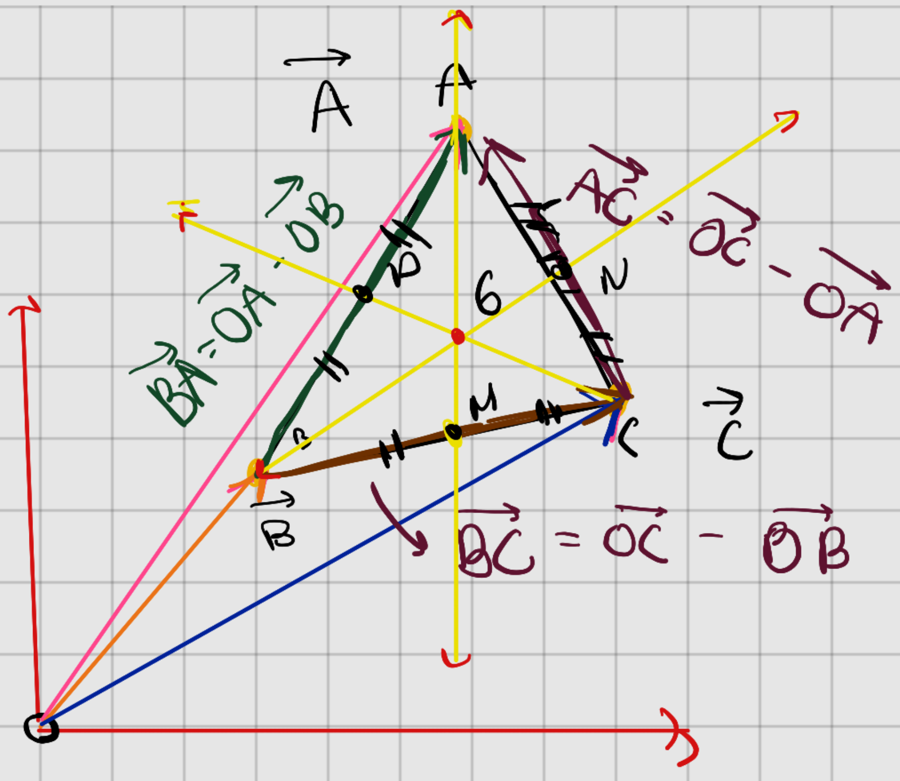


RECORDANDO QUE $\vec{OC} = c$, $\vec{OA} = a$, $\vec{OB} = b$

$\vec{BM} = \frac{\vec{BC}}{2}$, $\vec{BP} = \frac{\vec{BA}}{2}$, $\vec{AN} = \frac{\vec{AC}}{2}$

medianas del Triángulo con concurrentes en a

$\vec{B6} = \lambda \vec{BN}$ → TIENE SENTIDO!

$\vec{C6} = \mu \; \vec{CP}$

$A6 = \gamma \vec{AM}$

where $\lambda = \mu = r$ son $\boxed{\text{iguales}}$

$$\vec{B6} = \lambda \vec{BN}$$
$$\lambda(\vec{BC} + \vec{Cn})$$
$$\lambda(\vec{Oc} - \vec{OB} + \vec{Cn})$$
$$\lambda(c - b - \frac{1}{2}(\vec{AC})$$
$$\lambda(c - b - \frac{1}{2}(c - a)$$

Es decir $\vec{BG} = \lambda\left(c - b - \frac{c}{2} + \frac{a}{2}\right)$

$\vec{BG} = \lambda\left(\frac{c}{2} + \frac{a}{2} - b\right) \longrightarrow$ Tiene mucho sentido y no es difícil.

y ahora

$$\vec{CG} = \mu\,\vec{CP}$$
$$= \mu\left(\vec{CA} + \vec{AP}\right)$$
$$= \mu\left(a - c - \frac{1}{2}(a - b)\right)$$
$$= \mu\left(a - c - \frac{a}{2} + \frac{b}{2}\right)$$
$$= \left(\frac{a}{2} + \frac{b}{2} - c\right)$$

y $\vec{AG} = \gamma\,\vec{AM}$

$$\gamma\left(\vec{AB} + \vec{BM}\right)$$
$$\gamma\left(b - a + \frac{1}{2}(c - b)\right)$$
$$\gamma\left(\frac{b}{2} + \frac{c}{2} - a\right)$$

