MECH466 Automatic Control
Lab 2: Water Level Control in a Tank by the Proportional/Proportional/Integral Controller

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Summary:

In this lab we implemented P and PI controller using LabVIEW to control the water level of a one-tank system and a two-tank system. We observed the step response of the system with P and PI controllers at different values. We also tested the response of the system with a disturbance input. In this lab report, we explained the effect of the magnitudes of proportional gains on the steady state error of the system and why such error exists. We compared the step response of P and PI controller and mached a response model for each. Also, we studied the effect of proportional gain and integral gain on the overshoot, response speed, and settling time of the transient response. The response curve of a two tank system,controlled by P controller with inlet at one tank, outlet and height sensor at the other tank, is also studied and modeled.

Report Requirements

1. **Part 1 of the procedure. Proportional control only with the disturbance valve closed, and then half-way open. Proportional gains of 1.0, 3.0, 5.0**

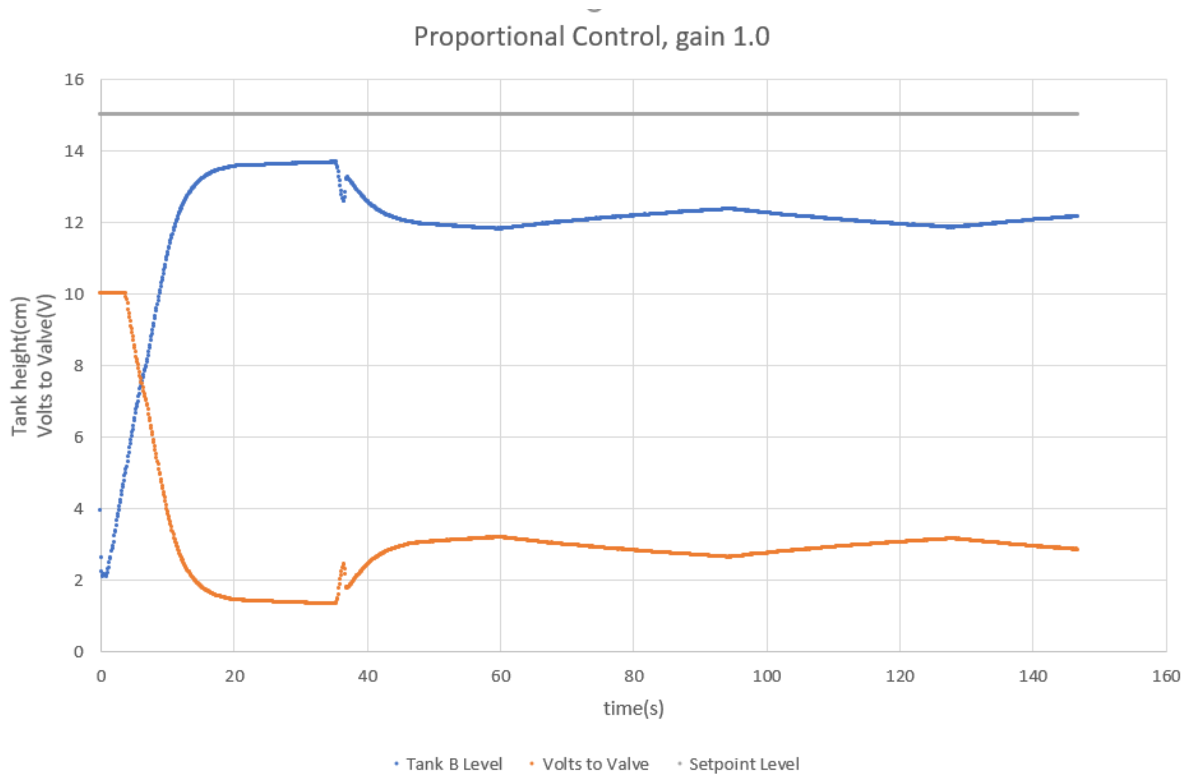

