Chap 4. Linked Lists (3)

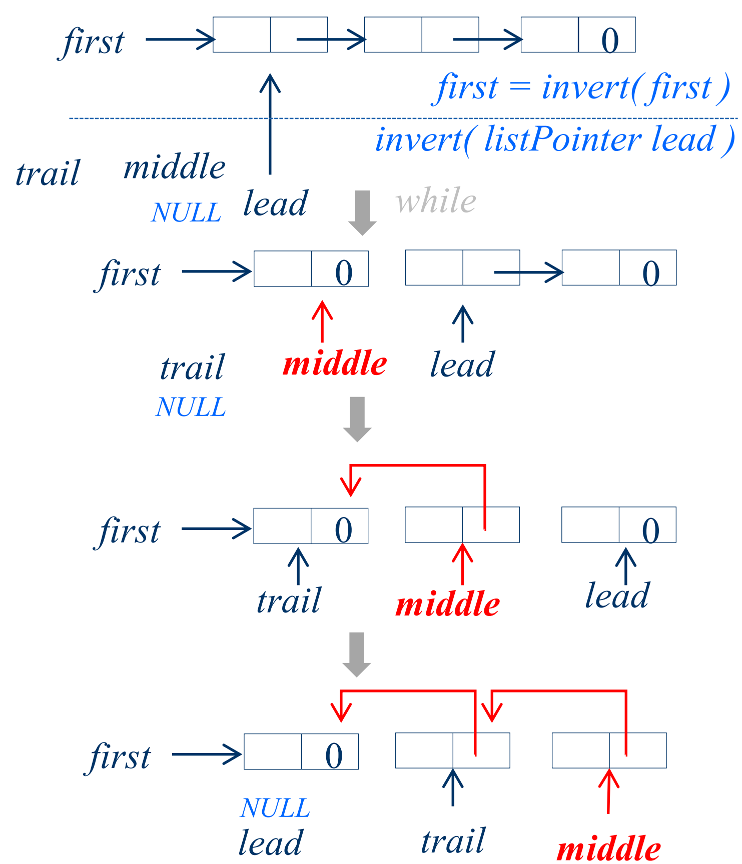
Contents

Contents

4.5 Additional List Operations (invert)

4.5.1 Operations For Chains

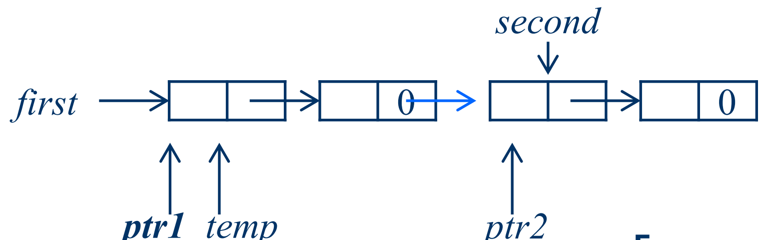
```
typedef struct listNode *listPointer;
typedef struct listNode {
        char data;
        listPointer link;
        }

listPointer invert(listPointer lead)
{/ * invert the list pointed to by lead * /
        listPointer middle,trail;
        middle = NULL;
        while (lead) {
                trail = middle;
                middle = lead;
                lead = lead->link;
                middle->link = trail;
        }
        return middle;
}
```



4.5 Additional List Operations (concatenate)

```
listPointer concatenate(listPointer ptrl, listPointer ptr2)
{/* produce a new list that contains the list
    ptrl followed by the list ptr2. The
    list pointed to by ptrl is changed permanently */
    listPointer temp;
    /* check for empty lists */
    if (!ptrl) return ptr2;
    if (!ptr2) return ptrl;
    /* neither list is empty, find end of first list */
    for (temp = ptrl; temp->link; temp = temp->link)
    /* link end of first to start of second */
    temp->link = ptr2;
    return ptrl;
}
```

concatList = concatenate(first, second)



Relation

- **Relation on Set A={a, b, c, d, e…}**
  - Notation : ${}_aR_b$ (a와 b는 관계 R)
  - Relation R = { (a, b), (c, d) ….} (set으로 나타냄)
  - Relation의 종류
    - 자연수 집합 : =, <, <=, >, >=
    - 사람의 집합 : 배우자, 가족, 동창, 동향, 원수