Podemos decir que:

$$\vec{AG} = \vec{AB} + \vec{BG}$$

2) Considere que:

$$r = x\hat{i} + y\hat{j} + z\hat{k} = x^i \hat{i}_j$$

$$a = a(r) = a(x,y,z) = a^i(x,y,z)\hat{i}_i \quad y \quad b = b^i(x,y,z)\hat{i}_i$$

$$\phi = \phi r = \phi(x,y,z), \quad \psi = \psi(r) = \psi(x,y,z)$$

a) $\nabla(\phi\psi) = \phi\nabla\psi + \psi\nabla\phi \longrightarrow$ Demostrar!

$$\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$$

$$\nabla_i, \quad \nabla_i \phi\psi = \frac{\partial \phi(x^i)\psi(x^i)}{\partial x^i} \rightarrow$$ Usamos la regla de la cadena

$$= \frac{\partial \phi(x^i)}{\partial x^i} \cdot \psi(x^i) + \phi(x^i)\frac{\partial \psi(x^i)}{\partial x^i}$$

$$\rightarrow \frac{\partial \phi(x^i)}{\partial x^i} = \frac{\partial \phi(x)}{\partial x} + \frac{\partial \phi(y)}{\partial y} + \frac{\partial \phi(z)}{\partial z}$$

Básicamente.

$$\phi(x^i)\nabla_i\psi(x^i) + \psi(x^i)\nabla_i\phi(x^i)$$

$$\boxed{\phi\nabla\psi + \psi\nabla\phi} \quad ✓✓✓$$

d) $\nabla \cdot (\nabla \times a)$ y $\nabla \times (\nabla \cdot a)$

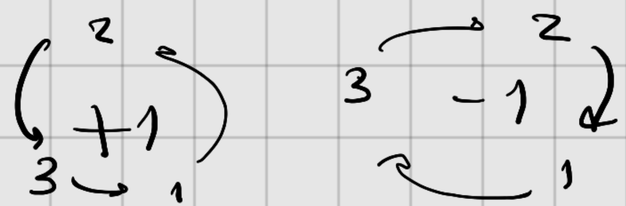
siendo a un campo vectorial

$a(r) = a(x,y,z) = a^i(x,y,z)\,\hat{\imath}_i$

1) Definir $\nabla \times A$.

$\longrightarrow \nabla \times A = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & y & z \end{vmatrix}$

- $\varepsilon_{ijk} = \begin{cases} 1 & \text{if } (i,j,k) \text{ is even permutation} \\ -1 & \text{if } (ijk) \text{ is odd permutation} \\ 0 & \text{if } i=j, \text{ or } j=k \text{ or } k=i \end{cases}$



$\varepsilon_{ijk} a_i b_j = c_k$

$\longrightarrow (\nabla \times A)_i = (\varepsilon_{ijk} \nabla_j A_k + \varepsilon_{ijk} \nabla_k A_j)\,\hat{e}_i$

$(\nabla \times A) = \boxed{\varepsilon_{ijk} \nabla_j A_k \, \hat{e}_i}$

A el dot product.

$a = (x, y, z)$. $b = (x_1, y_1, z_1)$

$a = x^i$ $b = x_1^i$

$\boxed{a \cdot b = x^i x_1^i}$

y $\nabla \cdot (\nabla \times A) = \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}$

$\nabla_i \, \varepsilon_{ijk} \nabla_j A_k \, \hat{e}_i = (0, 0, 0)$

↳ Básicamente esto es igual a 0 debido a que $\varepsilon_{ijk} \nabla_j A_k$ es un número que NO! depende de x (constante) y ∇_i por componentes $\frac{\partial}{\partial x}$ termina dando 0.

f) $\nabla \times \nabla \times A = \nabla(\nabla \cdot A) - \nabla^2 a$

$i = 1, 2, 3$

↳ $\nabla \times (\nabla \times A) = \nabla \times (\varepsilon_{ijk} \nabla_j A_k \hat{e}_i)$

y el rotacional de un vector es

$\ell = 1, 2, 3$

$\nabla \times A = \varepsilon_{\ell jk} \nabla_j A_\ell$

$= \boxed{\varepsilon_{\ell jk} \nabla_j \varepsilon_{\ell jk} \nabla_j A_k}$

Hasta aquí llego