Let $X_1, X_2, \ldots, X_n$ be a random sample from the normal distribution with mean μ and known variance σ^2.

Consider testing the simple versus simple hypotheses

$$H_0 : \mu = 3$$
$$H_1 : \mu = 5$$

$$H_0 : \mu = 3 \qquad H_1 : \mu = 5$$

Definition/Notation:

Let α = P(Type I Error)

= P(Reject H_0 when it's true)

= P(Reject H_0 when $\mu = 3$)

α is called the level of significance of the test.

It is also sometimes referred to as the size of the test.

$$H_0 : \mu = 3$$
$$H_1 : \mu = 5$$

Developing a Test

Step One:

Choose an estimator for μ.

$$\widehat{\mu} = \overline{X}$$

Step Two:

Give the “form” of the test.

The form of the test:

$$H_0 : \mu = 3$$
$$H_1 : \mu = 5$$

- We are looking for evidence that H_1 is true.
- The $N(3, \sigma^2)$ distribution takes on values from $-\infty$ to ∞.
- $\overline{X} \sim N(\mu, \sigma^2/n) \Rightarrow \overline{X}$ also takes on values from $-\infty$ to ∞.

The form of the test:

$$H_0 : \mu = 3$$
$$H_1 : \mu = 5$$

- It is entirely possible that $\overline{X}$ is very large even if the mean of its distribution is 3.
- However, if $\overline{X}$ is very large, it will start to seem more likely that μ is larger than 3.
- Eventually, a population mean of 5 will seem more likely than a population mean of 3.

Developing a Test

$$H_0 : \mu = 3$$
$$H_1 : \mu = 5$$

Step One:

Choose an estimator for μ.

$$\hat{\mu} = \overline{X}$$

Step Two:

Give the "form" of the test.

Reject H_0, in favor of H_1, if $\overline{X} > c$ for some c to be determined.

$$H_0 : \mu = 3$$
$$H_1 : \mu = 5$$

Developing a Test

Step Three:

Find c.

Reject H_0, in favor of H_1, if $\overline{X} > c$.

- If c is too large, we are making it difficult to reject H_0.

We are more likely to fail to reject when it should be rejected.

Type II Error

Developing a Test

$$H_0 : \mu = 3$$
$$H_1 : \mu = 5$$

Step Three:

Find c.

Reject H_0, in favor of H_1, if $\overline{X} > c$.

- If c is too small, we are making it to easy to reject H_0.

  We are more likely reject when it should not be rejected.

Type I Error

Developing a Test

$$H_0 : \mu = 3$$
$$H_1 : \mu = 5$$

Step Three:

Find c.

$\boxed{\text{Reject } H_0\text{, in favor of } H_1\text{, if } \overline{X} > c\,.}$

This is where α comes in.

$$\begin{aligned} \alpha &= \mathrm{P}(\text{Type I Error}) \\ &= \mathrm{P}(\text{Reject } H_0 \text{ when true}) \\ &= \mathrm{P}(\overline{X} > c \text{ when } \mu = 3) \end{aligned}$$

Developing a Test

$$H_0 : \mu = 3$$
$$H_1 : \mu = 5$$

Step Four:

Give a conclusion!

Example: $X_1, X_2, \ldots, X_{10} \overset{iid}{\sim} N(\mu, 4)$

Find a hypothesis test for

$$H_0 : \mu = 5 \quad \text{vs} \quad H_1 : \mu = 3$$

Use level of significance $\alpha = 0.05$.

Find a "test of size 0.05".

$$H_0 : \mu = 5$$
$$H_1 : \mu = 3$$

Example:

Step One:

Choose an estimator for μ.

$$\hat{\mu} = \overline{X}$$

Step Two:

Give the "form" of the test.

Reject H_0, in favor of H_1, if $\overline{X} < c$ for some c to be determined.

$$H_0 : \mu = 5$$
$$H_1 : \mu = 3$$

Example:

Step Three:

Find c.

$$0.05 = P(\text{Type I Error})$$
$$= P(\text{Reject } H_0 \text{ when true})$$
$$= P(\overline{X} < c \text{ when } \mu = 5)$$
$$= P\left(\frac{\overline{X} - \mu_0}{\sigma/\sqrt{n}} < \frac{c-5}{2/\sqrt{10}} \text{ when } \mu = 5\right)$$

