Chap 4. Linked Lists (2)

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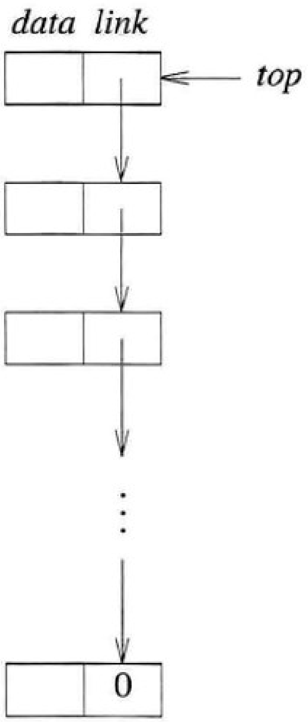
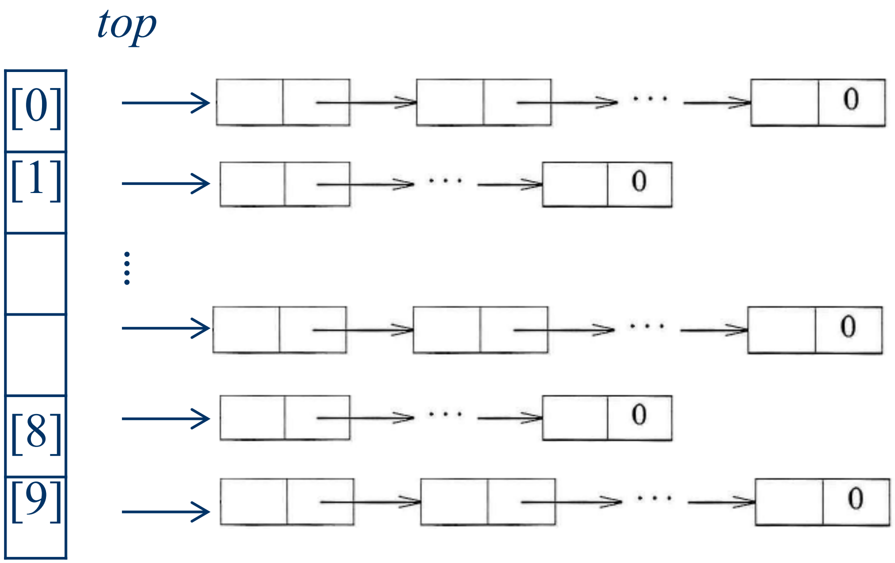
4.3 Linked Stacks And Queues



(a) Linked stack

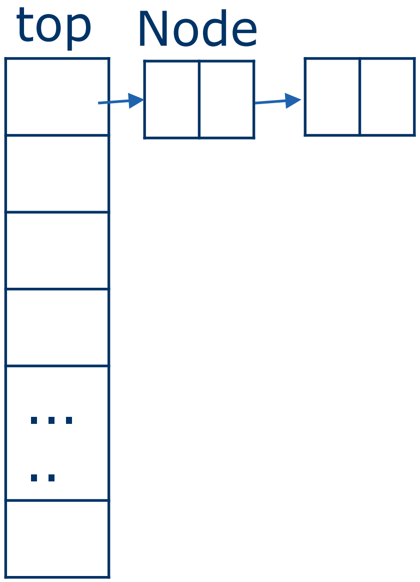


(b) Linked multi stack

4.3 Linked Stacks And Queues

- Representing *n≤MAX_STACKS* stacks simultaneously

```
#define MAX-STACKS 10 /* maximum number of stacks */
typedef struct {
        int key;
        /* other fields */
        } element;
typedef struct stack *stackPointer;
typedef struct stack {
        element data;
        stackPointer link;
        } ;
stackPointer top[MAX-STACKS];
```

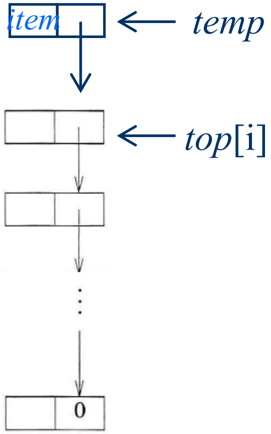


- *top [i] =NULL, 0≤ i <MAX –STACKS* //Initial conditions for the stacks
- *top* [i] ==*NULL* iff the ith stack is empty //Boundary condition for the *i*th stack

4.3 Linked Stacks And Queues

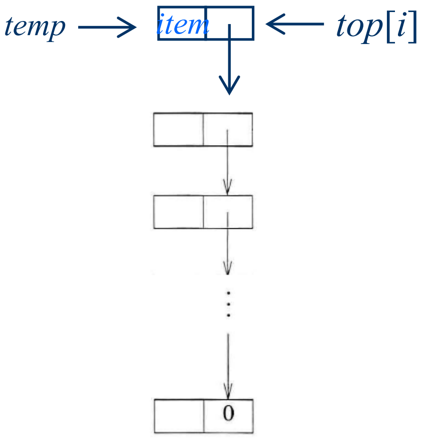
```
void push(int i, element item)
{/* add item to the ith stack */
        stackPointer temp;
        MALLOC(temp, sizeof(*temp) );
        temp->tdata = item;
        temp->tlink = top[i];
        top[i] = temp;
}
```

Program 4.5: Add to a linked stack



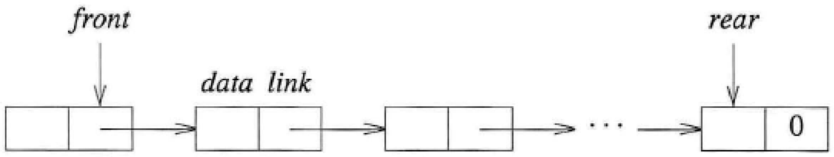
4.3 Linked Stacks And Queues

```
element pop(int i)
{/* remove top element from the ith stack */
        element item;
        stackPointer temp= top[i];
        if (!temp)
                return stackEmpty();
        item = temp->data;
        top[i] = temp->link;
        free (temp);
        return item;
}
```

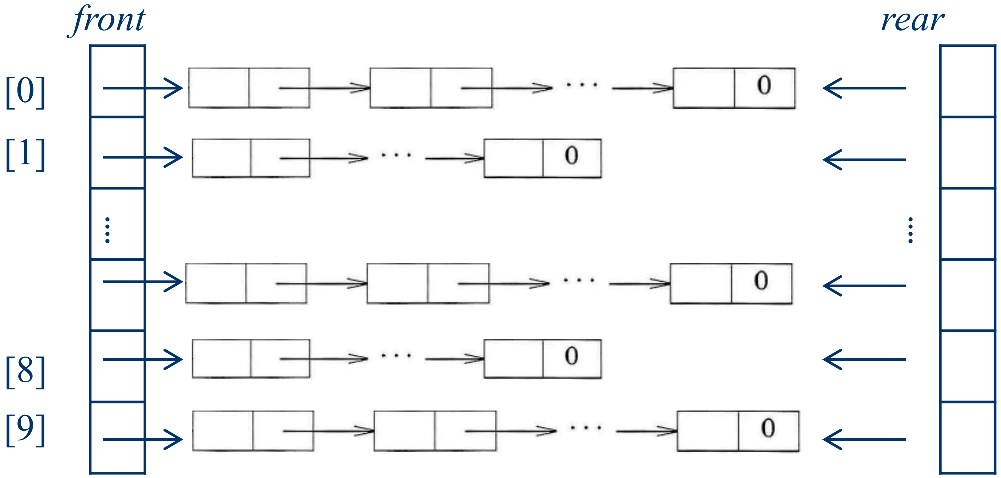


Program 4.6: Delete from a linked stack

4.3 Linked Stacks And Queues



(a) Linked queue



(b) Linked multi queue

4.3 Linked Stacks And Queues

- Representing *n≤MAX_QUEUES* queues simultaneously

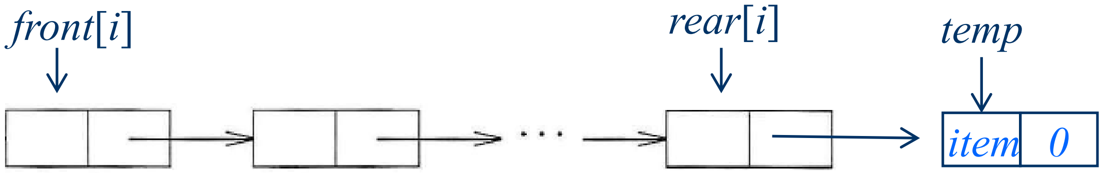
```
#define MAX-QUEUES 10 // maximum number of queues
typedef struct queue *queuePointer;
typedef struct queue {
       element data;
       queuePointer link;
       } ;
queuePointer front[MAX-QUEUES], rear[MAX-QUEUES];
```

- *front[i] =NULL,* 0 ≤ i <*MAX-QUEUES* Initial conditions for the queues
- *front[i]* =NULL iff the *ith* queue is empty Boundary condition for the *i*th queue

4.3 Linked Stacks And Queues

```
void addq(i, item)
{/* add item to the rear of queue i */
        queuePointer temp;
        MALLOC(temp, sizeof(*temp));
        temp->data = item;
        temp->link = NULL;
        if (front [i])
                rear[i]->link =temp;
        else
                front[i] =temp;
        rear[i] =temp;
}
```

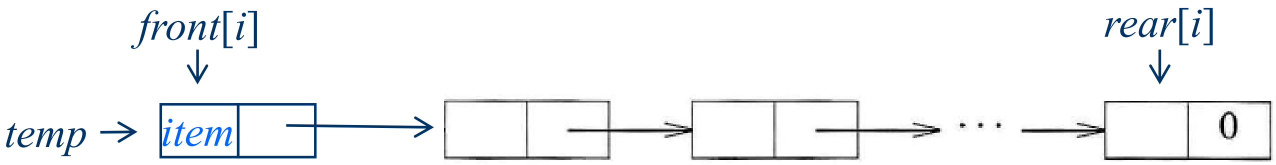
- **Program 4.7: Add to the rear of a linked queue**



4.3 Linked Stacks And Queues

```
element deleteq(int i)
{/* delete an element from queue i */
            element item;
      queuePointer temp= front[i];
      if (!temp)
            return queueEmpty();
      item = temp->data;
      front[i]= temp->link;
      free (temp) ;
      return item;
}
```

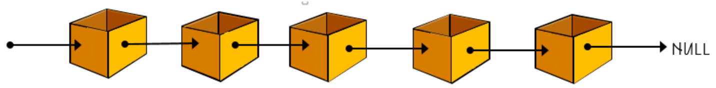
- **Program 4.8: Delete from the front of a linked queue**