Figure 1 - Proportional Control, gain 1.0

At around 35 seconds, the disturbance valve has been opened half way
Error analysis: With the disturbance valve closed, the error is 1.4cm below the target value
With the valve open, the error is 2.64 cm to 3.13 cm below the target. The error with the disturbance valve half open never really stabilized, thus the range of the error. I am assuming this is because the control valve is limited by a certain precision, and thus could not be operated at the exact flow-rate needed. I suspect that these are not control-induced oscillations, because for one, proportional control is not prone to slow oscillations, especially if the gain is low. Also, the fluctuation does not look like a sine wave, it is more of a straight line up, and then straight line down. This leads me to believe that there is a certain "step" in the accuracy of the output, and the true steady state value lies somewhere in between these steps.

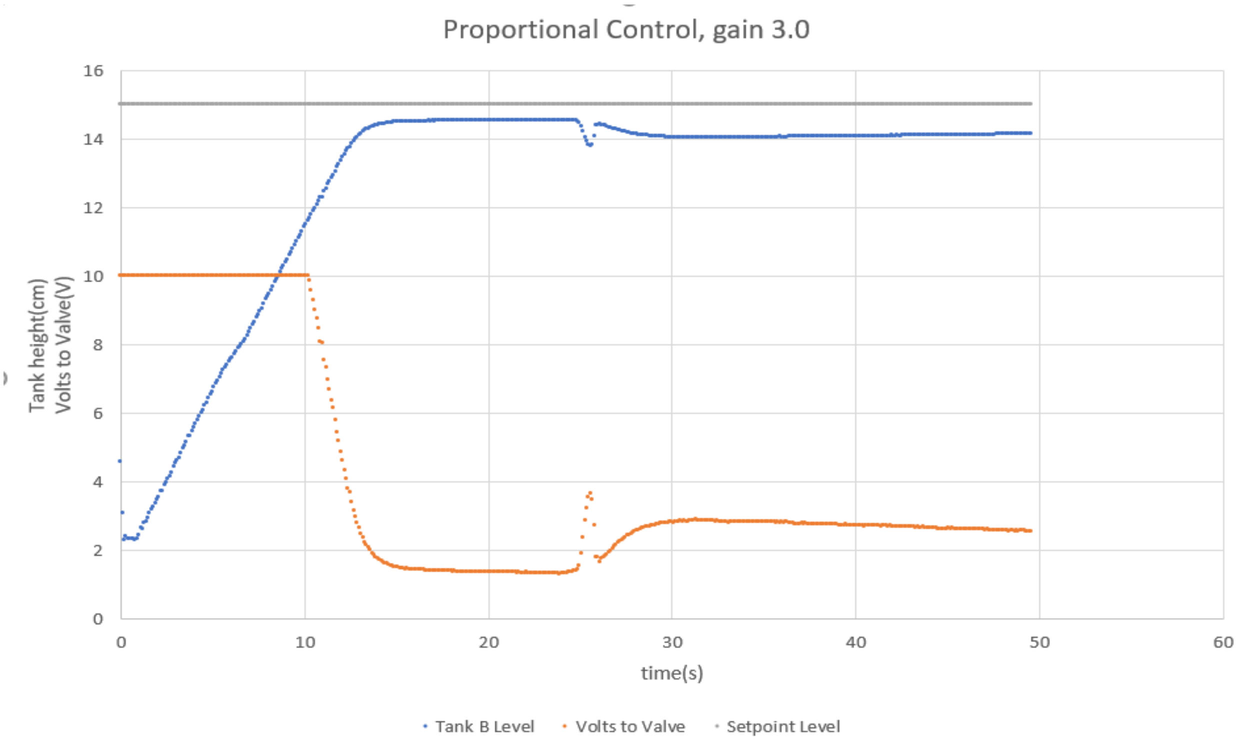

Figure 2 - Proportional Control, gain 3.0

At around 25 seconds the disturbance valve was open half way

Error analysis: With the disturbance valve closed the steady state error was 0.45 cm below target

