



Chap 5. Trees (5)

Contents

5.1 Introduction

5.2 Binary Trees

5.3 Binary Trees Traversals

5.4 Additional Binary Tree Operations

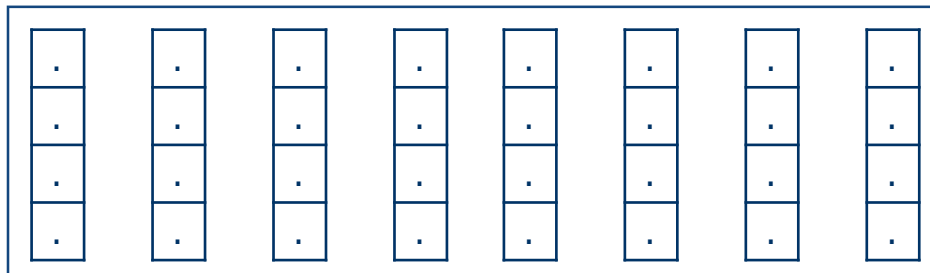
5.5 Threaded Binary Trees

5.6 Heaps

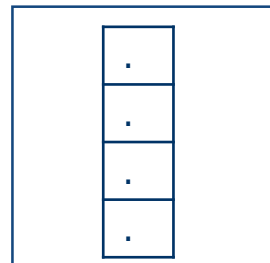
5.7 Binary Search Trees

5.8 Selection Trees

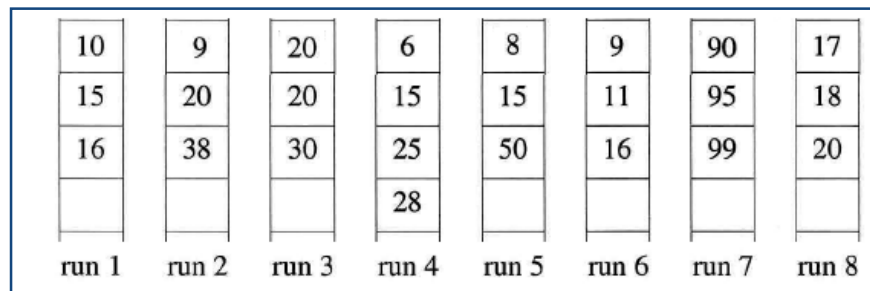
5.8 Selection Trees



External storage



Internal memory



External storage

ordered sequences

5.8 Selection Trees

5.8.1 Introduction

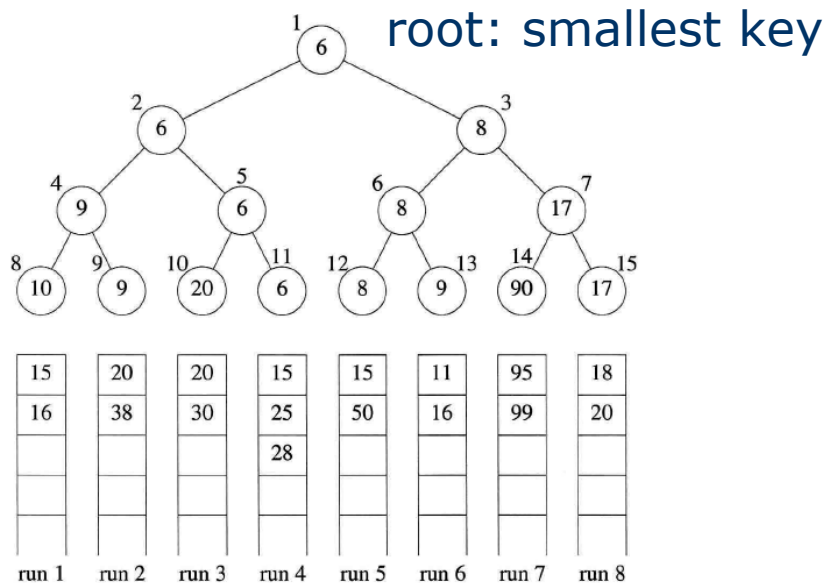
- ❖ **k ordered sequences, called *runs*, to be merged into a single ordered sequence.**

10	9	20	6	8	9	90	17
15	20	20	15	15	11	95	18
16	38	30	25	50	16	99	20
			28				
run 1	run 2	run 3	run 4	run 5	run 6	run 7	run 8