Circular List Representation

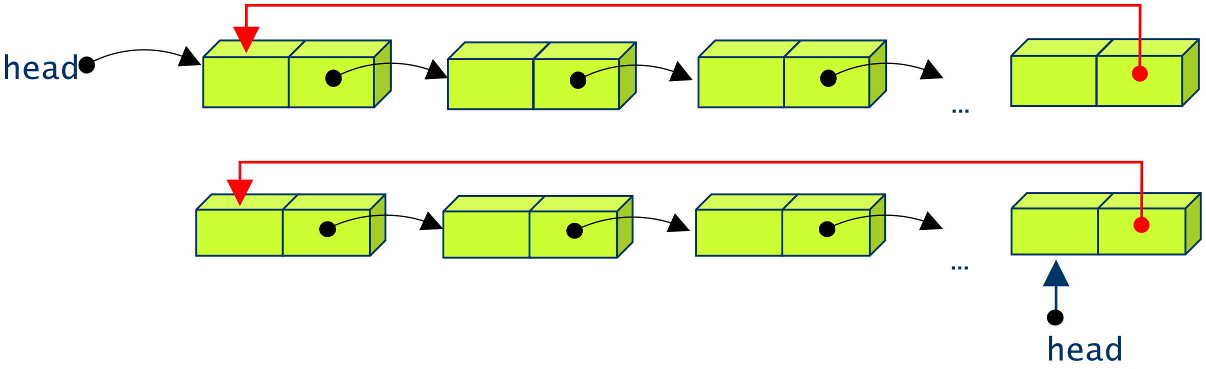
Chain

- A singly linked list in which the last node has a null link



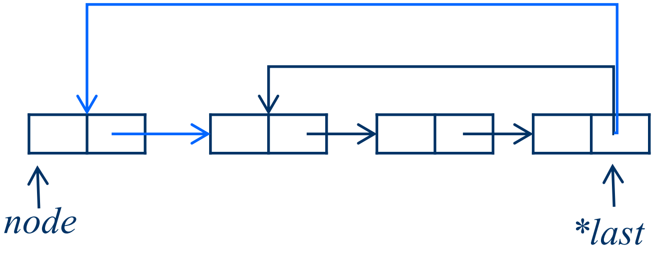
Circular list

- The link filed of the last node points to the first node in the list



Operations For Circularly Linked Lists

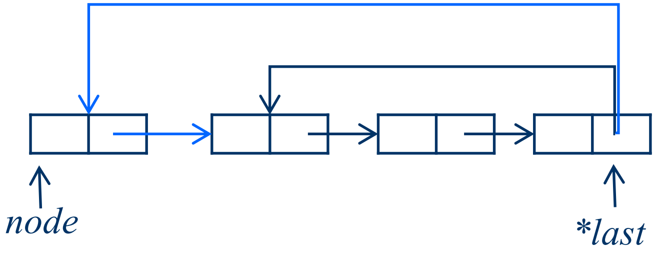
```
void insertFront(listPointer *last, listPointer node)
{/* insert node at the front of the circular list whose
     last node is last */
     if (!(*last)) {
     /* list is empty, change last to point to new entry */
          *last = node;
          node->link = node;
     }
     else {
     /* list is not empty, add new entry at front */
          node->link = (*last) ->link;
          (*last) ->link = node;
          }
}
```



Program 4.18: Inserting at the front of a list

Operations For Circularly Linked Lists

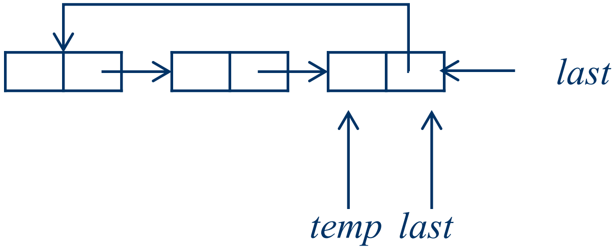
```
void insertLast(listPointer *last, listPointer node)
{/* insert node at the front of the circular list whose
        last node is last */
        if (!(*last)) {
        /* list is empty, change last to point to new entry */
                *last = node;
                node->link = node;
        }
        else {
        /* list is not empty, add new entry at front */
                node->link = (*last) ->link;
                (*last) ->link = node;
                *last = node;
}
```



Program 4.18: Inserting at the front of a list

Operations For Circularly Linked Lists

```
int length(listPointer last)
{/* find the length of the circular list last */
        listPointer temp;
        int count = 0;
        if (last)
                temp = last;
                do {
                count++;
                temp = temp->link;
                } while (temp !=last);
        }
        return count;
}
```



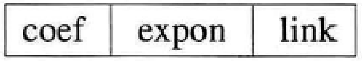
- **Program 4.19: Finding the length of a circular list**

4.4 Polynomials

4.4.1 Polynomial Representation

- **Polynomial**
- $A(x) = a_{m-1}x^{e_{m-1}} + \; \ldots\ldots \; + a_0x^{e_0}$
  - $e_{m-1} > e_{m-2} \ldots > e_1 > e_0 \geq 0$
  - a_i : nonzero coefficients
  - e_i : nonnegative integer exponents
- **Representation of Polynomial**

```
typedef struct polyNode *polyPointer;
typedef struct polyNode {
        int coef;
        int expon;
        polyPointer link;
        } ;
polyPointer a,b;
```



coef	expon	link

4.4 Polynomials

❖ Representation of polynomials

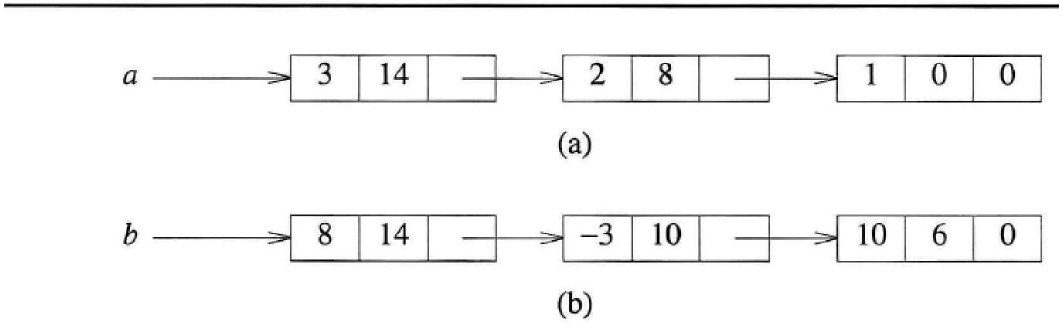


Figure 4.12: Representation of $3x^{14}+2x^{8}+1$ and $8x^{14}-3x^{10}+10x^{6}$

4.4.2 Adding Polynomials

$a = 3x^{14} + 2x^{8} + 1$

$b = 8x^{14} - 3x^{10} + 10x^{6}$

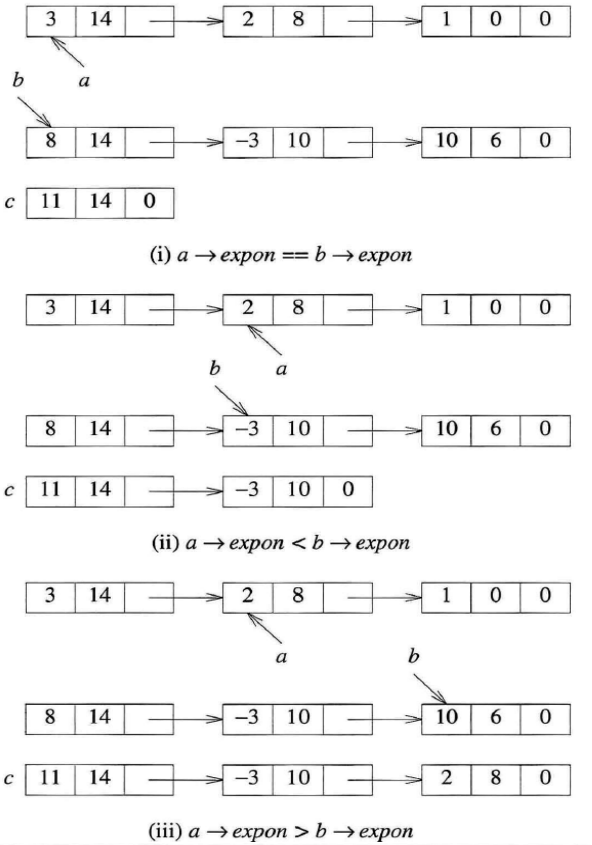
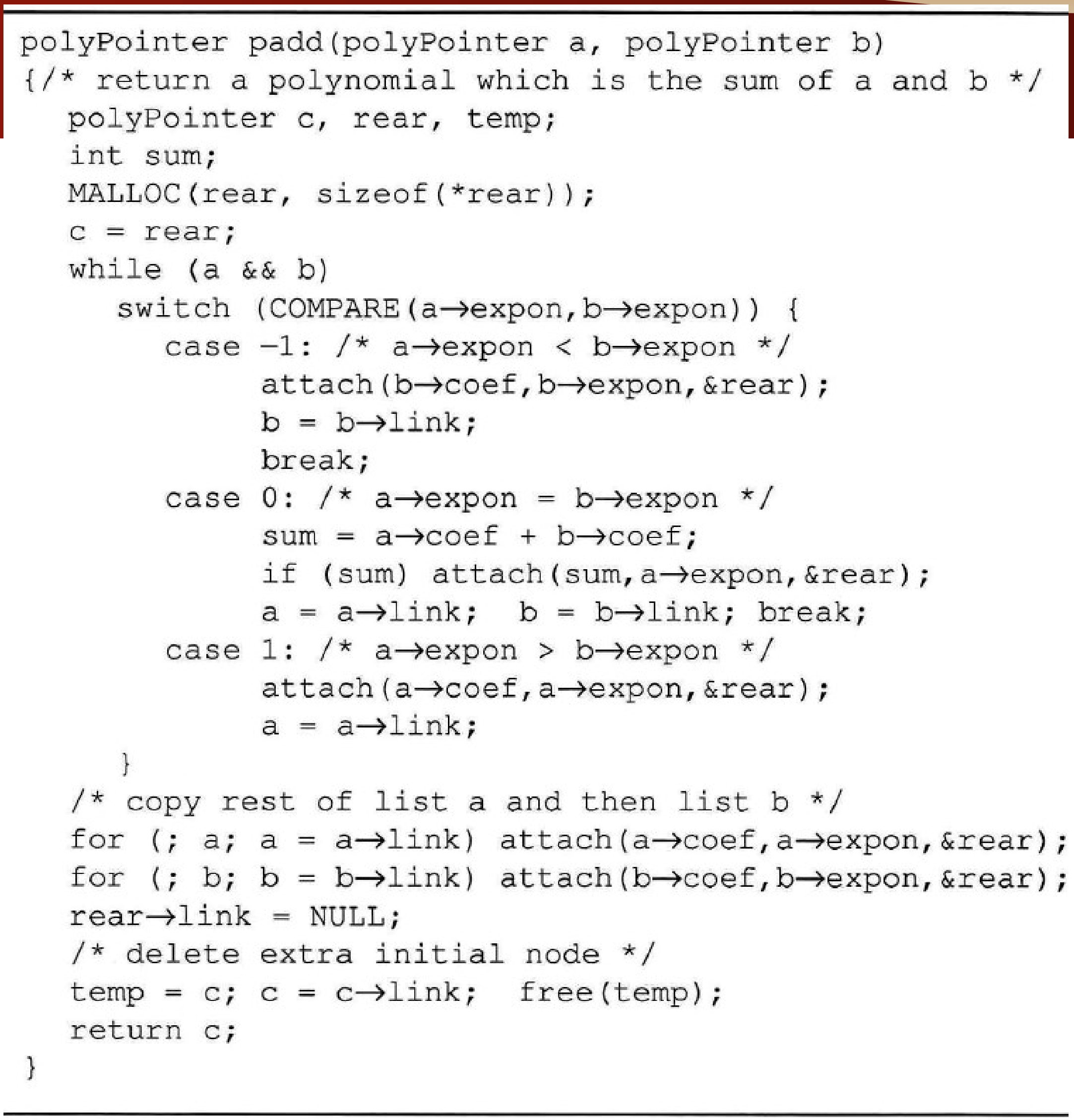


