ASEN 3112 Torsion Experimental Lab 1

Group Leader: Gerardo Pulido*
Justin Alvey † Mattia Astarita‡ Jose Feu Vidal§
Matthew Stoffle¶ Nicholas Taylor‖
Section 012, Group 9, ASEN 3112

November 26, 2016

This laboratory experiment was performed to compare the experimental results from the procedure and from the CTW(Closed Thin Wall)/OTW(Open Thin Wall) calculations. The experiment consisted of placing an open and closed cylinder, at a time, in the Instron torsion machine. The torque in the machine was controlled and started from approximately 0lb-in and was steadily increased to the desired torque, 400lb-in for the closed cylinder and 20lb-in for the open cylinder. The experimental data that was captured was the shear strain and the applied torque. This data was then used against exact and approximate calculations for shear strain. A theoretical analysis was performed to derive and present the equations that were used for the thin wall theories and the measured results. The approximate and exact strain values were then used to calculate twist angles, twist angle ratios, and torsional rigidities. The torque and shear strain results displayed a linear relationship, while the twist ratio vs twist angle and torque vs torsional rigidity displayed a constant relationship. At the end of the report, a final error analysis that compared maximum shear strain values was performed to verify exactly how far off the values from the thin wall theories were from the ones that were measured.

Nomenclature

τ	Shear Stress
T	Torque
R	Radius
R_e	Outer Radius
R_i	Inner Radius
J	Polar Moment of Inertia
γ	Shear Strain
G	Shear Modulus
q	Shear Flow
t	Thickness
A_e	Cross Sectional Area
ϕ	Twist Angle
ϕ_{rat}	Ratio of twist angle to shear strain
$\alpha\beta$	Dimensionless Coefficients

*SID: 100628458
†SID: 103265867
‡SID: 102966194
§SID: 103048835
¶SID: 103182665
‖SID: 104302919

Contents

I. Introduction

The purpose of this laboratory is to first test the behavior under applied torque of two circular thin wall specimens, and then compare the measured values with the ones predicted using analytical theory. The two objects are tested in the Instron Torsion Testing Machine and the values being measured are the shear strain γ and the Torque T. The angle of twist ϕ and the torsional rigidity GJ are calculated through Thin Wall (TW) theory and then compared to the experimental values in order to verify the validity of TW theory. The first object to be tested is a stock aluminum tube given in figure (1) with an exterior radius of $R_e = 3/8in$, a uniform wall thickness of $t = 1/16in$, and a shear modulus of $G = 3.75 \times 10^6 psi$. Since the thickness is about 6% of the radius and since one cell shear-flow circuit can be identified in the cross section, the tube can be analyzed using Closed Thin Wall theory. The second object to test is also made out of stock aluminum but it has a thin wall circular cross section with cut of negligible size given by figure (2). Specimen 2 has the same dimensions as specimen 1, but due to the cut it has to be analyzed using Open Thin Wall theory. Real images of the closed and open cylinders can be seen below in figures (2) and (4) respectively:

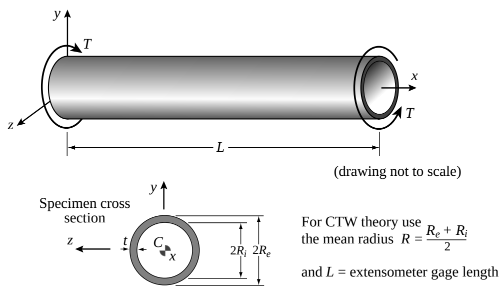

Figure 1: Stock aluminum tube tested in Instron Torsion testing machine. The closed circular cross section is shown at the bottom. This specimen obeys to CTW theory.

