

Problem 2:

2.2 Error Convergence Analysis

The obvious and most important result is, that the local error depends heavily on the step size, which is shown by the rate of algebraic convergence. The results are consistent, since all first order integrator (euler, symplectic euler & backwards euler) have an averaged rate of convergence of approximately 4 and the second order integrator (Midpoint Method) has an averaged convergence of approximately 8.

The damping adds no significant effects to the rate of convergence. The deviations in the convergence of the first order integrator (no damping vs damping as well as the difference between the different integrator) is due to the statistical properties of the computed average, a more practical approach could be the calculation of the median instead of the mean.

Current Settings:					
	-testcase	error_measurement			
	-method	symplectic_euler			
	-mass	0.1			
	-step	0.5			
	-stiff	10			
	-damp	0			
error measurement:					
	step	euler	symplectic_euler	midpoint	backwards_euler
	0.5	0.453931	1.77902	1.11648	0.108029
	0.25	0.0615007	0.392774	0.227137	0.0707123
	0.125	0.0114421	0.0713761	0.029967	0.0420407
	0.0625	0.00672574	0.0139788	0.00362653	0.0063294
	0.03125	0.00214926	0.00302688	0.00043881	0.000271321
	0.015625	0.000593283	0.000700751	5.37341e-05	0.00036252
	0.0078125	0.000155114	0.000168395	6.6406e-06	0.000127309
	0.00390625	3.96135e-05	4.12637e-05	8.25123e-07	3.62277e-05
	0.00195313	1.00068e-05	1.02125e-05	1.02825e-07	9.58996e-06
	0.000976563	2.51458e-06	2.54024e-06	1.28332e-08	2.46289e-06
rate of (algebraic) convergence					
		euler	symplectic_euler	midpoint	backwards_euler
		4.09864	4.48656	7.70731	5.32906

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	-stiff	10			
	-damp	0.5			
error measurement:					
	step	euler	symplectic_euler	midpoint	backwards_euler
	0.5	0.282399	0.958575	0.620487	0.0209923
	0.25	0.0607383	0.229782	0.14526	0.0388779
	0.125	0.00164035	0.0439014	0.0227709	0.0249359
	0.0625	0.00221801	0.00834724	0.00306461	0.00533366
	0.03125	0.000927804	0.00171351	0.000392853	0.00040355
	0.015625	0.000280593	0.000379736	4.95716e-05	9.3394e-05
	0.0078125	7.63205e-05	8.87617e-05	6.22058e-06	5.19682e-05
	0.00390625	1.98563e-05	2.14142e-05	7.78921e-07	1.67681e-05
	0.00195313	5.06137e-06	5.25626e-06	9.74443e-08	4.67312e-06
	0.000976563	1.27752e-06	1.30189e-06	1.21853e-08	1.22887e-06
rate of (algebraic) convergence					
		euler	symplectic_euler	midpoint	backwards_euler
		7.05767	4.50928	7.30582	4.0666

2.3 Stability Analysis

The results of the two measurements show that only one of the applied solver is stable. All the explicit methods which we used become unstable for an increasing step size. The only stable solver is the backwards Euler which is a semi-implicit method and due to its energy preserving properties stability analysis bounded.

The results of the measurements for with and without damping are rather similar. One could argue, that by introducing damping into the system, the rate of instability for the unstable solver slows down, which does make sense due to the nature of damping itself.

```
Current Settings:
-testcase stability_measurement
-method symplectic_euler
-mass 0.1
-step 0.001
-stiff 10
-damp 0
```

Max amplitude table:	step	euler	symplectic_euler	midpoint	backwards_euler	analytic
	0.001	2.25774	2.1962	2.1962	2.19467	2
	0.002	2.80799	2.1962	2.19622	2.19317	2.00002
	0.004	288.054	2.19622	2.19651	2.19025	2.00008
	0.008	6.90578e+12	2.19628	2.20129	2.18469	2.00031
	0.016	7.60715e+53	2.19652	2.3204	2.17463	2.00125
	0.032	4.07258e+210	2.19748	47226.9	2.15829	2.00498
	0.064	inf	2.20164	1.51951e+88	2.13789	2.01941
	0.128	inf	2.22577	inf	2.11901	2.06997
	0.256	inf	inf	6.88232e+306	2.10765	2.18007
	0.512	inf	inf	inf	2.10144	2.05921

```
Current Settings:
-testcase stability_measurement
-method symplectic_euler
-mass 0.1
-step 0.001
-stiff 10
-damp 0.5
```

Max amplitude table:	step	euler	symplectic_euler	midpoint	backwards_euler	analytic
	0.001	2.1424	2.14164	2.14169	2.14099	2
	0.002	2.14313	2.1416	2.14169	2.1403	2.00002
	0.004	2.14463	2.14151	2.14167	2.13896	2.00008
	0.008	2.14779	2.14134	2.14159	2.13645	2.00031
	0.016	2.15492	2.14102	2.14143	2.13195	2.00122
	0.032	2.17261	2.14075	2.141	2.12479	2.00473
	0.064	2.14875e+185	2.14172	2.1407	2.11573	2.01751
	0.128	inf	2.16073	2.16921	2.10686	2.05753
	0.256	inf	inf	5.4651e+306	2.1016	2.13065
	0.512	inf	inf	inf	2.0996	2.09832