



Chap 3. Stacks and Queues (2)

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Chapter 3. Stacks And Queues

3.1 Stacks

3.2 Stacks Using Dynamic Arrays

3.3 Queues

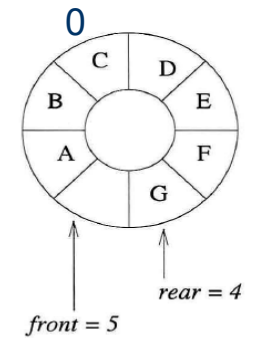
3.4 Circular Queues Using Dynamic Arrays

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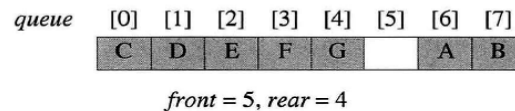
3.6 Evaluation of Expressions

3.7 Multiple Stacks and Queues

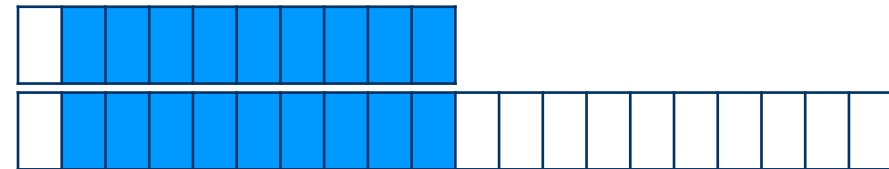
3.4 Circular Queues Using Dynamically Allocated Arrays



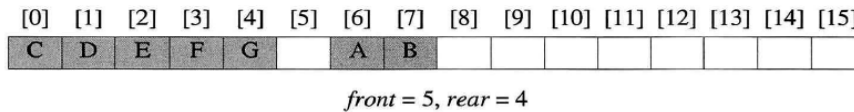
(a) A full circular queue



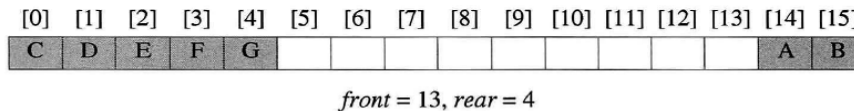
(b) Flattened view of circular full queue



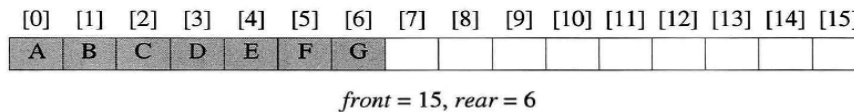
최악의 경우 이동



(c) After array doubling



(d) After shifting right segment



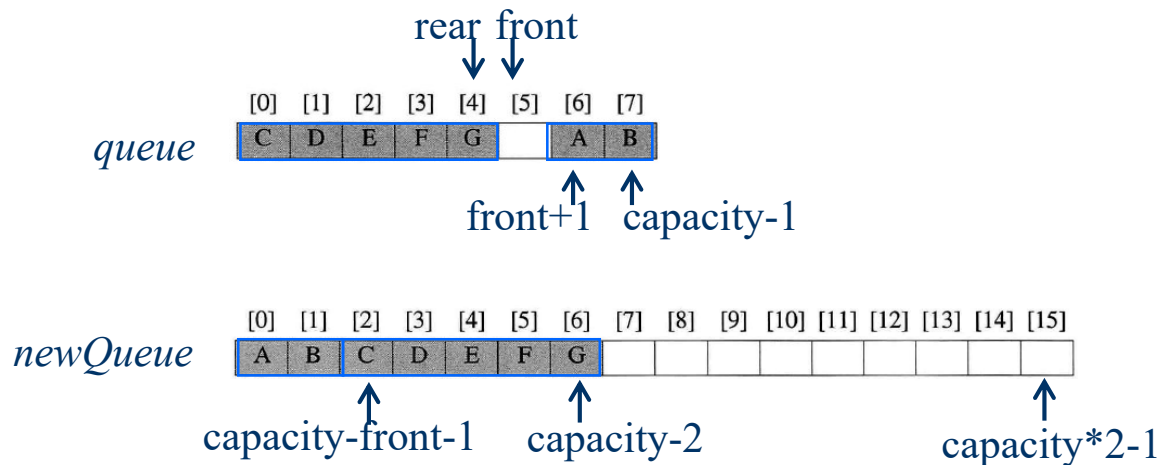
(e) Alternative configuration

Figure 3.7: Doubling queue capacity

3.4 Circular Queues Using Dynamically Allocated Arrays

❖ Figure 3.7 (b)→(e)

- (1) Create a new array *newQueue* of twice the capacity.
- (2) Copy the second segment (i.e., the elements *queue* [*front* + 1] through *queue* [*capacity* - 1]) to positions in *newQueue* beginning at 0.
- (3) Copy the first segment (i.e., the elements *queue* [0] through *queue* [*rear*]) to positions in *newQueue* beginning at *capacity* - *front* - 1.



