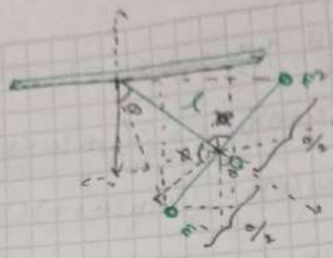
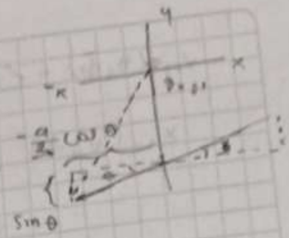
Ejercicio ①

- Plantear las ecuaciones de movimiento.





Consideremos la posición,

- Para la masa 1. - Consideramos punto a.

$(\ell\sin\theta, -\ell\cos\theta)$ ✓

→ El ángulo total de m_1 con respecto a y es $\theta + \phi$

$$x_1 = \ell\sin\theta - \frac{a}{2}\cos(\theta+\phi) \checkmark$$

$$y_1 = -\ell\cos\theta - \frac{a}{2}\sin(\theta+\phi \checkmark$$

→ El ángulo de m_2 con respecto a la vertical es $\theta+\phi$

$$x_{12} = \ell\sin\theta + \frac{a}{2}\cos(\theta+\phi)$$

$$y_{12} = -\ell\cos + \frac{a}{2}\sin(\theta+\phi).$$

Consideremos la velocidad.

$$\rightarrow m_1. \quad \left.\begin{aligned} \dot{x}_1 &= \frac{d}{dt}\left(\ell\sin\theta - \frac{a}{2}\cos(\theta+\phi)\right) \\ \dot{y}_1 &= \frac{d}{dt}\left(-\ell\cos\theta - \frac{a}{2}\sin(\theta+\phi)\right) \end{aligned}\right\}$$ la derivación en este caso: Trivial.

$$\rightarrow \dot{x}_1 = \frac{d}{dt}\ell\sin\theta - \frac{d}{dt}\frac{a}{2}\cos(\theta+\phi)$$

$$\dot{x}_1(\theta,\phi) = \ell\cos\theta\,\dot\theta - \frac{a}{2}\left[\frac{\partial}{\partial\theta}\frac{d\theta}{dt} + \frac{\partial}{\partial\phi}\frac{d\phi}{dt}\right]\left[\cos(\theta+\phi)\right]$$

$$= \ell\cos\theta\,\dot\theta - \frac{a}{2}\left(-\sin(\theta+\phi)\dot\theta + (-\sin(\theta+\phi)\dot\phi\right)$$

$$\dot{x}_1(\theta,\phi) = \ell\cos\theta\,\dot\theta + \frac{a}{2}\left(\sin(\theta+\phi)\dot\theta + \sin(\theta+\phi)\dot\phi\right)$$

$$= \ell\cos\theta\,\dot\theta + \frac{a}{2}\left(\sin(\theta+\phi)(\dot\theta+\dot\phi)\right)$$

$$\dot{y}_1(\theta,\phi) = \ell\sin\theta\,\dot\theta - \frac{a}{2}\left(\cos(\theta+\phi)\dot\theta + \cos(\theta+\phi)\dot\phi\right)$$

– Energía Cinética total:

$$T = T_1 + T_2 = \frac{1}{2} m_1 \left(\frac{\dot r^2}{\sin^2\alpha} + r^2 \dot\theta^2 \right) + \frac{1}{2} m_2 \dot r^2 \cot^2\alpha$$

– E. Potencial. m_1: $U_1 = m_1 g z_1 = m_1 g r\cot\alpha$

m_2: $U_2 = m_2 g z_2 = m_2 g (l - r\cot\alpha)$

$$U = U_1 + U_2 = m_1 g r\cot\alpha + m_2 g (l - r\cot\alpha)$$
$$= m_1 g r\cot\alpha + m_2 g l - m_2 g r\cot\alpha$$
$$= g r\cot\alpha (m_1 - m_2) + m_2 g l$$

→ $\mathcal{L} = T - U$

$$\frac{1}{2} m_1 \left(\frac{\dot r^2}{\sin^2\alpha} + r^2\dot\theta^2 \right) + \frac{1}{2} m_2 \dot r^2 \cot^2\alpha - \left(g r\cot\alpha (m_1 - m_2) + m_2 g l \right)$$

Para r: $\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot r}\right) - \frac{\partial \mathcal{L}}{\partial r} = 0$ → $\frac{\partial \mathcal{L}}{\partial \dot r} = \frac{1}{2} m_1 \frac{\ddot r^2}{\sin^2\alpha} + \frac{1}{2} m_2 \ddot r^2 \cot^2\alpha$?