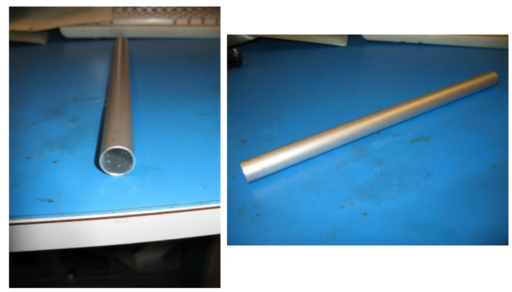

Figure 2: Real image of the aluminum tube with closed circular cross section

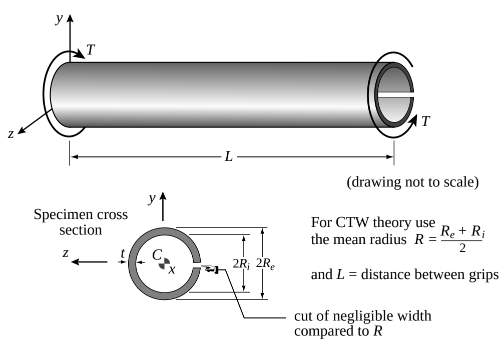

Figure 3: Stock aluminum tube tested in Instron Torsion testing machine. The circular cross section with its cut of negligible size is shown to the right. This specimen obeys to OTW theory.

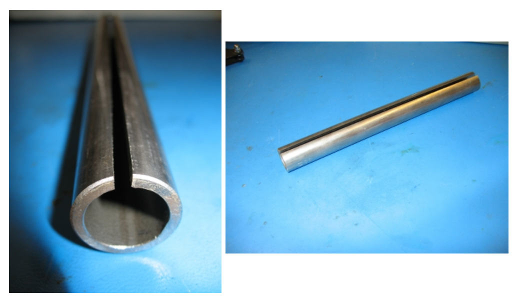

Figure 4: Real image of the aluminum tube with open circular cross section

II. Experimental Procedure

In order to obtain the necessary results from both OTW and CTW elements and be able to compare them to the expected results, the team used an Instron Torsion machine. The machine′s test section (shown in Figure 1) features two metal plugs along the same axis located at each end of the test section, each of which has a three teeth grip over it to ensure no slippage of the specimen. This machine applies torque to the tube placed between its grips and plots a graph of strain vs. torque in the user interface. To insert the specimen, the grips were loosed by inserting a chuck tool into them and rotating CCW, and the specimen was placed into the metal plugs and was secured with a small width piece of tape. The Closed Thin Wall tube was the first to be tested and by aligning the mark on the specimens surface with the grip, the team ensured that the specimen will extend about inch in, making it possible for the grip to only grab onto the aluminum tube instead of the plugs or the tape. In the case of the Open Thin Wall tube, this distance allowed space on either side between the grip and specimen to prevent premature buckling. When tightening up the grips, the left hand side was gripped first using a torque of 40 ft-lb and the right hand side followed as the mark on the specimen was again aligned with the inside edge of the grip. Because it is normal that the grips applied some torque to the specimen, the controlled torque was zeroed. Before running the Torsion machine, an Epsilon Torsional extensometer (depicted in Figure 2) was mounted on the specimen to record any change in shear strain through out the test. To do so, the device has four sharp contact points whose tips gently touched the specimen. The extensometer featured three different parts: Spring Clip assembly, Specimen Adjustment Thumb Screw, and Spring Pins; along with a custom alignment block. The Spring Pins worked as zero pins, thus the team made sure they were inserted into the extensometer up until the test was ready to be ran. By loosening up the Specimen Adjustment Thumb Screw the team was able to fit the extensometer into the specimens diameter. Then the Custom Alignment Block was placed underneath the specimen, allowing the extensometer to geometrically match with the tube. This last step was easily executed due to a color matching pattern that allowed the team to know the proper orientation of the extensometer. Following, the Spring Clip assembly was carefully attacked to the extensometer placing its arms parallel to each other. Lastly, all zero pins were removed carefully, while the shear strain due to placing of the tube was watched to not increase more than 0.5. If so, the zero all button was clicked and the test section lid could now be closed.

