

## Answers and Explanations: Probability and Distributions (Questions 1–5)

1. Distribution of  $X$ :

Given:  $f(x) = 4x^2 * e^{-2x}$ , for  $x > 0$

This matches the Gamma distribution: shape = 3, rate = 2

Answer:  $\text{Gamma}(3, 2)$

2. Probability  $X \leq 6$ :

PMF is piecewise:

From 0 to 5:  $\int (1/25)x \, dx = (1/25)*(25/2) = 0.5$

From 5 to 6:  $\int (2/5 - 1/25x) \, dx = \text{area} = 9/50$

Total:  $0.5 + 9/50 = 17/25$

Answer:  $17/25$

3. Exponential Distribution:

$P(X > x) = e^{-\lambda * x}$

For  $x = 1$ :  $P(X > 1) = e^{-\lambda}$

Answer:  $e^{-\lambda}$

4. Normal Distribution:

$P(|X| < 2) \approx 0.9545$

$P(X > 2) = (1 - 0.9545)/2 = 0.02275$

Answer:  $0.02275$

5. Find constant  $c$ :

$f(x) = c * e^x$  for  $x$  in  $[0, 1]$  and  $c * e^{(-x+2)}$  for  $x > 1$

Integrals:  $\int_0^1 c * e^x \, dx = c(e - 1)$ ,  $\int_1^\infty c * e^{(-x+2)} \, dx = c * e$

Total area =  $c(e - 1 + e) = 1$ , so  $c = 1 / (2e - 1)$

Answer:  $1/(2e - 1)$