$-\frac{d}{dt}(r$ $\qquad d = \ddot r^2 \left(\frac{m_1}{2\sin^2\alpha} + \frac{m_2 \cot^2\alpha}{2} \right)$?

Probablemente mal!

– Corrección.

$$\frac{\partial \mathcal{L}}{\partial \dot r} = \frac{1}{2} m_1 \left(\frac{2\dot r}{\sin^2\alpha} \right) + \frac{1}{2} m_2 2\dot r \cot^2\alpha$$
$$= \left[\frac{m_1 \dot r}{\sin^2\alpha} + m_2 \dot r \cot^2\alpha \right]$$

$$\frac{\partial \mathcal{L}}{\partial r} = \frac{1}{2} m_1 2 r \dot\theta^2 - g\cot\alpha (m_1 - m_2)$$

$$\frac{d}{d}\left(\frac{m_1 \dot r}{\sin^2\alpha} + m_2 \dot r \cot^2\alpha \right) - \longrightarrow = m_1 r \dot\theta^2 - g\cot\alpha (m_1 - m_2)$$

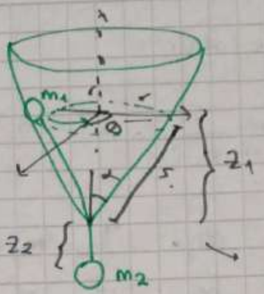
$$= \frac{m_1 \ddot r}{\sin^2\alpha} + m_2 \ddot r \cot^2\alpha - \left(m_1 r \dot\theta^2 - g\cot (m_1 - m_2) \right)$$

$$\left(\frac{m_1}{\sin^2\alpha} + m_2 \cot^2\alpha \right) \ddot r - m_1 r \dot\theta^2 + g\cot (m_1 - m_2)$$

y para θ: $\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot\theta}\right) - \frac{\partial \mathcal{L}}{\partial \theta} = 0$ → $\frac{\partial \mathcal{L}}{\partial \dot\theta} = \frac{1}{\cancel{2}} m_1 r^2 \cancel{2}\dot\theta = m_1 \dot\theta r^2$

$$\frac{d}{dt} \underbrace{m_1 \dot\theta r^2} = 0$$ → quiere decir que es constante (Conservación del momento angular)

Ejercicio #2



Ángulo de vértice $=\alpha$

r = distancia del eje del cono a m_1

θ = Ángulo de Rotación de m_1 alrededor del eje del cono.

z_1 = Altura Vertical del vértice del cono a m_1.

z_2 = longitud vertical cuerda a m_2.

1) Definimos z_1. $x = r\cos\theta$, $y = r\sin\theta$.

$\delta = \tan\alpha$. $\delta^2 = z^2 + r^2$. $\tan\alpha = \frac{r}{z}$ $\rightarrow$ $z = \frac{r}{\tan\alpha}$.

$\boxed{z_1 = r\cot\alpha}$ $\rightarrow$ Posición en z.

longitud cuerda $\ell = z_1 + z_2 = r\cot\alpha + z_2$

$z_2 = \ell - r\cot\alpha$

En cylindrical coordinates $v = \frac{dr}{dt} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta} + \dot{z}\hat{z}$

$x_m = (r\cos\theta, r\sin\theta, r\cot\alpha)$

Energía Cinética: $T_1 = \frac{1}{2}m_1(\dot{r}^2 + r^2\dot{\theta}^2 + \dot{z}_1^2)$