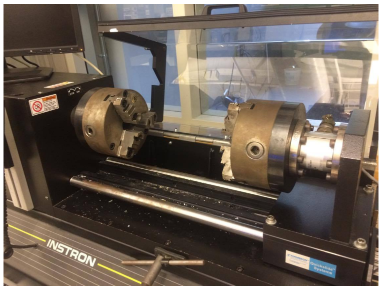

Figure 5: Instron Torsion Testing Machine

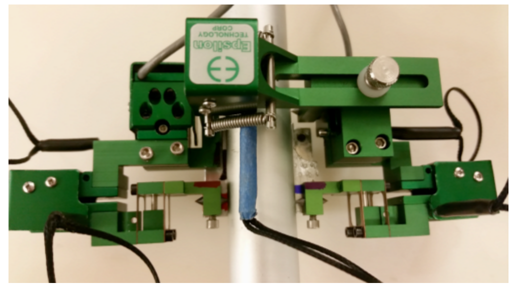
Figure 6: Extensometer attached to the aluminum cylinder

Once the machine was properly set up, the team ran the test while watching that the outputted plot (Strain vs. Torque) showed linearity and increase in Torque. In particular, the experiment twisted the left hand side by either 0.1 or 2deg/min (CTW or C-channel) until the torque applied reached 20 or 400 in-lb respectively. After reaching such torque the machine turned automatically CCW and return to the initial position. Once the data was saved and acquired, the file was opened to ensure a full data file. The specimen was removed when torque was no longer a high value and the lid of the test section was closed.

III. Theoretical Analysis

A. Shear strain vs. Torque

1. Closed Cylinder

Shear strain and torque were compared for the closed sample using exact cylinder theory and closed thin wall theory. For exact cylinder theory the maximum shear stress and annular polar moment of inertia are given below :

$$\tau_{max} = \frac{TR}{J} \qquad J = \frac{\pi}{2}(R_e^4 - R_i^4) \tag{1}$$

from lecture 7. Hookes law gives us shear strain

$$\gamma = \frac{\tau}{G} = \frac{2TR_e}{\pi(R_e^4 - R_i^4)G} \tag{2}$$

For closed thin wall theory, the maximum shear stress is given by

$$\tau_{max} = \frac{q}{t_{min}} = \frac{T}{2t_{min}A_e} \tag{3}$$

from lecture 9 equation 3. Assuming radius

$$R = \frac{R_e + R_i}{2} \tag{4}$$

the cross sectional area and constant thickness maximum shear of the cylinder are given as

$$A_e = \pi\left(\frac{R_e + R_i}{2}\right)^2 = \pi\frac{(R_e + R_i)^2}{4} \qquad \tau_{max} = \frac{T}{2t\pi\frac{(R_e+R_i)^2}{4}} = \frac{2T}{t\pi(R_e + R_i)^2} \tag{5}$$

Hookes law gives us shear strain

$$\gamma = \frac{\tau}{G} = \frac{2T}{Gt\pi(R_e + R_i)^2} \tag{6}$$

2. *Open Cylinder*

Shear strain and torque were also compared for the open sample using open thin wall theory. Open thin wall theory gives maximum shear stress and polar moment of inertia below

$$\tau_{max} = \frac{Tt}{J_\alpha} \qquad J_\alpha = \alpha b t^3 = \frac{1}{3} b t^3 \tag{7}$$

from lecture 8, since the circumference of the cylinder is significantly larger than the thickness, so alpha can be assumed to be equal to 1/3. Hookes law gives us shear strain

$$\gamma = \frac{\tau}{G} = \frac{3T}{Gbt^2} \tag{8}$$

B. Twist vs Torque

1. *Closed Cylinder*

Torque and Twist were compared for the closed wall using exact cylinder theory. Exact cylinder theory gives twist and annular polar moment of inertia below:

$$\phi = \frac{TL}{GJ} \qquad J = \frac{\pi}{2}(R_e^4 - R_i^4) \tag{9}$$

from lecture 7. Putting these equations together, twist can be written as

$$\phi = \frac{2TL}{\pi G(R_e^4 - R_i^4)} \tag{10}$$

For closed thin wall theory, twist and polar moment of inertia are given as:

$$\phi = \frac{TL}{GJ} \qquad J = \frac{4A_e^2 t}{p} \tag{11}$$

from lecture 9. The cross sectional area, perimeter, and polar moment of inertia of the cylinder are given as

$$A_e = \pi\Big(\frac{R_e + R_i}{2}\Big)^2 = \pi\frac{(R_e + R_i)^2}{4} \qquad p = \pi(R_e + R_i) \qquad J = (R_e + R_i)t \tag{12}$$

