TD régression data vehicles

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```
library(plsdepot)
data(vehicles)
```

Effectuons une régression linéaire:

```
modele=lm(price~.,data=vehicles)
summary(modele)
```

```
##
## Call:
## lm(formula = price ~ ., data = vehicles)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -2919.8 -1410.1  -355.5  1418.8  5241.7
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) -16321.524  42128.829  -0.387   0.7043
## diesel        1345.112   3988.023   0.337   0.7409
## turbo         1543.223   3167.584   0.487   0.6337
## two.doors      525.496   2182.582   0.241   0.8132
## hatchback    -7120.032   2830.591  -2.515   0.0247 *
## wheel.base      29.738    462.233   0.064   0.9496
## length        -371.484    254.303  -1.461   0.1662
## width          979.577    695.139   1.409   0.1806
## height        -494.126    695.891  -0.710   0.4893
## curb.weight     10.325     10.788   0.957   0.3548
## eng.size        83.775     96.394   0.869   0.3994
## horsepower      40.904     61.632   0.664   0.5177
## peak.rpm         2.265      1.954   1.159   0.2657
## symbol        1172.065    862.383   1.359   0.1956
## city.mpg       -89.821    545.087  -0.165   0.8715
## highway.mpg    128.438    464.388   0.277   0.7861
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2959 on 14 degrees of freedom
## Multiple R-squared:  0.9436, Adjusted R-squared:  0.8832
## F-statistic: 15.62 on 15 and 14 DF,  p-value: 3.19e-06
```

Le R^2 semble bon, proche de 1, alors que quasiment aucune variable n'est significative.

Examinons les VIFs :

```
library(car)
```

```
## Le chargement a nécessité le package : carData
```

```
vif(modele)
```

```
##      diesel       turbo   two.doors   hatchback  wheel.base      length
##    8.721236    6.151519    3.791296    5.766553   34.465577   42.429644
##       width      height curb.weight    eng.size  horsepower    peak.rpm
##    9.529732    6.434802  126.458500   44.285608   19.798272    3.009453
##      symbol    city.mpg highway.mpg
##    3.160566   41.026076   31.572018
```

Il y a en effet plusieurs variables qui sont corrélées entre elles.

```
library(MASS)
modele2=stepAIC(modele,direction = "both")
```

```
## Start:  AIC=488.68
## price ~ diesel + turbo + two.doors + hatchback + wheel.base +
##     length + width + height + curb.weight + eng.size + horsepower +
##     peak.rpm + symbol + city.mpg + highway.mpg
##
##               Df Sum of Sq       RSS    AIC
## - wheel.base   1     36232 122584300 486.69
## - city.mpg     1    237687 122785755 486.74
## - two.doors    1    507430 123055498 486.81
## - highway.mpg  1    669584 123217652 486.85
## - diesel       1    995819 123543886 486.93
## - turbo        1   2077686 124625754 487.19
## - horsepower   1   3855681 126403749 487.61
## - height       1   4413385 126961453 487.75
## - eng.size     1   6611702 129159770 488.26
## - curb.weight  1   8017751 130565819 488.59
## <none>                     122548068 488.68
## - peak.rpm     1  11764260 134312328 489.43
## - symbol       1  16168953 138717021 490.40
## - width        1  17382475 139930543 490.66
## - length       1  18678994 141227062 490.94
## - hatchback    1  55384497 177932565 497.87
##
## Step:  AIC=486.69
## price ~ diesel + turbo + two.doors + hatchback + length + width +
##     height + curb.weight + eng.size + horsepower + peak.rpm +
##     symbol + city.mpg + highway.mpg
##
##               Df Sum of Sq       RSS    AIC
## - city.mpg     1    201563 122785862 484.74
## - highway.mpg  1    634196 123218496 484.85
## - two.doors    1    660836 123245136 484.85
## - diesel       1   1193119 123777419 484.98
## - turbo        1   3167854 125752154 485.46
## - height       1   4517321 127101621 485.78
## - horsepower   1   4884503 127468803 485.87
## - eng.size     1   6667591 129251890 486.28
## <none>                     122584300 486.69
```