- ❖ The merging task can be accomplished by repeatedly outputting the record with the smallest key.
- ❖ For $k > 2$, we can *reduce the number of comparisons* by using the **selection tree; winner trees and loser trees.**

5.8.2 Winner Trees

- ❖ A **winner tree** is a complete binary tree in which each node represents the smaller of its two children.



sequential allocation
(complete binary tree)

Each node contains only a
pointer to the record

Leaf node: the first record
in the corresponding run

Runs : ordered sequences

Figure 5.32: Winner tree for $k=8$, showing the first three keys in each of the eight runs

5.8.2 Winner Trees

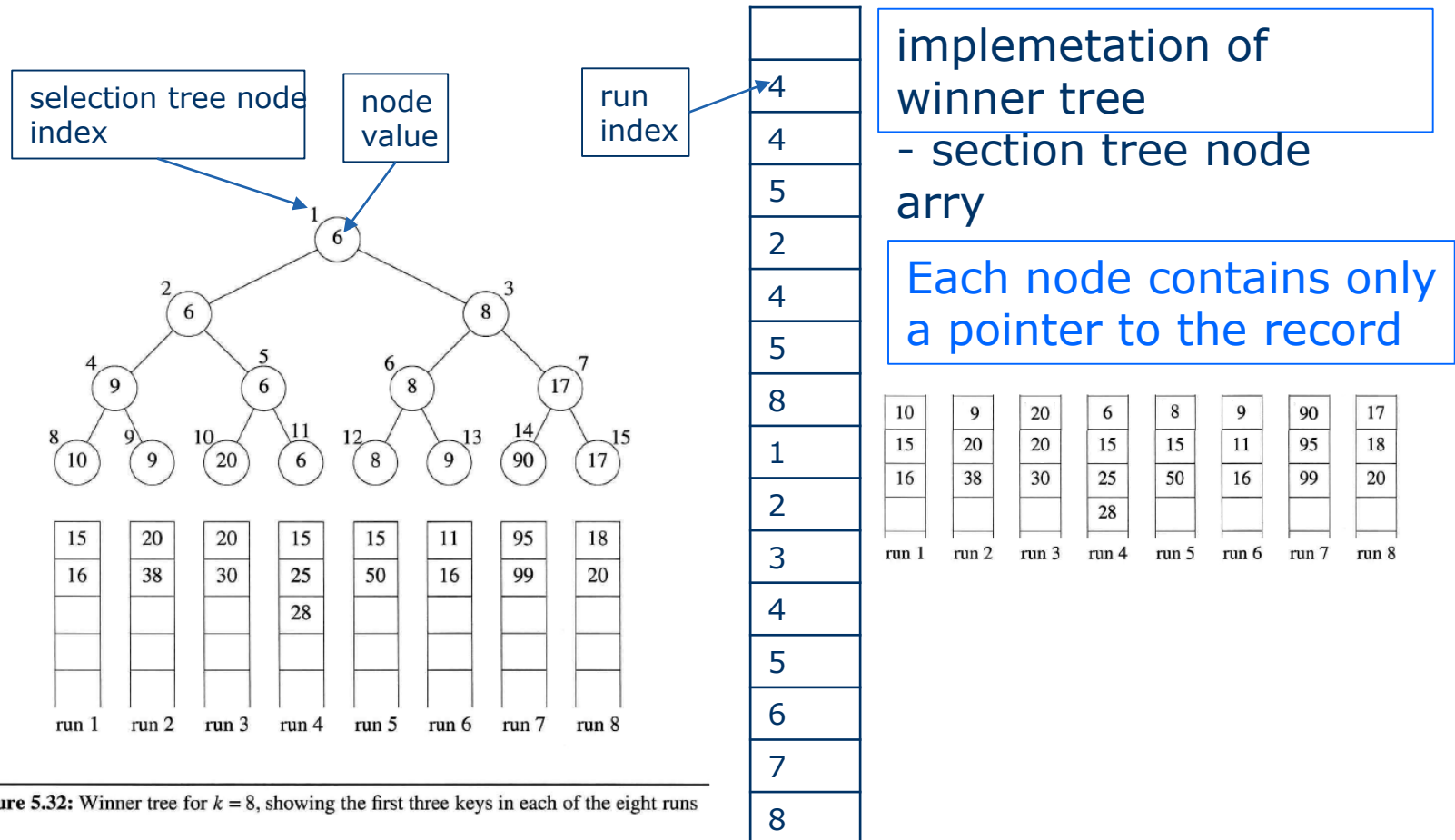
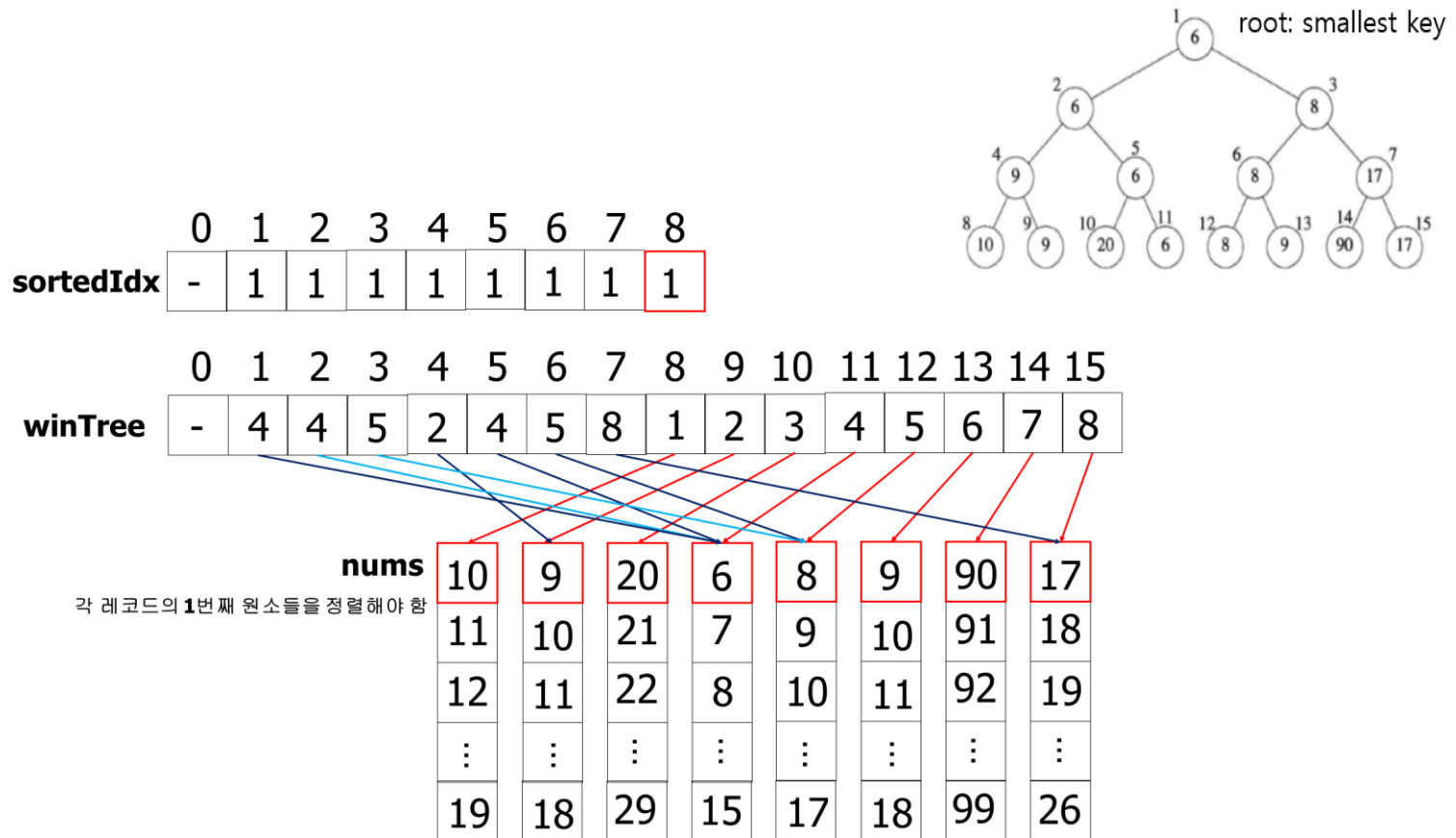


Figure 5.32: Winner tree for $k = 8$, showing the first three keys in each of the eight runs