3.4 Circular Queues Using Dynamically Allocated Arrays

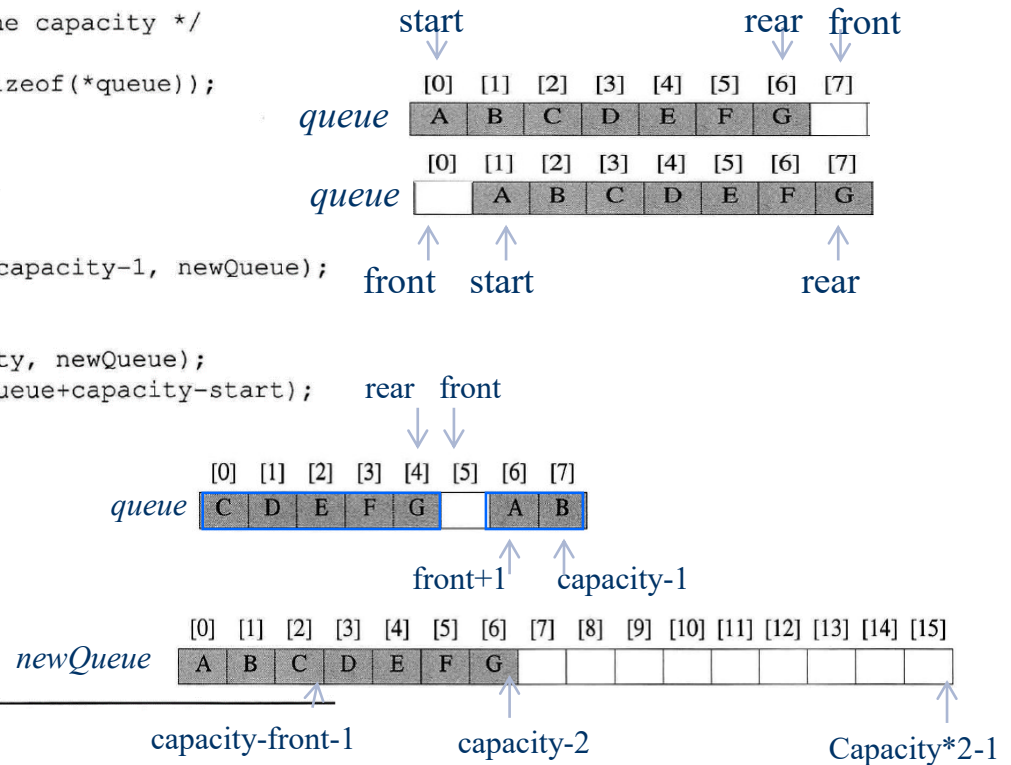
```

void queueFull()
{
    int start;
    /* allocate an array with twice the capacity */
    element* newQueue;
    MALLOC(newQueue, 2 * capacity * sizeof(*queue));

    /* copy from queue to newQueue */
    start = (front+1) % capacity;
    if (start < 2)
        /* no wrap around */
        copy(queue+start, queue+start+capacity-1, newQueue);
    else
        /* queue wraps around */
        copy(queue+start, queue+capacity, newQueue);
        copy(queue, queue+rear+1, newQueue+capacity-start);
    }

    /* switch to newQueue */
    front = 2 * capacity - 1;
    rear = capacity - 2;
    capacity *= 2;
    free(queue);
    queue = newQueue;
}

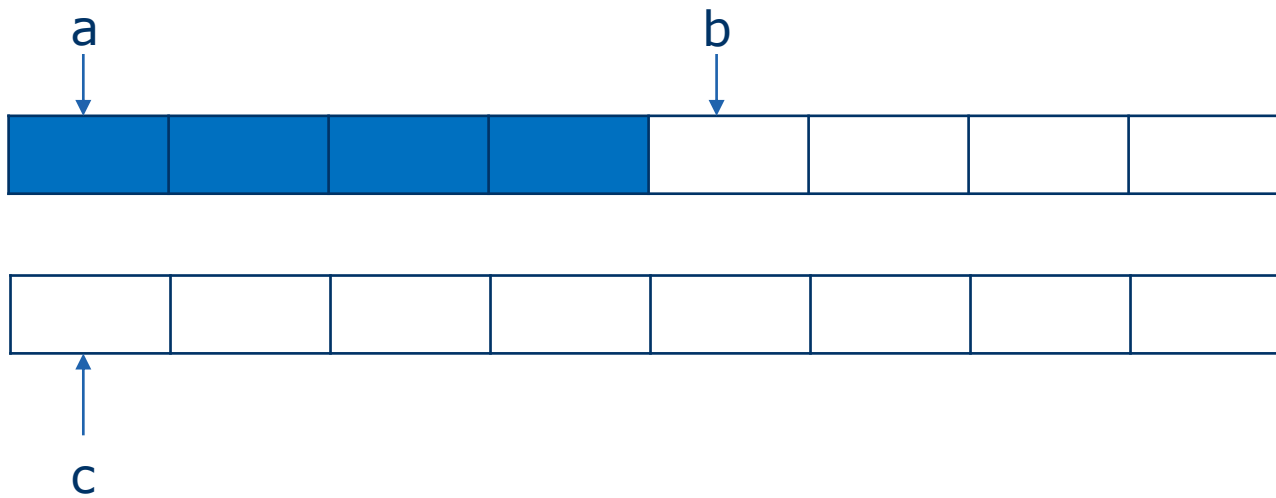
```



Program 3.10: Doubling queue capacity

3.4 Circular Queues Using Dynamically Allocated Arrays

```
void copy( element *a, element *b, element *c)
{
    while( a != b )
        *c++ = *a++;
}
```



3.4 Circular Queues Using Dynamically Allocated Arrays

Add to a circular queue

```
void addq(element item)
{
    /* add an item to the queue */
    rear = (rear + 1) % capacity;
    if (front == rear) {
        queueFull();                /* double capacity */
        queue[++rear] = item;
    }
    else queue[rear] = item;
}
```


Report (until 27th September)

1. What is problem when we make a new Circular Queue with front = -1 and rear = -1.
2. Even though Queue size is Max space in a Circular Queue, why dose the Circular Queue uses only Max-1 space

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3.5 A Mazing Problem

❖ Rat in a maze

- Experimental psychologists train rats to search mazes for food



❖ For us, a nice application of *stacks*

- Searching the maze for an entrance to exit path.

Implementation in C

❖ Representation of a maze

- A two-dimensional array, *maze*
- 0 : the open paths, 1 : the barriers

maze[12,15]

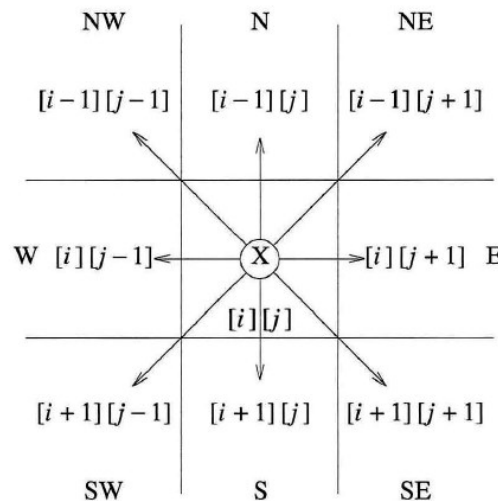
❖ Assumptions

- Rat starts at the top left
exits at the bottom right

entrance	0	1	0	0	0	1	1	0	0	0	1	1	1	1	1
	1	0	0	0	1	1	0	1	1	1	0	0	1	1	1
	0	1	1	0	0	0	0	1	1	1	1	0	0	1	1
	1	1	0	1	1	1	1	0	1	1	0	1	1	0	0
	1	1	0	1	0	0	1	0	1	1	1	1	1	1	1
	0	0	1	1	0	1	1	1	0	1	0	0	1	0	1
	0	0	1	1	0	1	1	1	0	1	0	0	1	0	1
	0	1	1	1	1	0	0	1	1	1	1	1	1	1	1
	0	0	1	1	0	1	1	0	1	1	1	1	1	0	1
	1	1	0	0	0	1	1	0	1	1	0	0	0	0	0
	0	0	1	1	1	1	1	0	0	0	1	1	1	1	0
	0	1	0	0	1	1	1	1	1	0	1	1	1	1	0
															exit

Implementation in C

- ❖ The current location of the rat in the maze
 - `maze[row][col]`
- ❖ The possible 8 moves from the current position



Implementation in C

❖ Not every position has eight neighbors.