```
## - peak.rpm     1  12023419 134607719 487.50
## - curb.weight  1  12395279 134979579 487.58
## - symbol       1  16204153 138788453 488.42
## + wheel.base   1     36232 122548068 488.68
## - length       1  19206645 141790945 489.06
## - width        1  23686779 146271079 489.99
## - hatchback    1  57070536 179654836 496.16
##
## Step:  AIC=484.74
## price ~ diesel + turbo + two.doors + hatchback + length + width +
##     height + curb.weight + eng.size + horsepower + peak.rpm +
##     symbol + highway.mpg
##
##               Df Sum of Sq       RSS    AIC
## - two.doors    1    632951 123418813 482.90
## - highway.mpg  1    732227 123518090 482.92
## - diesel       1   1161817 123947680 483.03
## - turbo        1   2966389 125752252 483.46
## - height       1   4994171 127780033 483.94
## - eng.size     1   6466362 129252225 484.28
## - horsepower   1   8212543 130998405 484.68
## <none>                     122785862 484.74
## - curb.weight  1  12248331 135034193 485.60
## - peak.rpm     1  12637444 135423306 485.68
## + city.mpg     1    201563 122584300 486.69
## + wheel.base   1       107 122785755 486.74
## - symbol       1  17855651 140641513 486.82
## - length       1  19623273 142409135 487.19
## - width        1  23490540 146276402 487.99
## - hatchback    1  57006309 179792171 494.18
##
## Step:  AIC=482.9
## price ~ diesel + turbo + hatchback + length + width + height +
##     curb.weight + eng.size + horsepower + peak.rpm + symbol +
##     highway.mpg
##
##               Df Sum of Sq       RSS    AIC
## - highway.mpg  1    897154 124315968 481.11
## - diesel       1   1412288 124831101 481.24
## - turbo        1   3443014 126861827 481.72
## - height       1   4490560 127909374 481.97
## - eng.size     1   8010335 131429149 482.78
## <none>                     123418813 482.90
## - horsepower   1   9327567 132746381 483.08
## - curb.weight  1  11672361 135091175 483.61
## - peak.rpm     1  13316947 136735761 483.97
## + two.doors    1    632951 122785862 484.74
## + city.mpg     1    173677 123245136 484.85
## + wheel.base   1     53356 123365457 484.88
## - length       1  20638640 144057454 485.54
## - width        1  22972587 146391400 486.02
## - symbol       1  25505587 148924401 486.53
## - hatchback    1  62356082 185774895 493.17
##
```

```
## Step:  AIC=481.11
## price ~ diesel + turbo + hatchback + length + width + height +
##     curb.weight + eng.size + horsepower + peak.rpm + symbol
##
##               Df Sum of Sq       RSS    AIC
## - diesel       1   2023223 126339190 479.60
## - turbo        1   2988529 127304497 479.83
## - height       1   4546668 128862636 480.19
## - eng.size     1   8100400 132416368 481.01
## - horsepower   1   8450644 132766612 481.09
## <none>                     124315968 481.11
## - curb.weight  1  11014306 135330273 481.66
## - peak.rpm     1  12606383 136922351 482.01
## + highway.mpg  1    897154 123418813 482.90
## + two.doors    1    797878 123518090 482.92
## + city.mpg     1    416982 123898985 483.01
## + wheel.base   1    212357 124103611 483.06
## - length       1  22840705 147156673 484.17
## - width        1  26571829 150887796 484.93
## - symbol       1  26972474 151288441 485.01
## - hatchback    1  64239297 188555265 491.61
##
## Step:  AIC=479.6
## price ~ turbo + hatchback + length + width + height + curb.weight +
##     eng.size + horsepower + peak.rpm + symbol
##
##               Df Sum of Sq       RSS    AIC
## - height       1   3898236 130237427 478.51
## - turbo        1   5523729 131862919 478.88
## - horsepower   1   6463352 132802542 479.10
## - eng.size     1   7012015 133351205 479.22
## <none>                     126339190 479.60
## - peak.rpm     1  10591565 136930756 480.01
## + diesel       1   2023223 124315968 481.11
## + highway.mpg  1   1508089 124831101 481.24
## + two.doors    1   1207507 125131684 481.31
## + city.mpg     1    857154 125482036 481.39
## + wheel.base   1     60620 126278570 481.58
## - curb.weight  1  21436497 147775687 482.30
## - width        1  35372225 161711416 485.00
## - symbol       1  43985240 170324430 486.56
## - length       1  49702694 176041884 487.55
## - hatchback    1 117434156 243773346 497.32
##
## Step:  AIC=478.51
## price ~ turbo + hatchback + length + width + curb.weight + eng.size +
##     horsepower + peak.rpm + symbol
##
##               Df Sum of Sq       RSS    AIC
## - horsepower   1   5222511 135459938 477.69
## - turbo        1   6935969 137173396 478.07
## <none>                     130237427 478.51
## - peak.rpm     1  12784771 143022198 479.32
## + height       1   3898236 126339190 479.60
```

```
## + highway.mpg  1   1447992 128789435 480.17
## + diesel       1   1374791 128862636 480.19
## - curb.weight  1  17658007 147895434 480.32
## + city.mpg     1    482395 129755032 480.40
## - eng.size     1  18143853 148381280 480.42
## + wheel.base   1    163775 130073652 480.47
## + two.doors    1      1918 130235508 480.51
## - width        1  35299696 165537122 483.71
## - symbol       1  40656541 170893968 484.66
## - length       1  56729208 186966635 487.36
## - hatchback    1 117588100 247825527 495.81
##
## Step:  AIC=477.69
## price ~ turbo + hatchback + length + width + curb.weight + eng.size +
##     peak.rpm + symbol
##
##               Df Sum of Sq       RSS    AIC
## <none>                     135459938 477.69
## + horsepower   1   5222511 130237427 478.51
## - curb.weight  1  14246825 149706763 478.69
## + height       1   2657396 132802542 479.10
## + wheel.base   1   2594735 132865203 479.11
## + city.mpg     1    662921 134797017 479.54
## + two.doors    1     81936 135378002 479.67
## + diesel       1     53426 135406511 479.68
## + highway.mpg  1     18454 135441484 479.69
## - turbo        1  20555961 156015899 479.93
## - width        1  32129527 167589465 482.07
## - peak.rpm     1  38526714 173986652 483.20
## - symbol       1  48434625 183894563 484.86
## - eng.size     1  49967934 185427871 485.11
## - length       1  51736255 187196193 485.39
## - hatchback    1 112522581 247982519 493.83
```

Examinons le modèle obtenu :