With the valve open, the error went to 0.86 cm below target

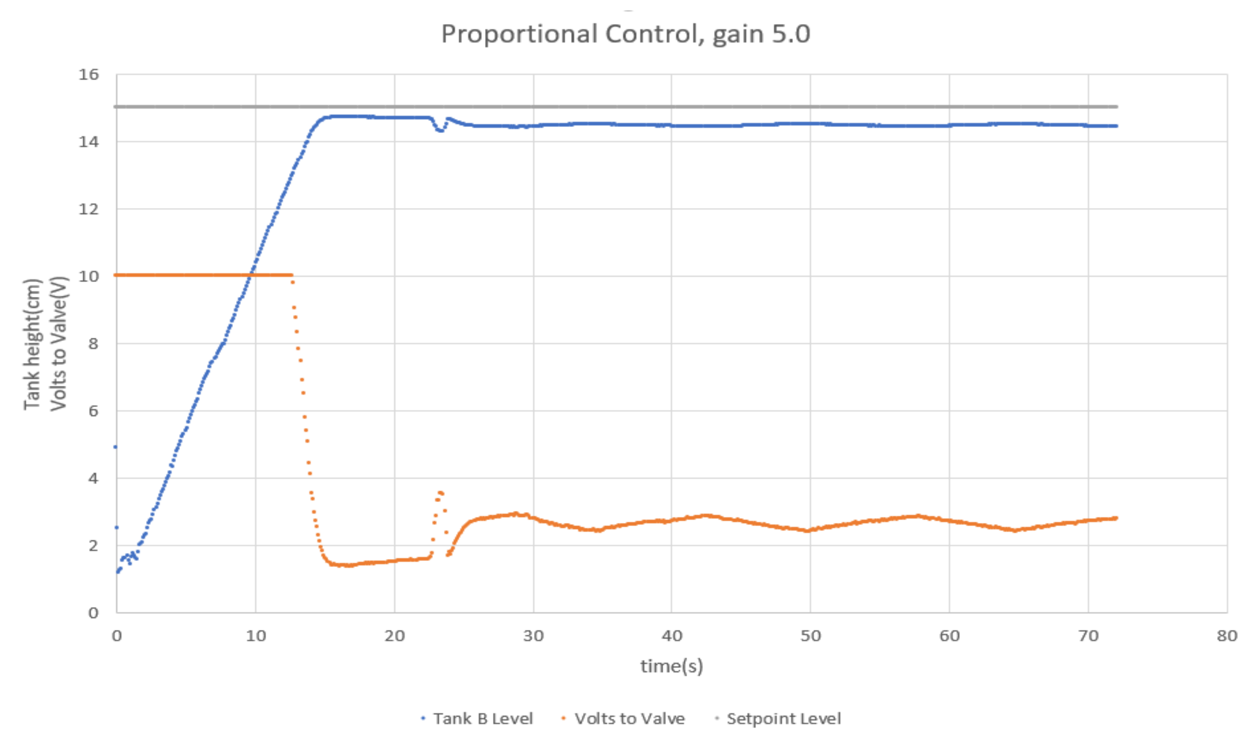

Figure 3 - Proportional Control, gain 5.0

At around 23 seconds the disturbance valve was open half way
Error analysis: With the disturbance valve closed, the error was 0.3 cm below target
With the disturbance open, the error was 0.49 cm to 0.57 cm below target, once again going up and down repeatedly.

2.

Effects of proportional gain on the error:

Higher gain reduces the steady state error. Proportional control corrects based on the value of the error. *Gain*error = correction*. In our case the correction is the flow rate. For steady state to be achieved, *flow rate in = flow rate out*. Flow rate out is fixed, determined by how open the valve is. *Flow rate in = gain*error*. Thus, if the gain is increased, the error must decrease to keep that product constant, and equal to the flow rate out. Intuitively, the higher gain controller "corrects more".

Why is the error larger if the disturbance vale is open:

Similar idea as with the previous question. *Flow in = gain*error*. As the disturbance valve is open, the flow rate out is increased by a certain value. This means that the flow rate in must increase as well. Since the gain remains unchanged during the opening of the valve, the error must increase to make the *gain*error* product increase to match the new flow rate out.