Then the twist angle can be written as

$$\phi = \frac{TL}{G(R_e + R_i)t} \tag{13}$$

2. *Open Cylinder*

The torque and twist for an open wall section is found using open wall theory, a slightly modified version of closed wall theory. Narrow open thin wall theory states:

$$\phi = \frac{TL}{GJ_\beta} \qquad J_\beta = \beta b t^3 \tag{14}$$

J in this situation is not necessarily the polar moment of intertia, but it still has the same units as length to the fourth power. Combining these two equations together results in:

$$\phi = \frac{TL}{G\beta b t^3} \tag{15}$$

The coefficient β is a numerical coefficient that depends on the ratio of $\frac{b}{t}$. The value of β can be found looking up in a table.

C. Twist Angle Ratio vs Twist Angle

1. *Closed Cylinder*

Twist angle ratio is defined as the twist angle between the grips divided by the shear strain of the outer surface for the closed specimen. As derived earlier, the twist angle and shear strain for a closed thin wall specimen are shown below:

$$\phi = \frac{TL}{G(R_e + R_i)t} \qquad \gamma = \frac{2T}{Gt\pi(R_e + R_i)^2} \tag{16}$$

Therefore the twist ratio can be found as:

$$\phi_{rat} = \frac{\phi}{\gamma} = \frac{(R_e + R_i)L\pi}{2} \tag{17}$$

2. Open Cylinder

As derived earlier, the twist angle and shear strain for an open thin wall specimen are shown below:

$$\phi = \frac{T}{G\beta bt^3} \qquad \gamma = \frac{3T}{Gbt^2} \tag{18}$$

Therefore the twist ratio can be found as:

$$\phi_{rat} = \frac{\phi}{\gamma} = \frac{L}{3\beta t} \tag{19}$$

IV. Data Reduction and Presentation

A. Experimental Data Results

Below are the tabulated data for measured and predicted shear strain. Table (1) has the predicted data for CTW theory, exact theory, and the measured experimental data, for the given torque values. Table (2) has the predicted data for OTW theory and the measured experimental data, for the given torque values. Both Tables hold the predicted and measured GJ values for torque rigidity for each specimen case, which reflects the quality of the torsion test. These predicted results for GJ in each case are constant.

Table 1: Experimental Data Results: CTW Specimen

Torsion Values	Predicted γ_{CTW}	Predicted γ_{Exact}	Measured γ	Predicted GJ	Measured GJ
100	0.0328	0.0355	0.0379	6031	5646
200	0.0658	0.0712	0.0741	6031	5802
300	0.0988	0.1069	0.1113	6031	5794
400	0.1317	0.1425	0.1484	6031	5794

Table 2: Experimental Data Results: OTW Specimen

Torsion Values	Predicted γ_{OTW}	Measured γ	Predicted GJ	Measured GJ
5	0.0250	0.0272	719	660.6
10	0.0500	0.0534	719	673.1
15	0.0748	0.0797	719	673.4
20	0.0997	0.1072	719	668.0