5.8.2 Winner Trees



5.8.2 Winner Trees

4(5)
4(2)
5
2
4
5
8
1
2
3
4
5
6
7
8

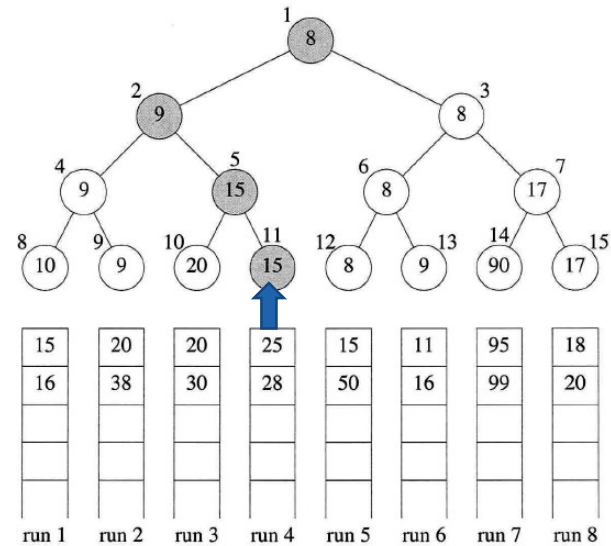
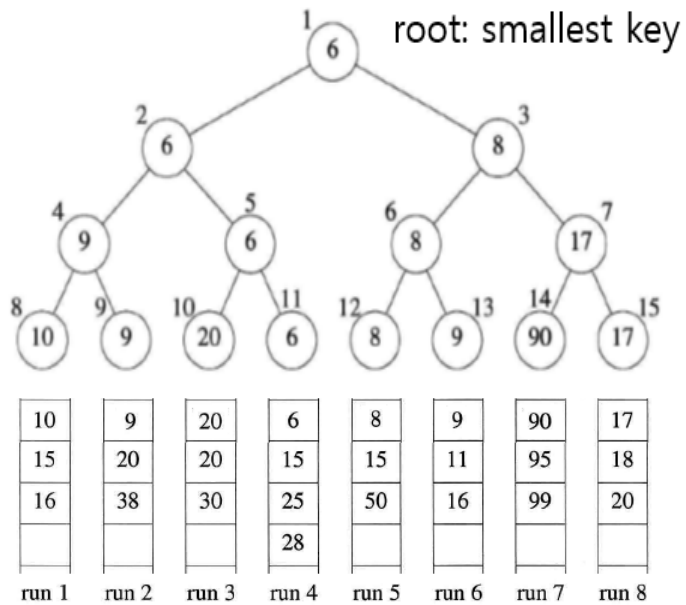
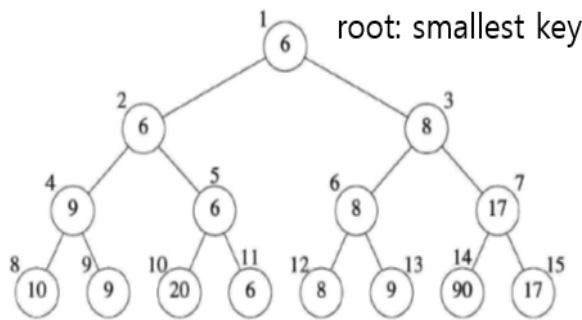


Figure 5.33: Winner tree of Figure 5.32 after one record has been output and the tree restructured (nodes that were changed are shaded)

5.8.2 Winner Trees

❖ Analysis of merging runs using winner trees

- Let n be the number of records in all k runs.
- The number of levels in the tree is $\lceil \log_2 k + 1 \rceil$
- The time to restructure the tree is $O(\log_2 k)$.
- The time required to merge all n records is $O(n \log_2 k)$.
- The time required to set up the selection tree the first time is $O(k)$.
- The total time needed to merge the k runs is $O(n \log_2 k)$.



5.8.3 Loser Trees

- ❖ A selection tree in which each nonleaf node retains a pointer to the loser is called a *loser tree*.