3. Following Are the plots of the step response with different P controller and PI controller values.

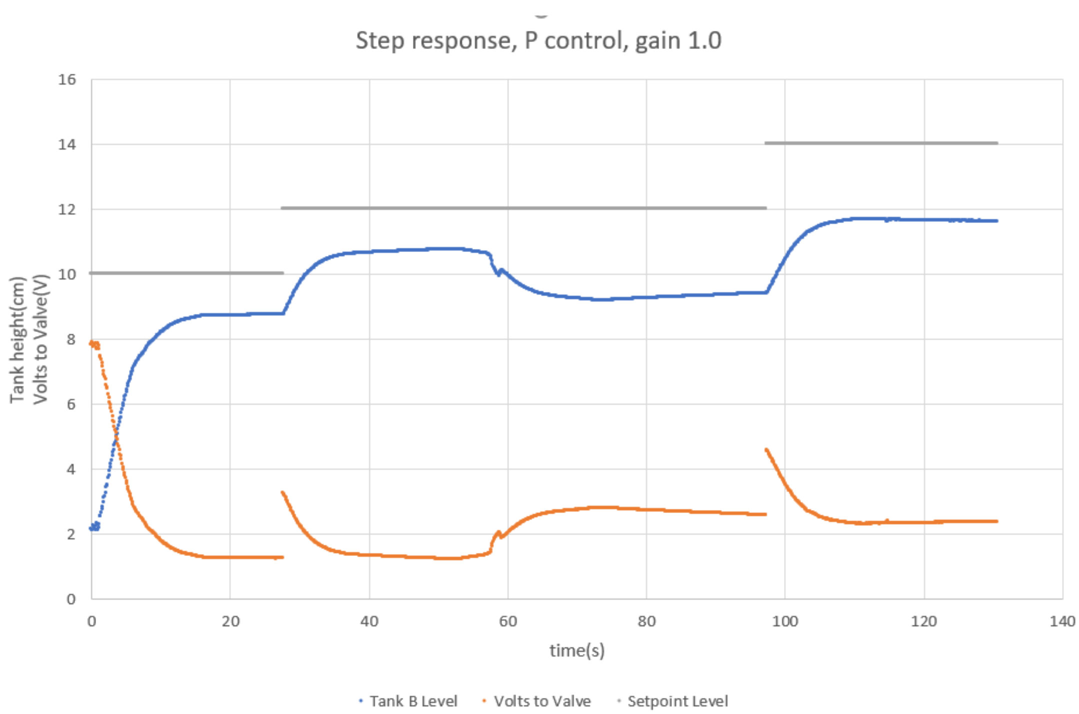

Figure 4 - Step Response Proportional Control, gain 1.0

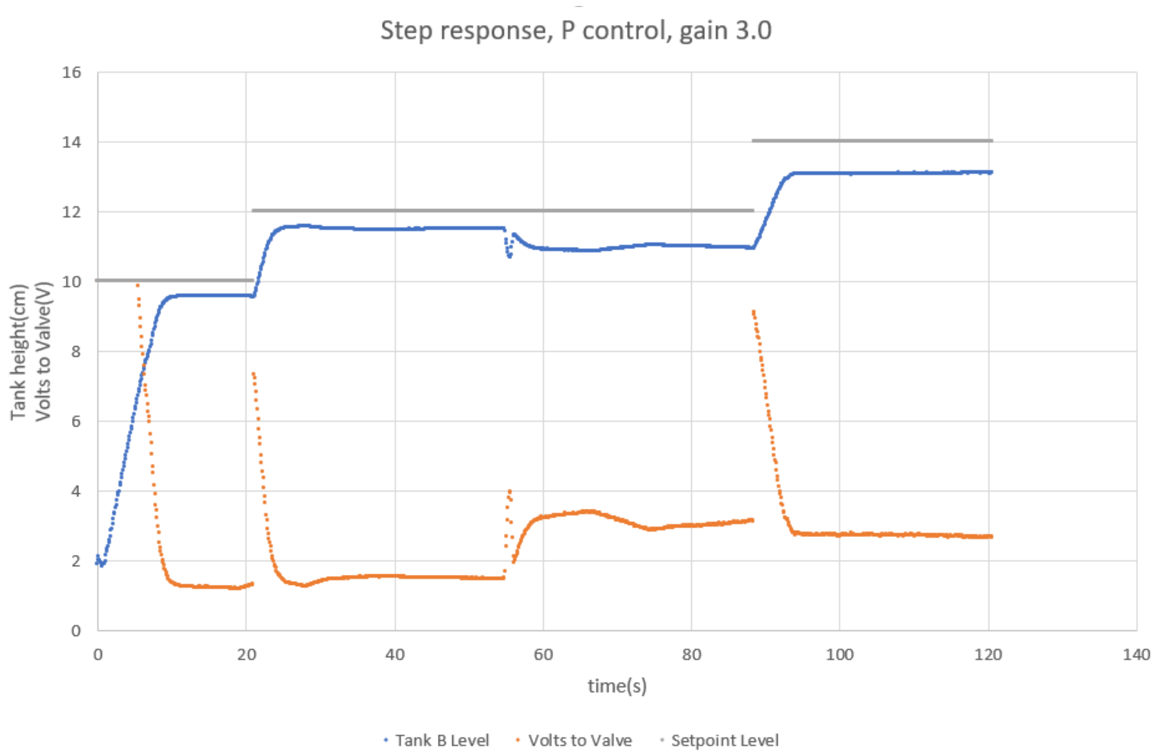

Figure 5 - Step Response Proportional Control, gain 3.0

Figure 6 - Step Response P Controller, gain 1.0, PI Controller, gain 0.6