- **Ex)**
  - S={“철수”, “영희”, “경자”, “순희”, “용필” …}
    - R={(“철수”, “영희”), (“용필”, “순희”,)…)} : 배우자

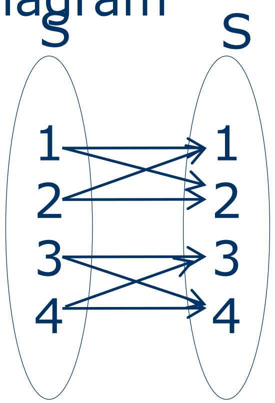
4.6 Equivalence Class

- **Definition : A relation, ≡, over a set, *S*, is said to be an *equivalence relation* over *S iff* it is *reflexive*, *symmetric* and *transitive* over S.**
  - Reflexive : $x \equiv x$
    - xRx
  - Symmetric : $x \equiv y \Rightarrow y \equiv x$
    - $xRy \Rightarrow yRx$
  - Transitive : $x \equiv y$ and $y \equiv z \Rightarrow x \equiv z$
    - xRy and $yRz \Rightarrow xRz$

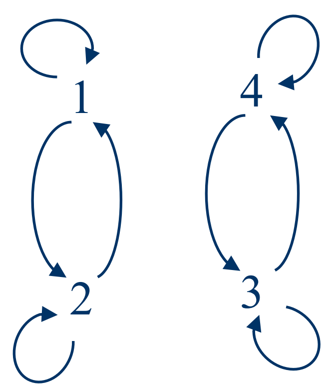
4.6 Equivalence Class

- **Example: A relation *R* of a set *S*={1, 2, 3, 4} is an *equivalence relation*, where *R* = {(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (3, 4), (4, 3), (4, 4)}.**

arrow diagram



directed graph



relation matrix

$$\begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

4.6 Equivalence Class

- **Definition : If *R* is an equivalence relation over a set S, *equivalence class* of *a* is defined as [a] = {x | (a, x)∈R}.**

- **Ex)**
  - S={“Kim”, “Park”, “Jon”, “Marry”, “Hong”, “Son”}
  - R(classmate)={(“kim”, “Kim”), ……(“Son”, “Son”), (“Kim”, “Park”), (“Park”, “Kim”), (“Jon”, “Marry”), (“Marry”, “Jon”), (“Hong”, “Son”), (“Son”, “Hong”), (“Kim”, “Hong”), (“Hong”, “Kim”), (“Kim”, “Son”), (“Son”, “Kim”), (“Park”, “Hong”), (“Hong”, “Park”), (“Park”, “Son”), (“Son”, “Park”)}
  - Equivalance Class [“Kim”] ={“Kim”, “Park”, “Hong”, “Son” }

4.6 Equivalence Class

- **Example: Consider an equivalence relation *R* = {(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (3, 4), (4, 3), (4, 4)} over a set *S*={1, 2, 3, 4}.**
  - The equivalence classes :

    [1] = [2] = { 1, 2 }, [3] = [4] = { 3, 4 }
  - The partition of S :{{1, 2}, {3, 4}}
  - *We can use an equivalence relation to partition a set S into equivalence classes* such that *two members x and y of S are in the same equivalence class iff* $x \equiv y$.

4.6 Equivalence Class

- **Example:**
  - For twelve polygons numbered 0 through 11

    S = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11}
  - If the following pairs overlap,

    0 = 4, 3 = 1, 6 = 10, 8 = 9, 7 = 4, 6 = 8, 3 = 5, 2 = 11, 11 = 0
  - then, the equivalence classes :
    - { 0, 2, 4, 7, 11 }, { 1, 3, 5}, { 6, 8, 9, 10}

Algorithm to determine equivalence classes

1. Read in and store the equivalence pairs <*i*, *j*>
2. Begin at 0 and find all pairs of the form <0, *j*>
3. *By transitivity*, all pairs of the form <*j*, *k*> imply that *k* is in the same equivalence class as 0.
4. Continue in this way, found, marked, and printed *the entire equivalence class containing 0*.
5. Then, continue on.

