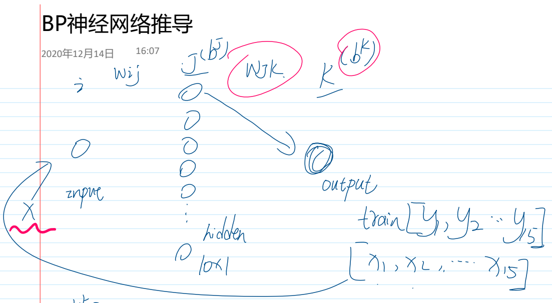
BP神经网络推导

2020年12月14日 16:07



符号

$I^{(i,j,k)} = Input^{(i,j,k)}$ $\quad O^{(i,j,k)} = Output^{(i,j,k)}$

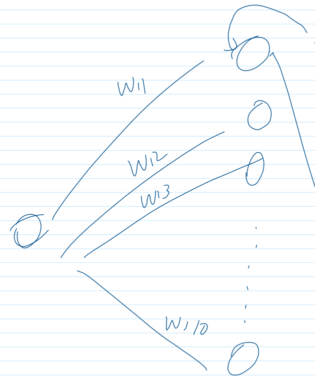
$Input^{(i)} = x = output^{(i)} = O^{(i)}$

$W_{ij\,10\times1} = \begin{bmatrix} W_{11} \\ W_{12} \\ W_{13} \\ \vdots \\ W_{1,10} \end{bmatrix}$

$I^{(j)} = W_{ij} \cdot O^{(i)} + b^{(j)}$

$O^{(j)} = sigmod(I^{(j)})_{(10\times1)}$

$sigmod(x)' = sigmod(x) \cdot (1 - sigmod(x))$



$I^{(j)} = \begin{bmatrix} W_{11} x \\ W_{12} x \\ W_{13} x \\ \vdots \\ W_{1,10} x \end{bmatrix} + b^{(j)}_{10\times1}$

$O^{(j)} = sigmod(I^{(j)})$

$W_{jk\,1\times10}$

$I^{(k)} = O^{(k)} = W_{jk\,1\times10} \cdot O^{(j)}_{10\times1} + b^{(k)}_{1\times1}$

$E = \frac{1}{2}(O^{(k)} - y)^2$

\# 正向传播结束

反向传播：

$\frac{\partial E}{\partial W_{jk}} \quad \frac{\partial E}{\partial W_{ij}} \quad \frac{\partial E}{\partial b_j} \quad \frac{\partial E}{\partial b_k}$ $\quad E \Rightarrow f(W_{jk}, W_{ik}, b_j, b_k)$

$\frac{\partial Ax}{\partial x} = A^T$

$\frac{\partial E}{\partial W_{jk}} = \frac{\partial E}{\partial O^{(k)}} \cdot \frac{\partial O^{(k)}}{\partial W_{jk}} = \frac{\frac{1}{2}(O^{(k)}-y)^2}{\partial O^{(k)}} \cdot \frac{\partial W_{jk} \cdot O^{(j)} + b_k}{\partial W_{jk}} = (O^{(k)} - y) \cdot O^{(j)T}$

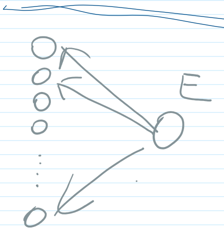
$\frac{\partial E}{\partial b_k} = \frac{\partial E}{\partial O^{(k)}} \cdot \frac{\partial O^{(k)}}{\partial b_k} = \sim \cdot \frac{\partial (W_{jk} \cdot O^{(j)} + b_k)}{\partial b_k} = (O^{(k)} - y) \cdot 1$

$E = f(W_{ij}, W_{jk}, b_j, b_k)$

E_{hidden} ∂b_k

$\frac{\partial E}{\partial W_{ij}} = \frac{\partial E}{\partial O^{(k)}} \cdot \frac{\partial O^{(k)}}{\partial O^{(j)}} \cdot \frac{\partial O^{(j)}}{\partial W_{ij}} = \underset{1\times1}{(O^{(k)} - y)} \cdot \underset{1\times10}{W_{jk}^T} \cdot sigmod(W_{ij} \cdot O^{(i)} + b_j)(1 - sigmod(W_{ij} \cdot O^{(i)} + b_j)) \cdot O^{(i)T}$

$\frac{\partial O^{(k)}}{\partial O^{(j)}} = \frac{\partial (W_{jk} \cdot O^{(j)} + b_k)}{\partial O^{(j)}} = W_{jk}^T$



$\frac{\partial E_{hidden}}{\partial W_{ij}}$

$\frac{\partial O^{(j)}}{\partial W_{ij}} = \frac{\partial sigmod(W_{ij} \cdot O^{(i)} + b_j)}{\partial W_{ij}} =$

$O^{(i)T} = x$

① $sigmod'(x) = sigmod(x)(1 - sigmod(x))$

② $f'(g(x)) = f'(u) \cdot g'(x)$ #: $u = g(x)$

$\frac{\partial E}{\partial b^{(j)}} = \frac{\partial E}{\partial O^{(k)}} \cdot \frac{\partial O^{(k)}}{\partial O^{(j)}} \cdot \frac{\partial O^{(j)}}{\partial b} = (O^{(k)} - y) \cdot W_{jk}^T \cdot sigmod(W_{ij} \cdot O^{(i)} + b_j)(1 - sigmod(W_{ij} \cdot O^{(i)} + b_j)) \cdot 1$

$$\frac{\partial E}{\partial b^{(J)}} = \frac{\partial E}{\partial o^{(K)}} \cdot \frac{\partial o^{(K)}}{\partial o^{(J)}} \cdot \frac{\partial o^{(J)}}{\partial b_J} = (o^{(K)} - y) \cdot w_{JK}^{T} \cdot \text{sigmoid}(w_{IJ} \cdot o^{(I)} + b_J)(1 - \text{sigmoid}(w_{IJ} \cdot o^{(I)} + b_J))$$

$$\frac{\partial o^{(J)}}{\partial b_J} = \frac{\partial \,\text{sigmoid}(w_{IJ} \cdot o^{(I)} + b_J)}{\partial b_J}$$

$$\text{New } w_{IJ} = \text{old } w_{IJ} - \alpha \cdot \frac{\partial E}{\partial w_{IJ}}$$

$$\text{New } w_{JK} = \text{old } w_{JK} - \alpha \frac{\partial E}{\partial w_{IJ}}$$

$$\text{New } b_J = \text{old } b_J - \alpha \frac{\partial E}{\partial b_J}$$

$$\text{New } b_K = \text{old } b_K - \alpha \frac{\partial E}{\partial b_K}$$