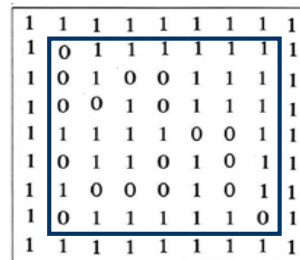
- If $[row, col]$ is on a border, then less than eight.



❖ To avoid checking for boarder conditions

- We can surround the maze by a boarder of ones.
- $\langle m \times p \text{ maze} \rangle$

- $(m+2) \times (p+2)$ array, *maze*
- entrance : *maze*[1][1]
- exit : *maze*[*m*][*p*]



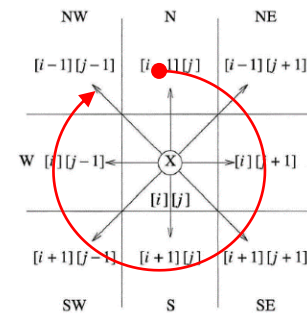
Implementation in C

❖ Predefining the possible directions to move in an array *move*

```
typedef struct {  
    short int vert;  
    short int horiz;  
} offsets;  
offsets move[8]; /*array of moves for each direction*/
```

The position of the next move, *maze[nextRow][nextCol]*
nextRow = row+ move[dir] .vert;
nextCol = col + move[dir] .horiz;

Name	Dir	<i>move[dir].vert</i>	<i>move[dir].horiz</i>
N	0	-1	0
NE	1	-1	1
E	2	0	1
SE	3	1	1
S	4	1	0
SW	5	1	-1
W	6	0	-1
NW	7	-1	-1



Implementation in C

- ❖ Since we do not know which choice is best,
 - we save our current position and
 - arbitrarily pick a possible move.
- ❖ By saving our current position,
 - we can *return to it* and *try another path* if we take a hopeless path.
- ❖ We examine the possible moves
 - starting from the north and moving clockwise.

Implementation in C

❖ Maintaining a second 2D array, *mark*

- to record the maze positions already checked
- initialize the *mark*'s entries to *zero*
- When we visit a position *maze[row][col]*, we change *mark[row][col]* to *one*

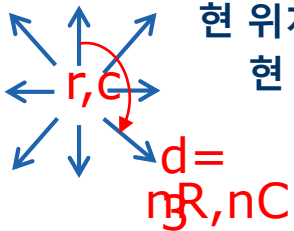
1	1	1	1	1	1	1	1	1
1	0	1	1	1	1	1	1	1
1	0	1	0	0	1	1	1	1
1	0	0	1	0	1	1	1	1
1	1	1	1	1	0	0	1	1
1	0	1	1	0	1	0	1	1
1	1	0	0	0	1	0	1	1
1	0	1	1	1	1	1	0	1
1	1	1	1	1	1	1	1	1

maze

0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

mark

Implementation in C



현 위치 (r, c) 에서 탐색방향 <8 이고 경로가 발견되지 않은 한 다음을 반복
현 위치 (r, c) 와 방향 d 에서 계산한 다음 위치 (nR, nC) 에 대해

- ① **if** 출구인 경우
 경로발견!
- ② **else if** 이동가능하고 이전에 방문하지 않은 경우
 push(백트래킹 후 탐색할 위치와 방향) // **push**($r, c, ++d$)
 다음위치(nR, nC)에 방문했음을 표시
 다음위치 (nR, nC) 으로 이동
- ③ **else**
 탐색방향증가 // $++d$

현 위치에서 탐색방향 $=8$ 이면 스택에서 돌아갈 위치를 가져와서
위의 과정을 반복, 스택이 **empty**이면 경로 발견 실패

Q1. 언제 **push**를 수행하는가?
Q2. 언제 **pop**을 수행하는가

Maze Search : Example

maze

entrance
[1,1]

0	1	1	1
0	1	1	0
1	0	0	1
1	1	0	1
1	0	1	0

exit
[5,4]