Algorithm to determine equivalence classes

Equivalence relation

out	0	1	2	3	4	5	6	7	8	9	10	11
T					1							1
T				1								
T												1
T		1				1						
T	1							1				
T				1								
T									1		1	
T					1							
T							1			1		
T									1			
T							1					
T	1		1									

0 = 4, 3 = 1, 6 = 10, 8 = 9, 7 = 4, 6 = 8, 3 = 5, 2 = 11, 11 = 0

Algorithm to determine equivalence classes

Equivalence relation

out	0	1	2	3	4	5	6	7	8	9	10	11
F					1							1
T				1								
T												1
T		1				1						
F	1							1				
T				1								
T									1		1	
T					1							
T							1			1		
T									1			
T							1					
F	1		1									

0 class : stack

11
4

0, 4, 11

0 = 4, 3 = 1, 6 = 10, 8 = 9, 7 = 4, 6 = 8, 3 = 5, 2 = 11, 11 = 0

Algorithm to determine equivalence classes

Equivalence relation

out	0	1	2	3	4	5	6	7	8	9	10	11
F					1							1
T				1								
F												1
T		1				1						
F	1							1				
T				1								
T									1		1	
T					1							
T							1			1		
T									1			
T							1					
F	1		1									

0 class : stack

11
4

2
4

0, 4, 11 , 2

0 = 4, 3 = 1, 6 = 10, 8 = 9, 7 = 4, 6 = 8, 3 = 5, 2 = 11, 11 = 0

Algorithm to determine equivalence classes

Equivalence relation

out	0	1	2	3	4	5	6	7	8	9	10	11
F					1							1
T				1								
F												1
T		1				1						
F	1							1				
T				1								
T									1		1	
F					1							
T							1			1		
T									1			
T							1					
F	1		1									

0 class : stack

11
4

2
4

7

0, 4, 11 , 2, 7

0 = 4, 3 = 1, 6 = 10, 8 = 9, 7 = 4, 6 = 8, 3 = 5, 2 = 11, 11 = 0

Algorithm to determine equivalence classes

Equivalence relation

out	0	1	2	3	4	5	6	7	8	9	10	11
F					1							1
T				1								
F												1
T		1				1						
F	1							1				
T				1								
T									1		1	
F					1							
T							1			1		
T									1			
T							1					
F	1		1									

0 class : stack

11
4

2
4

0, 4, 11 , 2, 7

0 = 4, 3 = 1, 6 = 10, 8 = 9, 7 = 4, 6 = 8, 3 = 5, 2 = 11, 11 = 0

Algorithm to determine equivalence classes

Equivalence relation

out	0	1	2	3	4	5	6	7	8	9	10	11
F					1							1
F				1								
F												1
F		1				1						
F	1							1				
F				1								
F									1		1	
F					1							
F							1			1		
F									1			
F							1					
F	1		1									

0 class : stack

{ 0, 2, 4, 7, 11 },
{ 1, 3, 5},
{ 6, 8, 9, 10}

0 = 4, 3 = 1, 6 = 10, 8 = 9, 7 = 4, 6 = 8, 3 = 5, 2 = 11, 11 = 0

Representations for this algorithm

- **An 2D *Boolean* array *pairs*[*n*][*n*]**
  - *n* : the number of objects
  - *pairs*[*i*][*j*] = *TRUE iff* the pair <*i*, *j*> is in the input.
  - potentially wasteful of space
  - $\Theta(n^2)$ time, just to initialize the array

- **A linked representation**
  - *seq*[*n*] holds the header nodes of the *n* lists
  - *out*[*n*] tells us whether or not the object *i* has been printed

Algorithm to determine equivalence classes

```
void equivalence()
{
    initialize seq to NULL and out to TRUE;
    while (there are more pairs) {
        read the next pair, <i,j>;
        put j on the seq[i] list;
        put i on the seq[j] list;
        }
    for (i = 0; i < n; i++)
        if (out [i]) {
            out[i]=FALSE;
            output this equivalence class;
        }
}
```