B. Torque and Shear Strain

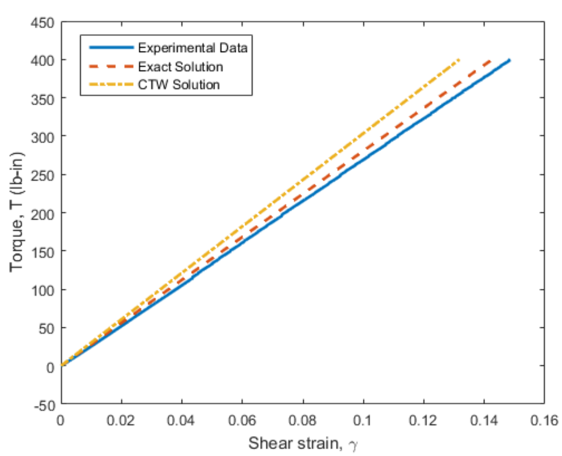

Figure 7: Torque vs. Shear strain for the closed thin wall specimen.

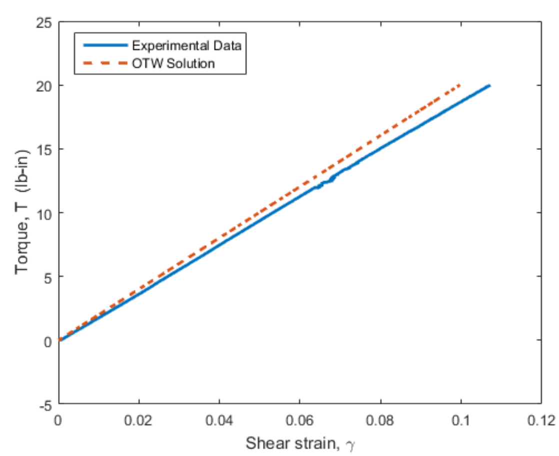

Figure 8: Torque vs. Shear strain for the open thin wall specimen.

C. Torque and Twist Angle

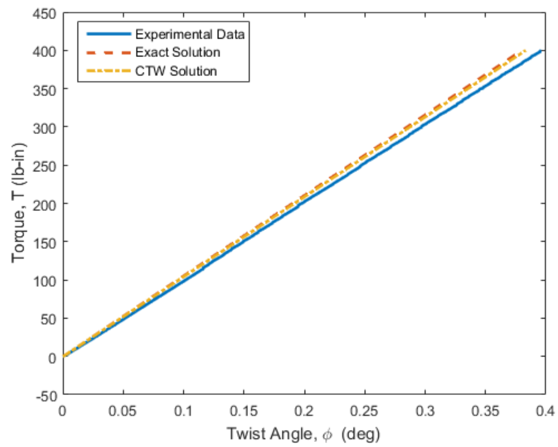

Figure 9: Torque vs. Twist Angle for the closed thin wall specimen.

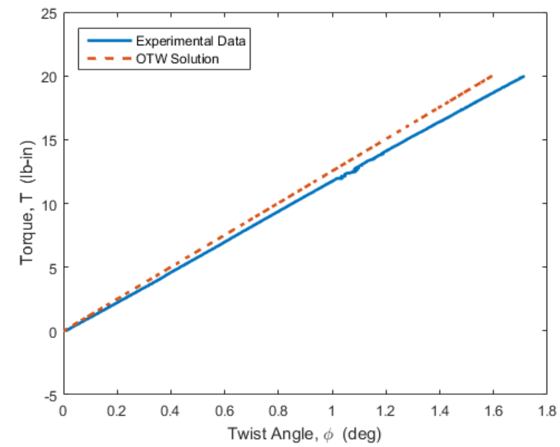

Figure 10: Torque vs. Twist Angle for the open thin wall specimen.

D. Twist Angle Ratio and Twist Angle

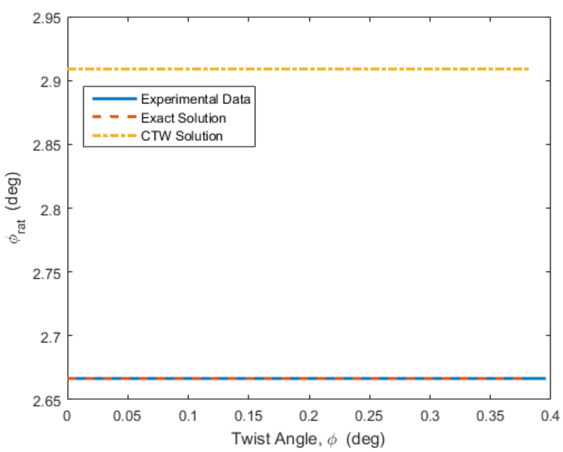

Figure 11: Twist Angle Ratio vs. Twist Angle for the closed thin wall specimen.