mark

0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0

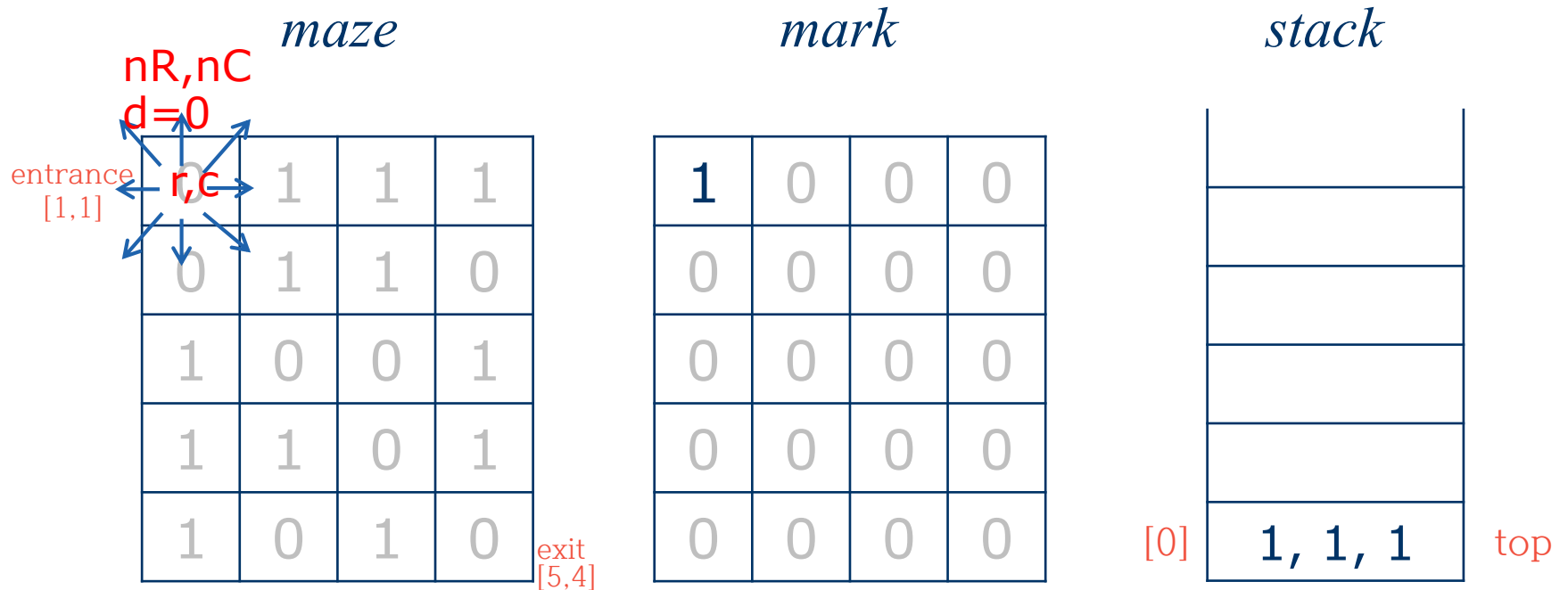
stack

[0]

