Comp Lab 4: Flow Over Thin Airfoils

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I. Introduction and Theory

The Vortex Panel Method is an accurate numerical method for estimating the flow over an airfoil. As thin airfoil theory allowed us to induce lifting flow over a cylinder, a vortex sheet can be used to induce lifting flow over an airfoil. This can be done by superimposing uniform flow and a vortex sheet onto a body of given shape, subject to the Kutta Condition.

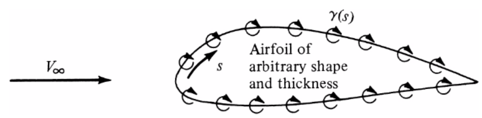


Figure 1: Depiction of a thin, cambered airfoil subject to lifting flow. This has been done by superimposing uniform flow and a vortex sheet onto an airfoil body. Per this depiction, the Vortex Panel Method will be used to calculate lift and other aerodynamic quantities for a number of airfoil geometries.

To solve a theoretical problem like this numerically, three steps need to be taken into account:

1. Discretize the geometry
2. Discretize the solution field
3. Discretize the governing equation[1]

These three steps form the basis of a numerical method known as the Vortex Panel method. These steps produce a linear system of m+1 equations and m+1 unknowns. Thus, the system $A\vec{\mathbf{x}} = \vec{\mathbf{b}}$[1] can be solved for the strength of each vortex, where $\vec{\mathbf{x}}_j = \gamma_j$ for every panel j = 1,..., M. From this relationship, the velocity can be calculated at every point, and then used to calculate the coefficient of pressure

$$C_{p,i} = 1 - \left(\frac{V_i}{V_\infty}\right)^2 \tag{1}$$

Further, the strength of the entire vortex sheet can be calculated using

$$\Gamma = -\oint_C \mathbf{V} \cdot \mathbf{ds} = \sum_{j=1}^{m} V_j S_j \tag{2}$$

to ultimately find the sectional coefficient of lift

$$c_l = \frac{L'}{q_\infty c} = \frac{\rho_\infty V_\infty \Gamma}{0.5\rho_\infty V_\infty^2 c} = \frac{2\Gamma}{V_\infty c} \tag{3}$$

II. A Note on Computational Accuracy

Using the Vortex Panel numerical method introduced a certain degree of error in producing a "realistic" accuracy in the results. Due to the nature of numerical methods, error cannot be totally eliminated from estimations. However, it can be limited greatly. A nominal number of panels N were used throughout these calculations to produce a "standard" solution - a solution close enough to the actual solution that the relative error is negligible. For this procedure, the definition of a desired quantitative level of accuracy was left to the student. Below are tabulated results of the coefficient lift for a NACA 0012 airfoil at 0^o angle of attack for varying panel resolutions.

C_l	**Panels (N)**	**Error**
6.7654e-16	100	80.2%
-2.1727e-15	200	>100%
1.3015e-15	300	62%
3.4220e-15	500	29%

Table 1: Values for sectional coefficient of lift vs number of panels N. From symmetric airfoil theory, it is known that the coefficient of lift at $\alpha = 0^o$ for a symmetric airfoil is 0. Thus, the error for these calculations is negligible.

The error varies from N = 100 to 500, having at most a relative error over 100%. This error seems high, but it is known from the thin airfoil theory, the coefficient of lift for a symmetric airfoil is zero at $\alpha = 0^o$. Since there is no camber in the airfoil geometry, a symmetric airfoil will not generate lift at zero angle of attack. With this in mind, the relative error between 100 and even 500 panels is negligible, as the precision is ever so close to the actual value. Also, from calculating the sectional coefficient of lift for a cambered airfoil, the relative error between 100 and the "actual value" at 2500 panels is barely 0.11%![a] Ultimately, to maintain a qualitative level of accuracy, N was taken for 250 panels. This number was high enough to ensure a qualitative accuracy for lifting flow over the airfoil, while still maintaining a low computational cost. The computations that follow all use this nominal number of panels for the Vortex Panel method.

III. Results

A. Lifting Flow for NACA 0012 Airfoil

For a NACA 0012 airfoil, the relationships of c_p vs α and c_l vs α can be depicted graphically from the results of the Vortex Panel Method. Below are c_p vs α and c_l vs α for chosen angles of attack, $\alpha = -5^o, 0^o, 5^o$, and 10^o.