```
summary(modele2)
```

```
##
## Call:
## lm(formula = price ~ turbo + hatchback + length + width + curb.weight +
##     eng.size + peak.rpm + symbol, data = vehicles)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -3989.6 -1391.4  -480.5  1219.1  5306.4
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) -35109.053  23964.875  -1.465 0.157727
## turbo         3140.737   1759.375   1.785 0.088691 .
## hatchback    -6760.217   1618.589  -4.177 0.000426 ***
## length         -433.600    153.104  -2.832 0.009984 **
## width          1049.308    470.161   2.232 0.036654 *
## curb.weight       8.848      5.953   1.486 0.152099
```

```
## eng.size       145.526     52.287   2.783 0.011140 *
## peak.rpm         2.848      1.166   2.444 0.023444 *
## symbol        1503.603    548.720   2.740 0.012264 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2540 on 21 degrees of freedom
## Multiple R-squared:  0.9377, Adjusted R-squared:  0.914
## F-statistic:  39.5 on 8 and 21 DF,  p-value: 5.974e-11
```

La variable curb.weight n'est pas significative, je l'enlève

```
modele3=lm(price ~ turbo + hatchback + length + width + eng.size + peak.rpm + symbol, data = vehicles)
summary(modele3)
```

```
##
## Call:
## lm(formula = price ~ turbo + hatchback + length + width + eng.size +
##     peak.rpm + symbol, data = vehicles)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -3282.5 -1270.8  -730.7  1712.3  5787.4
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) -54010.276  20862.105  -2.589 0.016751 *
## turbo         4648.590   1476.308   3.149 0.004661 **
## hatchback    -5686.831   1487.765  -3.822 0.000929 ***
## length        -263.948    104.792  -2.519 0.019548 *
## width         1086.271    482.227   2.253 0.034593 *
## eng.size       217.736     19.838  10.976 2.16e-10 ***
## peak.rpm         2.797      1.197   2.337 0.028928 *
## symbol        1247.061    534.977   2.331 0.029314 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2609 on 22 degrees of freedom
## Multiple R-squared:  0.9311, Adjusted R-squared:  0.9092
## F-statistic:  42.5 on 7 and 22 DF,  p-value: 2.442e-11
```

Tout est significatif. On peut comparrer les critères AIC (attention, avec la même fonction, par exemple AIC, car suivant les fonctions de R certains variantes du critères AIC peuvent être implémentée.)

```
AIC(modele2)
```

```
## [1] 564.826
```

```
AIC(modele3)
```

```
## [1] 565.8261
```

Même si le AIC est légérement moins bon pour le modèle3, toutes les variables sont significatives donc je le préférerai.

Si on veut fixer le seuil maximum des pvalues conserver, on peut faire cela en spécifiant k:

```
modele4=stepAIC(modele,direction = "both", k = qchisq(0.05, 1, lower.tail = F),trace = F)
summary(modele4)
```

```
##
## Call:
## lm(formula = price ~ turbo + hatchback + length + width + eng.size +
##     peak.rpm + symbol, data = vehicles)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -3282.5 -1270.8  -730.7  1712.3  5787.4
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) -54010.276  20862.105  -2.589 0.016751 *
## turbo         4648.590   1476.308   3.149 0.004661 **
## hatchback    -5686.831   1487.765  -3.822 0.000929 ***
## length        -263.948    104.792  -2.519 0.019548 *
## width         1086.271    482.227   2.253 0.034593 *
## eng.size       217.736     19.838  10.976 2.16e-10 ***
## peak.rpm         2.797      1.197   2.337 0.028928 *
## symbol        1247.061    534.977   2.331 0.029314 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2609 on 22 degrees of freedom
## Multiple R-squared:  0.9311, Adjusted R-squared:  0.9092
## F-statistic:  42.5 on 7 and 22 DF,  p-value: 2.442e-11
```

On obtient le mème modèle que modèle3.

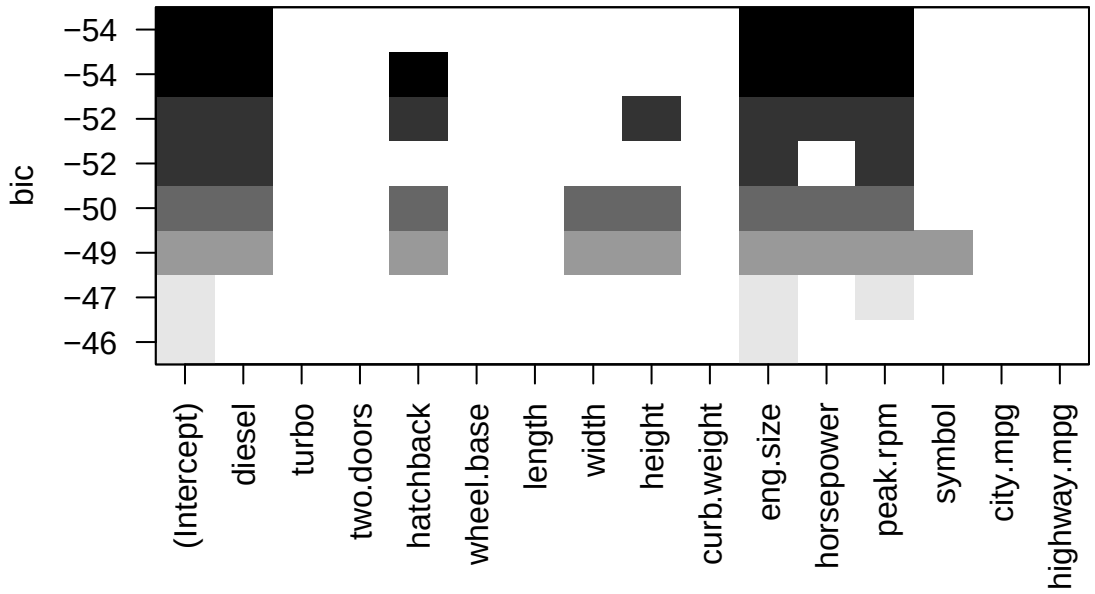
Examinons pour finir les VIF, ils sont corrects.

