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Leg: 13567
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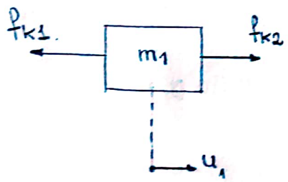
(1)

$\boxed{\alpha = 14}$

a) No considero las fuerzas verticales que actúan sobre las masas m_1 y m_2, sumatoria de las cuales es cero (las fuerzas verticales están equilibradas).

$\underline{m_1}$. (DCL). (Considero oscilaciones libres)

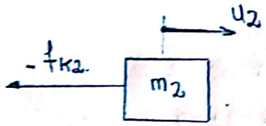


* Segunda Ley de Newton:

$$m_1\ddot{u}_1 = f_{k2} - f_{k1} = k_2\cdot(u_2 - u_1) - k_1u_1$$

$$m_1\ddot{u}_1 = -(k_1+k_2)u_1 + k_2u_2$$

$\underline{m_2}$. (DCL) (Considero oscilaciones libres)



* Segunda Ley de Newton:

$$m_2\ddot{u}_2 = -f_{k2} = -k_2(u_2 - u_1) = k_2u_1 - k_2u_2$$

Se obtiene el sistema de ecuaciones diferenciales ordinarias de segundo orden:

$$\begin{cases} m_1\ddot{u}_1 + (k_1+k_2)u_1 - k_2u_2 = 0 \\ m_2\ddot{u}_2 - k_2u_1 + k_2u_2 = 0 \end{cases}$$

En forma matricial se expresa:

$$\underbrace{\begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix}}_{M}\underbrace{\begin{bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{bmatrix}}_{\ddot{U}} + \underbrace{\begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}}_{K}\underbrace{\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}}_{U} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\boxed{M \cdot \ddot{U} + K \cdot U = \bar{0}}$$

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1.) ~~Se proponen~~

b) Proponemos soluciones de la forma:

$U = \cos(\omega t)\bar{v}$; $U = \operatorname{sen}(\omega t)\bar{v}$; donde ω es una constante y $\bar{v}$ es un vector constante.

$\ddot{U} = -\omega^2\cos(\omega t)\bar{v}$ (se obtiene para $U = \cos(\omega t)\bar{v}$)

Al reemplazar en la EDO matricial se obtiene:

$$M(-\omega^2\cos(\omega t)\bar{v}) + K(\cos(\omega t)\bar{v}) = \bar{0}.$$

$$\cos(\omega t)[-\omega^2 M + K]\cdot\bar{v} = \bar{0} \quad (1)$$

Para que (1) sea válida para todo t, se debe cumplir:

$$[-\omega^2 M + K]\bar{v} = 0.$$

M es invertible. Luego:

$$(-\omega^2 M^{-1}M + M^{-1}K)\bar{v} = 0.$$

$$(M^{-1}K - \omega^2 I)\bar{v} = 0.$$

Llamamos $B = M^{-1}K$ y ~~$\lambda = \omega^2$~~ $\lambda = \omega^2$

$$(B - \lambda I)\bar{v} = 0.$$

Encontramos entonces los valores y vectores propios de B.

$$B = \begin{bmatrix} \frac{k_1+k_2}{m_1} & -\frac{k_2}{m_1} \\ -\frac{k_2}{m_2} & \frac{k_2}{m_2} \end{bmatrix} = \quad B = \begin{bmatrix} \frac{\alpha k_2 + k_2}{\alpha m_2} & -\frac{k_2}{\alpha m_2} \\ -\frac{k_2}{m_2} & \frac{k_2}{m_2} \end{bmatrix}$$

$$B = \frac{k_2}{m_2}\cdot\begin{bmatrix} \frac{\alpha+1}{\alpha} & -\frac{1}{\alpha} \\ -1 & 1 \end{bmatrix}$$

Se tiene que: $Av = \lambda v$

$(cA)\bar{v} = (c\lambda)\bar{v}$, $c \in \mathbb{R}$; $c \neq 0$.

