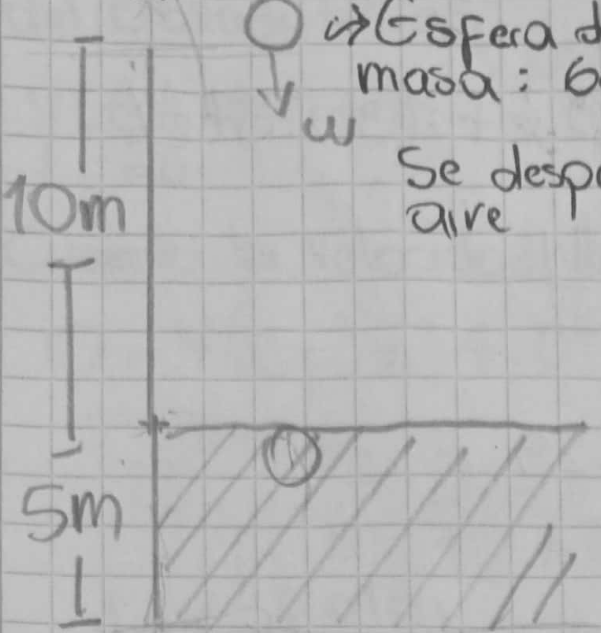
1 - A



$\Rightarrow$ Esfera de radio 0.35m
masa : 60kg

Se desprecia el rozamiento con el aire

Se necesita hallar la Velocidad con la que el cuerpo entra al agua.

$$\Rightarrow y(t) = y_0 + v_0 t - \frac{1}{2} g t^2$$

$$10 = 0 + 0t - \frac{1}{2} g t^2$$

$$t = \sqrt{\frac{20m}{9.81\frac{m}{s^2}}} = 1.42s \Rightarrow V(1.42) = -13.93$$

Diagrama de cuerpo libre al entrar en el agua

$\uparrow F_F + F_{empuje}$

$\downarrow w$

$$\sum F_y = 1000\frac{kg}{m^3}\left(\frac{4}{3}\pi r^3\right)\left(9.81\frac{m}{s^2}\right) - 60kg\left(9.81\frac{m}{s^2}\right) + \frac{1}{2}(0.47)\left(1000\frac{kg}{m^3}\right)(2\pi r^2)\frac{dx}{dt} = m\frac{d^2x}{d^2t}$$

Por lo cual, obtenemos lo siguiente

$$m\frac{d^2x}{d^2t} = 5285.46 - 588.6 + 180.87\frac{dy}{dt}$$

$$\frac{d^2y}{d^2t} = 78.28 + 3.01\frac{dy}{dx}$$

$$\Rightarrow \frac{d^2y}{d^2t} - 3.01\frac{dy}{dx} = 78.28$$

Solucionamos la EDO

$$y'' - 3.01y' = 0$$

$$m^2 - 3.01m = 0 \Rightarrow m(m-3.01) = 0$$

$$m_1 = 0,\ m_2 = 3.01$$

solución complementaria $y_c(t) = a_1 e^{\cancel{0}t} + a_2 e^{3.01t}$

$$y_p(t) = At + b$$

$$-3.01A = 78.28 \Rightarrow A = -26.01$$

$$y_p(t) + y_c(t) = a_2 e^{3.01t} - 26.01t + a_1$$

Sustituimos las condiciones iniciales

$$y(0) = 5 = a_2 + a_1$$

$$\frac{dy(t)}{dt} = 3.01a_2 e^{3.01t} - 26.01 = V(t)$$

$$V(0) = -13.93 = 3.01a_2 - 26.01$$

$$a_2 = 4.01,\ 5 - 4.01 = a_1 = 0.99$$

La expresión que describe la posición vertical del clavadista es

$$y(t) = 4.01e^{3.01t} - 26.01t + 1$$

Cuando la velocidad del atleta sea 0

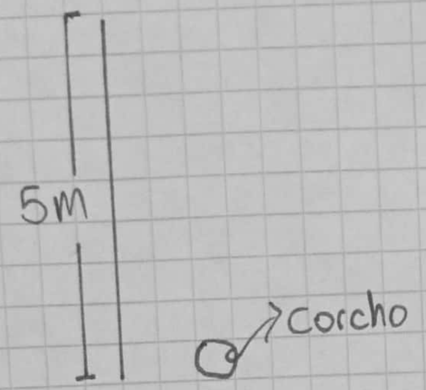
$$0 = v(t_f) = 4.01\cdot(3.01)e^{3.01t} - 26.01$$