```
vif(modele4)
```

```
##     turbo hatchback    length     width  eng.size  peak.rpm    symbol
##  1.718858  2.049228  9.267903  5.899295  2.412832  1.452460  1.564567
```

On peut aussi utiliser la librairie suivante, qui permet notamment de tracer pour les meilleurs modèles parcourus l'importance de chaque variable :

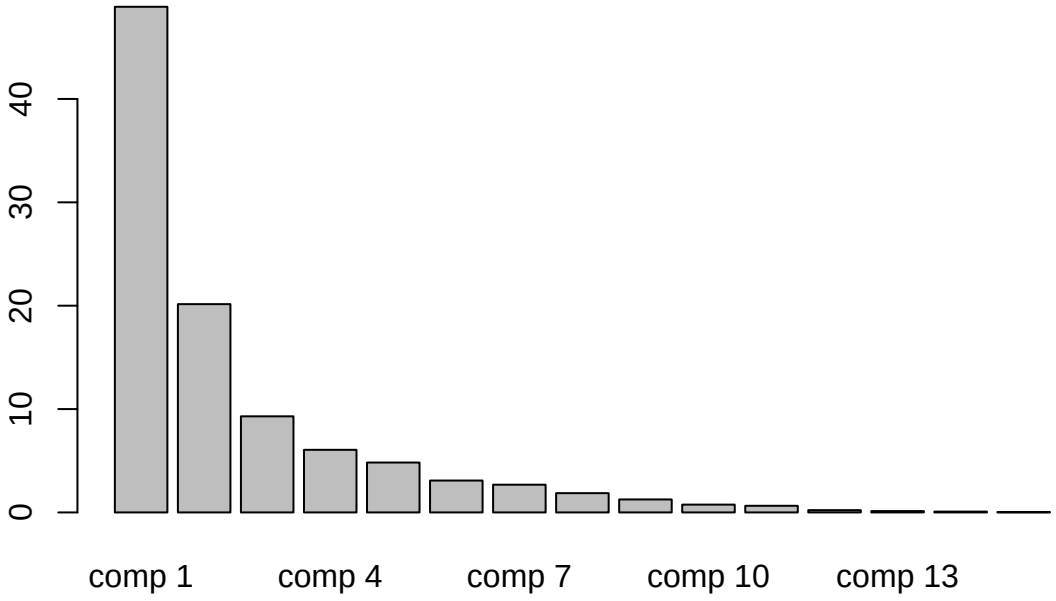
```
library(leaps)
modele4bis=regsubsets(price ~ .,method="forward", data = vehicles)
plot(modele4bis)
```



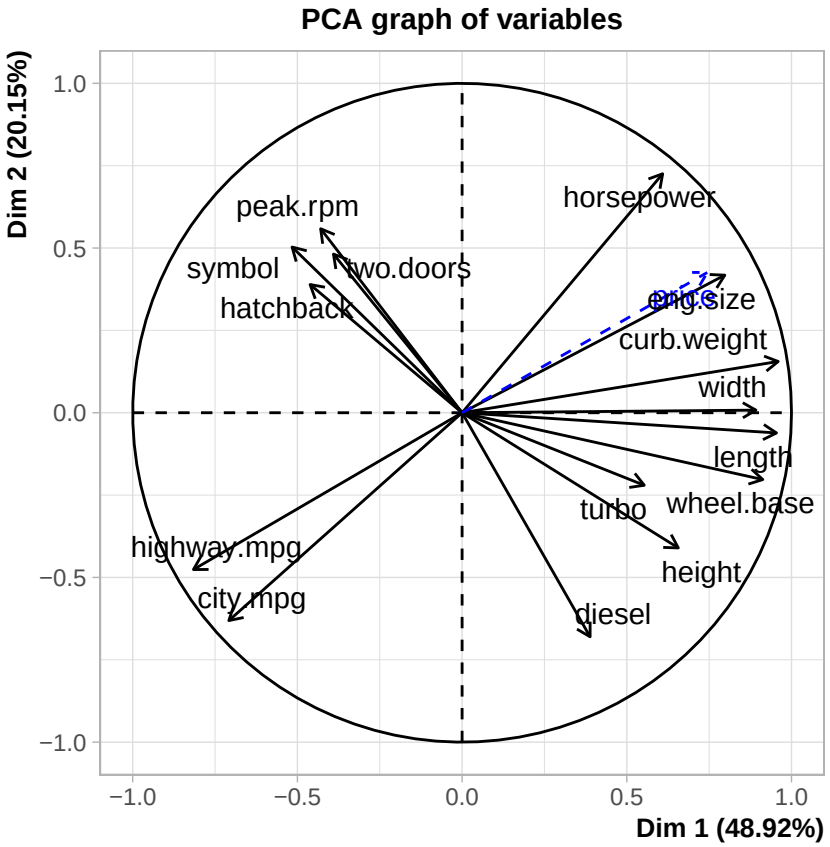
Regression sur composantes principales

On peut commencer par effectuer une ACP :