Figure 4.13: Generating the first three terms of $c = a + b$



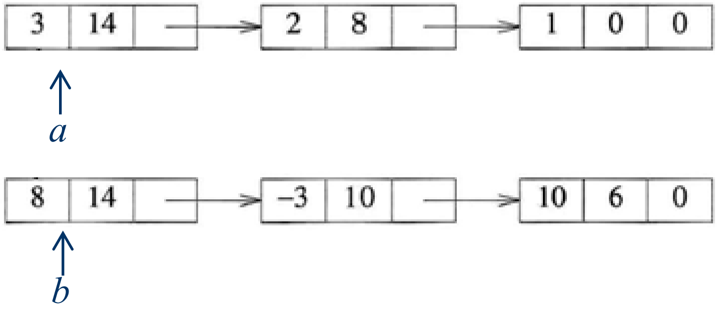
```
polyPointer padd(polyPointer a, polyPointer b)
{/* return a polynomial which is the sum of a and b */
  polyPointer c, rear, temp;
  int sum;
  MALLOC(rear, sizeof(*rear));
  c = rear;
  while (a && b)
    switch (COMPARE(a→expon,b→expon)) {
      case -1: /* a→expon < b→expon */
           attach(b→coef,b→expon,&rear);
           b = b→link;
           break;
      case 0: /* a→expon = b→expon */
           sum = a→coef + b→coef;
           if (sum) attach(sum,a→expon,&rear);
           a = a→link;  b = b→link; break;
      case 1: /* a→expon > b→expon */
           attach(a→coef,a→expon,&rear);
           a = a→link;
    }
  /* copy rest of list a and then list b */
  for (; a; a = a→link) attach(a→coef,a→expon,&rear);
  for (; b; b = b→link) attach(b→coef,b→expon,&rear);
  rear→link = NULL;
  /* delete extra initial node */
  temp = c; c = c→link;  free(temp);
  return c;
}
```

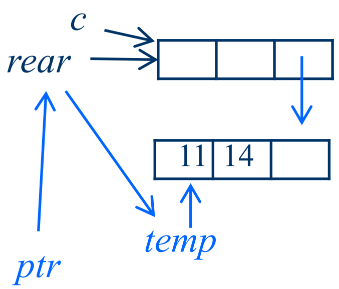
Program 4.9: Add two polynomials

4.4.2 Adding Polynomials

```
void attach(float coefficient, int exponent,
                  polyPointer *ptr)
{/* create a new node with coef = coefficient and expon =
    exponent, attach it to the node pointed to by ptr.
    ptr is updated to point to this new node */
   polyPointer temp;
   MALLOC(temp, sizeof(*temp));
   temp→coef = coefficient;
   temp→expon = exponent;
   (*ptr)→link = temp;
   *ptr = temp;
}
```

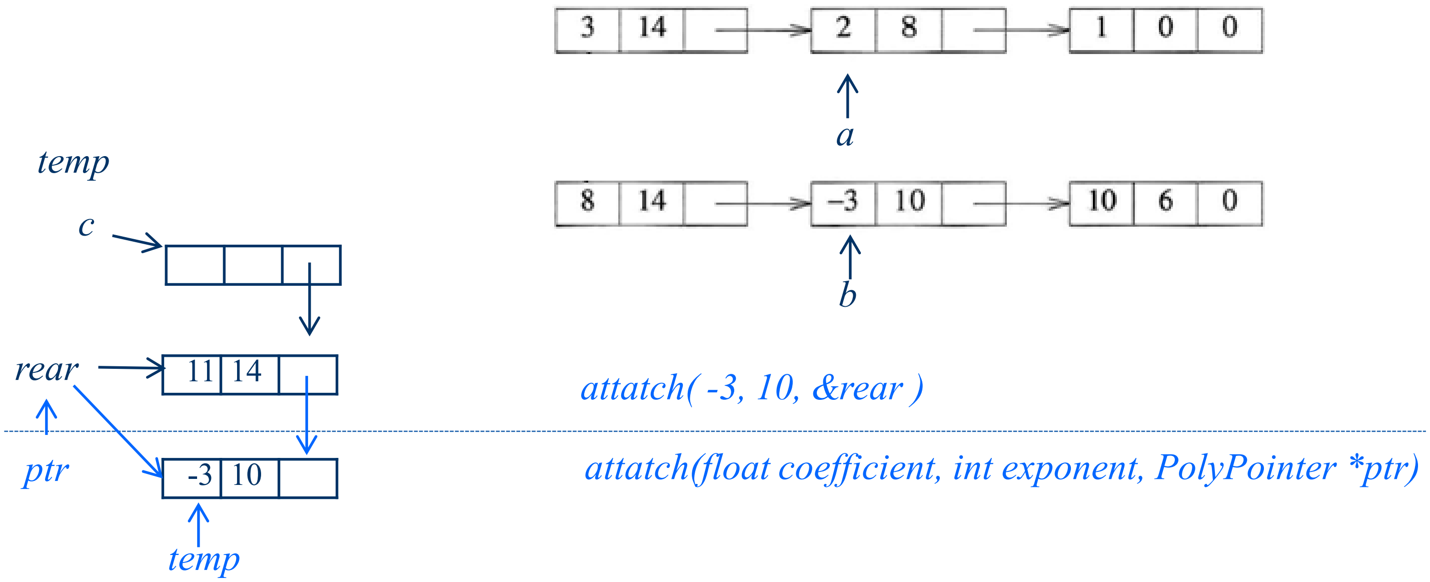
Program 4.10: Attach a node to the end of a list





attatch(11, 14, &rear)

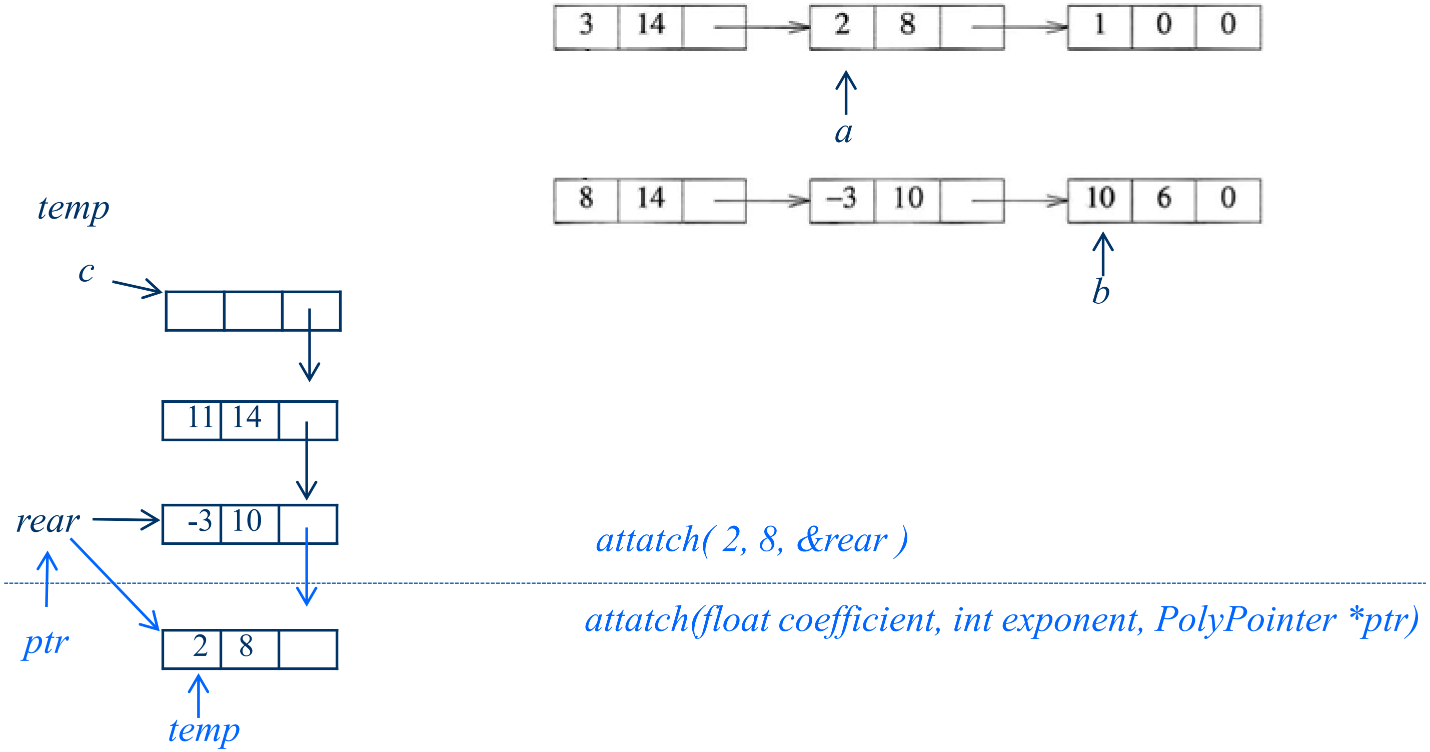
4.4.2 Adding Polynomials



attatch(-3, 10, &rear)