$\dot{z}_1 = \dot{r}\cot\alpha$

$T_1 = \frac{1}{2}m_1(\dot{r}^2 + r^2\dot{\theta}^2 + (\dot{r}\cot\alpha)^2$

$T_1 = \frac{1}{2}m_1(\dot{r}^2 + r^2\dot{\theta}^2 + \dot{r}^2\cot^2\alpha)$

$1+\cot^2\alpha = \csc^2\alpha$ $\leftarrow$ $T_1 = \frac{1}{2}m_1(\dot{r}^2(1+\cot^2\alpha) + r^2\dot{\theta}^2)$

$\frac{1}{\sin^2\alpha} = \csc^2\alpha$ $\quad T_1 = \frac{1}{2}m_1\left(\frac{\dot{r}^2}{\sin^2\alpha} + r^2\dot{\theta}^2\right)$

$T_2 = \frac{1}{2}m_2\dot{z}_2^2$ $\rightarrow$ $z_2 = \ell - r\cot\alpha$
$\dot{z}_2 = -\dot{r}\cot\alpha$

$= \frac{1}{2}m_2(\dot{r}^2\cot^2\alpha)$

→ $\dot{y}^2 + y\ddot{y} - \frac{1}{2}\dot{y}^2 - yg = 0$ → Tenemos:

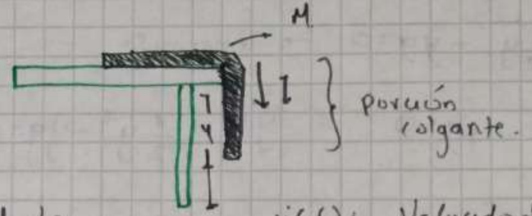
$\ddot{y}y + \underbrace{\dot{y}^2 - \frac{1}{2}\dot{y}^2} - yg = 0 \quad = \quad \ddot{y}y + \frac{1}{2}\dot{y}^2 - yg = 0$ → Resolver E.D.

De cinemática, sabemos que $v_f = \sqrt{2gh}$ (la podemos obtener)

entonces $\dot{y}(t) = \sqrt{2g(l-y(t))}$ → $\frac{d}{d}$

$\int \sqrt{2g(l-y(t))}\; d$

4)



$y(t)$: longitud de la cuerda en el instante t.

$\dot{y}(t)$: Velocidad de la cuerda.

$y(0) = l$.

1) Energía Cinética:

→ Densidad lineal M.C. (Masa Cuerda)

$$\lambda = \frac{M}{L} \begin{matrix} \to \text{masa} \\ \to \text{longitud.} \end{matrix}$$

→ Energía cinética porción colgante → $m_c = \lambda y$

$$T = \frac{1}{2} m_{colgante} \cdot \dot{y}^2 = \frac{1}{2} y\lambda \cdot \dot{y}^2 = \boxed{\frac{1}{2}\frac{M}{L} y \dot{y}^2}$$

→ Energía potencial g.

$$U = \int_0^y \underbrace{\lambda}_{\text{"masa"}} \underbrace{g}_{\text{gravedad}} \overbrace{y'}^{\text{Altura}} dy' = \frac{M}{L} g \int_0^y y' dy' = \frac{M}{L} g \left[\frac{y'^2}{2}\right]_0^y = \frac{M}{L} g \frac{y^2}{2}$$

$$U = \frac{Mgy^2}{2L}$$

Lagrangiano: $\mathcal{L} = T - U \to \frac{1}{2}\frac{M}{L} y\dot{y}^2 - \frac{Mgy^2}{2L}$

$$= \frac{1}{2}\frac{M}{L} y\left(\dot{y}^2 - y\right) = \frac{1}{2}\frac{M}{L}\left(y\dot{y}^2 - y^2\right)$$

$$\to \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{y}}\right) - \frac{\partial \mathcal{L}}{\partial y} = 0$$

1) $\frac{\partial \mathcal{L}}{\partial y} = \frac{1}{2}\frac{M}{L}\dot{y}^2 - \frac{1}{2}\frac{M}{L}2y = \frac{1}{2}\frac{M}{L}\dot{y}^2 - \frac{Myg}{L}$