```
library(FactoMineR)
acp=PCA(vehicles,quanti.sup = 13,graph = FALSE)
barplot(acp$eig[,2])
```




```
plot(acp,choix="var")
```



Des stratégies classiques de sélection du nombre d'axe en ACP nous conduirait à choisir 3 axes. On peut aussi sélectionner ce nombre en fonction de la qualité du modèle de régression.

La régression sur composante principale peut-être réalisée directement

```
library(pls)
```

```
##
## Attachement du package : 'pls'
```

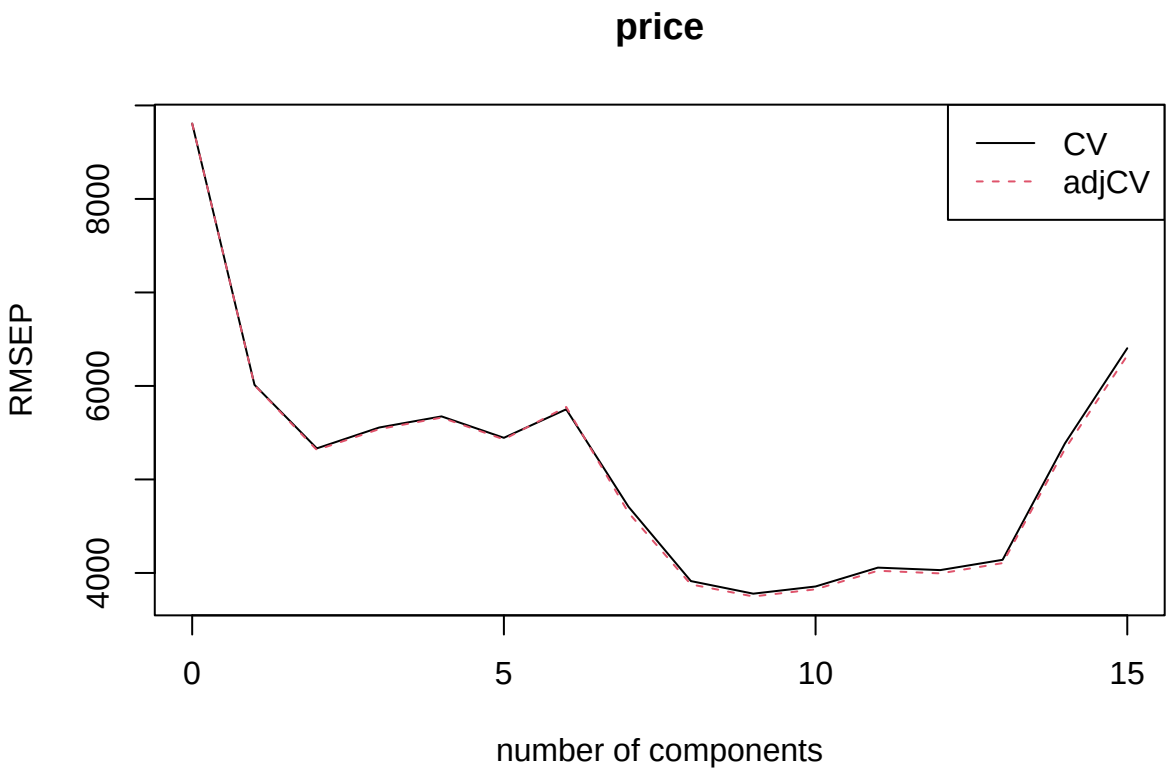
```
## L'objet suivant est masqué depuis 'package:stats':
##
##     loadings
```

```
modele5=pcr(price~.,data=vehicles,validation='LOO',scale=TRUE)
summary(modele5)
```