Luego si λ es valor propio de A. $c\lambda$ es valor propio de cA (para los mismos autovectores).

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Luego:

$\bar{v}_\lambda = (v_1, (15-14\lambda)v_1) = v_1 \cdot (1, (15-14\lambda))$; $v_1 \in \mathbb{R}-\{0\}$.

Se obtienen los autovectores.

$v_1 = (1; 15-14 \cdot 0{,}7661) = (1; 4{,}2749)$ (c)

$v_2 = (1; 15-14 \cdot 1{,}3053) = (1; -3{,}2749)$ (D)

Al normalizarlos (norma 2) se obtienen.

~~$v_1 = (0{,}0519; 0{,}2218)$~~

~~$v_2 = (0{,}0853;$~~)

$\boxed{v_1 = (0{,}2278; 0{,}9737)}$

~~$v_2 = (0{,}290$~~ $\boxed{v_2 = (0{,}2920; -0{,}9564)}$

c)

$U =$ ~~$c_1 \cos(\omega_1 t)v_1 + c_2 \cos$~~

$U = c_1 \cos(\omega_1 t)v_1 + c_2 \operatorname{sen}(\omega_1 t)v_1 + c_3 \cos(\omega_2 t)v_2 + c_4 \operatorname{sen}(\omega_2 t)v_2$.

c1) $U(0) = c_1 v_1 + c_3 v_2$

$\dot{U} = -c_1\omega_1 \operatorname{sen}(\omega_1 t)v_1 + \omega_1 c_2 \cos(\omega_1 t)v_1 - c_3\omega_2 \operatorname{sen}(\omega_2 t)v_2$
$+ \omega_2 c_4 \cos(\omega_2 t)v_2$.

$\dot{U}(0) = \omega_1 c_2 v_1 + \omega_2 c_4 v_2$.

~~$\Rightarrow$ Hacemos $v_1 = \begin{pmatrix} a \\ b \end{pmatrix}$~~

~~$v_2 = \begin{pmatrix} c \\ d \end{pmatrix}$~~

~~Luego: $(\omega_1 v_1 + \omega_2 v_2)\begin{bmatrix} c_2 \\ c_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$~~

~~Hacemos $v_1 = \begin{pmatrix} a \\ b \end{pmatrix}$; $v_2 = \begin{pmatrix} c \\ d \end{pmatrix}$~~

~~$\begin{pmatrix} \omega_1 a + \omega_2 c \\ \omega_1 b + \omega_2 d \end{pmatrix}$~~

Luego:

~~$-\omega_1 c_2 v_1 = \omega_2 c_4 v_2$~~

~~$\begin{pmatrix} -\omega_1 c_2 a = \omega_2 c_4 c \\ -\omega_1 c_2 b = \omega_2 c_4 d \end{pmatrix}$~~

~~$-\omega_1 c_2 a = \omega_2 c_4 c$~~

~~$c_2 = -\dfrac{\omega_2 c_4 c}{a\omega_1}$~~

~~$c_2 = -\dfrac{\omega_2 c_4 d}{\omega_1 b}$~~ ~~$\Rightarrow \dfrac{\omega_2 c_4 c}{a\omega_1} = \dfrac{\omega_2 c_4 d}{\omega_1 b} \Rightarrow$~~

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Para $A = \begin{bmatrix} \frac{\alpha+1}{\alpha} & -\frac{1}{\alpha} \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

Se obtiene la ecuación característica.

