Comp Lab 4: Flow Over Thin Airfoils

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I. Introduction and Theory

The Vortex Panel Method is an accurate numerical method for estimating the flow over an airfoil. Just as thin airfoil theory allowed us to induce lifting flow over a cylinder, a vortex sheet can be used to induce lifting flow over an airfoil. This can be done by superimposing uniform flow and a vortex sheet onto a body of given shape, subject to the Kutta Condition.

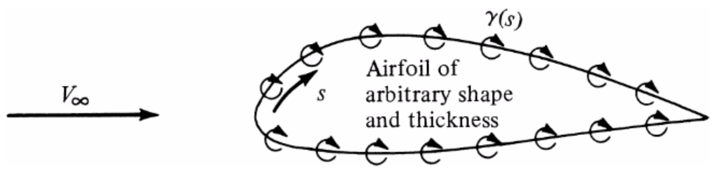


Figure 1: Depiction of a thin, cambered airfoil subject to lifting flow. This has been done by superimposing uniform flow and a vortex sheet onto an airfoil body. Per this depiction, the Vortex Panel Method will be used to calculate lift and other aerodynamic quantities for a number of airfoil geometries.

To solve a theoretical problem like this numerically, three steps need to be taken into account:

1. Discretize the geometry
2. Discretize the solution field
3. Discretize the governing equation[1]

These three steps form the basis of a numerical method known as the Vortex Panel method. These steps produce a linear system of m+1 equations and m+1 unknowns. Thus, the system $A\vec{\mathbf{x}} = \vec{\mathbf{b}}$[a] can be solved for the strength of each vortex, where $\vec{\mathbf{x}}_j = \gamma_j$ for every panel j = 1,..., M. From this relationship, the velocity can be calculated at every point, and then used to calculate the coefficient of pressure

$$C_{p,i} = 1 - \left(\frac{V_i}{V_\infty}\right)^2 \tag{1}$$

Further, the strength of the entire vortex sheet can be calculated using

$$\Gamma = -\oint_C \mathbf{V} \cdot \mathbf{ds} = \sum_{j=1}^{m} V_j S_j \tag{2}$$

to ultimately find the sectional coefficient of lift

$$c_l = \frac{L^{'}}{q_\infty c} = \frac{\rho_\infty V_\infty \Gamma}{0.5\rho_\infty V_\infty^2 c} = \frac{2\Gamma}{V_\infty c} \tag{3}$$

[a]see Appendix B-A: Defining the System

II. A Note on Computational Accuracy

Using the Vortex Panel numerical method introduced a certain degree of error to produce these results. Due to the nature of numerical methods, error cannot yet be totally eliminated from estimations, but it can be limited greatly. A nominal number of panels N were used throughout these calculations to produce a "standard" solution - a solution close enough to the actual solution, where the relative error is negligible. For this procedure, the definition of a desired quantitative level of accuracy was left to the student. Below are tabulated results of the coefficient lift for a NACA 0012 airfoil at 0^o angle of attack for varying number of panels

C_l	**Panels (N)**	**Accuracy**	**Run Time**
1.6878e-16	100	1.12%	3s
8.0785e-17	1000	51.6%	25s
1.9087e-16	2500	14.4%	6m
-1.3249e-16	3500	20.6%	41m
1.6691e-16	5000	0	50m

Table 1: Lift and Drag approximations for several different values of N. Note that increasing the number of ite+rations increases the accuracy of the model.

By increasing the number of panels for this method, the overall run time required increases. From taking N = 100, 1000, 2500, and 3500, the associated error compared to the N = 5000 "exact" case[b] can be seen as decreasing as N approaches 5000. To maintain a qualitative level of accuracy, N was taken for 1000 panels. The computations that follow all use this nominal case for the Vortex Panel method.

III. Results

A. Lifting Flow for NACA 0012 Airfoil

For a NACA 0012 airfoil, the relationships of c_p vs α and c_l vs α can be depicted graphically from the results of the Vortex Panel Method. Below are c_p vs α and c_l vs α for chosen angles of attack, $\alpha = -5^o, 0^o, 5^o$, and 10^o.