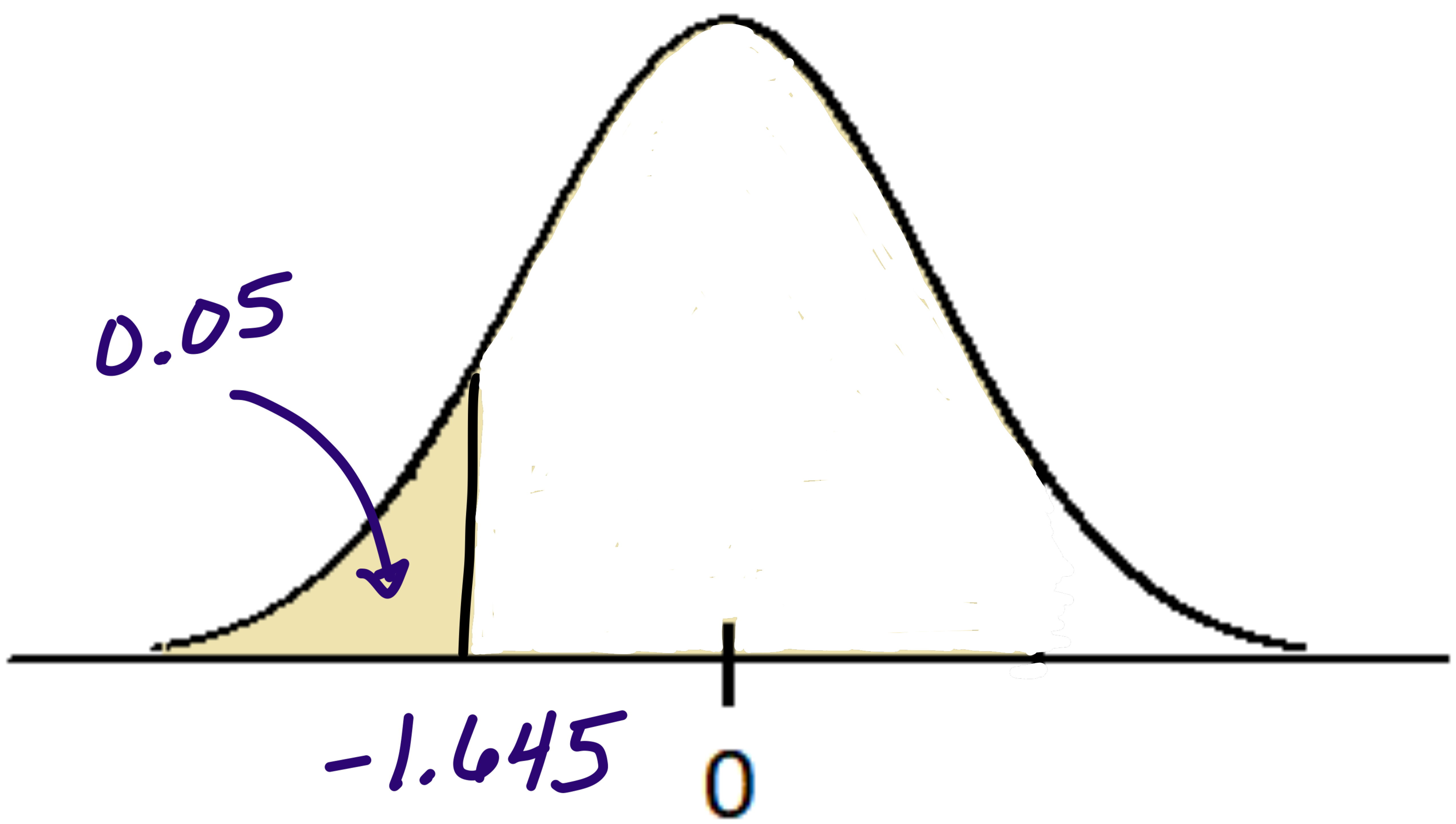
Example:

$$H_0 : \mu = 5$$
$$H_1 : \mu = 3$$

Step Three:

Find c.

$$0.05 = P\left(Z < \frac{c-5}{2/\sqrt{10}}\right)$$



$z_{0.95}$

qnorm(0.05)

Example:

$$H_0 : \mu = 5$$
$$H_1 : \mu = 3$$

Step Three:

Find c.

$$0.05 = P\left(Z < \frac{c - 5}{2/\sqrt{10}}\right)$$

$$\Rightarrow \quad \frac{c - 5}{2/\sqrt{10}} = -1.645$$

$$\Rightarrow \quad c = 3.9596$$

$$H_0 : \mu = 5$$
$$H_1 : \mu = 3$$

Example:

Step Four:

Give a conclusion.

Reject H_0, in favor of H_1, if

$$\overline{X} < 3.9596$$