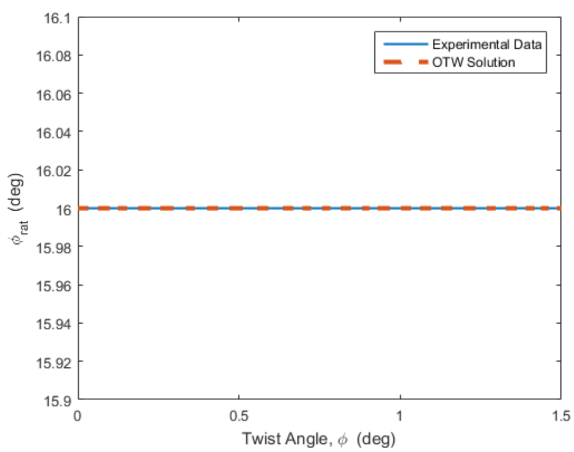

Figure 12: Twist Angle Ratio vs. Twist Angle for the open thin wall specimen.

E. Torque and Torsional Rigidity

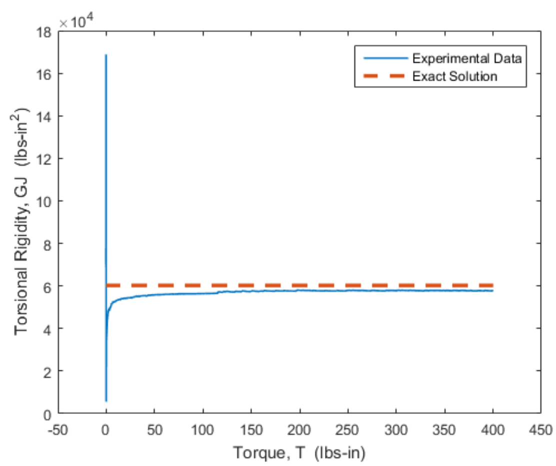

Figure 13: Torsional Rigidity vs. Torque for the closed thin wall specimen.

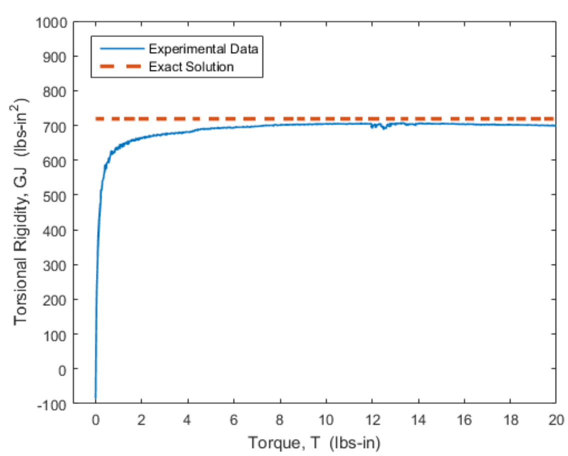

Figure 14: Torsional Rigidity vs. Torque for the open thin wall specimen.

V. Discussion and Error Analysis

The relationship studied here is between the torque and the shear strain. The torque and shear strain values are from the measured results (experimental data), exact theory, CTW theory, and OTW thoery. As given by Figures (7) and (8), shear strain varies linearly with torque. Similarly, Figures (9) and (10) show that the twist angle varies linearly with torque. Twist angle can be rewritten as the ratio between phi and gamma, ϕ_{rat}. The twist angle is constant with respect to ϕ_{rat} for both specimen. As mentioned, torsional rigidity is an indiciation of how well an experiment has run, given by the data. This quantity is constant with torque in the theoretical case, but is shown as an increasing function by the experimental data in Figures (13) and (14), up until a constant GJ.