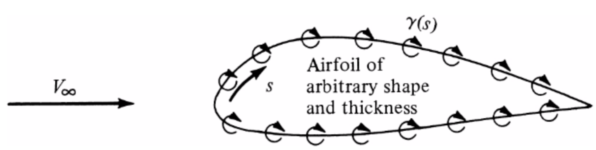


Figure 2: The coefficient of pressure vs alpha for the NACA 0012 airfoil.

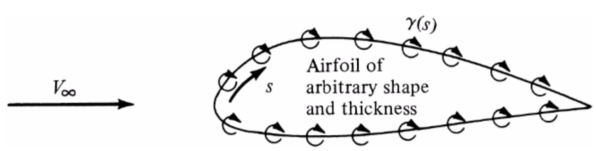


Figure 3: The coefficient of lift vs alpha for the NACA 0012 airfoil.

[b]Note that this case is not in fact exact, but for this method produces a well-defined level of accuracy in itself to compare against. This is also the largest level of accuracy available to the student, per the cost of computation.

B. Lifting Flow for Other Airfoils

The general form of c_l vs α for varying α can be depicted for any airfoil geometry. Results of the Vortex Panel Method for airfoils NACA 0012, 2212, 4412, and 2430[c] are provided below in Figure (4).

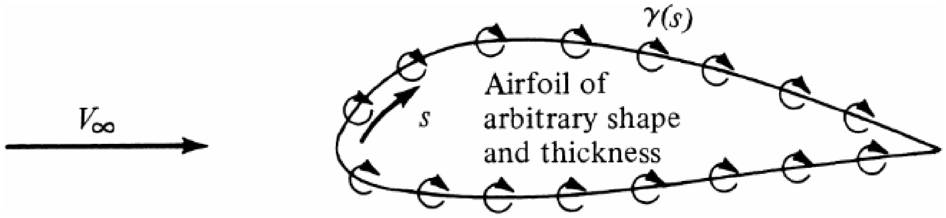


Figure 4: The sectional coefficient of lift vs alpha for the NACA 0012, 2212, 4412, and 2430 airfoils.

C. Estimating Lift Slope and $\alpha_{L=0}$

From the plots of c_l vs α, the lift slope for each airfoil can be estimated,[1] given the expression

$$a_0 = \frac{\partial c_l}{\partial \alpha} \tag{4}$$

Simply, the lift slope is the time rate of change of the coefficient of lift with respect to alpha. Another useful aerodynamic quantity is the

