TD 9 : Systèmes de numération flottante

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Contents

1 Exercice 1 :

Donnez la représentation flottante, en simple précision des nombres suivants :

1.1 128

1. $S = 0$
2. $128 = (10000000)_2 = (1, 0) \times 2^7$
3. $M = 0$ et $e = 7$
4. $E = e + biais = 7 + 127 = 6 + 128$
   $E = (10000110)_2$
5. $128 \implies (01000011000000000000000000000000)_2 \implies (43000000)_{16}$

1.2 -32.75

1. $S = 1$
2. $|-32.75| = (00100000, 11)_2 = (1, 0000011)x2^4$
3. $M = 0000011$ et $e = 4$
4. $E = 4 + 127 = (10000100)_2$
5. $-32.75 \implies (11000010000000110000000000000000)_2 = (C2060000)_{16}$

1.3 18.125

1. $S = 0$
2. $18.125 = (00010010, 001)_2 = (1, 0010001)x2^4$
3. $M = 0000011$ et $e = 4$
4. $E = 4 + 127 = (10000100)_2$
5. $18.125 \implies (11000010000000110000000000000000)_2 = (41910000)_{16}$

1.4 0.0625

1. $S = 0$
2. $0.0626 = (0, 0001)_2 = (1, 0)x2^{-4}$
3. $M = 0$ et $e = -4$
4. $E = -4 + 127 = (01111011)_2$
5. $0.0625 \implies (00111101100000000000000000000000)_2 = (3D800000)_{16}$

2 Exercice 2:

2.1 1

1. $S = 0$
2. $1 = (1)_2 = (1,0) \times 2^0$
3. $M = 0$ et $e = 0$
4. $E = 0 + 1023 = (011\,1111\,1111)_2$
5. $\begin{array}{rcl} 1 & \Longrightarrow & (0\,01111111111\,0000000000000000000000000000000000000000000000000000)_2 \\ & \Longrightarrow & (3FF0000000000000)_{16} \end{array}$

2.2 -64

1. $S = 1$
2. $|-64| = (100\,0000)_2 = (1,0) \times 2^6$
3. $M = 0$ et $e = 6$
4. $E = 6 + 1023 = (100\,0000\,0101)_2$
5. $\begin{array}{rcl} -64 & \Longrightarrow & (1\,10000000101\,0000000000000000000000000000000000000000000000000000)_2 \\ & \Longrightarrow & (C050\,0000\,0000\,0000)_{16} \end{array}$

2.3 12.06640625

1. $S = 0$
2. $12.06640625 = (1100,0001\,0001)_2 = (1,1000\,0010\,001) \times 2^3$
3. $M = 1000\,0010\,001$ et $e = 3$
4. $E = 2 + 1023 = (100\,0000\,0010)_2$
5. $\begin{array}{rcl} 12.06640625 & \Longrightarrow & (0\,10000000010\,1000000010000000000000000000000000000000000000000000)_2 \\ & \Longrightarrow & (4028\,2200\,0000\,0000)_{16} \end{array}$

2.4 0.2734375

1. $S = 0$
2. $0.2734375 = (0,010001)_2 = (1,00011) \times 2^{-2}$
3. $M = 00011$ et $e = 3$
4. $E = -2 + 1023 = (01111111101)_2$
5. $\begin{array}{rcl} 0.2734375 & \Longrightarrow & (0\,01111111101\,0001100000000000000000000000000000000000000000000000)_2 \\ & \Longrightarrow & (3FD1\,8000\,0000\,0000)_{16} \end{array}$

3 Exercice 3 :

3.1 1011 1101 0100 0000 0000 0000 0000 0000

Donc : S = 1 E = 0111 1010 e = E - biais = $(01111010)_2 - (01111111)_2 = -5$
$m_2 = (1, M)_2 = (1, 1)_2 = (11)_2 \times 2^{-1}$
Cl : $-m_2 \times 2^e = -(1, 1)_2 \times 2^{-5} = -(11)_2 \times 2^{-6} = -3 \times 2^{-6} = -0,046875$

3.2 0101 0101 0110 0000 0000 0000 0000 0000

Donc : S = 0 E = $(10101010)_2 - 127 = 43$
$m_2 = (1, M)_2 = (1, 11)_2 = (111)_2 \times 2^2$
Cl : $m_2 = (1, 11)_2 \times 2^{43} = (111)_2 \times 2^2 \times 2^{41} = 7 \times 2^{41} = -0,046875$

3.3 1100 0001 1111 0000 0000 0000 0000 0000

Donc : $S = 1$ $E = 10000011 = 131$ $e = E - biais = 131 - 127 = 4$
$m_2 = (1, M)_2 = (1, 111)_2 \times 2^4$
$Cl : -m_2 \times 2^e = -(1, 111)_2 \times 2^4 = -(1111)_2 \times 2 = -30$

3.4 1111 1111 1000 0000 0000 0000 0000 0000

Donc : $S = 1$
$E = 1111\,1111$
$M = 0 \times 23 \implies -\infty$

3.5 0000 0000 0100 0000 0000 0000 0000 0000

Donc : S = 0 E = 0 donc si M = 0 alors 0 sinon nb denormalisé
donc e = 1 - biais = - 126
et m = $(0, M)_2 = (0, 1)_2 = (1, 0)_2 \times 2^{-1}$
CL $(1, 0)_2 = 2^{-1} \times 2^{-126} = 2^{-127}$

4 Exercice 4 :

4.1 1. $(4030480000000000)_{16}$

$\implies (0\ 10000000011\ 110101000000...000)_2$
$S = 0 \implies positif$

$$\begin{aligned} e = E - biais &= (10000000011 - 1023)_{16} \\ e &= 1027 - 1023 = 4 \end{aligned}$$

$$\begin{aligned} m_2 &= (1, M)_2 \\ &= (1, 1101010010000000.....0000)_2 \\ &= (1110101001)_2 \times 2^9 \\ &= 937 \times 2^{-9} \end{aligned}$$

$$937 \times 2^{-9} * 2^4 = 937 \times 2^{-5} = 29,28125$$

4.2 2. $(C040000000000000)_{16}$

$\implies (1|10000000100|000000000000...000)_2$
$s = 1 \implies negatif$

$$\begin{aligned} e = E - biais &= (10000000100)_2 - (1023)_{16} \\ e &= 1028 - 1023 = 5 \end{aligned}$$

$$\begin{aligned} m_2 &= (1, M)_2 \\ &= (1, 0)_2 \end{aligned}$$

$$-1 \times 2^5 = -32$$

4.3 3. $(BFC0000000000000)_2$

$\implies (1|01111111100|000000000000...000)_2$
$s = 1 \implies negatif$

$$\begin{aligned} e = E - biais &= (01111111100)_2 - (1023)_{16} \\ e &= 1020 - 1023 = -3 \end{aligned}$$

$$\begin{aligned} m_2 &= (1, M)_2 \\ &= (1, 0)_2 \end{aligned}$$

$$-1 \times 2^{-3} = -0,125$$

4.4 4. $(8000000000000000)_{16}$

$\implies (1|00000000000|000000000000...000)_2$
$s = 1 \implies negatif\ E = 0 et M) 0$ donc on a (-)0

4.5 5. $(FFF0000100000000)_2$

$\implies (1|11111111111|00000000000000000100...00)_2$
$s = 1 \implies negatif\ E = 11111111111 et M \neq 0$ donc on a NaN