```
## Data:    X dimension: 30 15
##  Y dimension: 30 1
## Fit method: svdpc
## Number of components considered: 15
##
## VALIDATION: RMSEP
## Cross-validated using 30 leave-one-out segments.
##        (Intercept)  1 comps  2 comps  3 comps  4 comps  5 comps  6 comps
## CV            8806     6012     5332     5556     5674     5445     5750
## adjCV         8806     6007     5315     5538     5662     5429     5775
##        7 comps  8 comps  9 comps  10 comps  11 comps  12 comps  13 comps
## CV        4707     3913     3778      3855      4057      4030      4141
## adjCV     4643     3877     3749      3825      4023      3995      4105
```

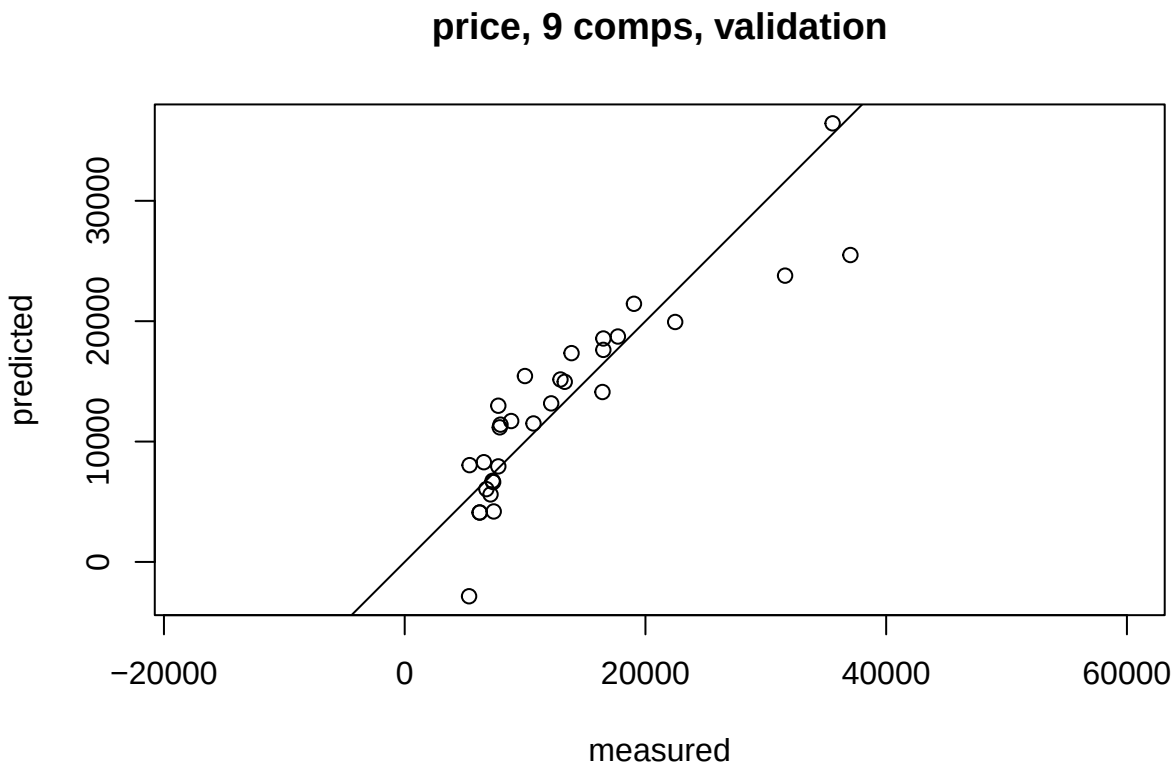
```
##        14 comps  15 comps
## CV         5386      6403
## adjCV      5325      6326
##
## TRAINING: % variance explained
##        1 comps  2 comps  3 comps  4 comps  5 comps  6 comps  7 comps  8 comps
## X        48.92    69.06    78.36    84.42    89.25    92.33    95.01    96.88
## price    55.39    73.52    73.56    75.97    78.87    82.64    92.29    92.85
##        9 comps  10 comps  11 comps  12 comps  13 comps  14 comps  15 comps
## X        98.13     98.89     99.53     99.75     99.88     99.96    100.00
## price    93.21     93.22     93.28     93.93     93.93     94.19     94.36
```

