

4.5.2

$$|\uparrow\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$|\Psi_1\rangle = C_{10}|\uparrow\rangle + C_{11}|\downarrow\rangle = \begin{bmatrix} C_{10} \\ C_{11} \end{bmatrix} \in \mathbb{C}^2$$

$$|\downarrow\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$|\Psi_2\rangle = C_{20}|\uparrow\rangle + C_{21}|\downarrow\rangle = \begin{bmatrix} C_{20} \\ C_{21} \end{bmatrix} \in \mathbb{C}^2$$

$$|\Psi_1\rangle \otimes |\Psi_2\rangle = \begin{bmatrix} C_{10} & C_{20} \\ C_{10} & C_{21} \\ C_{11} & C_{20} \\ C_{11} & C_{21} \end{bmatrix} \in \mathbb{C}^4$$

$$= C_{10}C_{20}|\uparrow\rangle \otimes |\uparrow\rangle + C_{10}C_{21}|\uparrow\rangle \otimes |\downarrow\rangle + C_{11}C_{20}|\downarrow\rangle \otimes |\uparrow\rangle + C_{11}C_{21}|\downarrow\rangle \otimes |\downarrow\rangle$$

En general un vector con n partículas genérico permanecerá a \mathbb{C}^{2^n}

4.5.3

$$|\phi\rangle = |x_0\rangle \otimes |\psi_1\rangle + |x_1\rangle \otimes |\psi_1\rangle$$

$$|\Psi_1\rangle = C_0|x_0\rangle + C_1|x_1\rangle = \begin{bmatrix} C_0 \\ C_1 \end{bmatrix}$$

$$|\Psi_2\rangle = C_0'|y_0\rangle + C_1'|y_1\rangle = \begin{bmatrix} C_0' \\ C_1' \end{bmatrix}$$

$$|\phi\rangle = |\Psi_1\rangle \otimes |\Psi_2\rangle = \begin{bmatrix} C_0 & C_0' \\ C_0 & C_1' \\ C_1 & C_0' \\ C_1 & C_1' \end{bmatrix}$$

$$|\phi\rangle = C_0 C_0' |x_0\rangle \otimes |y_0\rangle + C_0 C_1' |x_0\rangle \otimes |y_1\rangle + C_1 C_0' |x_1\rangle \otimes |y_0\rangle + C_1 C_1' |x_1\rangle \otimes |y_1\rangle$$

Para que se cumpla $|\phi\rangle$ inicial

$$C_0 C_0' = 0$$

$$C_0 C_1' = 1$$

$$C_1 C_0' = 0$$

$$C_1 C_1' = 1$$

por lo que

$$C_0' = 0$$

$$C_0 = 1$$

$$C_1 = 1$$

$$C_1' = 1$$

Por lo tanto son separables de esta forma y una solución sería:

$$|\psi_1\rangle = |x_0\rangle + |x_1\rangle = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$|\psi_2\rangle = |y_1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$