Program 4.21: A more detailed version of the equivalence algorithm

Representations for this algorithm

❖ Phase one

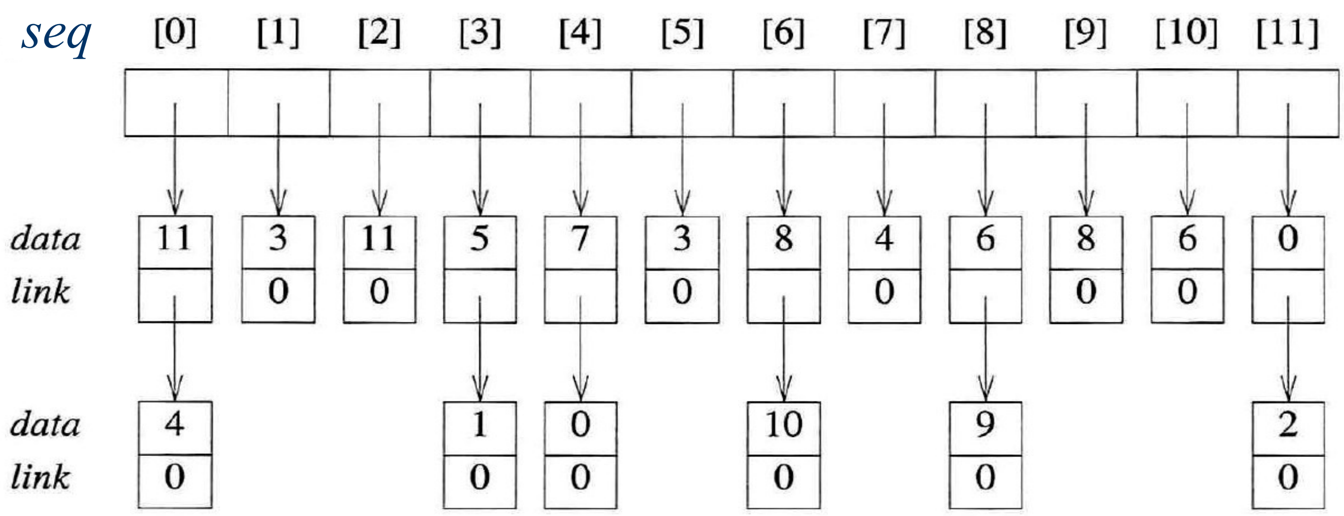


Figure 4.16: Lists after pairs have been input

Input pairs : (0, 4), (3, 1), (6, 10), (8, 9), (7, 4), (6, 8), (3, 5), (2, 11), (11, 0)

Representations for this algorithm

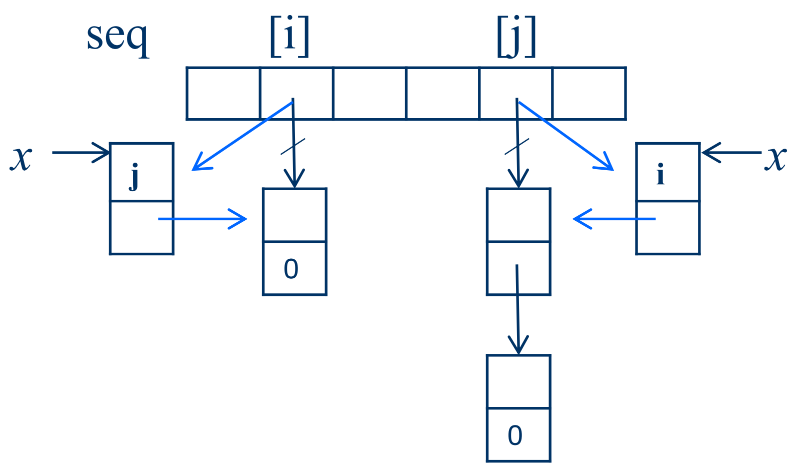
- **Phase two**
  - we scan the `seq` array for the first i, $0 \le i \le n$, such that `out`[i] = TRUE. Each element in the list `seq`[i] is printed.
  - To process the remaining lists which, by transitivity, belong in the same class as i, we create a stack of their nodes.
  - We do this by changing the link fields so that they point in the reverse direction.