[a]To see this calculation and the relative error associated with a cambered airfoil at zero angle of attack, see Appendix B

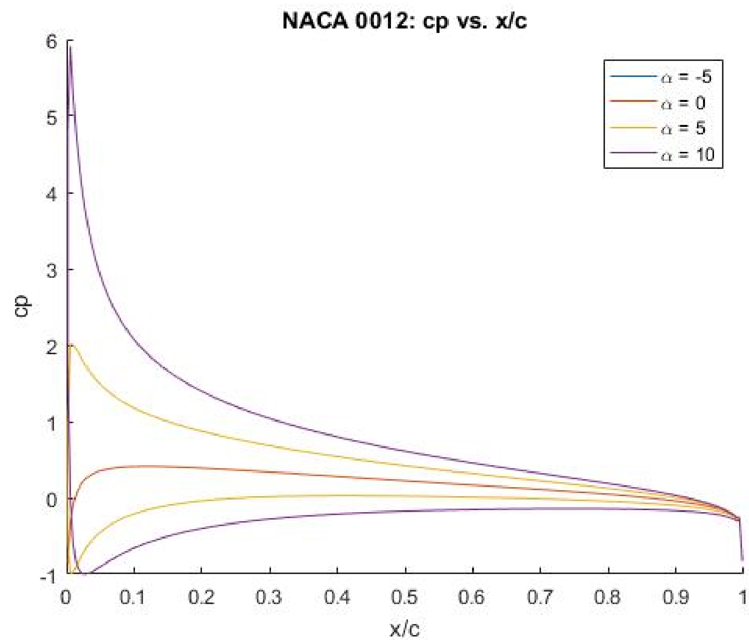


Figure 2: The coefficient of pressure vs alpha for NACA 0012 airfoil.

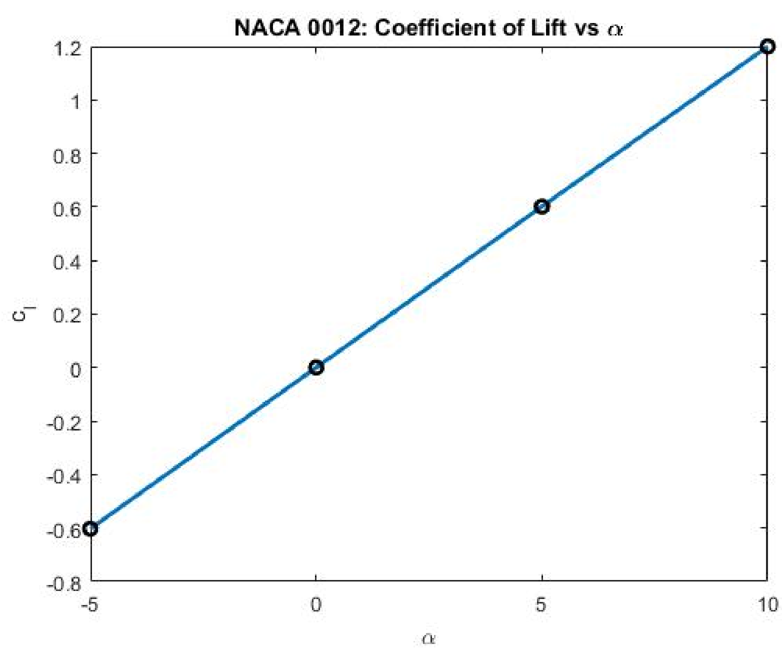


Figure 3: The sectional coefficient of lift vs alpha for the NACA 0012 airfoil.

B. Lifting Flow for Other Airfoils

The general form of c_l vs α for varying α can be depicted for any airfoil geometry. Results of the Vortex Panel Method for airfoils NACA 0012, 2212, 4412, and 2430[b] are provided below in Figure (4).