A final calculation of relative error for both open and closed cylinders was performed to verify how far off the results were from the exact theory, CTW theory, and open thoery, to the experimental results. The maximum values for the shear strain, with its respected torque value, were used. The set of equations are given below:

$$Error_{CTW} = \frac{|\gamma_m - \gamma_{CTW}|}{\gamma_m} \qquad Error_{Exact} = \frac{|\gamma_m - \gamma_{Exact}|}{\gamma_m} \qquad Error_{OTW} = \frac{|\gamma_m - \gamma_{OTW}|}{\gamma_m} \tag{20}$$

Tables (3) and (4) given below display numerical values for the error between the maximum shear strain from the thin wall solutions and the shear strain measured. Maximum values for the shear are taken to observe the maximum error involved, as the difference between the predicted and measured results for each case increases with torque.

Table 3: Relative error for the closed cylinder

γ_m	γ_{CTW}	Error %
0.1484	0.1317	11.23%

γ_m	γ_{Exact}	Error %
0.1484	0.1425	3.95%

Table 4: Relative error for the open cylinder

γ_m	γ_{OTW}	Error %
0.1072	0.0997	7.02%

While it is clear that it is physically impossible to obtain a perfect experiment, it is important to realize where possible sources of error originate to know how to improve for later tests. One source of error is the slippage that occurs between the grips and the specimen. Slippage can be seen as very jagged changes in torque as the machine rotates the bar and after occuring many times, small amounts of error can build up for what the actual torque values should be. Another source of error would be the fact that the stress concentration is assumed to be constant throughout the specimen. In reality, stress concentrations can be very complex, therefore this lab does not yet mention variable stress concentrations, however making this assuming will allow error to build over time.

VI. Conclusions

In conclusion, the results for all of the comparisons were as expected. The plots for the torque and shear strain displayed a linear relationship. The CTW solution was a bit off from the exact and measured results. Although, the open cylinder OTW solution was not as far off from the measured results. Similar behavior for the other required plots was displayed with the CTW theory being bit off from the measured results. For the open cylinder, the results for the OTW theory also showed a small deviation from the measured results. Noticeable slippage also occurred when applying torque to each specimen, which caused the great drop in data in Figure (13). A final error analysis was performed for the shear strain values due to the fact that those are the first values to be calculated. The error analysis was a final piece of evidence that displayed the accuracy of the thin wall theories. Although the results for the OTW and CTW solutions displayed a decent approximation, the error does increase as the torque applied increases, especially for the open cylinder due to the accuracy at a low applied torque.

References

[1]Trudy Schwartz, *Torsional Testing Machine Procedure* [*Desire2Learn Course Webpage*]. Accessed on 09/26/16
https://learn.colorado.edu/d2l/le/content/181955/viewContent/2642754/View

[2]Carlos Felippa, *ASEN 3112 Lab 1 Description* [*Desire2Learn Course Webpage*]. Accessed on 10/07/16
https://learn.colorado.edu/d2l/le/content/181955/viewContent/2642754/View

[3]Carlos Felippa, *Lecture 7* [*Desire2Learn Course Webpage*]. Accessed on 10/07/16
https://learn.colorado.edu/d2l/le/content/181955/viewContent/2627072/View

[4]Carlos Felippa, *Lecture 8* [*Desire2Learn Course Webpage*]. Accessed on 10/07/16
https://learn.colorado.edu/d2l/le/content/181955/viewContent/2627074/View

[5]Carlos Felippa, *Lecture 9* [*Desire2Learn Course Webpage*]. Accessed on 10/07/16
https://learn.colorado.edu/d2l/le/content/181955/viewContent/2642767/View

Acknowledgments

We want to thank the TA's for the help during the lab experiment for the support during the lab. We would also like to thank Trudy for the experimental lab demo. A final thanks goes to Professor Felippa.