Representations for this algorithm

```
#include <stdio.h>
#include <alloc.h>
#define MAX-SIZE 24
#define FALSE 0
#define TRUE 1
typedef struct node *nodePointer;
typedef struct node {
        int data;
        nodePointer link;
} ;
void main(void)
{
        short int out[MAX-SIZE];
        nodePointer seq[MAX-SIZE];
        nodePointer x,y,top;
        int i,j,n;
        printf ("Enter the size (<= %d) ",MAX-SIZE);
        scanf ( "%d", &n);
        for (i = 0; i < n; i++)
        /* initialize seq and out */
                out[i] =TRUE; seq[i] =NULL;
```

Representations for this algorithm

```
/* Phase 1: Input the equivalence pairs: */
printf ("Enter a pair of numbers (-1 -1 to quit): ");
scanf("%d%d",&i,&j);
while (i >= 0) {
        MALLOC(x, sizeof(*x));
        x->data = j; x->link = seq[i]; seq[i]=x;
        MALLOC(x, sizeof(*x));
        x->data = i; x->link=seq [ j ]; seq[j]=x
      printf ("Enter a pair of numbers (-1 -1 to quit): ");
      scanf("%d%d",&i,&j);
}
```



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Representations for this algorithm

❖ Phase one

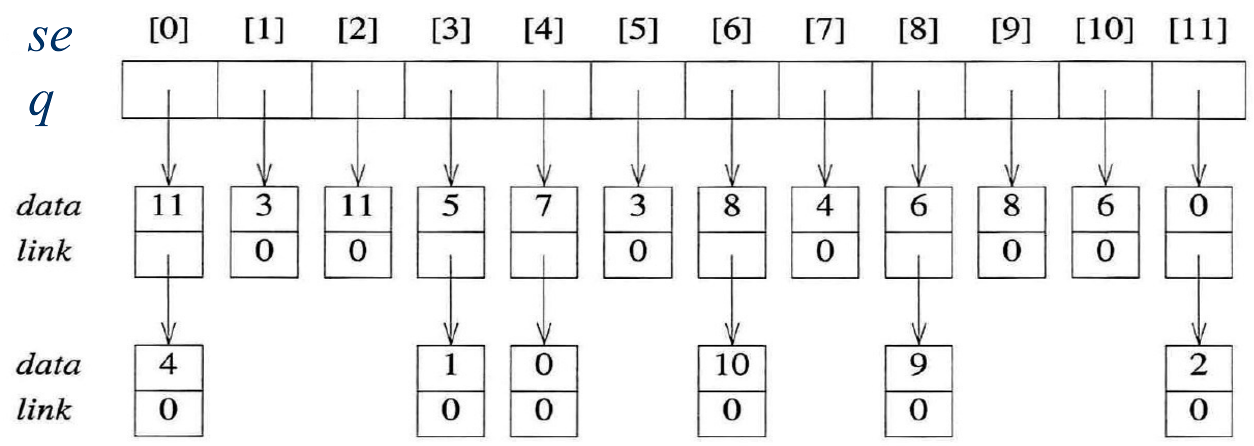
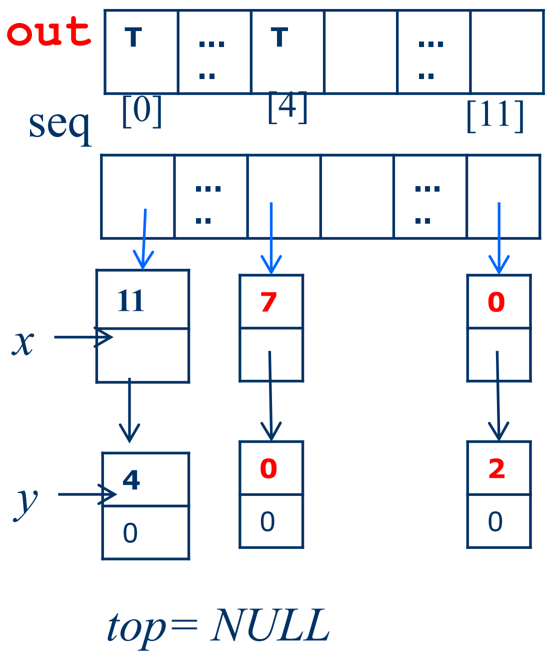