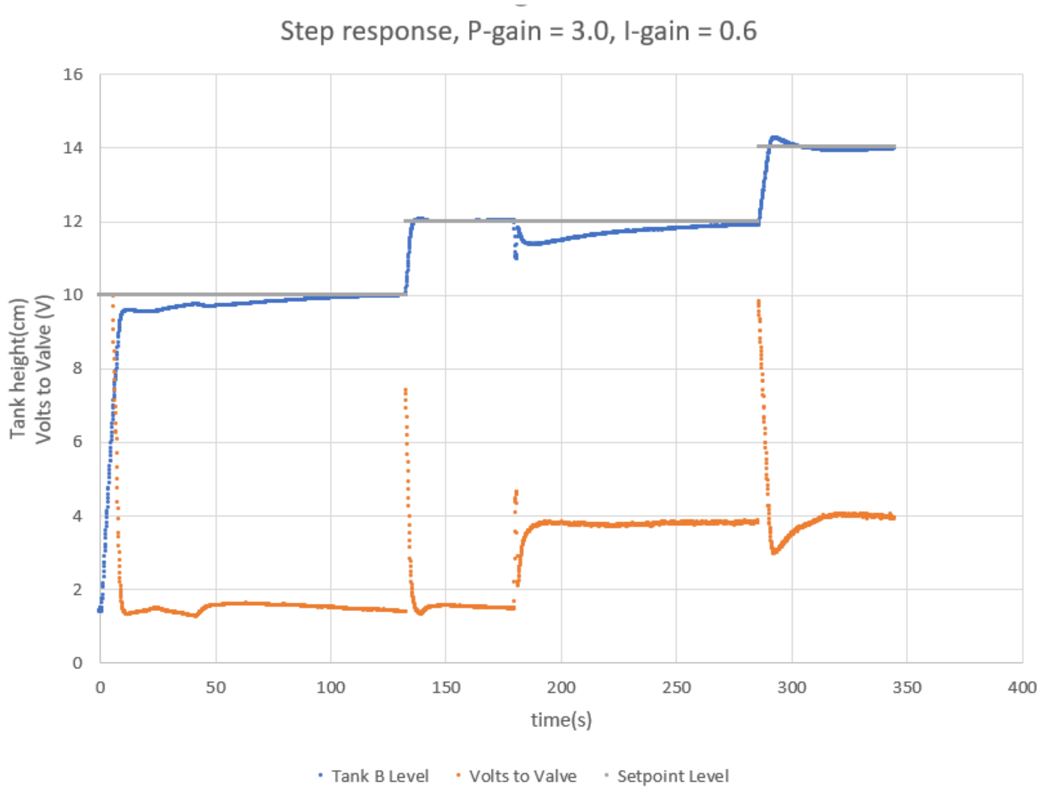

Figure 7 - Step Response P Controller, gain 3.0, PI Controller, gain 0.6

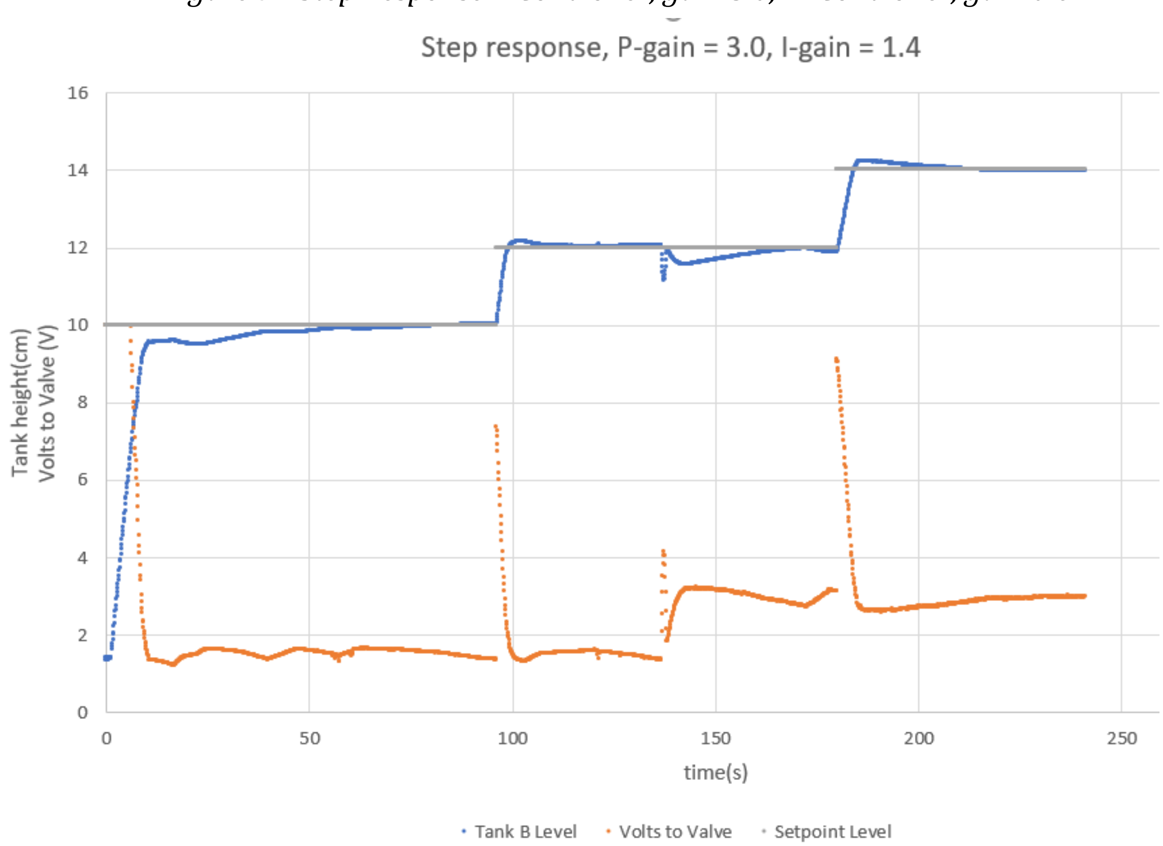

Figure 8 - Step Response P Controller, gain 3.0, PI Controller, gain 1.4

What order of system response can you use to model these systems? Explain.
From the plots we can see that the step responses of the P controller behave similar to both 1st order system and 2nd order overdamped system. However, since the P controller has similar response curves no matter how Kp is changed, we can conclude that the response can be modeled by 1st order system. Meanwhile, the responses of the PI controller behave similar to an underdamped 2nd order system.

