110, 101\}$
- B = $\{010, 001\}$, C = $\{011\}$, A = $\{111\}$, Outside A \cup B \cup C = $\{000\}$

Key Takeaways

- Understand population vs. sample
- Learn the **structure of probability** (experiments, sample spaces, events)
- Use **set operations** and **Venn diagrams** to describe relationships between events
- Foundation for future topics: prediction, inference, modeling