$(a_{11}-\lambda)(a_{22}-\lambda) - a_{21}a_{12} = 0.$

$a_{11}a_{22} - \lambda a_{11} - \lambda a_{22} + \lambda^2 - a_{21}a_{12} = 0.$

$\lambda^2 - \lambda(a_{11}+a_{22}) - a_{21} a_{12} + a_{11}a_{22} = 0.$

$\lambda^2 - \lambda\left(\frac{\alpha+1}{\alpha}+1\right) - (-1)\cdot\left(\frac{-1}{\alpha}\right) + \left(\frac{\alpha+1}{\alpha}\right) = 0.$

$\lambda^2 - \lambda\left(\frac{2\alpha+1}{\alpha}\right) - \cancel{\frac{1}{\alpha}} + \frac{\alpha+\cancel{1}}{\alpha} = 0$

$\lambda^2 - \lambda\left(\frac{2\alpha+1}{\alpha}\right) + 1 = 0.$

$\lambda^2 - \lambda\left(29/14\right) + 1 = 0.$

Se obtienen los autovalores:

$\begin{cases} \lambda_1 = 0{,}7661 \\ \lambda_2 = 1{,}3053 \end{cases}$

Para los valores así calculados tenemos:

$\omega^2 = \frac{k_2}{m_2}\lambda \Rightarrow \omega = \sqrt{\frac{k_2}{m_2}}\sqrt{\lambda}$

Así

$\boxed{\omega_1 = \sqrt{\frac{k_2}{m_2}} \cdot 0{,}8752}$

$\boxed{\omega_2 = \sqrt{\frac{k_2}{m_2}} \cdot 1{,}1425}$

Ahora encontramos los autovectores

$A - \lambda I = \begin{bmatrix} 15/14 - \lambda & -1/14 \\ -1 & 1-\lambda \end{bmatrix} =$ De la primera fila obtenemos para: $(A-\lambda I)\bar{v} = \bar{0}$

$\left(\frac{15}{14}-\lambda\right)v_1 - \frac{v_2}{14} = 0.$

$\boxed{v_2 = 14\left(\frac{15}{14}-\lambda\right)v_1}$

Hacemos: $U(0)=\begin{pmatrix}U_1\\U_2\end{pmatrix}$.

$$\begin{pmatrix}U_1\\U_2\end{pmatrix}=c_1\overset{}{V_1}+c_3\overset{}{V_2}=c_1\underbrace{\begin{pmatrix}a\\b\end{pmatrix}}_{V_1}+c_3\underbrace{\begin{pmatrix}c\\d\end{pmatrix}}_{V_2}$$

$$\begin{pmatrix}U_1\\U_2\end{pmatrix}=\begin{pmatrix}a & c\\b & d\end{pmatrix}\begin{pmatrix}c_1\\c_3\end{pmatrix}$$

$$\left(\begin{array}{cc|c}a & c & u_1\\b & d & u_2\end{array}\right)\rightarrow$$

Se obtienen los valores: $\boxed{c_1=0{,}1956.}$ $\boxed{c_3=0{,}084370}$

$$U(0)=\begin{pmatrix}14\cdot 0{,}02\\14\cdot 0{,}04\end{pmatrix}$$

$$\tilde{U}(0)=[\omega_1V_1 \quad +\omega_2V_2]\begin{bmatrix}c_2\\c_4\end{bmatrix}$$

$$\tilde{U}(0)=\begin{pmatrix}\omega_1 a & \omega_2 c\\\omega_1 b & \omega_2 d\end{pmatrix}\begin{pmatrix}c_2\\c_4\end{pmatrix}$$

$$\begin{pmatrix}\omega_1 a & \omega_2 c\\\omega_1 b & \omega_2 d\end{pmatrix}\begin{bmatrix}c_2\\c_4\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$$

$$\sqrt{\frac{k_2}{m_2}}\begin{pmatrix}\sqrt{\lambda_1}a & \sqrt{\lambda_2}c\\\sqrt{\lambda_1}b & \sqrt{\lambda_2}d\end{pmatrix}\begin{bmatrix}c_2\\c_4\end{bmatrix}=\bar{0}.$$

$$\begin{pmatrix}\sqrt{\lambda_1}a & \sqrt{\lambda_2}c\\\sqrt{\lambda_1}b & \sqrt{\lambda_2}d\end{pmatrix}\begin{bmatrix}c_2\\c_4\end{bmatrix}=\begin{bmatrix}\bar{0}\end{bmatrix}\rightarrow \boxed{c_2=c_4=0}$$

Se obtienen las soluciones.