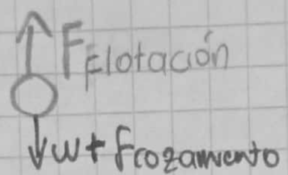
$$e^{3.01t} = \frac{26.01}{12.07} = 2.15$$

$$\ln(e^{3.01t}) = 3.01t = \ln(2.15)$$

$$t_f = \ln(2.15)/3.01) = 0.255$$

$y(0.25) = 3.01$ metros





Volumen del corcho: $\frac{4}{3}\pi(0.025)$

Densidad: $256.74\ Kg/m^3$

masa corcho: $0.02Kg$

$$\sum f_y = F_{Flotación} - w - f_r = m\frac{d^2y}{dt}$$

$$= 1000\frac{kg}{m^3}\cdot\left(\frac{4}{3}\pi(0.025m)^3\right)\left(9.81\frac{m}{s^2}\right) - g\frac{4}{3}\pi(0.025)^3\left(256.74\frac{Kg}{m^3}\right)$$

$$-\frac{10.47}{2}\left(1000\frac{kg}{m^3}\right)\left(\pi(0.025m)^2\right)\left(\frac{dy}{dt}\right)$$

$$m\frac{d^2y}{dt^2} = 0.64 - 0.19 - 1.84\left(\frac{dy}{dt}\right)$$

$$\frac{d^2y}{dt^2} = 22.5 - 23.1\left(\frac{dy}{dt}\right)$$

$y'' + 23.1y' = 22.5 \Rightarrow$ Solucionamos la EDO

$\Rightarrow m^2 + 23.1m = 0 \Rightarrow m(m+23.1) = 0 \mid m_1 = 0,\ m_2 = -23.1$

por lo cual $y_c(t) = a_1 + a_2e^{-23.1t}$

$y_p(t) = At + B \Rightarrow y_p'' + 23.1y' = 22.5 \Rightarrow 23.1A = 22.5$

$A = 0.97$

$y(t) = y_p(t) + y_c(t) = a_2e^{-23.1t} + 0.97t + a_1$

evaluamos las condiciones iniciales

$y(0) = a_2 + a_1 = -5,\ V(0) = -23.1a_2 + 0.97 = 0$

$a_2 = 0.04,\ a_1 = -5.04$

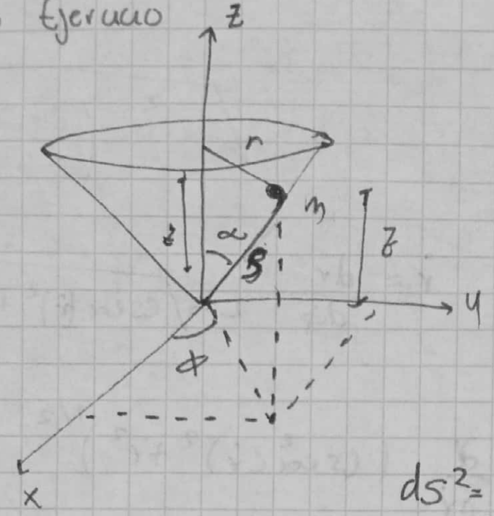
$$y(t_F) = 0 = 0.04e^{-23.1t} + 0.97t - 5.04$$

$$126 = e^{-23.1t} + 24.25t$$

$$t = 5.19,\ V = 5.7\frac{cm}{s}$$

$$V(t) = -23.1(0.04)e^{-23.1t} + 0.97t - 5.04$$

Solución Ejercicio
2.



1) Definimos las coordenadas generalizadas:

$q_1 = \phi$ y $q_2 = r$

$x = r\cos\phi$

$y = r\sin\phi$

$z = r\cot\alpha$

$S^2 = z^2 + r^2$

$\tan\alpha = \frac{r}{z}$

$z = \frac{r}{\tan\alpha} = r\cot\alpha$

$ds^2 = dr^2 + r^2 d\phi^2 + dz^2 \rightarrow$ cilyndrical

$dz^2 = d(r^2\cot^2\alpha) = dr^2\cot^2\alpha.$ $\quad \alpha = cte.$

$ds^2 = dr^2 + r^2 d\phi^2 + dr^2\cot^2\alpha$

$\rightarrow dr^2(1+\cot^2\alpha) + r^2 d\phi^2$ $\quad 1+\cot^2 = \csc^2\alpha.$

$ds^2 = \csc^2\alpha\, dr^2 + r^2 d\phi^2$

$\boxed{ds^2 =}$ c

2) Definimos la longitud de la curva.