4. The effect of the integrator on steady state error
The integrator annihilates the steady state error. The integrator control corrects based on the integral of the error signal. *Flow rate in = gain*integral(error)dt,* where the "gain" is the integral gain of course. A steady state error integrates to a forever increasing ramp, so the only way to get a steady signal out of an integrator is to have zero error; zero being the only signal that just integrates to zero. As soon as there is any deviation from the target value, the integrated error starts to grow and forces the system to go right back to equilibrium at target.
For a system with a step input to achieve zero steady-state error, at least 1 integrator is needed.

5. Explain the effect of the proportional gain kP and the integrator gain kI on the transient response (rise time, overshoot etc.)

The system can be modelled using water level h (ρgh as the pressure head that affects the flow rate at the outlet) and the change of tank water level $\frac{dh}{dt}$ ($A\frac{dh}{dt}$ as the sum of the flow rate). Therefore, the system can be modeled with $G(s) = \frac{1}{as+b}$

For P controller:
For a P only controller, with $G(s) = \frac{1}{as+b}$ and negative control, the transfer function will be similar to $T(s) = \frac{Kp/(b+Kp)}{[a/(b+Kp)]s+1}$ where $Kp/(b+Kp)$ is the K and $a/(b+Kp)$ is the T for the step response of a 1st order system. Therefore with a larger Kp, the steady state value is higher, which makes it closer to the set value. Also, with a larger Kp value, the system has a smaller time constant; thus, the system responses faster. And there is overshoot in a 1st order response system. From the plots in part 1 (Figure 1, 2, and 3), we also observed that with larger Kp, the steady state value is higher and the system response is faster.

For PI controller:
For the PI controller system, the transfer function will be similar to $T(s) = \frac{Kp*s+Ki}{as^2+(Kp+b)s+Ki}$.
With the same Kp, a higher Ki results in a lower damping ratio; the system is less damped and has a larger overshoot. This agrees with our experiment results in Figure 7 and 8.

With the same Ki, a higher Kp results in a higher damping ratio, shorter settling time and faster response; this system is more damped and has a smaller overshoot. This agrees with our experiment results in Figure 6 and 7.
In summary for PI controller, a higher Kp results in a more damped system with smaller overshoot, shorter settling time and faster response; a higher Ki results in a less damped system with larger overshoot.

6. comment on the response of the system to the step input. What response model can you use to describe this two tank system? Explain.
As shown in the Figure below, the the curve of the water levels in Tank A and B have similar shapes while approaching the set value. The overshoots in both curves are attributed to the delay in the response of the flow rate into Tank B because it is through the tube from Tank A. On the other hand, the water levels in Tank A and B behave oppositely in response to the opening of the disturbance valve. The water level in Tank A gives a relatively smoother response to the opening of disturbance valve comparing to the water level in Tank B; also, the water level in Tank A raised slightly while the water level in Tank B has a steep drop when the disturbance valve was opened. Since the water is draining from Tank B, it is reasonable that when the disturbance valve is suddenly opened, there is a significant drop in Tank B. Also, since the height difference between water level and set value is calculated using the water level in Tank B, in response to the drop, the inlet flow rate suddenly increased and that is why the water level in Tank A had a slight increase. One thing to notice is that the water level in Tank A is slightly higher (1.75cm higher when the disturbance valve is opened) than the level in Tank B based on observation, but the plots indicates the opposite. This is attributed the calibration error in the system because we used the calibration data from Lab 1. It make sense that the water level in Tank A should be higher because it requires a small pressure head for the water to overcome the head loss in tubes and keep flowing from Tank A to Tank B. The calibration error can be roughly corrected by shifting the water level in Tank A up by 1.75cm. Therefore, the steady state errors of Tank A with and without disturbance valve opened are +0.35cm and +0.3cm respectively; the steady state errors of Tank B with and without disturbance valve opened are -0.67cm and -1.45cm respectively. The response can still be considered as a 1st order step response.