*attatch(float coefficient, int exponent, PolyPointer *ptr)*

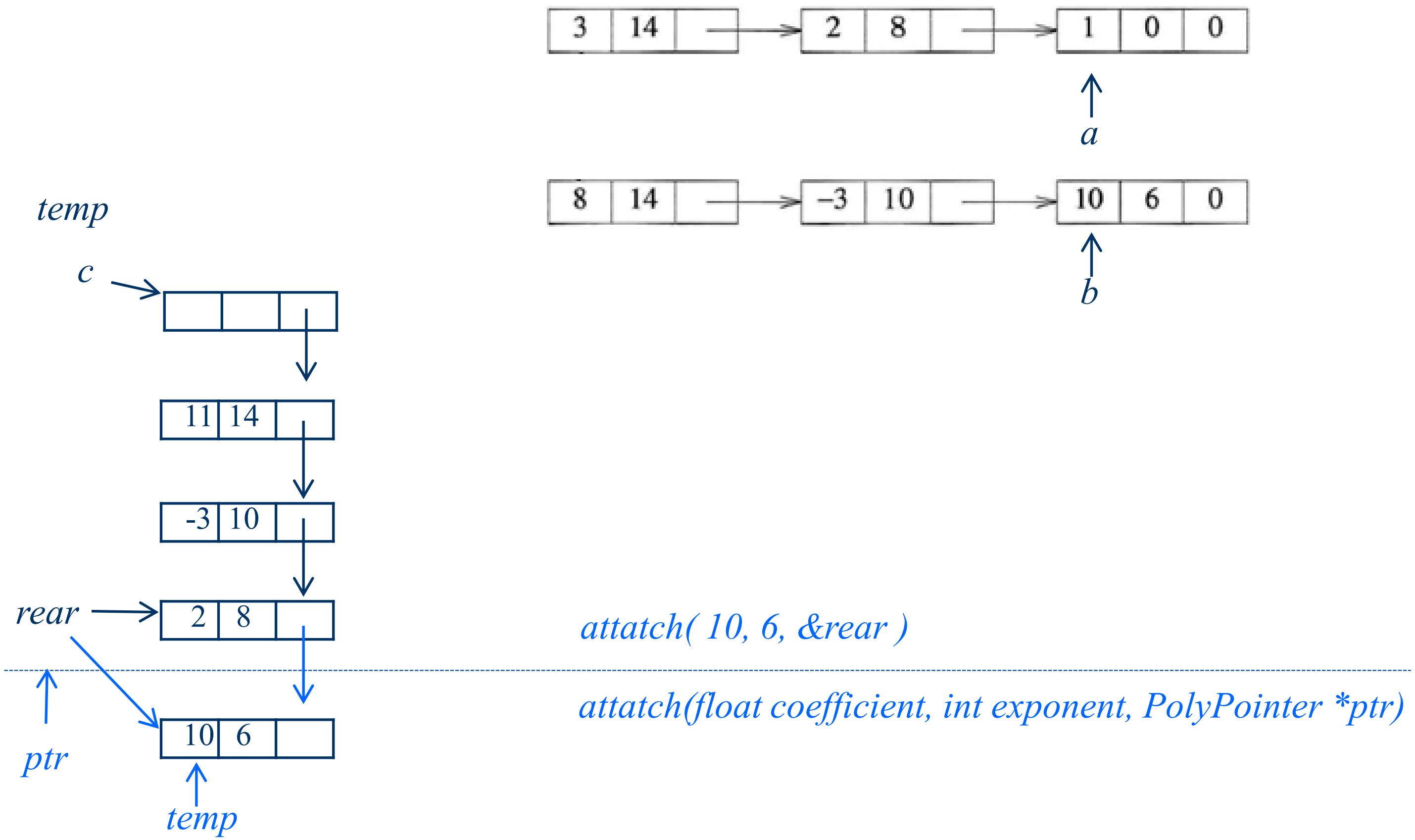
4.4.2 Adding Polynomials



attatch(2, 8, &rear)

*attatch(float coefficient, int exponent, PolyPointer *ptr)*

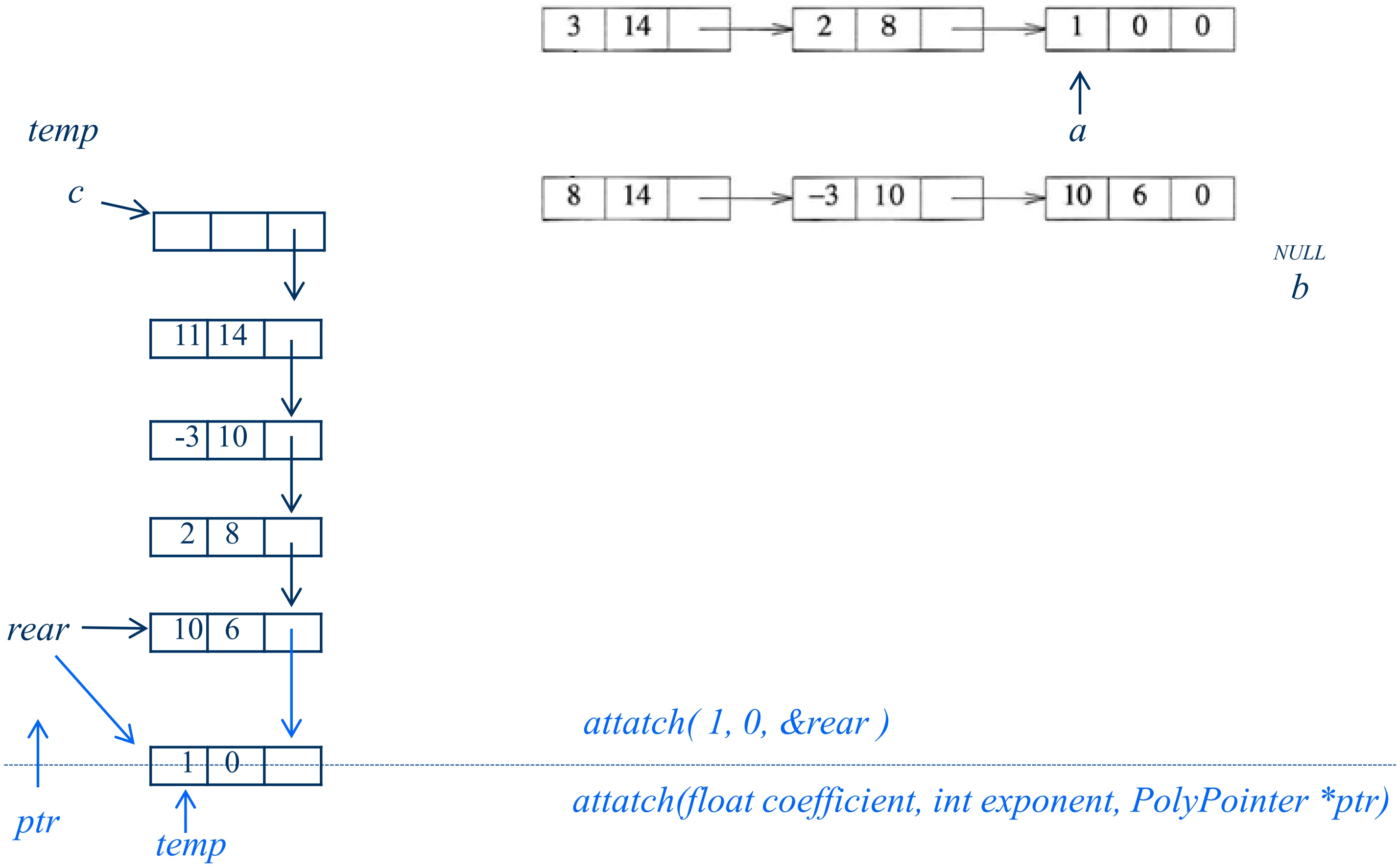
4.4.2 Adding Polynomials



attatch(10, 6, &rear)

*attatch(float coefficient, int exponent, PolyPointer *ptr)*

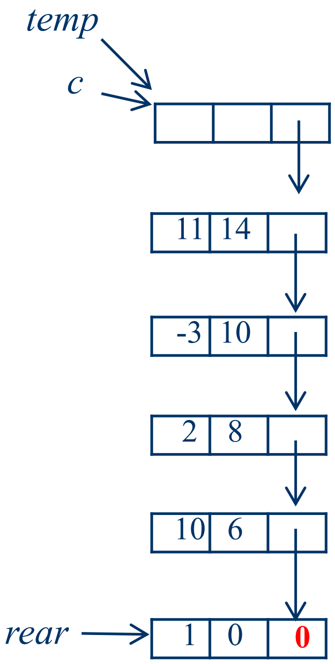
4.4.2 Adding Polynomials

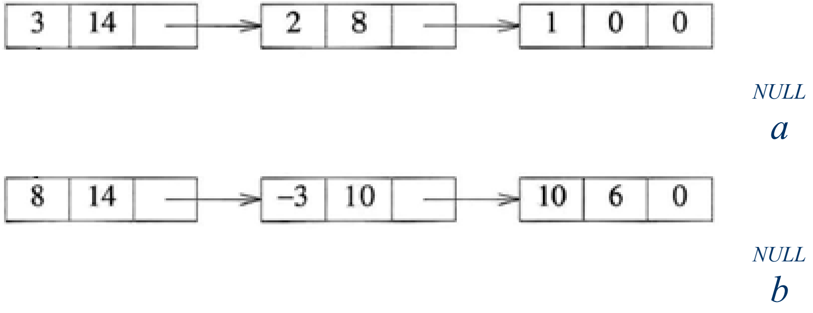


attatch(1, 0, &rear)