```
plot(RMSEP(modele5), legendpos = "topright")
```



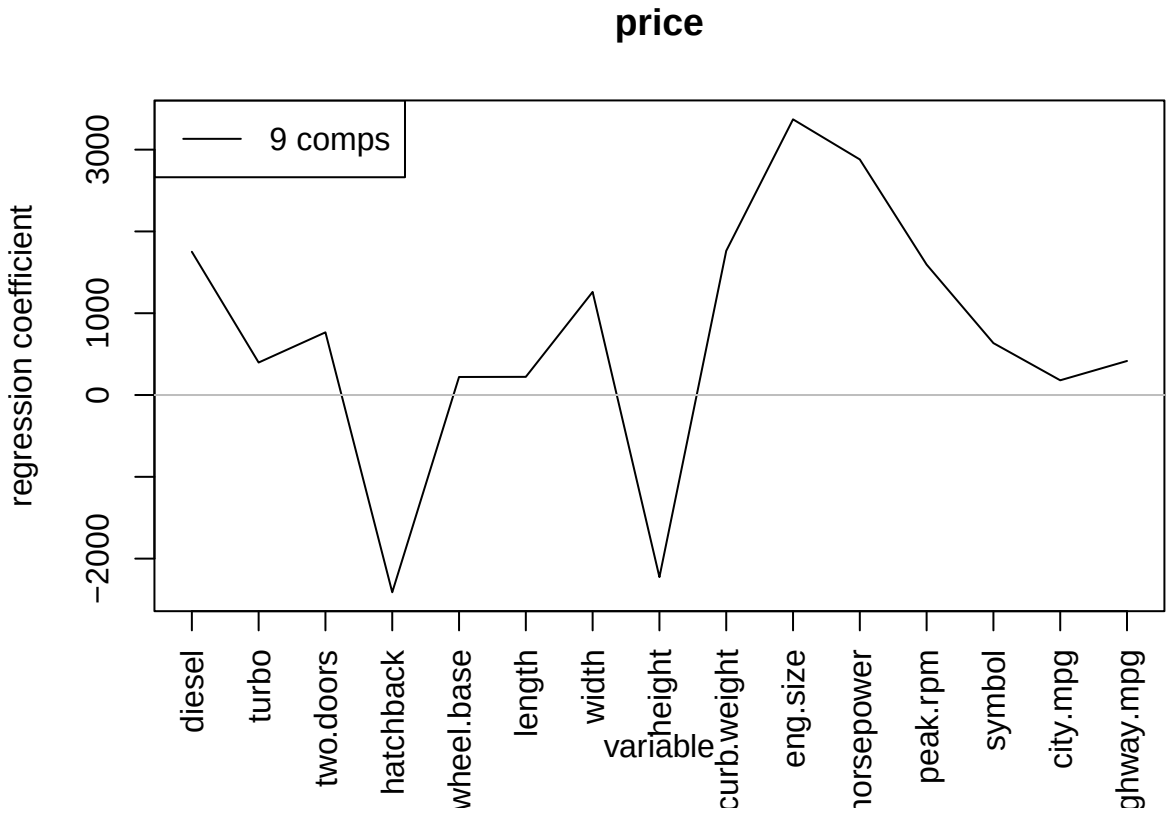
Le bon nombre de composantes semble être 9. On peut regarder la qualité de la prédiction de ce modèle en traçant les valeurs prédites en fonctions des valeurs mesurées :

```
plot(modele5, ncomp = 9, asp = 1, line = TRUE)
```



L'examen des coefficients de régression des composantes principales ne nous sera d'aucune utilité. Par contre, ces composantes étant elles-mêmes des combinaisons linéaires des variables initiales, on peut reconstruire les coefficients de régressions sur les variables initiales :

```
plot(modele5, plottype = "coef", ncomp=9, legendpos = "topleft",xaxt='n')
axis(1,at=1:15,labels=colnames(vehicles)[-13],las = 2)
```



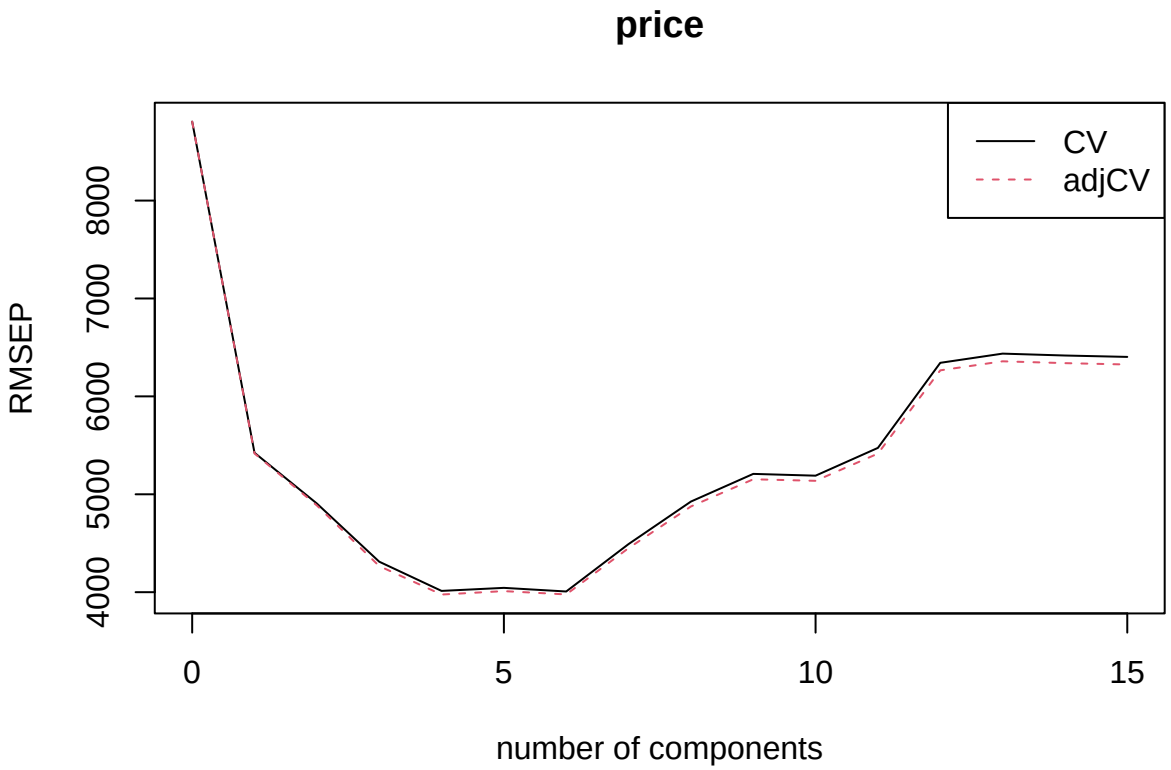
Regression sur composantes PLS

