

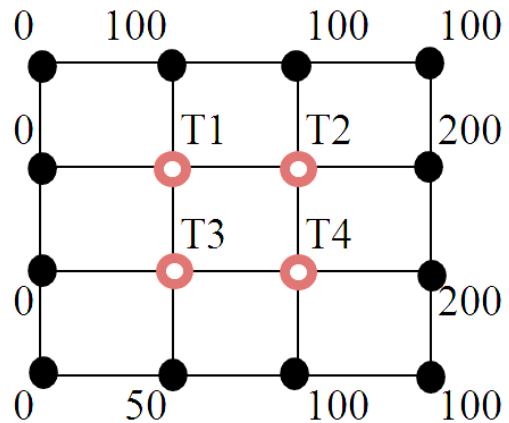
# Assignment#7

## Numerical Sol<sup>n</sup>. of Partial Differential Equations

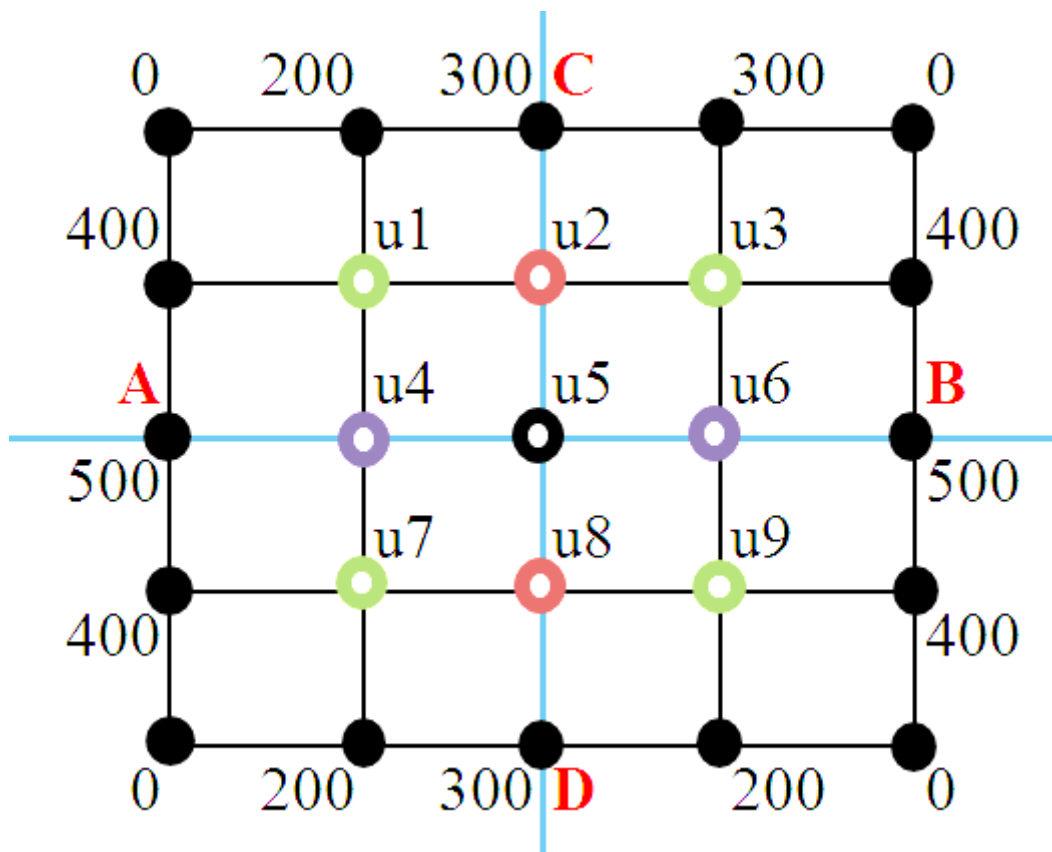
1. The steady state two dimensions heat-flow in a metallic plate is given by

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$

Given the boundary conditions as shown in the figure below, find the temperatures T1, T2, T3 & T4. Solve the equations using Gauss-Seidel method.

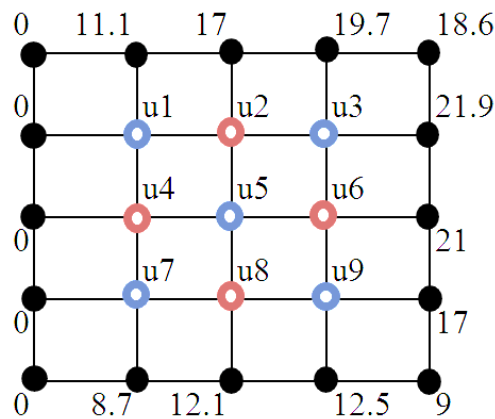


2. Torsion on a rectangular bar subject to twisting is governed by  $\nabla^2 T = -4$ . Given conditions:  $T = 0$  on boundary, find  $T$  over a cross section of a bar of size **9cm x 9cm**, use the small grid size of **3cm x 3cm**.
3. Solve the Laplace equation  $U_{xx} + U_{yy} = 0$  given that



4. Solve for the steady-state temperature in rectangular plate **8cm x 10cm**, if one 10cm side is held at 50C, and the other 10cm side held at 30C and the other sides are held at 10C. Assume square grids of **2cm x 2cm**. [Hint: 4x5 data grid]

5. Solve the Laplace equation  $U_{xx} + U_{yy} = 0$  given that



6. Solve the equation  $\nabla^2 f = F(x, y)$  with  $F(x, y) = xy$  and  $f = 0$  on boundary. The domain is a square with corners at  $(0, 0)$  &  $(3, 3)$ . Use  $h=1$
7. Given  $\frac{\partial^2 f}{\partial x^2} - \frac{\partial f}{\partial t} = 0$ ;  $f(0, t) = f(5, t) = 0$ ,  $f(x, 0) = x^2(25-x^2)$ ; find the values of  $f$  for  $x = ih$  ( $i = 0, 1, \dots, 5$ ) and  $t = jk$  ( $j = 0, 1, \dots, 6$ ) with  $h = 1$  and  $k = \frac{1}{2}$ , using the explicit method.
8. Estimate the values at grid points of the following equations using recurrence formula [ $h = 1$ ]:
- $f_{xx} - 0.5f_t = 0$   
Given:  $f(0, t) = 0$ ;  $f(5, t) = 0$ ;  $f(x, 0) = x(5-x)$ ;
  - $9f_{xx} = f_t$   
Given:  $f(0, t) = -5$ ;  $f(5, t) = 5$ ;  
$$f(x, 0) = \begin{cases} -5 & \text{for } 0 \leq x \leq 2.5 \\ 5 & \text{for } 2.5 \leq x \leq 5 \end{cases}$$
9. Solve by relaxation method, the equation  $\nabla^2 u = 0$  in the square region with square meshes starting with the initial values  $u_1 = u_2 = u_3 = u_4 = 1$ .

[Ans:  $u_1 = 1$ ,  $u_2 = 1.3$ ,  $u_3 = 0.7$ ,  $u_4 = 1$ ]