2) $\frac{\partial \mathcal{L}}{\partial \dot{y}} = \frac{M}{L} y\dot{y} \to \frac{d}{dt}\frac{M}{L} y\dot{y} = \frac{M}{L}\frac{d}{dt} y\dot{y} = \frac{M}{L}\left[\dot{y}^2 + y\ddot{y}\right]$

$\mathcal{L}$: $\frac{M}{L}\left[\dot{y}^2 + y\ddot{y}\right] - \left[\frac{1}{2}\frac{M}{L}\dot{y}^2 - \frac{Myg}{L}\right]$

$\mathcal{L}$: $\left(\frac{M}{L}\dot{y}^2\right) \to \frac{M}{L}\left[\dot{y}^2 + y\ddot{y}\right] - \frac{M}{L}\left[\frac{1}{2}\dot{y}^2 - yg\right] = 0$

λ_n nuestra fuerza normal, podemos decir que cuando $\lambda_n - 0$

continuamos

$$\mathcal{L} = \Big(\frac{1}{2}m_1\big(\ell^2\cos^2\theta_1\dot\theta_1^2 - \cancel{2}\ell\cos\theta_1\dot\theta_1\frac{a}{\cancel{2}}\sin(\theta_2)\dot\theta_2 + \frac{a^2}{4}\sin^2(\theta_2)\dot\theta_2^2$$

$$+\ell^2\sin^2(\theta_1)\dot\theta_1^2 - \cancel{2}\ell\sin\theta_1\dot\theta_1\frac{a}{\cancel{2}}\cos(\theta_2)\dot\theta_2 + \frac{a^2}{4}\cos^2(\theta_2)\dot\theta_2^2\big)\Big)$$

$$+\Big(\frac{1}{2}m_2\big(\ell^2\cos^2\theta_1\dot\theta_1^2 + \cancel{2}\ell\cos\theta_1\dot\theta_1\frac{a}{\cancel{2}}\sin(\theta_2)\dot\theta_2 + \frac{a^2}{4}\sin^2(\theta_2)\dot\theta_2^2$$

$$+\ell^2\sin^2(\theta_1)\dot\theta_1^2 - \cancel{2}\ell\sin\theta_1\dot\theta_1\frac{a}{\cancel{2}}\cos\theta_2\dot\theta_2 + \frac{a^2}{4}\cos^2(\theta_2)\dot\theta_2^2\big)\Big)$$

$$+\Big(-\ell\cos\theta_1 - \frac{a}{2}\sin\theta_2\Big)\big(-m_1 g - m_2 g\big)$$

$$= \frac{1}{2}m_1\Big(\ell^2\dot\theta_1^2 + \ell\dot\theta_1\dot\theta_2 a(-\cos(\theta_1)\sin(\theta_2) - \sin\theta_1\cos\theta_2) + \frac{a^2}{4}\dot\theta_2^2\Big)$$

$$+\frac{1}{2}m_2\Big(\ell^2\dot\theta_1^2 + \ell\dot\theta_1\dot\theta_2 a(\cos(\theta_1)\sin(\theta_2) - \sin(\theta_1)\cos(\theta_2)) + \frac{a^2}{4}\dot\theta_2^2\Big)$$

$$+\ell m_1 g\cos\theta_1 + m_2\ell g\cos\theta_1 + \frac{a}{2}\sin\theta_2 m_1 g + m_2 g\frac{a}{2}\sin\theta_2$$

$$= \ell^2\dot\theta_1^2\Big(\frac{1}{2}m_1 + \frac{1}{2}m_2\Big) + \ell\dot\theta_1\dot\theta_2 a\big(\underbrace{-\cos(\theta_1)\sin(\theta_2) - \sin\theta_1\cos\theta_2}_{-\sin(\theta_1+\theta_2)}\big)\Big(\frac{1}{2}m_1\Big)$$

$$+\frac{1}{2}m_2\Big(\ell\dot\theta_1\dot\theta_2 a\big(\underbrace{\cos(\theta_1)\sin(\theta_2) - \sin(\theta_1)\cos(\theta_2)}_{\sin(\theta_1-\theta_2)}\big)\Big)$$