D. Comparison with Thin Airfoil Theory

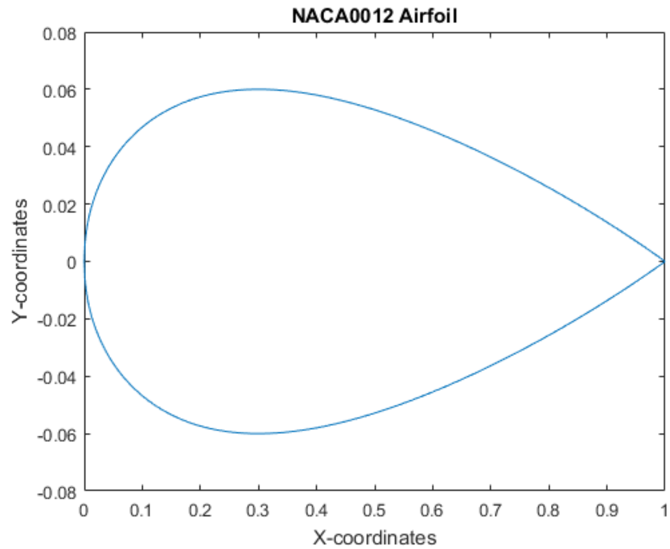
IV. Conclusions

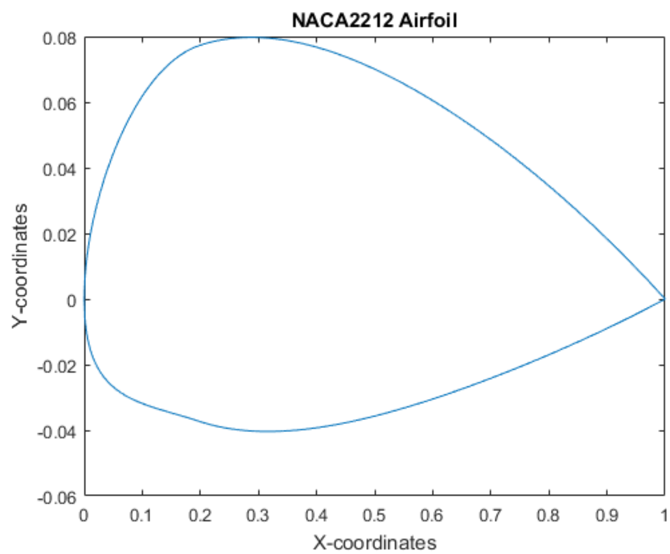
[c]See Appendix A for airfoil geomtries

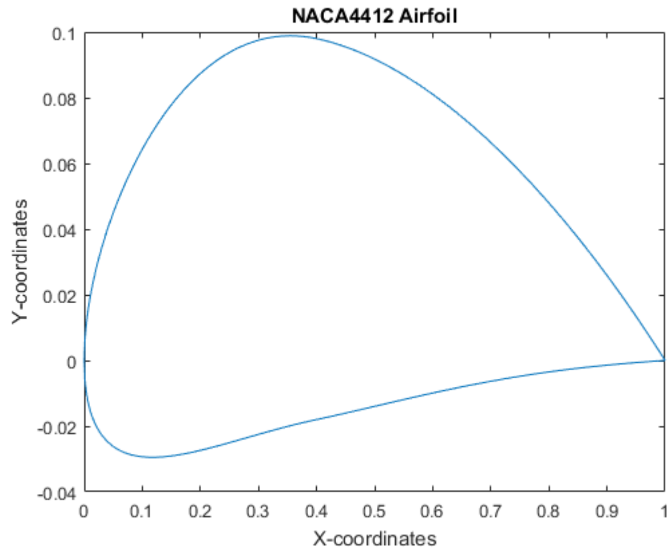
References

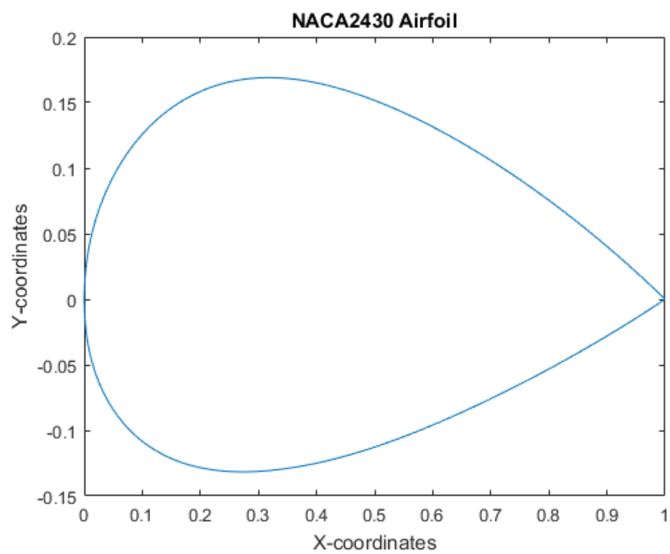
[1] Evans, *ASEN 3111 Computational Lab #3: Flow Over Thin Airfoils,* [cited September 23 2016].
[2] Evans, *MATLAB FILE: "Lifting_Cylinder.m",* [cited September 23 2016].

Appendix A: Airfoil Geometries






NACA4412 Airfoil
0.1
0.08
0.06
0.04
0.02
0
-0.02
-0.04
Y-coordinates
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
X-coordinates


NACA2430 Airfoil
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
Y-coordinates
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
X-coordinates

Appendix B: Derivations

A. Vortex Panel Method: Defining the system

Appendix C: Other Useful Plots