(r, c, d) top

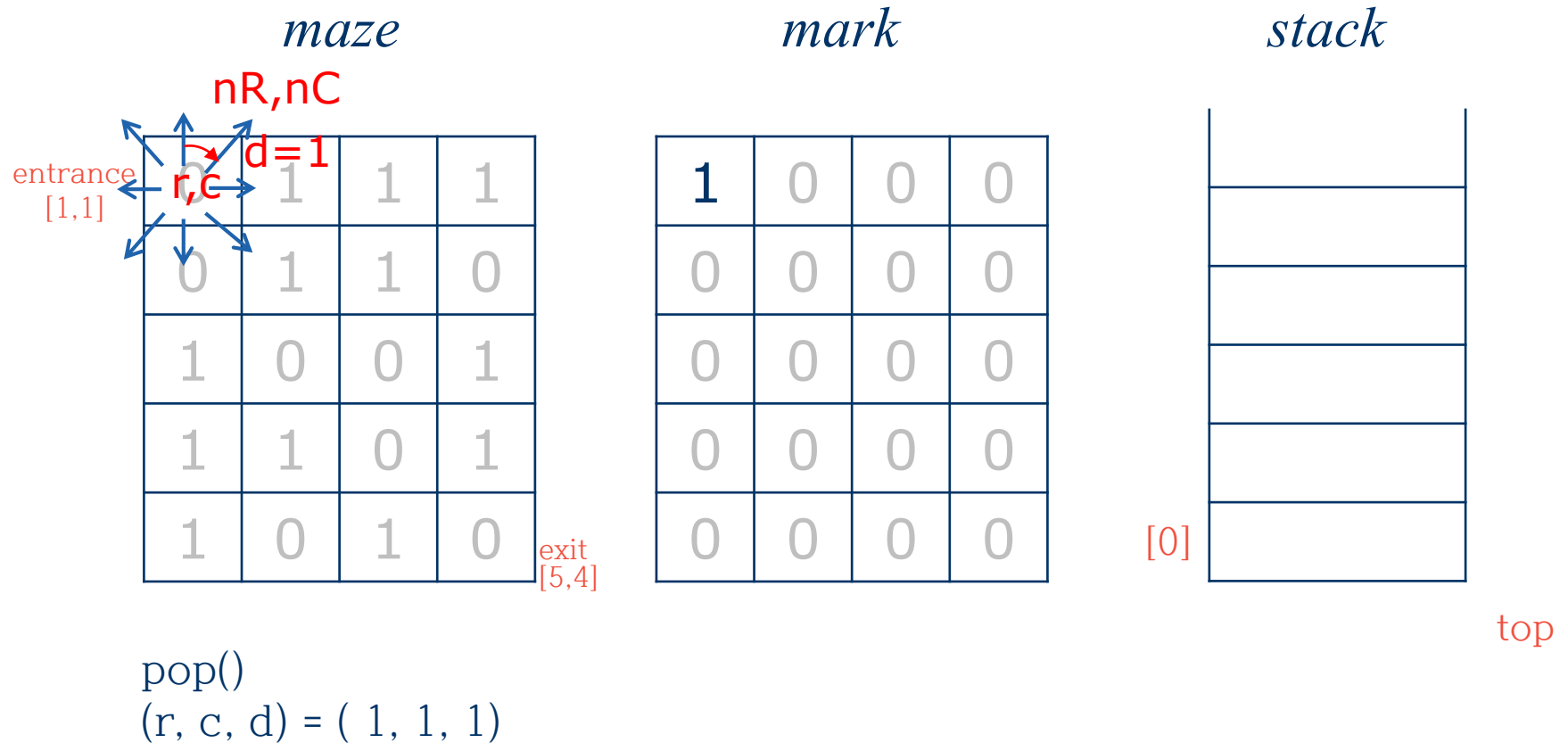
Program initialization

Maze Search : Example

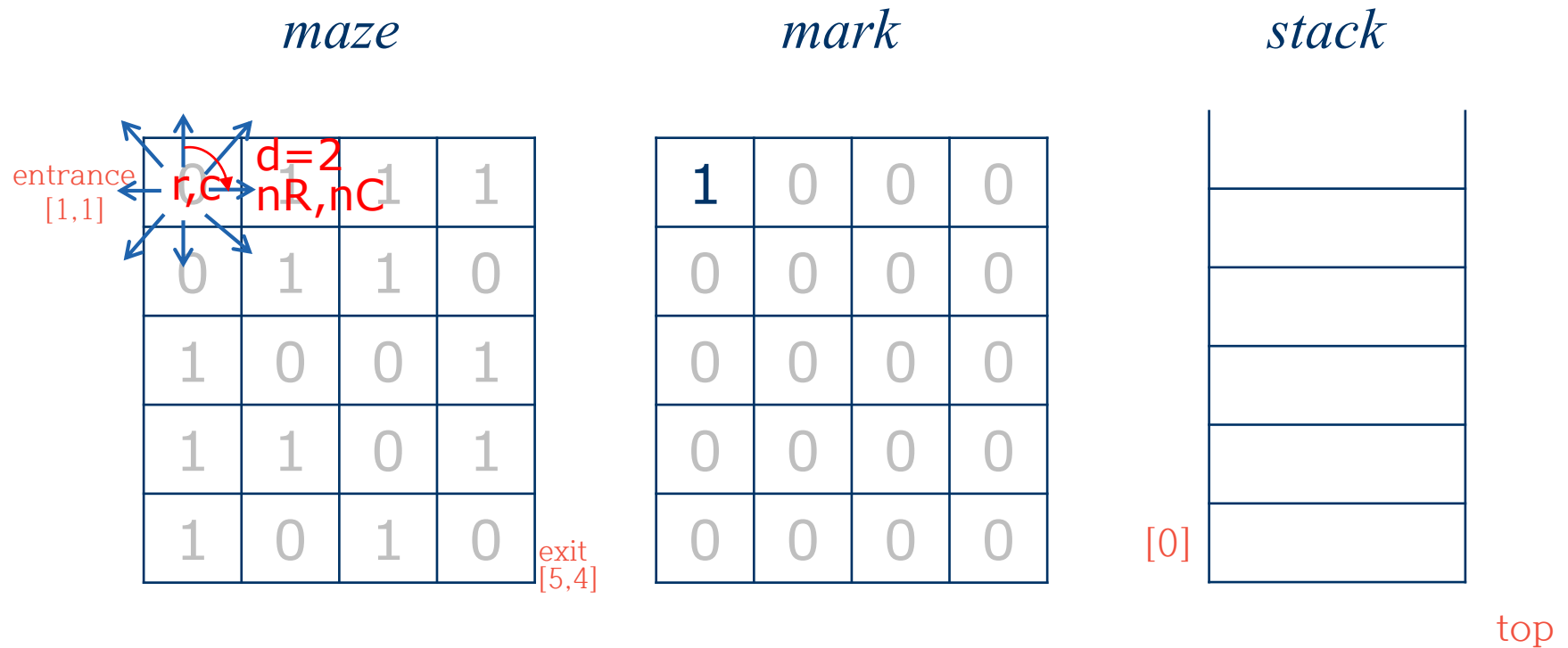


Initialization of function path()