$S = \int\sqrt{ds^2} = \int\sqrt{\csc^2\alpha\, dr^2 + r^2 d\phi^2}$ $\rightarrow \left(\frac{dr}{d\phi}\right)^2 = \frac{dr^2}{d\phi^2}$

$= \int\sqrt{\csc^2\alpha\left(\frac{dr^2}{d\phi^2}\right) + r^2 d\phi^2}$

3) Definimos el punto: $P_1(r_1, \phi_1)$ $P_2(r_2, \phi_2)$ y definimos el parámetro, este va a ser ϕ.

$r = r(\phi) \rightarrow$ Integramos sobre ϕ

$dr^2 = \frac{dr^2}{d\phi^2} d\phi^2$

$S = \int\sqrt{ds^2} = \int\sqrt{\csc^2\alpha\, dr^2 d\phi + r^2 d\phi^2} = \int\sqrt{\csc^2\alpha\frac{dr^2}{d\phi^2} + r^2}\, d\phi$

Fijamos ϕ_1 a ϕ_2

$$S = \int_{\phi_1}^{\phi_2}\sqrt{\csc^2\alpha\left(\frac{dr^2}{d\phi^2}\right) + r^2}\, d\phi,$$

3) Función lagrangiana.

$$L = \sqrt{\csc^2(\alpha)\left(\frac{dr}{d\phi}\right)^2 + r^2}$$

3. Ecuaciones de Euler langrange:

$$S = \int_{\phi_1}^{\phi_2} L \, d\phi \qquad L = \sqrt{\csc^2\alpha \left(\frac{dr}{d\phi}\right)^2 + r^2}$$

$$\rightarrow \frac{d}{d\phi}\left(\frac{\partial L}{\partial\left(\frac{\partial r}{\partial \phi}\right)}\right) - \frac{\partial L}{\partial r} = 0 \qquad \dot{r} = \frac{dr}{d\phi} \quad L = \sqrt{\csc\alpha (\dot{r})^2 + r^2}$$

$$\frac{\partial L}{\partial \dot{r}} = \frac{\partial}{\partial \dot{r}} \sqrt{\csc^2\alpha (\dot{r})^2 + r^2} = \frac{\partial}{\partial \dot{r}} \left(\csc^2\alpha (\dot{r})^2 + r^2 \right)^{1/2}$$

$$= \frac{\partial}{\partial \dot{r}} \frac{1}{2\sqrt{\csc^2\alpha (\dot{r})^2 + r^2}} \cdot 2\dot{r} \csc^2(\alpha) = \frac{\dot{r} \csc^2(\alpha)}{\sqrt{\csc^2\alpha (\dot{r})^2 + r^2}}$$

$$\frac{d}{d\phi}\left(\frac{\left(\frac{dr}{d\phi}\right) \csc^2(\alpha)}{\sqrt{\csc^2\alpha \left(\frac{dr}{d\phi}\right)^2 + r^2}} \right) \rightarrow \frac{dr^2}{d^2\phi} \frac{\csc^2(\alpha)}{\sqrt{\csc^2\alpha \left(\frac{dr}{d\phi}\right)^2 + r^2}}$$

$$+ \frac{dr}{d\phi} \csc^2(\alpha) \cdot \frac{1}{2\sqrt{\csc^2\alpha \left(\frac{dr}{d\phi}\right)^2 + r^2}} \cdot 2 \left(\frac{dr}{d\phi}\right) \cdot \frac{d^2 r}{d\phi^2}$$

$$= \frac{dr^2}{d^2\phi} \frac{\csc^2(\alpha)}{\sqrt{\csc^2\alpha \left(\frac{dr}{d\phi}\right)^2 + r^2}} + \cancel{\frac{dr}{d\phi}} \frac{\csc^2(\alpha) \frac{dr}{d\phi} \cdot \frac{d^2 r}{d\phi^2}}{\sqrt{\csc^2\alpha \left(\frac{dr}{d\phi}\right)^2 + r^2}}$$

$$\frac{dr}{d^2\phi} \frac{\csc^2(\alpha)}{\sqrt{\csc^2\alpha (\dot{r})^2 + r^2}} + \frac{\dot{r} \csc^2(\alpha) \dot{r} \ddot{r}}{\sqrt{\csc^2\alpha (\dot{r})^2 + r^2}}$$

$$\frac{\ddot{r} \csc^2(\alpha)}{\sqrt{\csc^2\alpha (\dot{r})^2 + r^2}} + \frac{(\dot{r})^2 \csc^2(\alpha) \ddot{r}}{\sqrt{\csc^2\alpha (\dot{r})^2 + r^2}}$$