*attatch(float coefficient, int exponent, PolyPointer *ptr)*

4.4.2 Adding Polynomials





4.4.2 Adding Polynomials

Analysis of *padd*

- Three cost measures for this algorithm

(1)Coefficient additions

$A(x) = a_{m-1}x^{e_{m-1}} + \quad ..... \quad +a_0x^{e_0}$

$\mathrm{B}(x) = b_{n-1}x^{f_{n-1}} + \quad ..... \quad +b_0x^{f_0}$

where a_i , b_i $\neq 0$ *and* $e_{m-1} > e_{m-2} \ldots > e_1 > e_0 \geq 0,$ $f_{n-1} > f_{n-2} \ldots > f_1 > f_0 \geq 0$

$0 \leq \text{number of coefficient additions} \leq min\{m, n\}$

4.4.2 Adding Polynomials

(2) Exponent comparisons

- One comparison on each iteration of the `while` loop
- The number of iterations is bounded by *m* + *n*

  ex) *m*+*n*-1 iterations, for example *m* == *n*

  $e_{m-1} > f_{n-1} > e_{m-2} > f_{n-2} \ldots > e_1 > f_1 > e_0 > f_0$

  다항식 a의 지수

  다항식 b의 지수

(3) Creations of new nodes for *c*

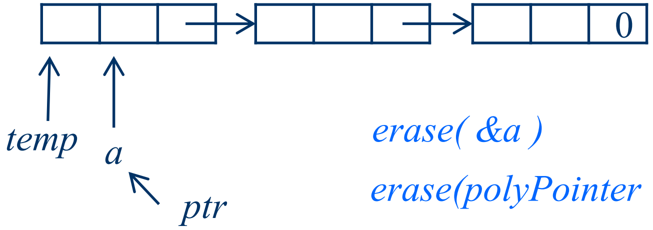
- The maximum number of terms in *c* is *m* + *n*

From (1)~(3),

- the total time complexity is **O (*m* + *n*)**

4.4.3 Erasing Polynomials

```
void erase(polyPointer *ptr)
{/* erase the polynomial pointed to by ptr */
      polyPointer temp;
      while ( *ptr) {
            temp = *ptr;
            *ptr = (*ptr)->link;
            free (temp);
      }
}
```



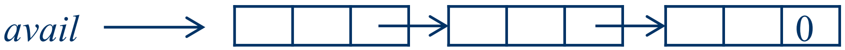
erase(&a)

*erase(polyPointer *ptr)*

4.4.4 Circular List Representation of Polynomials

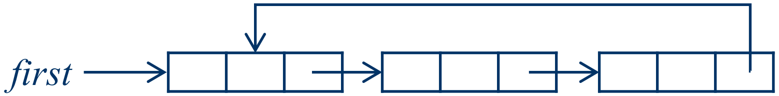
Available space list

- *A chain of nodes that have been "freed"*
- Use *getNode* and *retNode*, instead of *malloc* & *free*



When maintaining it,

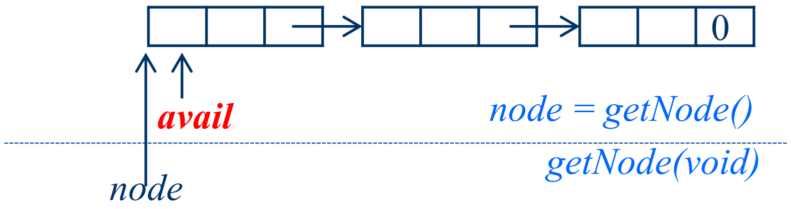
- we can obtain an efficient erase algorithm for circular list.



4.4.4 Circular List Representation of Polynomials

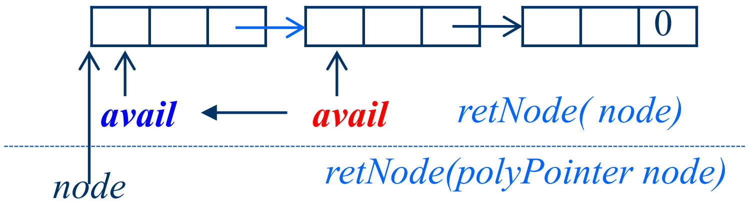
```
polyPointer getNode(void)
{/* provide a node for use */
        polyPointer node;
        if (avail) {
                node = avail;
                avail = avail->link;
        else
        }
                MALLOC(node, sizeof(*node));
        return node;
}
```

Program 4.12: *getNode* function



```
void retNode(polyPointer node)
{/* return a node to the available list */
        node->link = avail;
        avail = node;
}
```

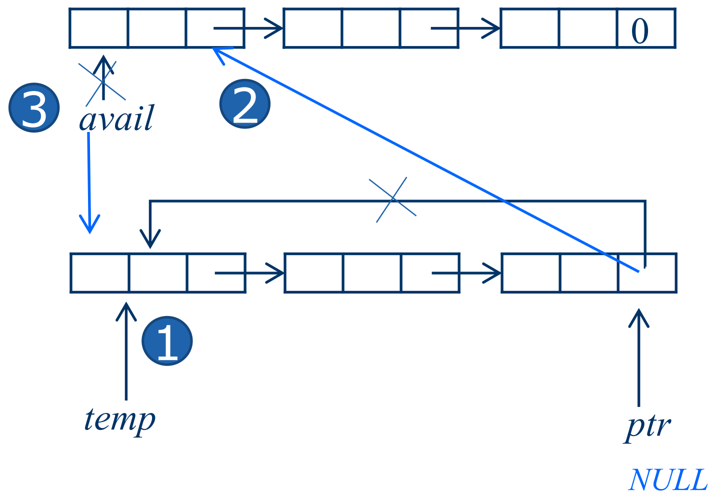
Program 4.13: *retNode* function



4.4.4 Circular List Representation of Polynomials

```
void cerase(polyPointer *ptr)
{/* erase the circular list pointed to by ptr */
        polyPointer temp;
        if ( *ptr) {
                temp = (*ptr)->link;
                (*ptr)->link = avail;
                avail = temp;
                *ptr = NULL;
        }
}
```

Program 4.14: Erasing a circular list



cerase(&first)

4.4.4 Circular List Representation of Polynomials

- A header node is added to easily handle polynomial addition in a circular

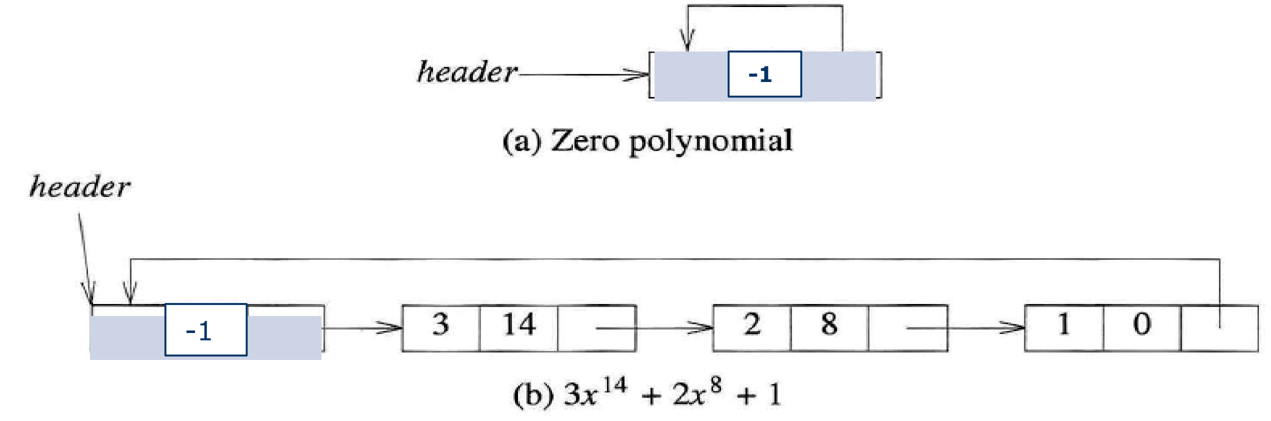
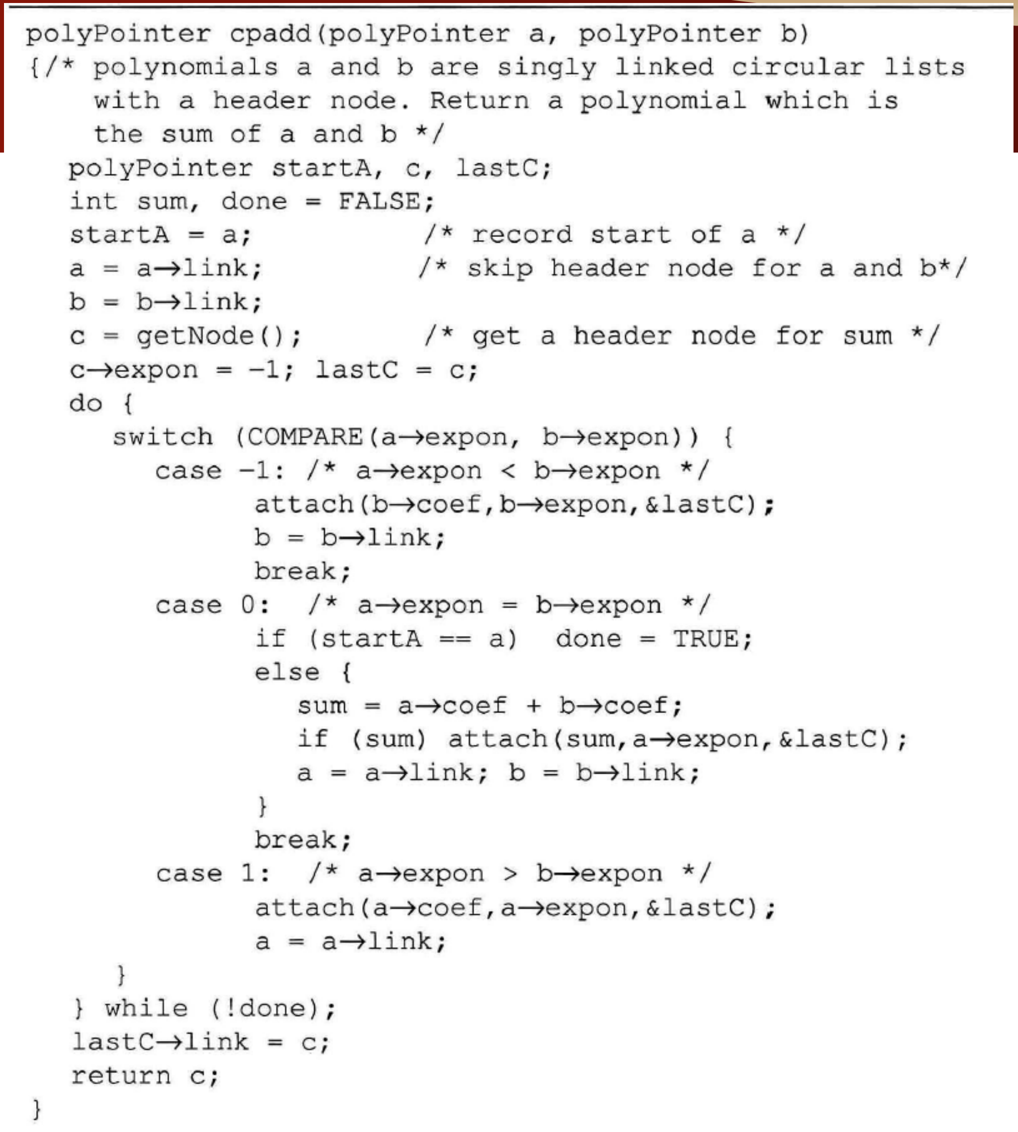