```
library(pls)
modele6=plsr(price~.,data=vehicles,validation='LOO',scale=TRUE)
summary(modele6)
```

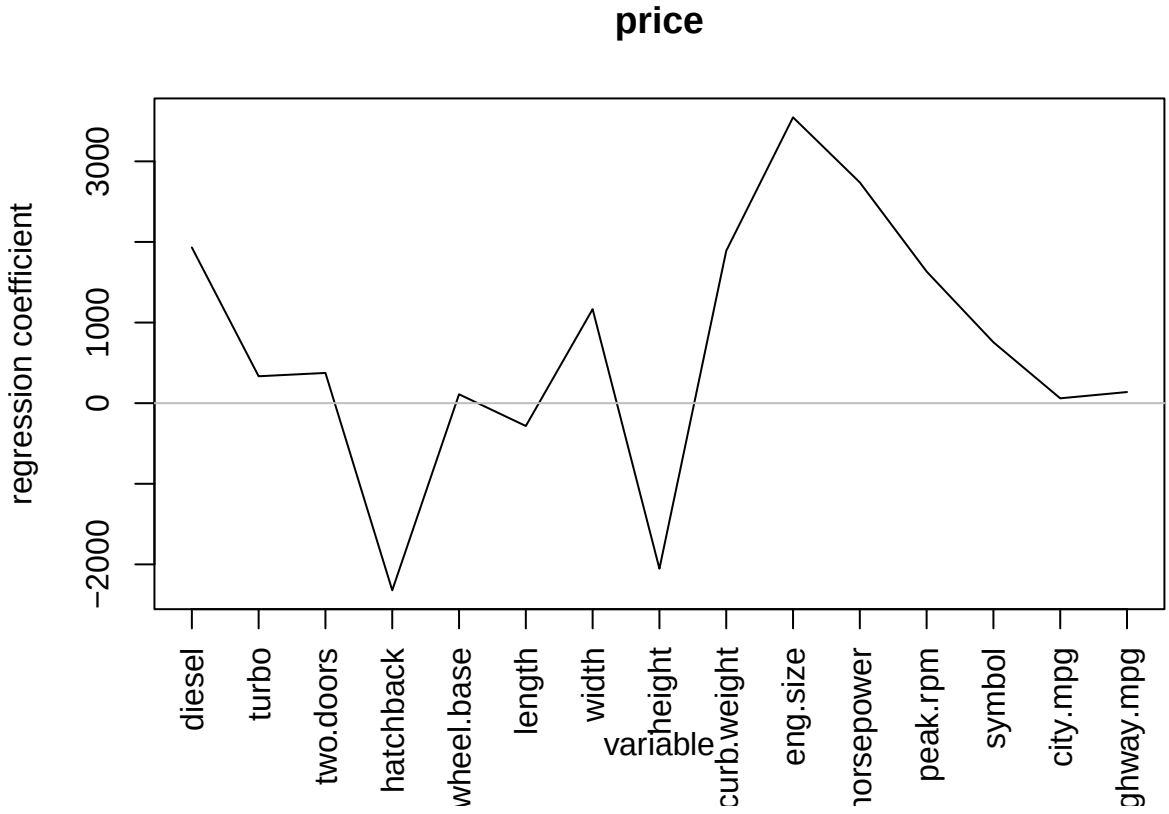
```
## Data:    X dimension: 30 15
##  Y dimension: 30 1
## Fit method: kernelpls
## Number of components considered: 15
##
## VALIDATION: RMSEP
## Cross-validated using 30 leave-one-out segments.
##        (Intercept)  1 comps  2 comps  3 comps  4 comps  5 comps  6 comps
## CV            8806     5424     4906     4312     4013     4044     4007
## adjCV         8806     5417     4886     4267     3976     4011     3977
##        7 comps  8 comps  9 comps  10 comps  11 comps  12 comps  13 comps
## CV        4491     4926     5208      5190      5474      6343      6437
## adjCV     4450     4876     5153      5137      5416      6266      6358
##        14 comps  15 comps
## CV         6417      6403
## adjCV      6339      6326
##
## TRAINING: % variance explained
##        1 comps  2 comps  3 comps  4 comps  5 comps  6 comps  7 comps  8 comps
## X        47.32    67.78    73.14    77.87    84.81    89.60    92.93    94.71
## price    70.47    82.18    92.06    93.32    93.46    93.64    93.84    94.07
##        9 comps  10 comps  11 comps  12 comps  13 comps  14 comps  15 comps
```

```
## X        96.58     98.07     98.84     99.09     99.78     99.87    100.00
## price    94.17     94.21     94.26     94.34     94.35     94.36     94.36
```

```
plot(RMSEP(modele6), legendpos = "topright")
```




```
plot(modele6, plottype = "coef", ncomp=4,xaxt='n')
axis(1,at=1:15,labels=colnames(vehicles)[-13],las = 2)
```


price
regression coefficient
3000
1000
0
−2000
diesel
turbo
two.doors
hatchback
wheel.base
length
width
height
curb.weight
eng.size
horsepower
peak.rpm
symbol
city.mpg
ghway.mpg
variable