$$+\frac{a^2}{4}\dot\theta_2^2\Big(\frac{1}{2}m_1 + \frac{1}{2}m_2\Big) + \ell g\cos\theta_1(m_1+m_2) + \frac{a}{2}\sin\theta_2 g(m_1+m_2)$$

utilizamos la Ec Euler-Lagrange

$$\frac{d}{dt}\Big(\frac{\partial \mathcal{L}}{\partial\dot\theta_1}\Big) - \frac{\partial \mathcal{L}}{\partial\theta_1} + \sum_{i=1}^{2}\lambda_i\frac{\partial F_i}{\partial\theta_1} = 0 \qquad -\sin(\theta_1+\theta_2)$$

$$\frac{\partial \mathcal{L}}{\partial\dot\theta_1} = \dot\theta_1\ell^2(m_1+m_2) - \ell\dot\theta_2 a(\sin(\theta_1+\theta_2))\Big(\frac{1}{2}m_1\Big)$$

$$+\frac{1}{2}m_2\ell\dot\theta_2 a\sin(\theta_1-\theta_2)$$

$$\frac{d}{dt}\Big(\frac{\partial \mathcal{L}}{\partial\dot\theta_1}\Big) = \ddot\theta_1\ell^2(m_1+m_2) - \ell\ddot\theta_2 a(\cos(\theta_1+\theta_2)(\dot\theta_1+\dot\theta_2))\Big(\frac{1}{2}m_1\Big)$$

$$+\frac{1}{2}m_2\ell\ddot\theta_2 a\cos(\theta_1-\theta_2)(\dot\theta_1-\dot\theta_2)$$

$$\frac{\partial L}{\partial\theta_1} = -\ell\dot\theta_1\dot\theta_2 a\cos(\theta_1+\theta_2)\Big(\frac{1}{2}m_1\Big) + \frac{1}{2}m_2\big(\ell\dot\theta_1\dot\theta_2 a(\cos(\theta_1-\theta_2))$$

$$-\ell g\,\mathrm{sen}\,\theta_1(m_1+m_2)$$

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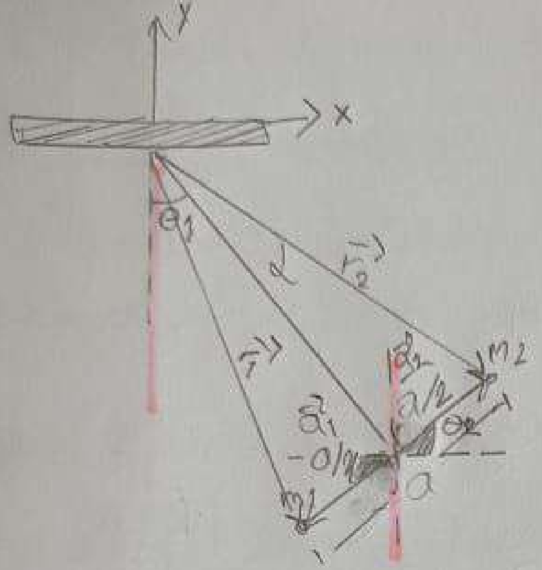
1)



Funciones Restrictivas...

$\vec{r_1} = \vec{l} + \vec{a_1}$

$\vec{r_2} = \vec{l} + \vec{a_2}$

$2(2) - 2 = 2$ coordenadas generalizadas

Definiendo $\vec{l}$, $\vec{a_1}$ y $\vec{a_2}$: $\vec{l} = l\sin\theta_1\hat{\imath} - l\cos\theta_1\hat{\jmath}$

$\vec{a_1} = \frac{a}{2}\cos\theta_2\hat{\imath} - \frac{a}{2}\sin\theta_2\hat{\jmath}$; $\vec{a_2} = -\frac{a}{2}\cos\theta_2\hat{\imath} - \frac{a}{2}\sin\theta_2\hat{\jmath}$

Re-escribiendo $\vec{r_1}$ y $\vec{r_2}$

$\vec{r_1} = (l\sin\theta_1 + \frac{a}{2}\cos\theta_2)\hat{\imath} + (-l\cos\theta_1 - \frac{a}{2}\sin\theta_2)\hat{\jmath}$

