

Assume a solid ball of mass m and radius r_b at radius r from a pivot point attached by a massless arm with length r. The angle of the arm is θ , a function of time. The torque applied to the arm is τ , positive in the counterclockwise direction. By the parallel axis theorem, the moment of inertia is given by

$$I = m\left(r^2 + \frac{2}{5}r_b^2\right).$$

The angular version of Newton's second law is

$$\tau = I\ddot{\theta}.$$

The torque due to gravity, assuming it is applied only at the center of the ball, is given by

$$\tau_q = -mrg\cos\theta,$$

where $g=9.81~{\rm m/s^2}$ is the acceleration of gravity at sea level. Hence, we have:

$$\ddot{\theta} = \frac{\tau + \tau_g}{I} = \frac{\tau - mrg\cos\theta}{m\left(r^2 + \frac{2}{5}r_b^2\right)}$$