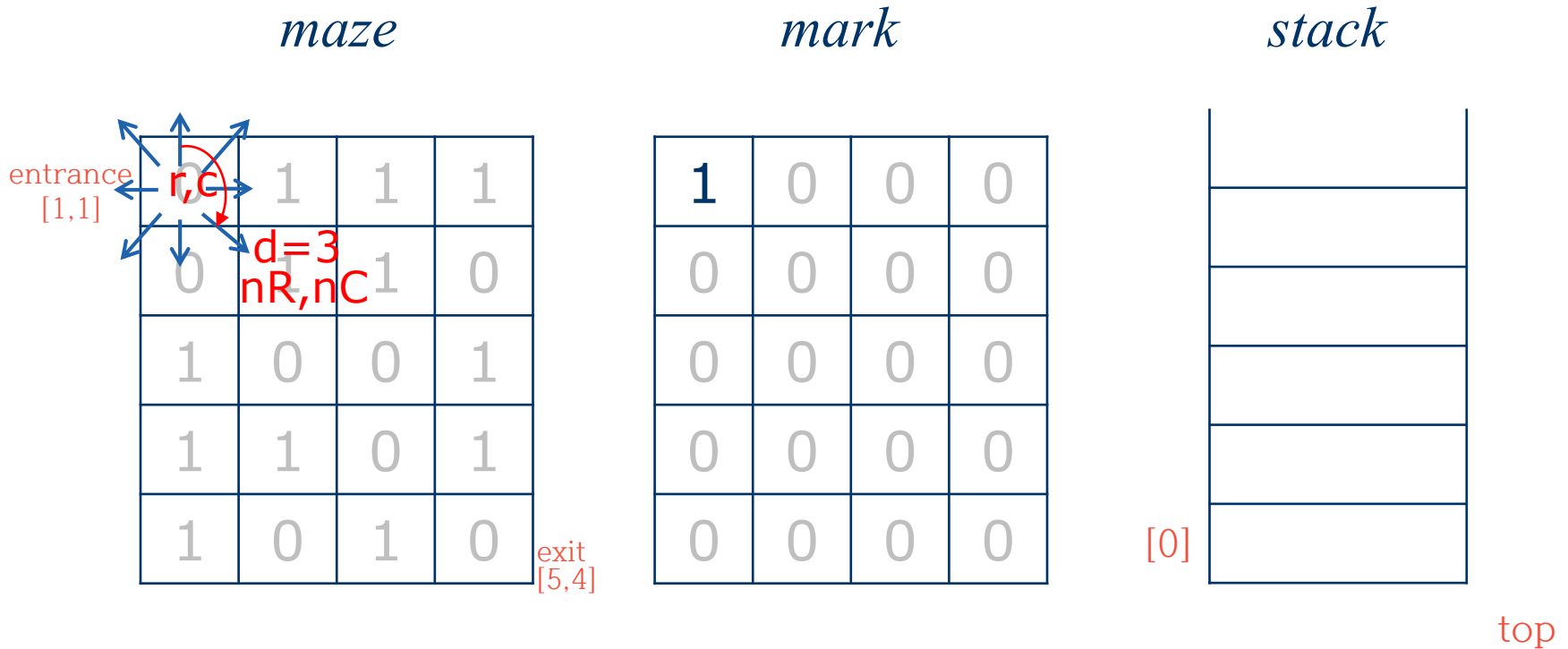
Maze Search : Example



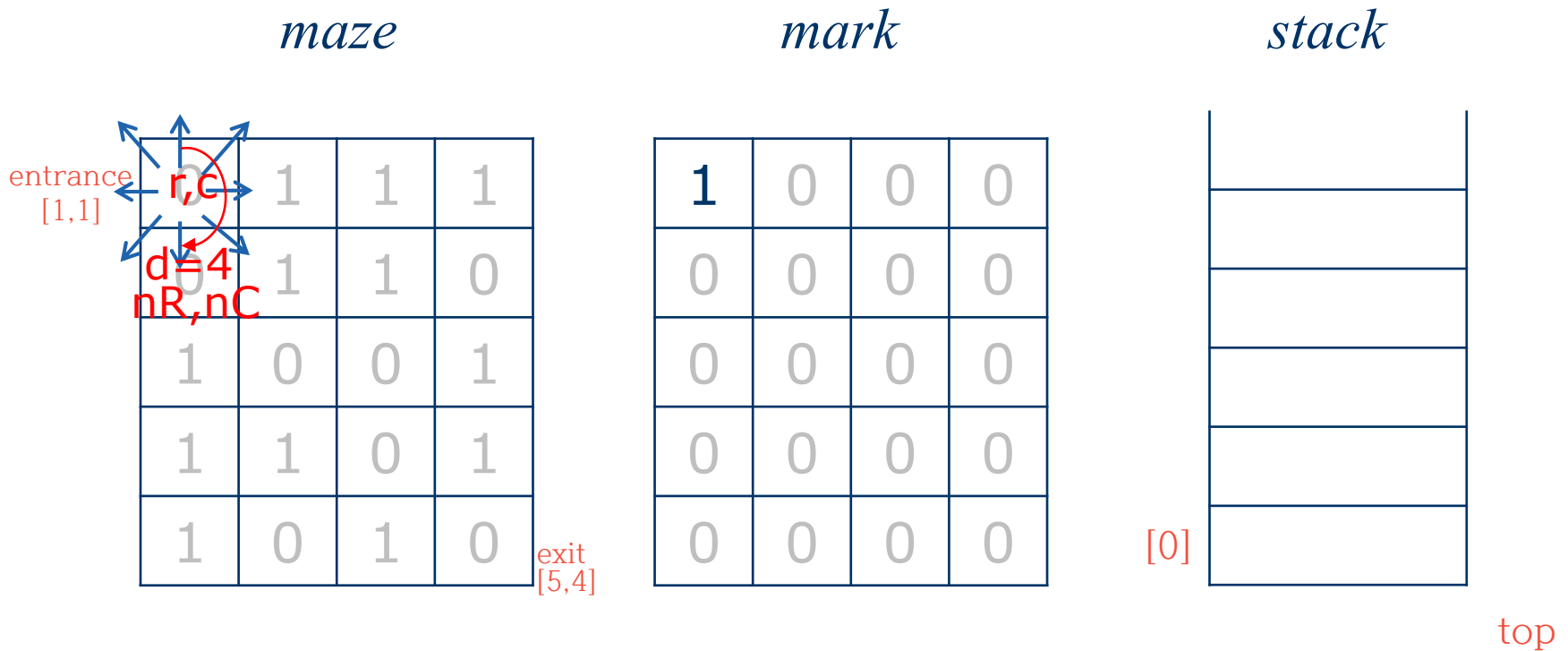
Maze Search : Example



Maze Search : Example



Maze Search : Example



Maze Search : Example

maze

entrance
[1,1]

$d=0$
 nR, nC

r, c

0	1	1	1
0	1	1	0
1	0	0	1
1	1	0	1
1	0	1	0

exit
[5,4]

mark

1	0	0	0
1	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0

stack

[0] 1, 1, 5 top

push(1, 1, 5)

(1, 2) 방문, $(r, c, d) = (1, 2, 0)$

Maze Search : Example

maze

entrance
[1,1]

0	1	1	1
0	1	1	0
1	0	0	1
1	1	0	1
1	0	1	0

exit
[5,4]

nR, nC
 $d=1$
 r, c

mark

1	0	0	0
1	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0

stack

[0]	1, 1, 5	top

Maze Search : Example

maze

entrance
[1,1]

0	1	1	1
0	1	1	0
1	0	0	1
1	1	0	1
1	0	1	0

exit
[5,4]

$d=2$
 r, c
 nR, nC

mark

1	0	0	0
1	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0

stack

[0] 1, 1, 5 top

Maze Search : Example

maze

entrance
[1,1]

0	1	1	1
1	0	1	0
1	0	0	1
1	1	0	1
1	0	1	0

exit
[5,4]

r, c

$d=3$
 nR, nC

mark

1	0	0	0
1	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0

stack

[0] 1, 1, 5 top

Maze Search : Example

maze

entrance
[1,1]

0	1	1	1
0	$d=0$ nR, nC	1	0
1	r, c	0	1
1	1	0	1
1	0	1	0

exit
[5,4]

mark

1	0	0	0
1	0	0	0
0	1	0	0
0	0	0	0
0	0	0	0

stack

2, 1, 4
1, 1, 5

[0] top

push(2, 1, 4)

(3, 2) 방문, (r, c, d) = (3, 2, 0)

Maze Search : Example

maze

entrance
[1,1]

0	1	1	1
0	1	1	0
1	0	0	1
1	1	0	1
1	0	1	0

exit
[5,4]

Diagram illustrating the maze search process. The maze is a 5x4 grid. The entrance is at [1,1] (row 1, column 1). The exit is at [5,4] (row 5, column 4). The current position is at [3,2] (row 3, column 2), marked with 'r,c'. The distance from the entrance is 'd=1'. The next position is at [3,3] (row 3, column 3), marked with 'nR,nC'. Blue arrows indicate the possible moves from the current position.

mark

1	0	0	0
1	0	0	0
0	1	0	0
0	0	0	0
0	0	0	0

stack

2, 1, 4
1, 1, 5

[0] top

Maze Search : Example

maze

entrance
[1,1]

0	1	1	1
0	1	1	0
1	0	0	1
1	1	0	1
1	0	1	0

exit
[5,4]

mark

1	0	0	0
1	0	0	0
0	1	0	0
0	0	0	0
0	0	0	0

stack

2, 1, 4
1, 1, 5

[0] top

Maze Search : Example

maze

entrance
[1,1]

0	1	1	1
0	1	$d=0$ nR, nC	0
1	0	r, c	1
1	1	0	1
1	0	1	0

exit
[5,4]

mark

1	0	0	0
1	0	0	0
0	1	1	0
0	0	0	0
0	0	0	0

stack

3, 2, 3
2, 1, 4
1, 1, 5

[0] top

push(3, 2, 3)

(3, 3) 방문, $(r, c, d) = (3, 3, 0)$

Maze Search : Example

maze

entrance
[1,1]

0	1	1	1
0	1	1	0
1	0	0	1
1	1	0	1
1	0	1	0

exit
[5,4]

nR,nC
d=1
r,c

mark

1	0	0	0
1	0	0	0
0	1	1	0
0	0	0	0
0	0	0	0

stack

3, 2, 3
2, 1, 4
1, 1, 5

top

[0]

Maze Search : Example

maze

entrance
[1,1]

0	1	1	1
0	1	1	0
1	0	0	1
1	1	0	1
1	0	1	0

exit
[5,4]

$d=0$
 nR, nC
 r, c

mark

1	0	0	0
1	0	0	1
0	1	1	0
0	0	0	0
0	0	0	0

stack

3, 3, 2
3, 2, 3
2, 1, 4
1, 1, 5

top

[0]

push(3, 3, 2)

(2, 4) 방문, $(r, c, d) = (2, 4, 0)$

Maze Search : Example

maze

entrance
[1,1]