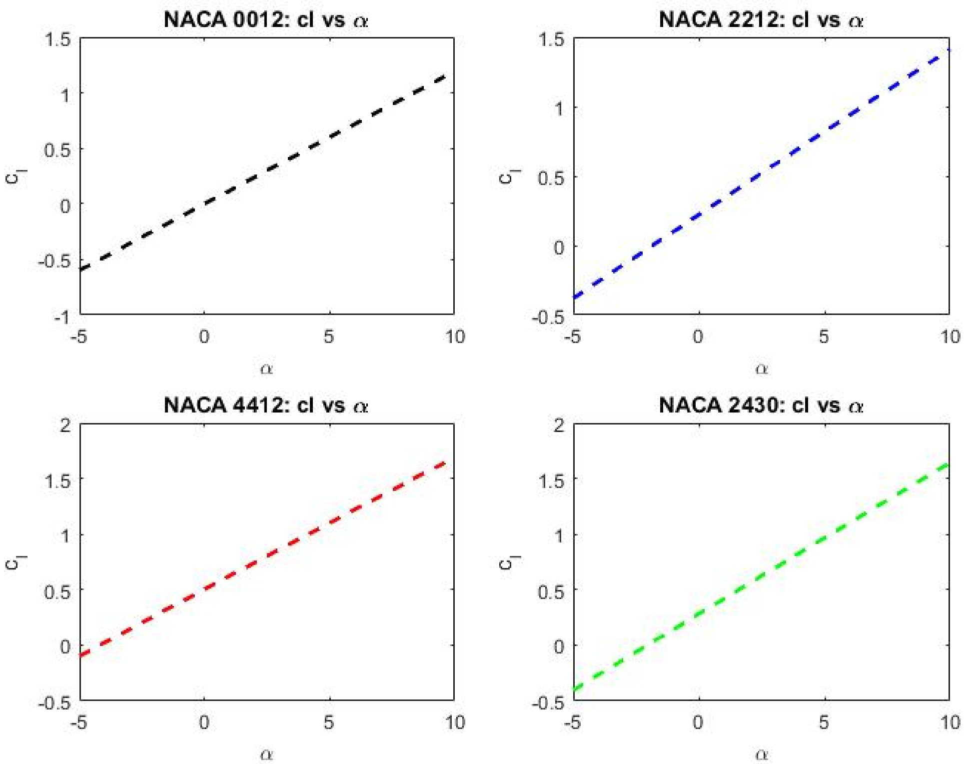


Figure 4: The sectional coefficient of lift vs alpha for the NACA 0012, 2212, 4412, and 2430 airfoils.

C. Estimating Lift Slope and $\alpha_{L=0}$

From the plots of c_l vs α, the lift slope for each airfoil can be estimated,[1] given the expression

$$a_0 = \frac{\partial c_l}{\partial \alpha} \tag{4}$$

Simply, the lift slope is the time rate of change of the coefficient of lift with respect to angle of attack. Another useful aerodynamic quantity is the angle of attack where lift is zero, $\alpha_{L=0}$.

Airfoil	Estimated a_o	Theoretical a_o	Estimated $\alpha_{L=0}$	Theoretical $\alpha_{L=0}$
NACA 0012	6.84	6.28	0.0	0.0
NACA 2212	6.84	6.28	-1.82	-4.94
NACA 4412	6.83	6.28	-4.09	-4.98
NACA 2430	7.81	6.28	-1.97	-4.94

Table 2: Values for zero-lift angle of attack and lift slope for each NACA airfoil. The estimated values are calculated from the Vortex Panel Method. For verification, these values are calculated against Thin Airfoil Theory.

[b]See Appendix A for airfoil geometries

Thin airfoil theory says, that an airfoil will have a lift slope of 2π, regardless if it is symmetric or cambered. The values obtained from the Vortex Panel Method are each off by about 0.6. This could be due to a rounding error, but is within the bounds of 10% off from the actual. Even those these are incorrect, I recognize that the actual lift slope should be equal to 2π for all four airfoils. Also, the calculated results for $\alpha_{L=0}$ look correct when compared against experimental data for the given airfoils, but do not match up with Thin Airfoil Theory calculations. The equations must not have been used correctly to verify the results of the Vortex Panel Method. Intuitively, however, the results c_l, c_p, and $\alpha_{L=0}$ are at least consistent with reality.