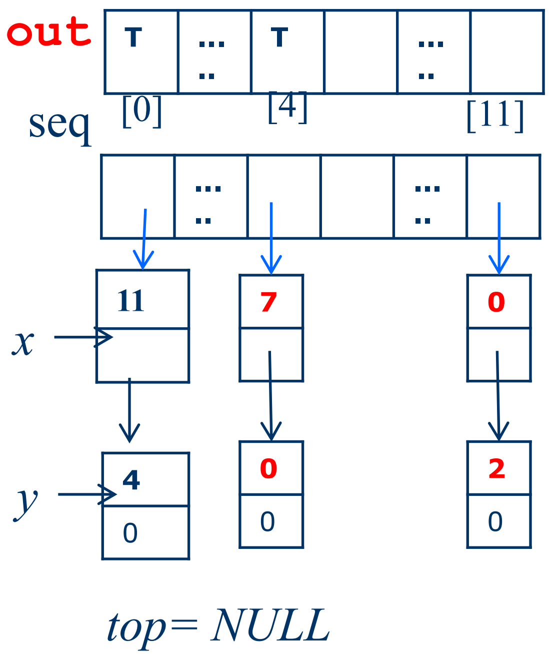
Figure 4.16: Lists after pairs have been input

Input pairs : (0, 4), (3, 1), (6, 10), (8, 9), (7, 4), (6, 8), (3, 5), (2, 11), (11, 0)

```
/* Phase 2: Output the equivalence classes */
        for ( i = 0; i < n; i++)
        {
                if( out[i] == TRUE )
                {
                        printf("\nNew class: %5d", i);
                        out[i] = FALSE;                         /* set class to true */
                        x = seq[i]; top = -1;  // top = NULL;          /* initialize stack */
                        for (;;)                /* find rest of class */
                        {
                            while(x)            /* process list */
                            {
                                    j = x->data;
                                    if(out[j])
                                        {
                                         printf("%5d", j); out[j] = FALE;
                                         push(j);
                                         x = x->link;
                                         //y = x->link; x->link = top; top = x; x = y;
                                         }
                                    else
                                         {
                                          x = x->link;
                                          }
                             } // while

                        if ( top == -1) break; // top = NULL;
                        x = seq[pop()];         /* unstack */
                         // x = seq[top->data]; top = top->link;
                        }// for
                }// if
        }// for
```

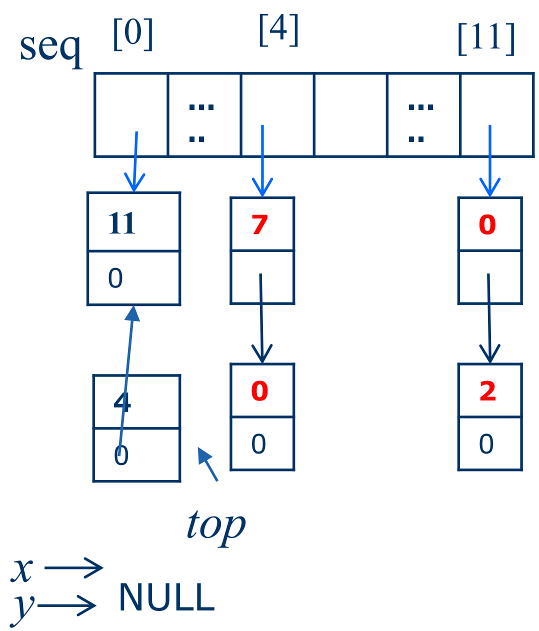




/* Phase 2: output the equivalence classes */

```
for (i = 0; i < n; i++)
    if (out[i]) {
        printf("\nNew class: %5d",i);
        out[i] =FALSE; /* set class to false */
        x = seq[i]; top= NULL; /*initialize stack*/
        for (;;) { /* find rest of class *I
            while (x) /* process list */
            {
                j = x->data;
                if (out[j]) {
                    printf("%5d",j); out[j] =FALSE;
                    y = x->link; x->link = top; top = x; x=y;  //push
                }
                else x = x->link;
            } // while
         if (!top) break; //empty stack
         x = seq[top->data]; top top->link;  //pop
         } //for
    } //if
} for
```