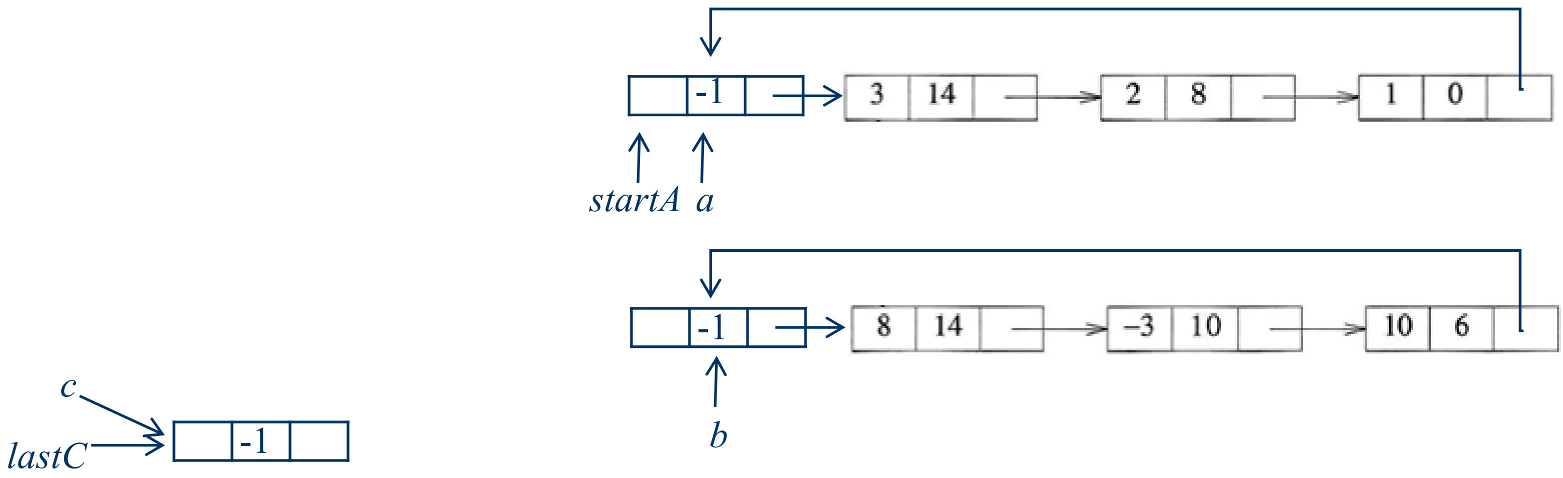
Figure 4.15: Example polynomials with header nodes



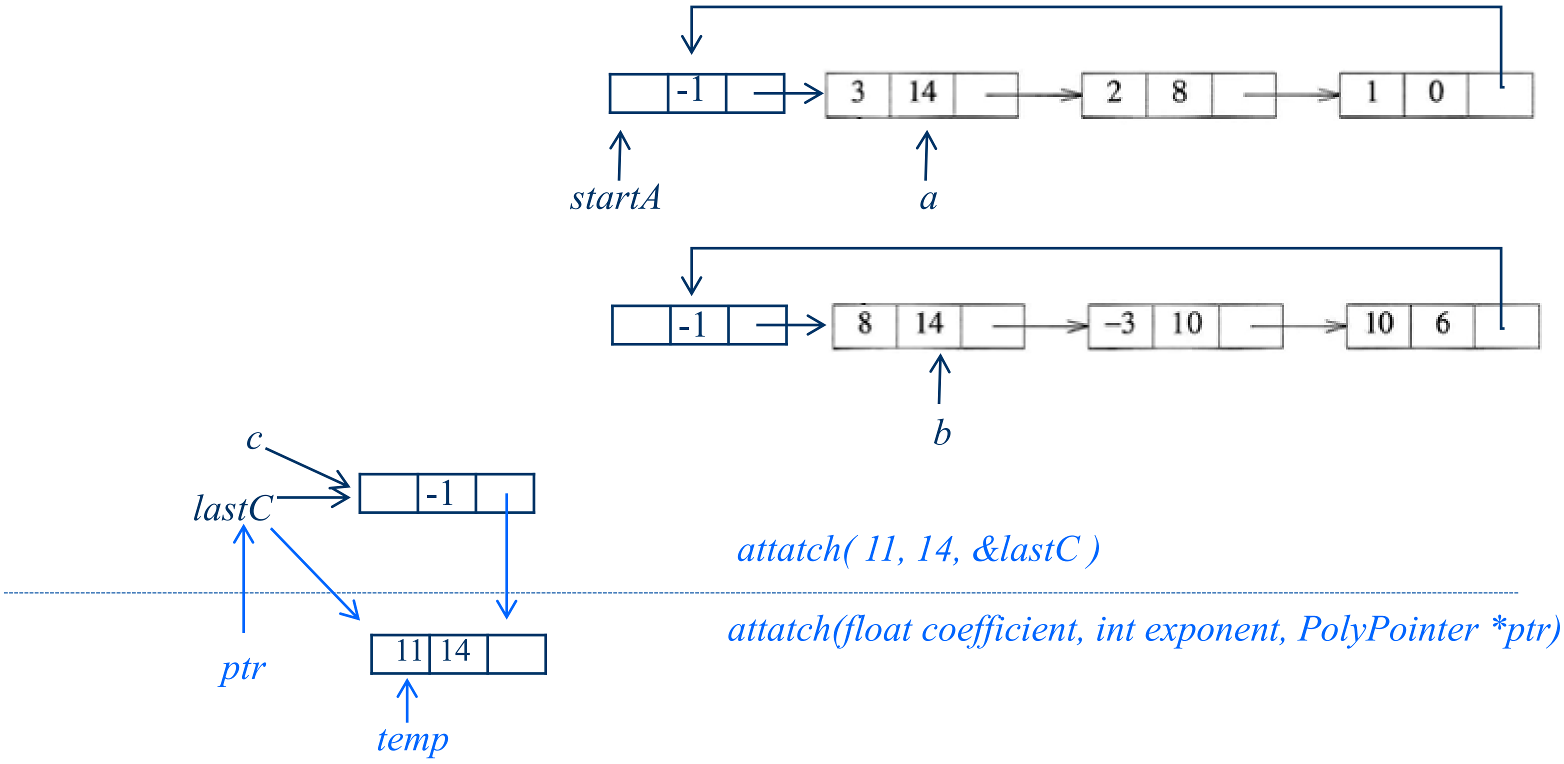
```
polyPointer cpadd(polyPointer a, polyPointer b)
{/* polynomials a and b are singly linked circular lists
    with a header node. Return a polynomial which is
    the sum of a and b */
   polyPointer startA, c, lastC;
   int sum, done = FALSE;
   startA = a;           /* record start of a */
   a = a→link;           /* skip header node for a and b*/
   b = b→link;
   c = getNode();        /* get a header node for sum */
   c→expon = -1; lastC = c;
   do {
      switch (COMPARE(a→expon, b→expon)) {
         case -1: /* a→expon < b→expon */
               attach(b→coef,b→expon,&lastC);
               b = b→link;
               break;
         case 0:  /* a→expon = b→expon */
               if (startA == a)  done = TRUE;
               else {
                  sum = a→coef + b→coef;
                  if (sum) attach(sum,a→expon,&lastC);
                  a = a→link; b = b→link;
               }
               break;
         case 1:  /* a→expon > b→expon */
               attach(a→coef,a→expon,&lastC);
               a = a→link;
      }
   } while (!done);
   lastC→link = c;
   return c;
}
```

Program 4.15: Adding two polynomials represented as circular lists with header nodes

4.4.4 Circular List Representation of Polynomials



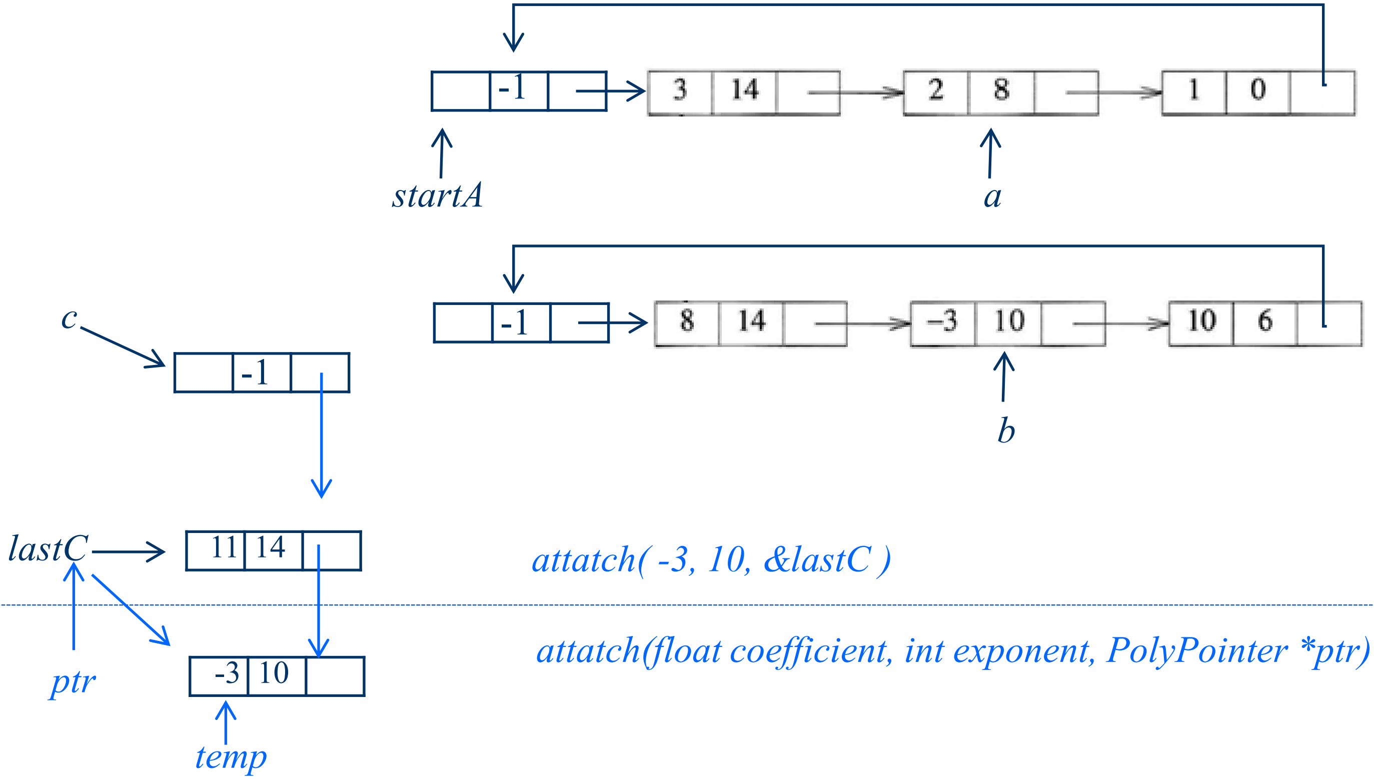
4.4.4 Circular List Representation of Polynomials



attatch(11, 14, &lastC)

