Notes: Introduction to Random Variables and Discrete Distributions

What is a Random Variable?

- A random variable assigns numerical values to outcomes of a probabilistic experiment.
- Example: Flip a coin. Define variable X:
  - $X = 1$ for heads, $X = 0$ for tails.
- Before flipping, X is unknown - it's a random variable.
- Convention: Capital letters (e.g., X) are used to denote random variables.

Probability Mass Function (PMF):

- PMF, $f(x)$, gives the probability that a discrete random variable takes a specific value.
  - For fair coin:
    - $f(1) = 0.5$, $f(0) = 0.5$, $f(x) = 0$ for other x.
- PMFs describe how probability is distributed across possible values.

Bernoulli Distribution:

- Models a single trial with success (1) and failure (0).
- Notation: $X \sim \text{Bern}(p)$
- PMF: $f(x) = p^x * (1-p)^{(1-x)}$, for $x = 0$ or 1

Indicator Functions:

- $I_A(x) = 1$ if x in A, else 0.
- Useful for writing PMFs compactly.
- Example: $f(x) = p^x * (1-p)^{(1-x)} * I_{\{0,1\}}(x)$

Geometric Distribution:

- Models number of trials until first success.

Two forms:

1. Trials until success (values 1, 2, 3, ...)

   - PMF: $f(x) = (1-p)^{(x-1)} * p$

2. Failures before success (values 0, 1, 2, ...)

   - PMF: $f(x) = (1-p)^x * p$

- Notation: $X \sim Geom(p)$, sometimes with subscripts like $Geom_1$ or $Geom_0$

Summary:

- Random variables turn outcomes into numbers.

- PMFs define distributions of discrete random variables.

- Bernoulli and Geometric distributions are foundational.

- Indicator functions are concise and powerful tools in probability.