/* Phase 2: output the equivalence classes */




```
for (i = 0; i < n; i++)
    if (out[i]) {
        printf("\nNew class: %5d",i);
        out[i] =FALSE; /* set class to false */
        x = seq[i]; top= NULL; /*initialize stack*/
        for (;;) { /* find rest of class *I
            while (x) /* process list */
            {
                j = x->data;
                if (out[j]) {
                    printf("%5d",j); out[j] =FALSE;
                    y = x->link; x->link = top; top = x; x=y;  //push
                }
                else x = x->link;
            } // while
            if (!top) break; //empty stack
            x = seq[top->data]; top top->link;  //pop
        } //for
    } //if
} //for
```

Analysis of the equivalence program

Analysis of the equivalence program

- *m*: the number of input pairs
- *n*: the number of objects (the size of a set S)
- Time complexity : O(m+n)
  - initialize : n(seq, out)+2m(node)
  - second path : n + 2m
  - 2n+4m => O(m+n)
- Space complexity : O(m+n)
  - seq, out : 2n
  - node link to seq : 2m
  - stack : reuse node

4.8 Doubly Linked Lists

- **Limitation in *chains* and *singly linked circular lists***
  - The only way to find a specific node *p* or the node that precedes the node *p* is to start at the beginning of the list.
  - Easy deletion of an arbitrary node requires knowing the preceding node.

4.8 Doubly Linked Lists

- **It is useful to have *doubly linked lists,* for a problem that**
  - need to move in either directions
  - must delete an arbitrary node

4.8 Doubly Linked Lists

```
typedef struct node *nodePointer;
typedef struct node  {
        nodePointer llink;
        element data;
        nodePointer rlink;
};
```

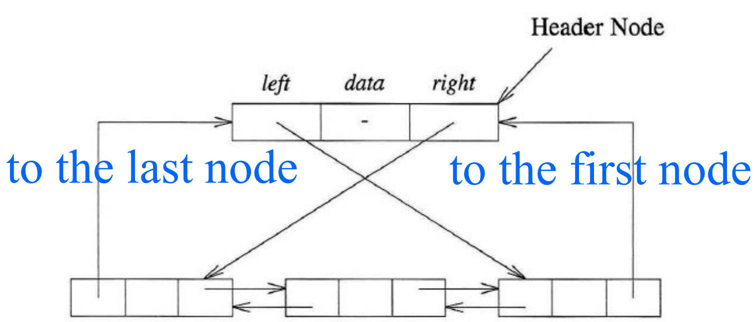


Figure 4.21: Doubly linked circular list with header node

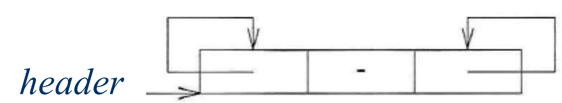


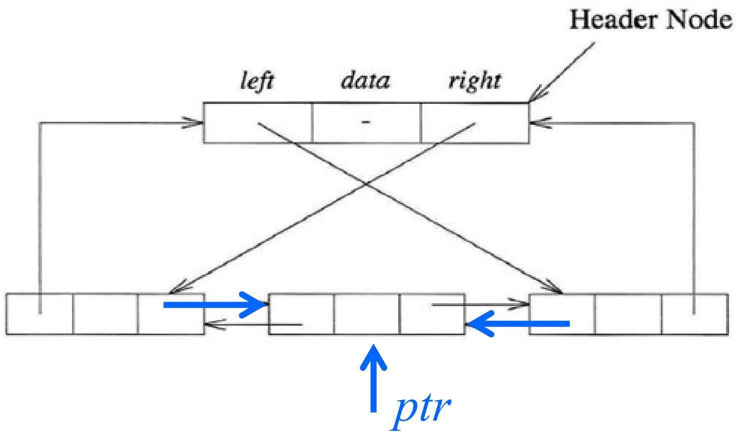
Figure 4.22: Empty doubly linked circular list with header node