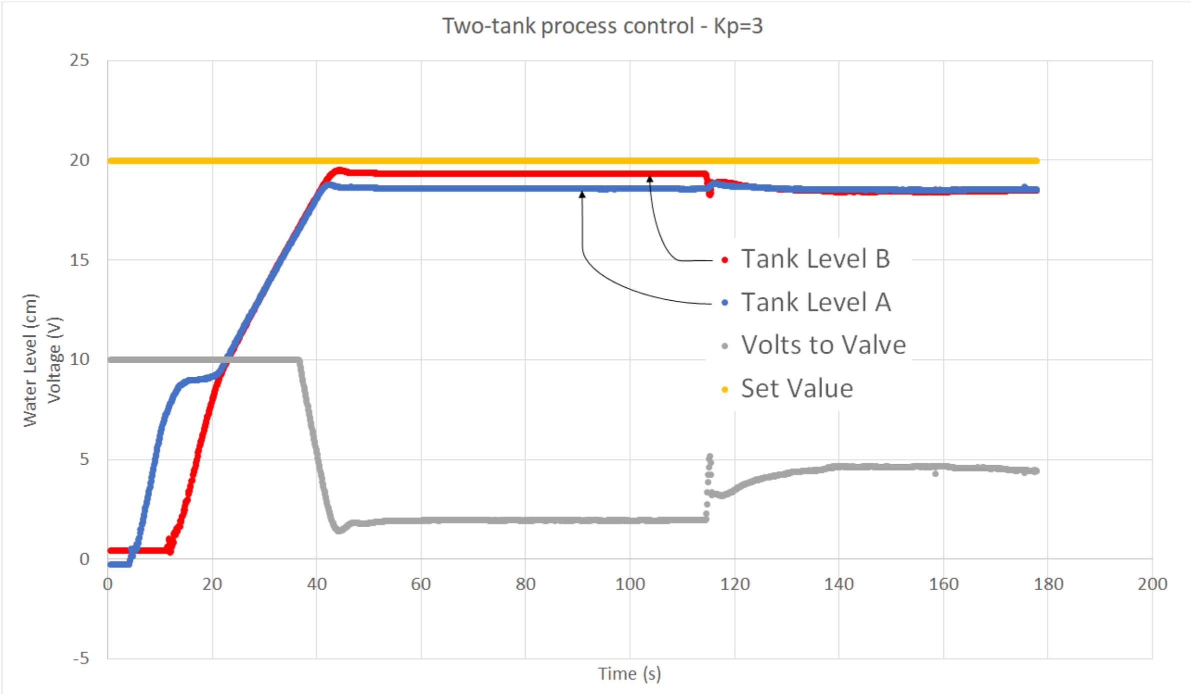

Figure 9 - Two Tank Process Control

7. As long as a non-zero error is observed, a proportional controller will send a non-zero control signal (voltage) to the plant. Explain then why is it possible for steady-state errors to exist, for example, when a proportional controller is used to track a step-input?
Hint: From your experiments, you noticed that the control signals (voltage) to the valve are not zero at steady state. If the control signals are not zero, then why do they appear to have no effect in reducing the steady-state errors?

With respect to our experiment, the proportional controller produced steady-state error because of the process and disturbance valves. There was a steady flow of water out of the tank from these valves. The controller output (input valve) attempted to force the water level to the set point.

The steady state error was visible when the output flow of the tank was equal to the input flow to the tank, causing the tank level to remain constant. This always occurred below the actual set point. This is because the error, and thus the controller's output signal, would be zero at the set point due to the nature of proportional control. Conversely, the output flow from the process/disturbance valves is always non-zero if the tank is not empty. Thus the level at which the input flow counteracts the output flow must be below the set point, corresponding to a non-zero steady-state error.

In general, for any system where a finite disturbance exists, a proportional controller will be unable to eliminate steady-state error. As the gain Kp is increased, the steady-state error will decrease; this was reflected in our experiment, where the equilibrium point of the tank moved closer to the set point as we increased Kp. For the steady-state error to go to zero, however, Kp must go to infinity. This is impossible in real systems, and thus proportional controllers produce non-zero steady-state errors.