0	1	1	1
0	1	1	0
1	0	0	1
1	1	0	1
1	0	1	0

nR,nC
d=1
r,c
exit
[5,4]

mark

1	0	0	0
1	0	0	1
0	1	1	0
0	0	0	0
0	0	0	0

stack

3, 3, 2
3, 2, 3
2, 1, 4
1, 1, 5

top
[0]

Maze Search : Example

maze

entrance
[1,1]

0	1	1	1
0	1	1	0
1	0	0	1
1	1	0	1
1	0	1	0

exit
[5,4]

mark

1	0	0	0
1	0	0	1
0	1	1	0
0	0	0	0
0	0	0	0

stack

3, 3, 2
3, 2, 3
2, 1, 4
1, 1, 5

top

[0]

Maze Search : Example

maze

entrance
[1,1]

0	1	1	1
0	1	1	1
1	0	0	1
1	1	0	1
1	0	1	0

exit
[5,4]

$d=3$
 nR, nC

mark

1	0	0	0
1	0	0	1
0	1	1	0
0	0	0	0
0	0	0	0

stack

3, 3, 2
3, 2, 3
2, 1, 4
1, 1, 5

top

[0]

Maze Search : Example

maze

entrance
[1,1]

0	1	1	1
0	1	1	0
1	0	0	0
1	1	0	1
1	0	1	0

exit
[5,4]

Diagram details: A red circle highlights the cell at row 3, column 4 (0-indexed). Blue arrows point from this cell to its four adjacent cells (up, down, left, right). Red text labels the current position as r, c and the distance as $d=4$, with nR, nC indicating the next step.

mark

1	0	0	0
1	0	0	1
0	1	1	0
0	0	0	0
0	0	0	0

stack

3, 3, 2
3, 2, 3
2, 1, 4
1, 1, 5

top

[0]

Maze Search : Example

maze

entrance
[1,1]

0	1	1	1
0	1	1	0
1	0	0	1
1	1	0	1
1	0	1	0

exit
[5,4]

d=5
nR,nC

r,c

mark

1	0	0	0
1	0	0	1
0	1	1	0
0	0	0	0
0	0	0	0

stack

3, 3, 2
3, 2, 3
2, 1, 4
1, 1, 5

top

[0]

Maze Search : Example

maze

entrance
[1,1]

0	1	1	1
0	1	1	1
1	0	0	1
1	1	0	1
1	0	1	0

exit
[5,4]

$d=6$
 nR, nC
 r, c

mark

1	0	0	0
1	0	0	1
0	1	1	0
0	0	0	0
0	0	0	0

stack

3, 3, 2
3, 2, 3
2, 1, 4
1, 1, 5

top

[0]

Maze Search : Example

maze

entrance
[1,1]

0	1	1	1
0	1	1	1
1	0	0	1
1	1	0	1
1	0	1	0

exit
[5,4]

$d=7$
 nR, nC

r, c

mark

1	0	0	0
1	0	0	1
0	1	1	0
0	0	0	0
0	0	0	0

stack

3, 3, 2
3, 2, 3
2, 1, 4
1, 1, 5

top

[0]

Maze Search : Example

maze

entrance
[1,1]

0	1	1	1
0	1	1	1
1	0	0	1
1	1	0	1
1	0	1	0

exit
[5,4]

$d=8$

r,c

mark

1	0	0	0
1	0	0	1
0	1	1	0
0	0	0	0
0	0	0	0

stack

3, 3, 2
3, 2, 3
2, 1, 4
1, 1, 5

top

[0]

($d < 8$ && !found) ? No!

(top > -1 && !found) ? Yes!

Maze Search : Example

maze

entrance
[1,1]

0	1	1	1
0	1	1	0
1	0	0	1
1	1	0	1
1	0	1	0

exit
[5,4]

$d=2$
 r, c
 nR, nC

mark

1	0	0	0
1	0	0	1
0	1	1	0
0	0	0	0
0	0	0	0

stack

3, 2, 3
2, 1, 4
1, 1, 5

top

[0]

pop()
(r, c, d) = (3, 3, 2)

Maze Search : Example

maze

entrance
[1,1]

0	1	1	1
0	1	1	0
1	0	0	1
1	1	0	1
1	0	1	0

exit
[5,4]

$d=3$
 nR, nC

mark

1	0	0	0
1	0	0	1
0	1	1	0
0	0	0	0
0	0	0	0

stack

3, 2, 3
2, 1, 4
1, 1, 5

top

[0]

Maze Search : Example

maze

entrance
[1,1]

0	1	1	1
0	1	1	0
1	0	0	1
1	1	0	1
1	0	1	0

exit
[5,4]

Diagram details: A red circle is centered at the cell (3,3) with the label r,c in red. Eight blue arrows point outwards from this circle to the adjacent cells. A red arrow points from the cell (4,3) to the cell (4,4), with the label $d=4$ and nR,nC in red.

mark

1	0	0	0
1	0	0	1
0	1	1	0
0	0	0	0
0	0	0	0

stack

3, 2, 3
2, 1, 4
1, 1, 5

top

[0]

Maze Search : Example

maze

entrance
[1,1]

0	1	1	1
0	1	1	0
1	0	0	1
1	1	0	1
1	0	1	0

exit
[5,4]

$d=0$
 nR, nC
 r, c

mark

1	0	0	0
1	0	0	1
0	1	1	0
0	0	1	0
0	0	0	0

stack

3, 3, 5
3, 2, 3
2, 1, 4
1, 1, 5

[0] top

push(3, 3, 5)

(4, 3) 방문, $(r, c, d) = (4, 3, 0)$

Maze Search : Example

maze

entrance
[1,1]

0	1	1	1
0	1	1	0
1	0	0	1
1	1	0	1
1	0	1	0

exit
[5,4]

nR,nC
d=1
r,c

mark

1	0	0	0
1	0	0	1
0	1	1	0
0	0	1	0
0	0	0	0

stack

3, 3, 5
3, 2, 3
2, 1, 4
1, 1, 5

top

[0]

Maze Search : Example

maze

entrance
[1,1]

0	1	1	1
0	1	1	0
1	0	0	1
1	1	0	1
1	0	1	0

exit
[5,4]

$d=2$
 $r, c \rightarrow nR, nC$

mark

1	0	0	0
1	0	0	1
0	1	1	0
0	0	1	0
0	0	0	0

stack

3, 3, 5
3, 2, 3
2, 1, 4
1, 1, 5

top

[0]

Maze Search : Example

maze

entrance
[1,1]