$\vec{r_2} = (l\sin\theta_1 - \frac{a}{2}\cos\theta_2)\hat{\imath} + (-l\cos\theta_1 - \frac{a}{2}\sin\theta_2)\hat{\jmath}$

$\dot{\vec{r_1}} = (l\cos\theta_1\dot{\theta}_1 - \frac{a}{2}\sin\theta_2\dot{\theta}_2)\hat{\imath} + (l\sin\theta_1\dot{\theta}_1 - \frac{a}{2}\cos\theta_2\dot{\theta}_2)\hat{\jmath}$

$\dot{\vec{r_2}} = (l\cos\theta_1\dot{\theta}_1 + \frac{a}{2}\sin\theta_2\dot{\theta}_2)\hat{\imath} + (l\sin\theta_1\dot{\theta}_1 - \frac{a}{2}\cos\theta_2\dot{\theta}_2)\hat{\jmath}$

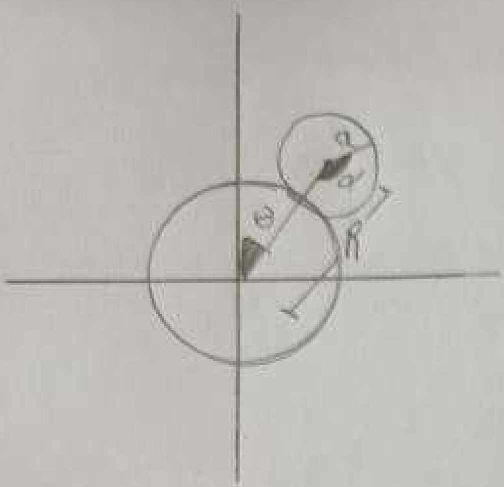
Definimos el Lagrangiano

$$\mathcal{L} = T - V = \frac{1}{2}m_1((l\cos\theta_1\dot{\theta}_1 - \frac{a}{2}\sin\theta_2\dot{\theta}_2)^2 + (l\sin\theta_1\dot{\theta}_1 - \frac{a}{2}\cos\theta_2\dot{\theta}_2)^2)$$

$$+\frac{1}{2}m_2((l\cos\theta_1\dot{\theta}_1 + \frac{a}{2}\sin\theta_2\dot{\theta}_2)^2 + (l\sin\theta_1\dot{\theta}_1 - \frac{a}{2}\cos\theta_2\dot{\theta}_2)^2)$$

$$+ m_1 g(-l\cos\theta_1 - \frac{a}{2}\sin\theta_2)$$

$$+ m_2 g(-l\cos\theta_1 - \frac{a}{2}\sin\theta_2)$$



Definimos la posicion de la rueda

$x = R\sin\theta$, $\dot x = \cos\theta R\dot\theta$

$y = R\cos\theta$, $\dot y = -\sin\theta R\dot\theta$

Contamos con 2 ligaduras

F_2 ; $R-b-a=0$ Holonomica

F_1 ; $\theta R - \alpha a = 0$ No Holonomica

$\Rightarrow \alpha = \frac{R}{a}\theta$

Planteamos el Lagrangiano

$L = T - V = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 - mgy = \frac{1}{2}m(R^2\dot\theta^2) + \frac{1}{2}ma^2\dot\alpha^2 - mgR\cos\theta$

Planteamos las Ec de Lagrange para las coordenadas ↳ R en algun punto variaria

$\frac{d}{dt}\left(\frac{\partial L}{\partial\dot\alpha}\right) = \frac{d}{dt}(ma^2\dot\alpha) = ma^2\ddot\alpha$; $\frac{\partial L}{\partial\alpha} = 0$, $\frac{\partial F_1}{\partial\alpha} = -a$

$\Rightarrow m a^{\cancel{2}}\ddot\alpha = -\cancel{a}\lambda_1 = ma\ddot\alpha = -\lambda_1$ (1)

$\frac{d}{dt}\left(\frac{\partial L}{\partial\dot\theta}\right) = \frac{d}{dt}(mR^2\dot\theta) = mR^2\ddot\theta$, $\frac{\partial L}{\partial\theta} = -mgR(-\sin\theta) = mgR\operatorname{sen}\theta$