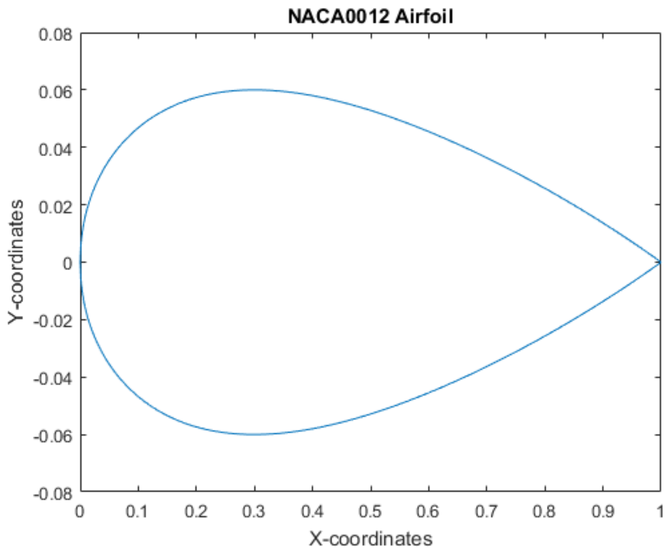
IV. Conclusions

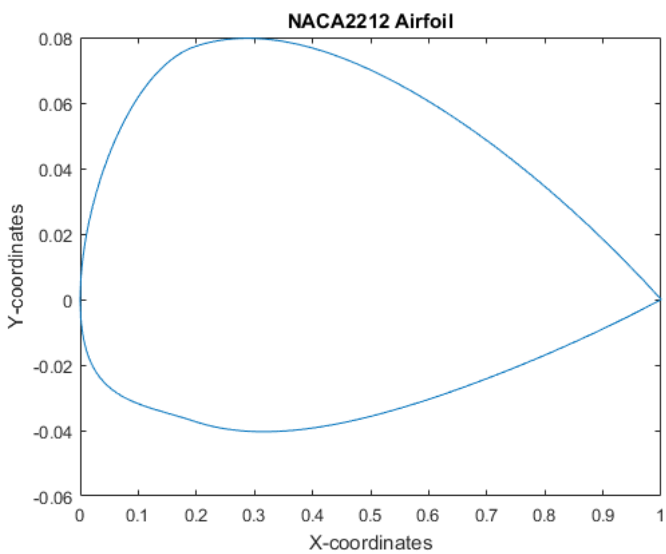
The Vortex Panel method is a fairly accurate method for modeling fluid flow over an airfoil body. From this method, pressure and lift can be calculated for the airfoil body. A nominal number of panels was used to achieve an accurate result, within 1% error. The NACA 0012 airfoil was plotted for the angles of attack -5, 0, 5, and 10, which showed a linear relationship along these angles of attack. Also, this airfoil's coefficient of pressure was plotted against percentage of chord length (x/c) to show increased values of pressure at higher angle of attack, a direct cause of the increase in lift as the angle of attack increases. The NACA 0012 airfoil was the only symmetric airfoil, and thus was the only airfoil that had a coefficient of lift of 0 at the angle of attack of 0. For the remaining NACA airfoils, the sectional coefficient of lift was plotted versus angle of attack for $\alpha = [-5, 10]$ to show a similar linear relation.

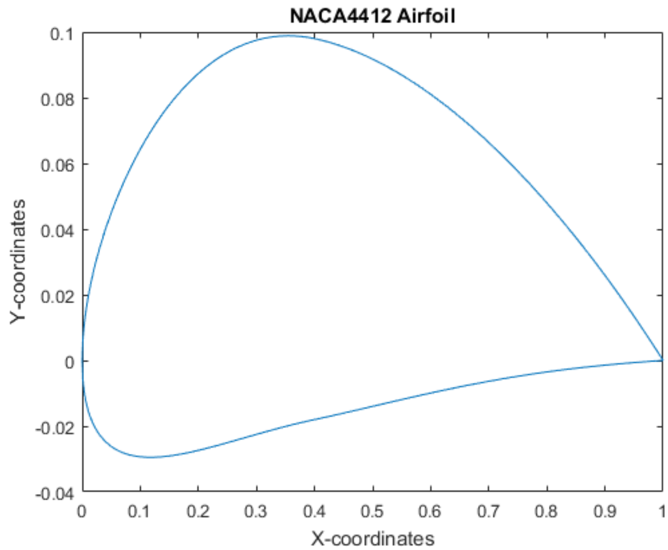
References

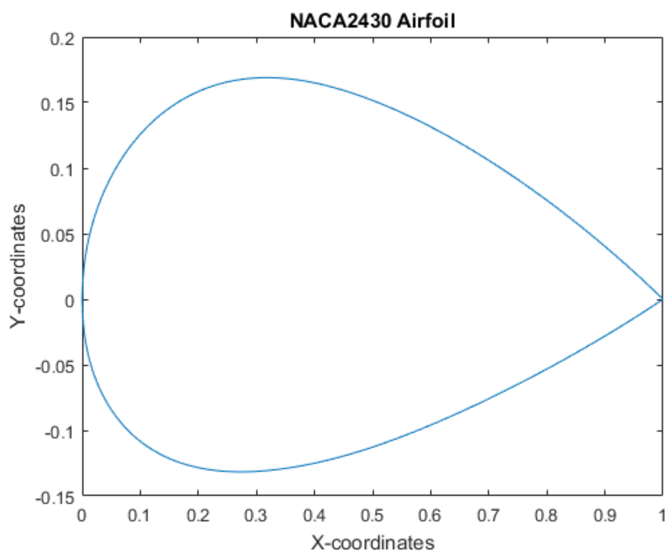
[1] Evans, *"Vortex_Panel_Method,"* [cited October 21 2016].
[2] Anderson, John D., *"Fundamentals of Aerodynamics,"* Fifth Edition [cited October 23 2016].
[3] Evans, *"VortexPanelFromKuetheAndChow,"* [cited October 24 2016].