0	1	1	1
0	1	1	0
1	0	0	1
1	1	0	1
1	0	1	0

d=3
nR,nC
exit
[5,4]

mark

1	0	0	0
1	0	0	1
0	1	1	0
0	0	1	0
0	0	0	0

stack

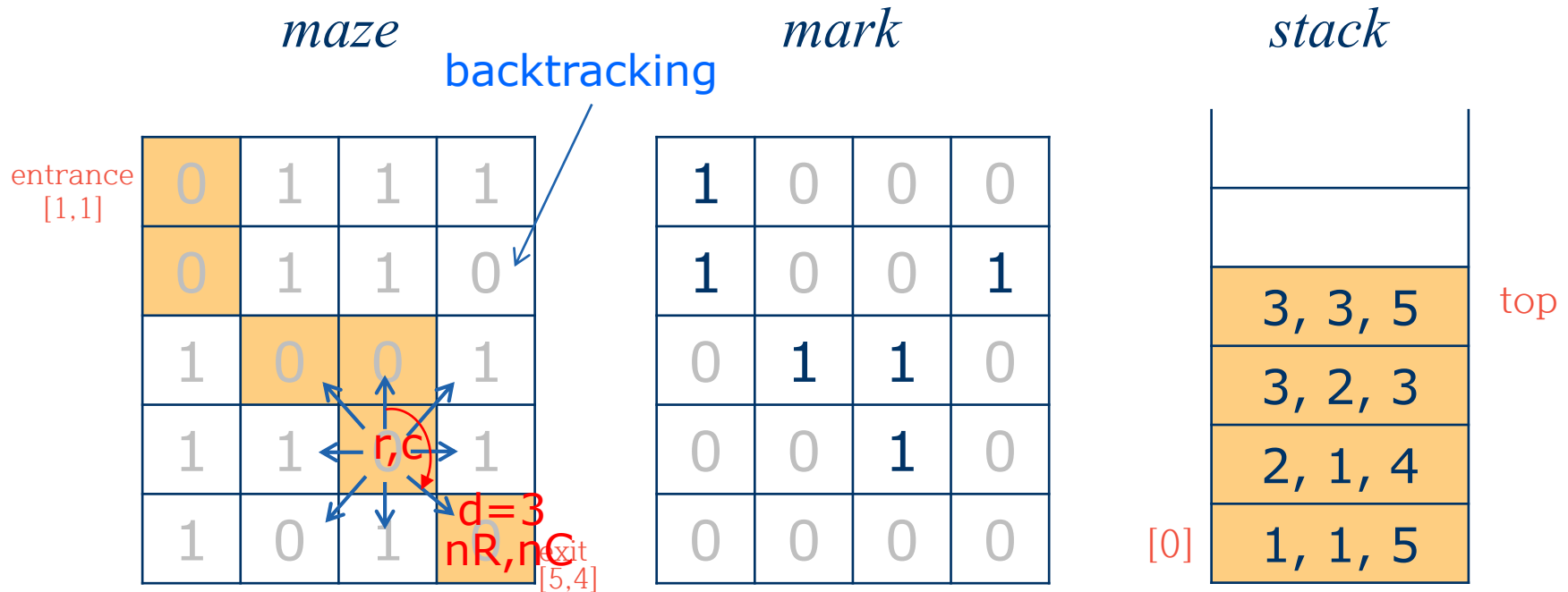
3, 3, 5
3, 2, 3
2, 1, 4
1, 1, 5

top

[0]

다음 위치 (nR, nC)가 출구(EXIT_ROW, EXIT_COL)임
경로발견!

Maze Search : Example



< 경로출력순서 >

① stack[0] → stack[top]

② 현재 위치 (r, c)

③ 출구 위치 (EXIT_ROW, EXIT_COL)

path: (1, 1), (2, 1), (3, 2), (3, 3), (4, 3), (5, 4)

Implementation in C

❖ Stack

- Use the implementation of section 3.1 or 3.2

```
typedef struct {  
    short int row;  
    short int col;  
    short int dir;  
} element;
```

- Capacity

- Each position in the maze is visited no more than once.
- An $m \times p$ maze has at most mp zeroes.
- ***mp*** is sufficient for the stack capacity.

Implementation in C

```
initialize a stack to the maze's entrance coordinates and
direction to north;
while (stack is not empty) {
    /* move to position at top of stack */
    <row,col,dir> = delete from top of stack;
    while (there are more moves from current position) {
        <nextRow, nextCol> = coordinates of next move;
        dir = direction of move;
        if ((nextRow == EXIT-ROW) && (nextCol == EXIT-COL))
            success;
        if (maze[nextRow][nextCol] == 0 &&
            mark[nextRow][nextCol] == 0) {
            /* legal move and haven't been there */
            mark[nextRow][nextCol] = 1;
            /* save current position and direction */
            add <row,col,dir> to the top of the stack;
            row = nextRow;
            col = nextCol;
            dir = north;
        }
    }
}
printf("No path found\n");
```

Program 3.11: Initial maze algorithm

Implementation in C

```
void path(void)
/* output a path through the maze if such a path exists */
int i, row, col, nextRow, nextCol, dir, found = FALSE;
element position;
mark[1][1] = 1; top = 0;
stack[0].row = 1; stack[0].col = 1; stack[0].dir = 1;
while (top > -1 && !found) {
    position = pop();
    row = position.row; col = position.col;
    dir = position.dir;
    while (dir < 8 && !found) {
        /* move in direction dir */
        nextRow = row + move[dir].vert;
        nextCol = col + move[dir].horiz;
        if (nextRow == EXIT-ROW && nextCol == EXIT-COL)
            found = TRUE;
        else if (!maze[nextRow][nextCol] &&
            ! mark[nextRow][nextCol]) {
            mark[nextRow][nextCol] = 1;
            position.row = row; position.col = col;
            position.dir = ++dir;
            push(position);
            row = nextRow; col = nextCol; dir = 0;
        }
        else ++dir;
    }
}
if (found) {
    printf("The path is:\n");
    printf("row  col\n");
    for (i = 0; i <= top; i++)
        printf("%2d%5d", stack[i].row, stack[i].col);
    printf("%2d%5d\n", row, col);
    printf("%2d%5d\n", EXIT-ROW, EXIT-COL);
}
else printf("The maze does not have a path\n");
}
```

Program 3.12: Maze search function

Analysis of *path*:

- each position within the maze is visited no more than once,
- worst case complexity : $O(mp)$, for $m \times p$ maze