*attatch(float coefficient, int exponent, PolyPointer *ptr)*

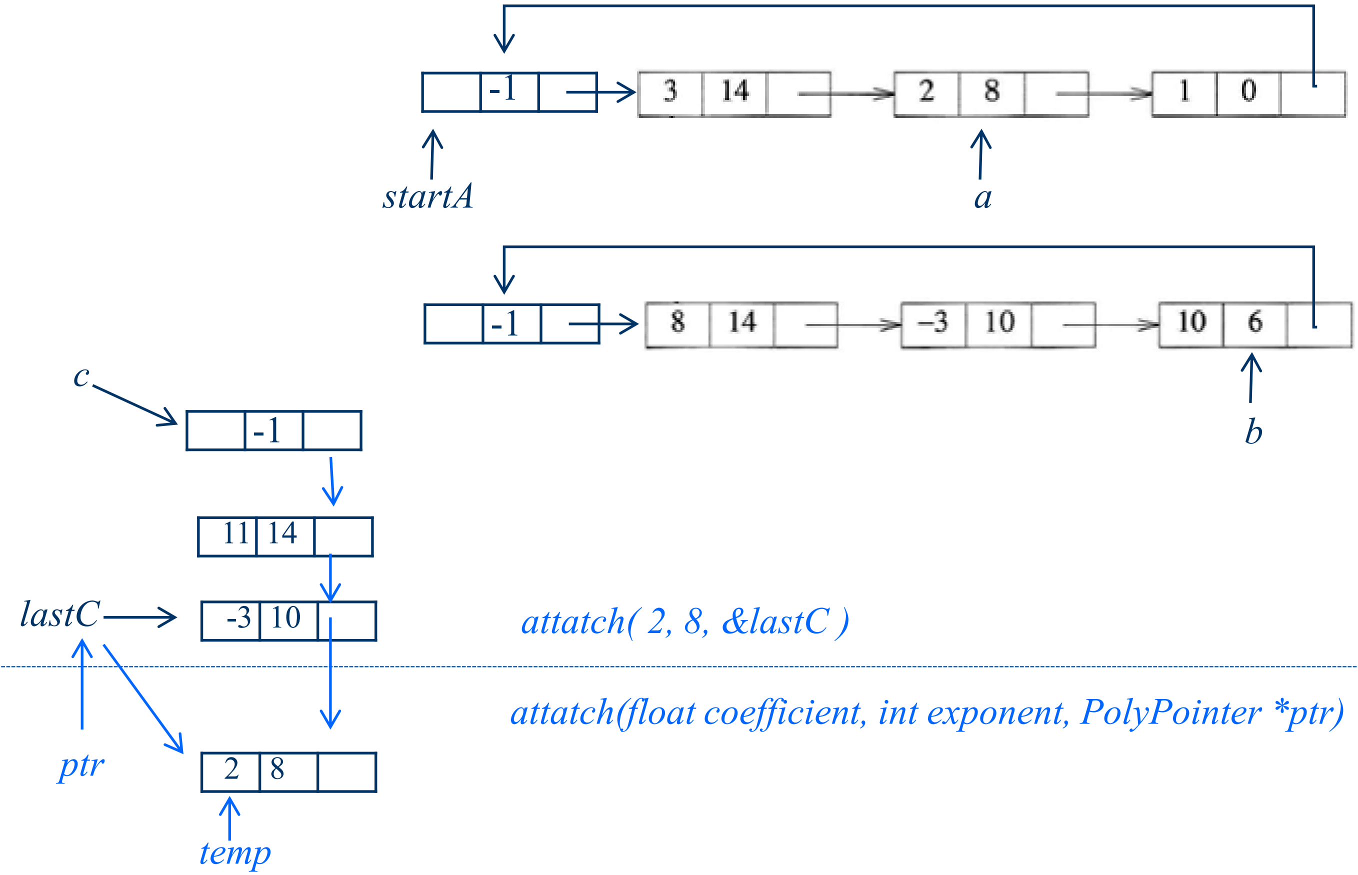
4.4.4 Circular List Representation of Polynomials



attatch(-3, 10, &lastC)

*attatch(float coefficient, int exponent, PolyPointer *ptr)*

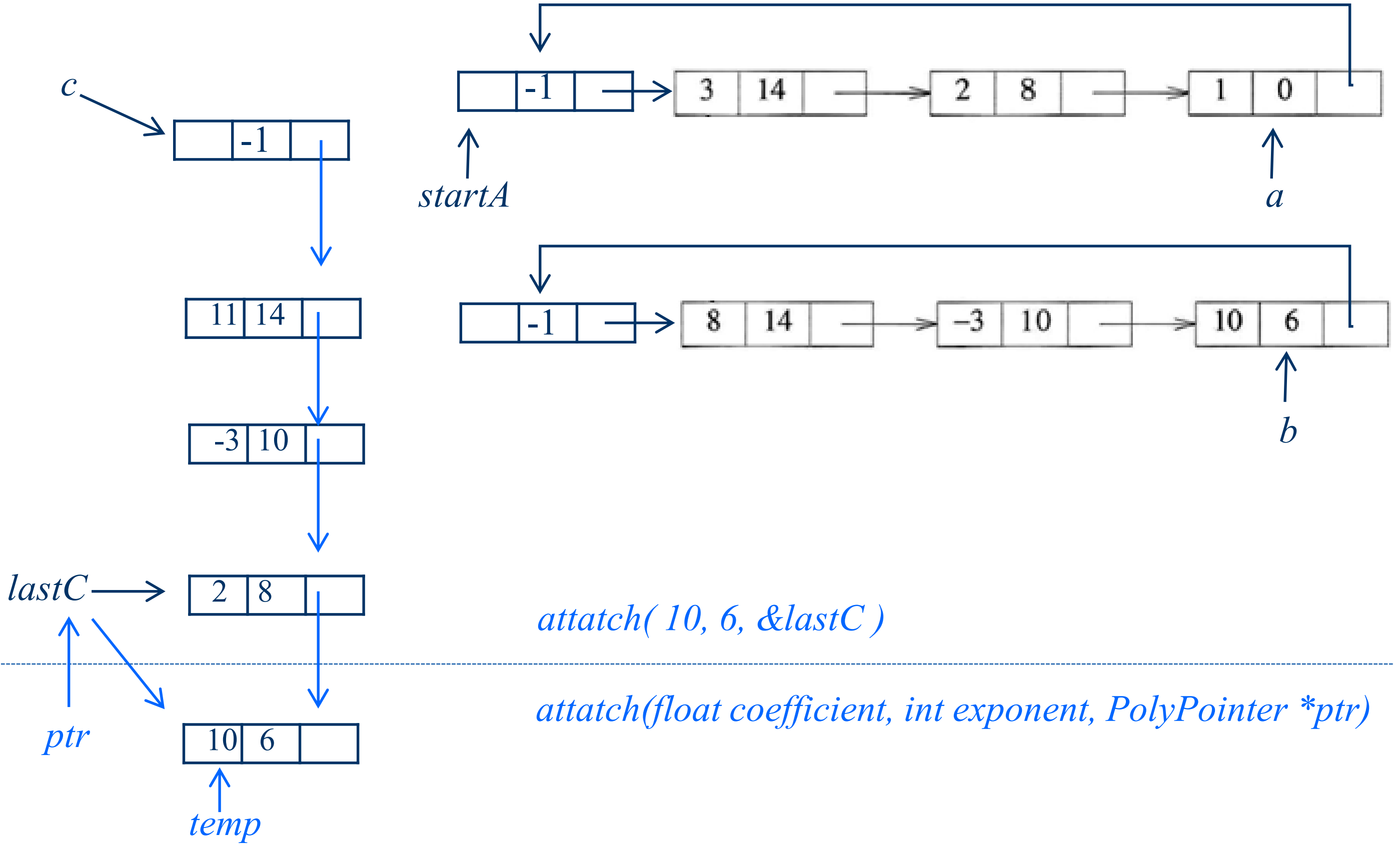
4.4.4 Circular List Representation of Polynomials



attatch(2, 8, &lastC)