$\frac{\partial F_1}{\partial\theta} = R \Rightarrow mR^2\ddot\theta - mgR\sin\theta\dot\theta = -R\lambda_1 \Rightarrow mR\ddot\theta - mg\sin\theta = -\lambda_1$ (2)

para R, $\frac{\partial L}{\partial\dot R} = 0$, $\frac{\partial L}{\partial R} = mR\dot\theta^2 - mg\cos\theta$, $\frac{\partial F_2}{\partial R} = 1$

$\Rightarrow -mR\dot\theta^2 + mg\cos\theta = -\lambda_2 \Rightarrow \lambda_2 = mR\dot\theta^2 + mg\cos\theta$ (3)

Tomando (1) y (2) e igualando $\cancel{m}a\ddot\alpha = -\cancel{m}R\ddot\theta + \cancel{m}g\sin\theta\dot\theta$, $\ddot\alpha = \frac{R}{a}\ddot\theta$

$\Rightarrow R\ddot\theta = -R\ddot\theta + g\sin\theta\dot\theta \Rightarrow 2R\ddot\theta = g\sin\theta\dot\theta \Rightarrow \dot\theta\frac{d\dot\theta}{dt} = \dot\theta g\frac{\sin\theta}{2R}$

$\Rightarrow \int\dot\theta\, d\dot\theta = \frac{\dot\theta^2}{2} = \int\frac{d\theta}{dt}\, g\frac{\sin\theta}{2R}\, dt = \frac{g(-\cos\theta)}{2R} + \frac{g}{2R}$ Reemplazamos en (3)

$\Rightarrow \lambda_2 = m\cancel{R}\left(\frac{g}{\cancel{R}}(1-\cos\theta)^2\right) + mg\cos\theta = \lambda_2$

Siendo λ_2 nuestra fuerza normal, podemos decir que cuando $\lambda_2 = 0$ los cuerpos se separaron.

$$0 = \lambda_2 = \cancel{m}R\left(\frac{g}{\cancel{R}}(1-\cos\theta)\right) + \cancel{m}g\cos\theta$$

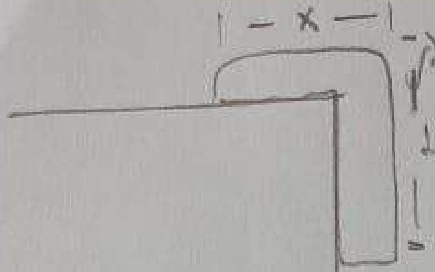
$$= \cancel{g}(1-\cos\theta) + \cancel{g}\cos\theta$$

$$= 1 - 2\cos\theta \Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{1}{3}\pi$$

$$\theta_1 l^2(m_1+m_2) - l\ddot{\theta}_2 a\cos(\theta_1+\theta_2)(\dot{\theta}_1+\dot{\theta}_2)\left(\frac{1}{2}m_1\right)$$

$$+\frac{1}{2}m_2 l\ddot{\theta}\, a\cos(\theta_1-\theta_2)(\dot{\theta}_1-\dot{\theta}_2) - \left(-l\dot{\theta}_1\dot{\theta}_2 a\cos(\theta_1+\theta_2)(\dot{\theta}_1-\dot{\theta}_2)\left(\frac{1}{2}m\right.\right.$$

$$\frac{1}{2}m_2(l\dot{\theta}_1\dot{\theta}_2 a(\cos(\theta_1-\theta_2)) - lg\sin\theta_1(m_1+m_2)) = 0$$



$\Rightarrow$ masa M, Densidad lineal $\lambda = \frac{M}{L} = \frac{dm}{dy}$

$L = L + x = y + x$

c) Dado que solo tenemos una restricción y es holonoma ; $2(1) - 1 = 1$

Planteamos el lagrangiano $\mathcal{L}_g$

$$\mathcal{L}_g = T - V = \frac{1}{2}M(\dot{y})^2 - V$$

$\dot{y}$ puede ser tomada como la Velocidad en la E_T dado que la magnitud de la Velocidad en $\dot{y}$ y $\dot{x}$ es la misma