4.8 Doubly Linked Lists

- If *ptr* points to any node in a doubly linked list, then

  ptr = ptr->llink- >rlink = ptr- >rlink- >llink

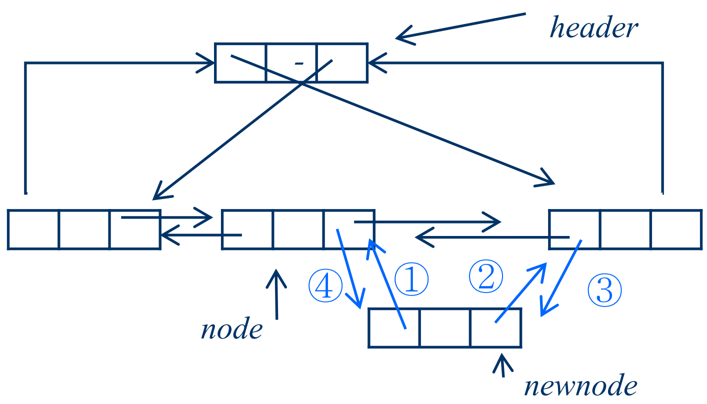
- *This formula reflects that we can go back and forth with equal ease.*



4.8 Doubly Linked Lists

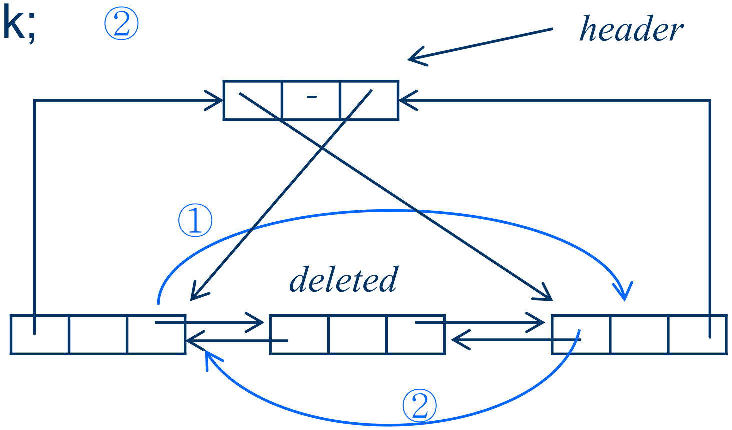
```
void dinsert(nodePointer node, nodePointer newnode)
{/* insert newnode to the right of node */
    newnode->llink = node;                    ①
    newnode- >rlink = node- >rlink;           ②
    node- >rlink- >llink = newnode;           ③
    node- >rlink = newnode;                   ④
}
```

Program 4.26: Insertion into a doubly linked circular list



4.8 Doubly Linked Lists

```
void ddelete(nodePointer node, nodePointer deleted)
{/* delete from the doubly linked list */
    if (node == deleted)
        printf("Deletion of header node not permitted.\n");
    else {
    deleted→llink → rlink  = deleted → rlink; ①
    deleted →rlink->llink = deleted →llink;   ②
    free(deleted);   ③
    }
}
```



- **Program 4.27: Deletion from a doubly linked circular list**

4.8 Doubly Linked Lists

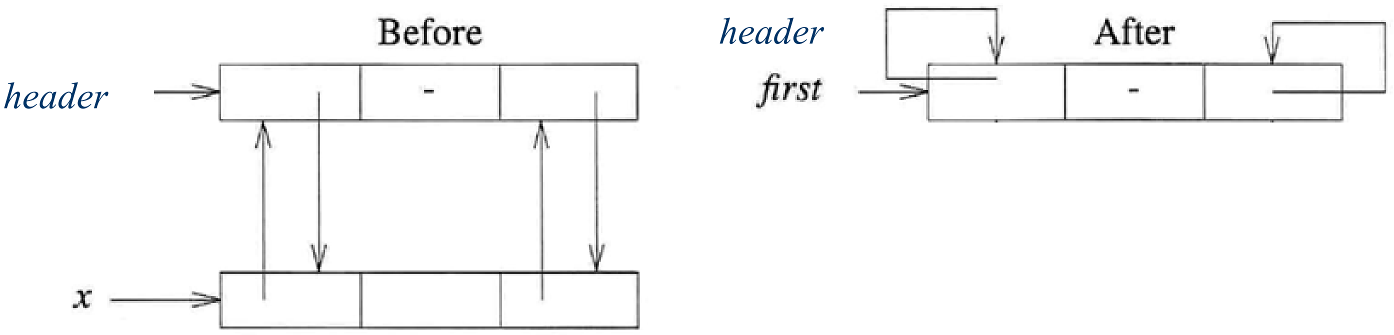


Figure 4.23: Deletion from a doubly linked circular list with a single node