*attatch(float coefficient, int exponent, PolyPointer *ptr)*

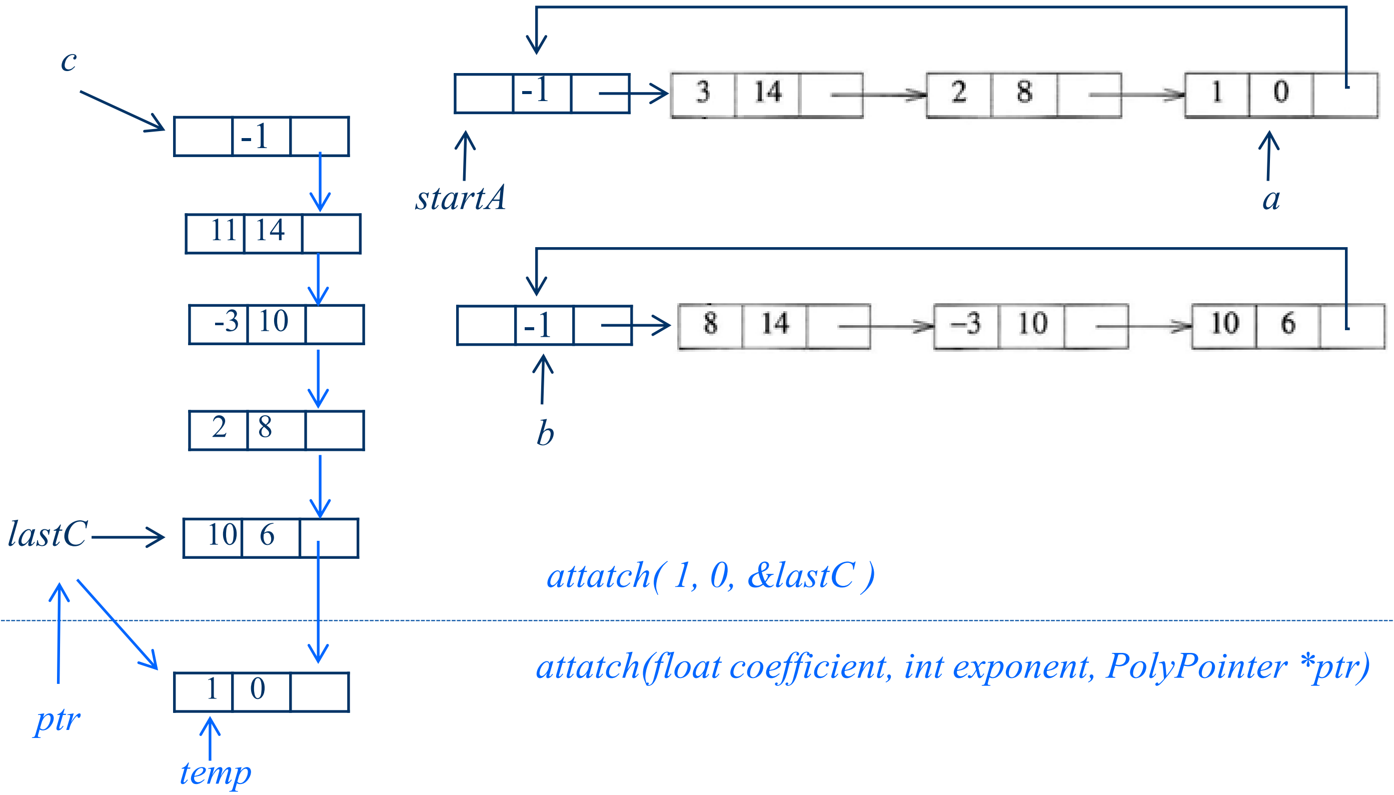
4.4.4 Circular List Representation of Polynomials



attatch(10, 6, &lastC)

*attatch(float coefficient, int exponent, PolyPointer *ptr)*

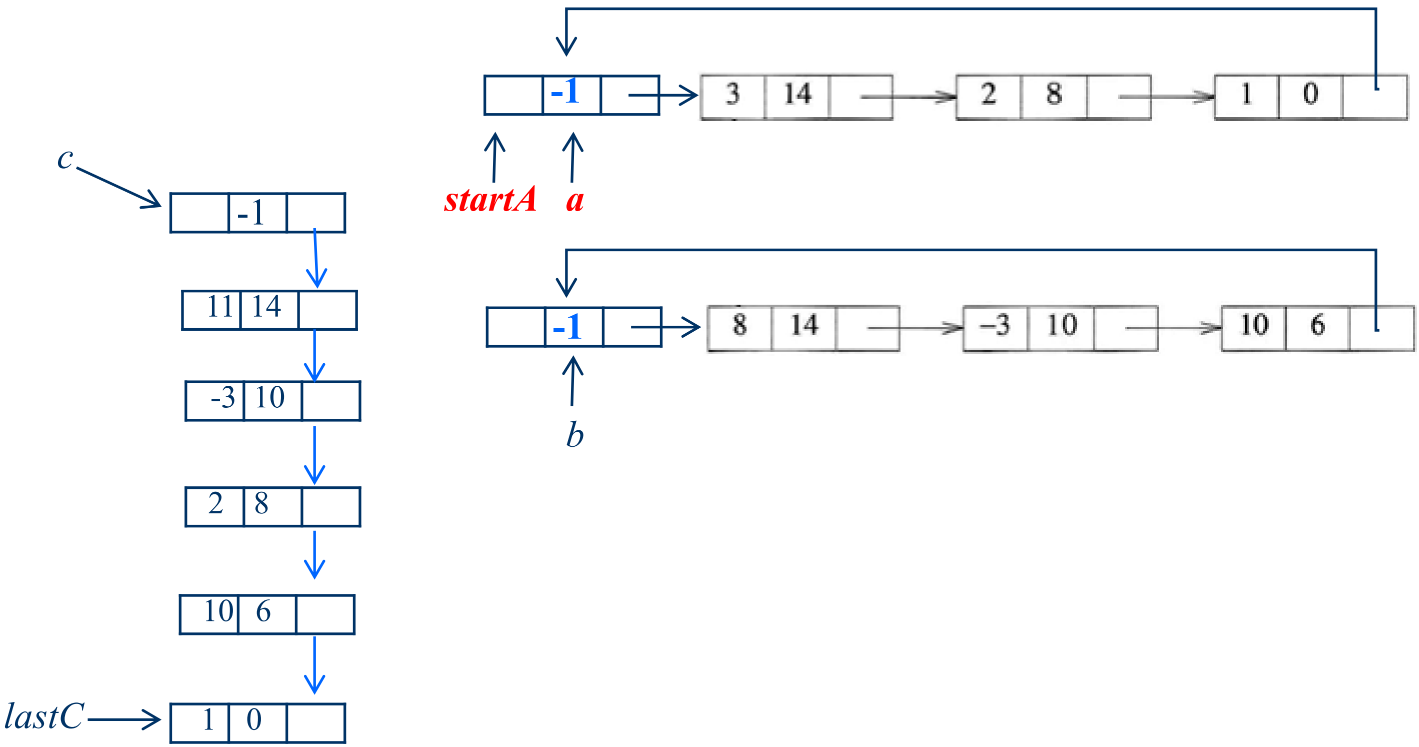
4.4.4 Circular List Representation of Polynomials



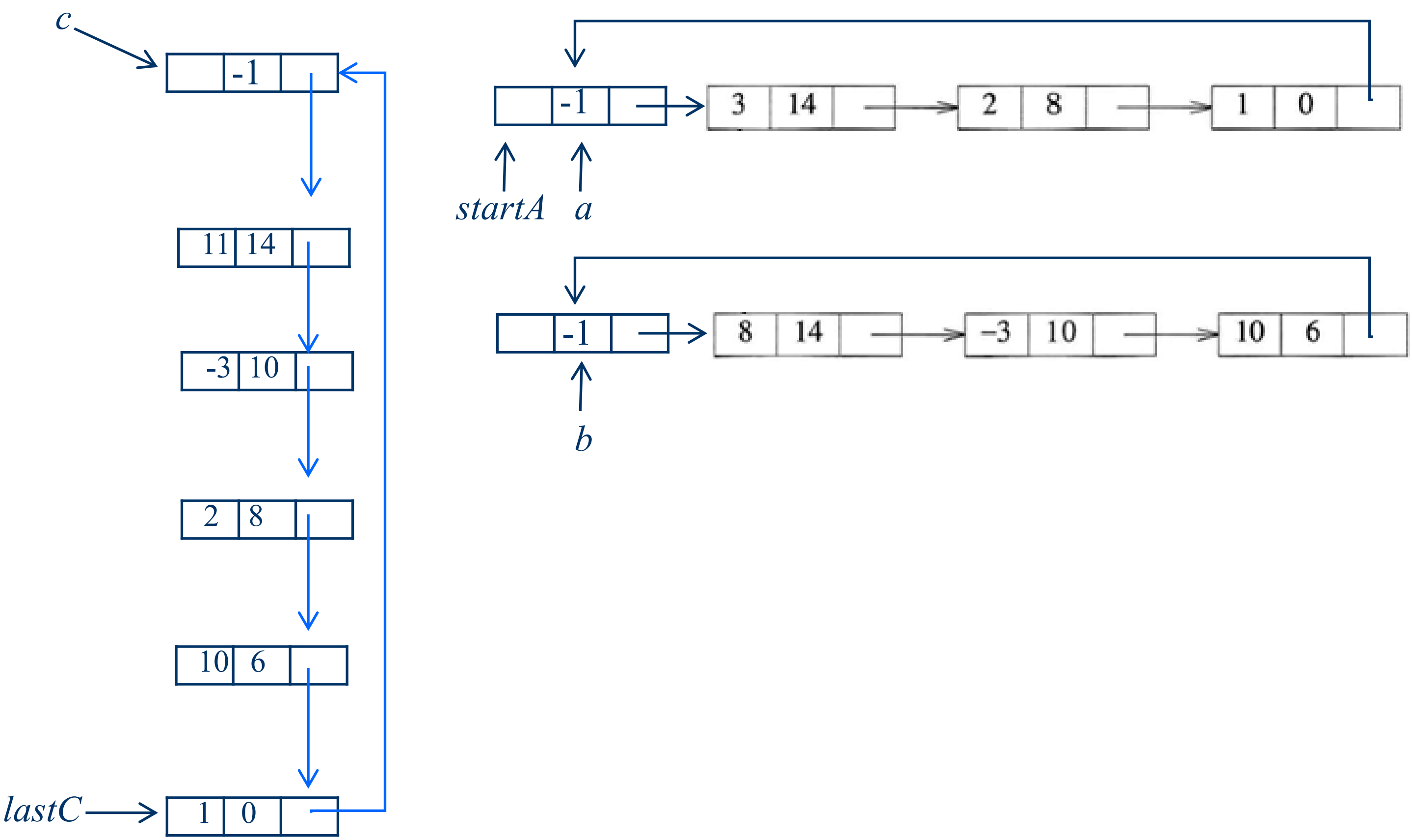
attatch(1, 0, &lastC)

attatch(float coefficient, int exponent, PolyPointer *ptr)

4.4.4 Circular List Representation of Polynomials



4.4.4 Circular List Representation of Polynomials


c
-1
11
14
-3
10
2
8
10
6
lastC
1
0
-1
3
14
2
8
1
0
startA
a
-1
8
14
–3
10
10
6
b