Appendix A: Airfoil Geometries






NACA4412 Airfoil
0.1
0.08
0.06
0.04
0.02
0
-0.02
-0.04
Y-coordinates
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
X-coordinates


NACA2430 Airfoil
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
Y-coordinates
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
X-coordinates

Appendix B: Further Error Calculations

The error associated with computing the sectional coefficient of lift for an airfoil was negligible for increasing panels, but I don't believe it accurately depicts the full precision capabilities of the Vortex Panel Method, or show how close to an exact solution can be found for N = 1500, when compared to N = 2500. As stated, since the sectional coefficient of lift is always 0 for a symmetric airfoil at 0 deg angle of attack, the precision isn't as important I'd say, since each panel case was at least on the order of 10^{-15}. Below are the calculated coefficient of lift values for the same panels, but instead for a cambered airfoil (NACA 2412) at 0 deg angle of attack.

C_l	**Panels (N)**	**Error**
0.2493	100	2.2%
0.2515	200	1.3%
0.2524	300	0.94%
0.2533	500	0.59%
0.2542	1000	0.24%
0.2545	1500	0.12%
0.2548	2500	N/A

Table 3: Sectional coefficient vs N for a NACA 2412 cambered airfoil. Note, these values can be verified for this airfoil on pgs. 324 and 325 in Anderson,[2] where c_l is equal to 0.25 at 0^o angle of attack.

These values more accurately depict the precision capabilities of the Vortex Panel Method, even for a difference of 1000 panels, from 1500 to 2500! If further labs are interested, I think asking students to calculate the error using a cambered airfoil at 0 deg angle of attack would be a better clue to the accuracy of their code. To this level of accuracy (0.12%) would be a bit extreme, so maybe not with as many panels. However, it is important for the student to notice that the sectional coefficient of lift for a symmetric airfoil at 0 deg angle of attack is 0. Both together could be useful, as a bridge between theory and numerical methods - each with their own place. Just thoughts on improving the lab.

Larger Plots

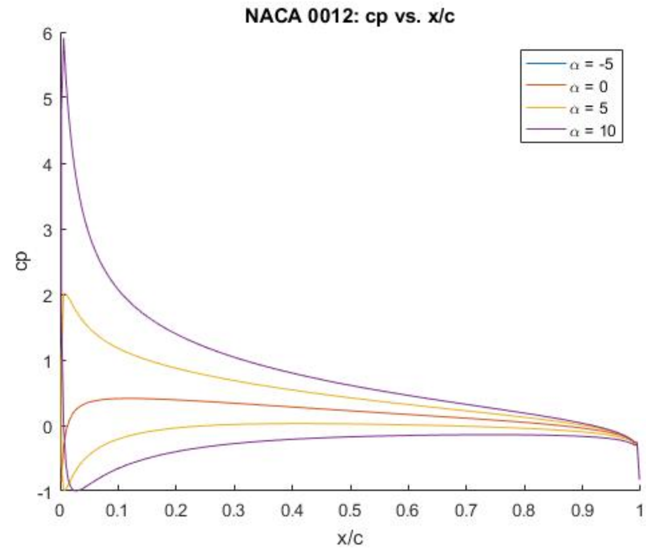


Figure 5: The coefficient of pressure vs alpha for NACA 0012 airfoil.

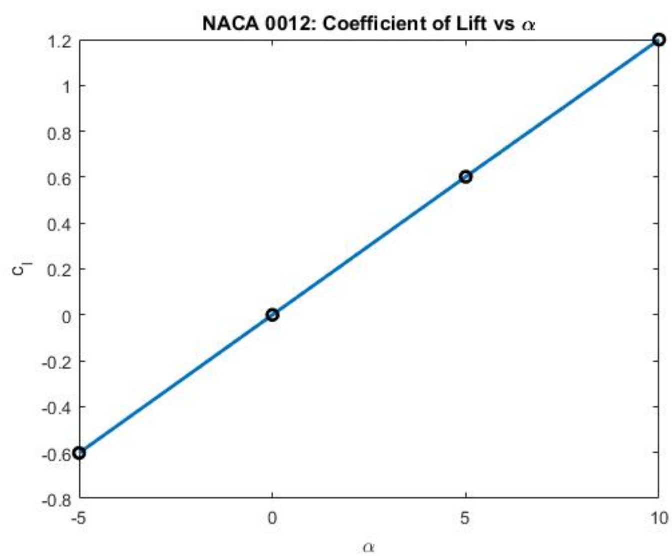


Figure 6: The sectional coefficient of lift vs alpha for the NACA 0012 airfoil.