$$dV = dm\, g y = \lambda\, dy\, g y \Rightarrow V = \frac{1}{2}y^2 g \lambda$$

$$\mathcal{L}_g = \frac{1}{2}M(\dot{y})^2 - \frac{1}{2}y^2 g\lambda$$

planteamos las Ec de Lagrange

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}_g}{\partial \dot{y}}\right) = \frac{d}{dt}(m\dot{y}) = m\ddot{y} \, , \; \frac{\partial \mathcal{L}_g}{\partial y} = -yg\lambda \Rightarrow m\ddot{y} + yg\lambda \Rightarrow m\ddot{y} - y(9.81)\lambda$$

$$\Rightarrow \frac{d^2y}{d^2t} = \frac{yg\lambda}{m} = \frac{d}{dt}(\dot{y}) = \frac{yg}{L} \Rightarrow \dot{y}\, d(\dot{y}) = \frac{dy}{dt}\frac{yg}{L}dt$$

$$\Rightarrow \int \dot{y}\, d\dot{y} = \int \frac{yg}{L} dy = \frac{\dot{y}^2}{2} = \frac{y^2}{2L}g + c \Rightarrow \dot{y}^2 = \frac{y^2 g}{2} + c$$

$$dy = \sqrt{\frac{y^2 g}{L}}\, dt \Rightarrow \frac{1}{y}dy = \sqrt{\frac{g}{L}}\, dt \Rightarrow \ln|y| = \sqrt{\frac{g}{L}}\, t + c$$

$$\Rightarrow e^{\ln|y|} = y = e^{\sqrt{\frac{g}{L}}t}m$$

$$y = e^{\sqrt{\frac{g}{L}}t}m$$

$x_{12}(\theta,\phi) = l\sin\theta + \frac{a}{2}\cos(\theta+\phi)$

$\dot{x}_{12}(\theta,\phi) = l(\cos\theta)\dot{\theta} + \frac{a}{2}\left(-\sin(\theta+\phi)\dot{\theta} - \sin(\theta+\phi)\dot{\phi}\right)$

$= l\cos(\theta)\dot{\theta} - \frac{a}{2}\sin(\theta+\phi)(\dot{\theta}+\dot{\phi})$

$y_{12}(\theta,\phi) = -l\cos\theta + \frac{a}{2}\sin(\theta+\phi)$

$= l\sin\theta\,\dot{\theta} + \frac{a}{2}\left[\cos(\theta+\phi)\dot{\theta} + \cos(\theta+\phi)\dot{\phi}\right)$

$= l\sin\theta\,\dot{\theta} + \frac{a}{2}\cos(\theta+\phi)(\dot{\theta}+\dot{\phi})$

→ Calculamos T. → $T = \frac{1}{2}m_1[\dot{x}_1^2 + \dot{y}_1^2] + \frac{1}{2}m_2[\dot{x}_{12}^2 + \dot{y}_{12}^2]$

$T = \frac{1}{2}m_1[((l\cos\theta\dot{\theta})^2 + (\frac{a}{2}\sin(\theta+\phi)(\dot{\theta}+\dot{\phi}))^2 + ((l\sin\theta\,\dot{\theta} - \frac{a}{2}\cos(\theta+\phi)(\dot{\theta}+\dot{\phi}))^2]$

$+ \frac{1}{2}m_2[(l\cos\theta\dot{\theta} - \frac{a}{2}\sin(\theta+\phi)(\dot{\theta}+\dot{\phi})]^2 + (l\sin\dot{\theta} + \frac{a}{2}\cos(\theta+\phi)(\dot{\theta}+\dot{\phi}))^2$

→ Calculamos U.

$U_{m_1} = m_1 g y_1 = m_1 g(y_1)$ → Reemp.

$U_{m_2} = m_2 g y_2 = m_1 g(y_2)$ → Reemp.

$\mathcal{L} = T - U$

$\frac{\partial}{\partial t}\left(\frac{\partial \mathcal{L}}{\partial \theta}\right) - \frac{\partial \mathcal{L}}{\partial \theta} = 0 \quad , \quad \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\phi}}\right) - \frac{\partial \mathcal{L}}{\partial \phi} = 0$