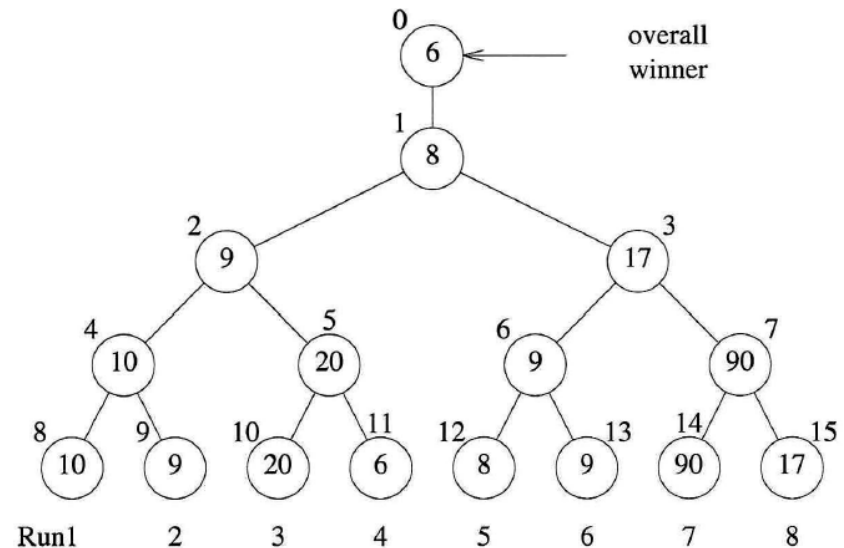
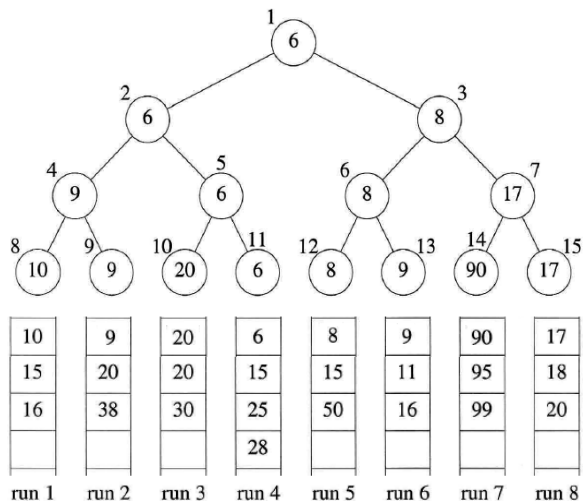


Figure 5.34: Loser tree corresponding to winner tree of Figure 5.32

5.8.3 Loser Trees

- ❖ In winner tree, following the output of the overall winner, the tree is restructured by playing tournaments along the path from node 11 to node 1.
- ❖ *In loser, the records with which the tournaments are to be played are readily available from the parent nodes.*
 - As a result, sibling nodes along the path from 11 to 1 are not accessed.

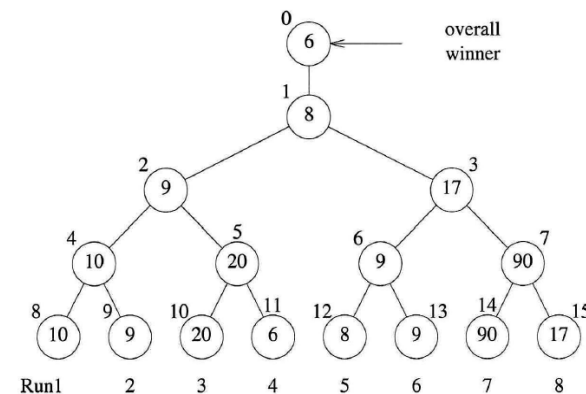
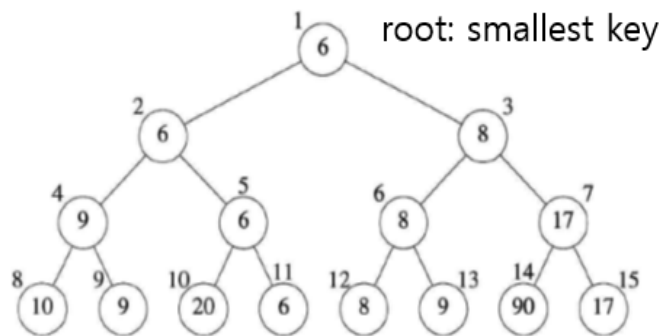


Figure 5.34: Loser tree corresponding to winner tree of Figure 5.32