Time Series

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What's that?

Time series analysis plays a crucial role in understanding and predicting economic phenomena. In economics, data is often collected over time, capturing the dynamics of various economic variables such as GDP, inflation rates, stock prices, and unemployment rates. These data points arranged chronologically create a time series. Time series analysis allows economists to explore patterns, trends, and relationships within these data sets, providing valuable insights into economic behavior.

The distinctive feature of time series data **is its temporal dependency**. Unlike cross-sectional data that captures observations at a specific point in time, time series data represents observations taken at regular intervals over a continuous time period. This temporal dimension introduces a unique set of challenges and opportunities for analysis.

One of the fundamental objectives of time series analysis is to uncover the underlying structure and patterns within the data. This involves identifying **trends, seasonality, cyclical movements, and irregular components**. By understanding these patterns, economists can make informed predictions about future behavior, develop forecasting models, and formulate effective policy decisions.

Time series analysis employs a range of statistical and econometric techniques to extract meaningful information from the data. These techniques include descriptive statistics, graphical methods, autocorrelation, spectral analysis, exponential smoothing, and various time series models such as autoregressive integrated moving average (ARIMA) and vector autoregression (VAR).

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ARIMA Model (Autoregressive Integrated Moving Average)

1. Autoregressive (AR) Component:

$$X_t = c + \phi_1 X_{t-1} + \phi_2 X_{t-2} + \ldots + \phi_p X_{t-p} + \varepsilon_t$$

where: - X_t is the value of the variable at time 't'. - c is a constant term. - $\phi_1, \phi_2, \ldots, \phi_p$ are the autoregressive coefficients. - $X_{t-1}, X_{t-2}, \ldots, X_{t-p}$ are the lagged values of the variable. - ε_t is the error term at time 't'.

2. Integrated (I) Component:

$$\nabla^d X_t = X_t - X_{t-d}$$

where: - $\nabla^d X_t$ represents the differenced time series of order 'd'. - X_t is the value of the variable at time 't'. - X_{t-d} represents the lagged value at time 't-d'.

3. Moving Average (MA) Component:

$$X_t = c + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \ldots + \theta_q \varepsilon_{t-q} + \varepsilon_t$$

where: - X_t is the value of the variable at time 't'. - c is a constant term. - $\theta_1, \theta_2, \ldots, \theta_q$ are the moving average coefficients. - $\varepsilon_{t-1}, \varepsilon_{t-2}, \ldots, \varepsilon_{t-q}$ are the past forecast errors. - ε_t is the error term at time 't'.

VAR Model (Vector Autoregression)

1. VAR Representation:

$$\begin{bmatrix} X_{1,t} \\ X_{2,t} \\ \vdots \\ X_{n,t} \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix} + \sum_{i=1}^{p} \begin{bmatrix} \Phi_{11}^{(i)} & \Phi_{12}^{(i)} & \dots & \Phi_{1n}^{(i)} \\ \Phi_{21}^{(i)} & \Phi_{22}^{(i)} & \dots & \Phi_{2n}^{(i)} \\ \vdots & \vdots & \ddots & \vdots \\ \Phi_{n1}^{(i)} & \Phi_{n2}^{(i)} & \dots & \Phi_{nn}^{(i)} \end{bmatrix} \begin{bmatrix} X_{1,t-i} \\ X_{2,t-i} \\ \vdots \\ X_{n,t-i} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \vdots \\ \varepsilon_{n,t} \end{bmatrix}$$

where: - $X_{i,t}$ is the value of variable 'i' at time 't'. - $\mathbf{c}$ is a vector of constant terms. - $\mathbf{\Phi}^{(i)}$ represents the coefficient matrix for lag 'i'. - $X_{i,t-i}$ represents the lagged value of variable 'i' at time 't-i'. - $\varepsilon_{\mathbf{t}}$ is a vector of error terms at time 't'.

2. Estimating VAR Model: Estimating the VAR model involves determining the order of the model, selecting appropriate lag lengths, and estimating the coefficient matrices. This is typically done using techniques such as least squares or maximum likelihood estimation.

3. Impulse Response Analysis: Once the VAR model is estimated, economists often conduct impulse response analysis to examine the dynamic response of the variables to a shock. This analysis helps understand the transmission of shocks across variables and their impact on the system.

4. Granger Causality: VAR models are also used to assess Granger causality, which explores the causal relationships among variables. By examining the significance of lagged variables in predicting the current values, economists can infer causal links between variables.

GARCH Model (Generalized Autoregressive Conditional Heteroskedasticity)

1. Conditional Variance: The GARCH model represents the conditional variance of the series as a function of its past variances and squared error terms. The GARCH(p, q) model combines autoregressive and moving average components to capture the time-varying volatility.

The equation for the conditional variance in the GARCH(p, q) model is:

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2$$

where: - σ_t^2 is the conditional variance of the series at time 't'. - ω is the constant term. - α_i are the autoregressive parameters for the squared error terms. - ε_{t-i}^2 are the lagged squared error terms. - β_j are the moving average parameters for the past conditional variances. - σ_{t-j}^2 are the lagged conditional variances.

2. Estimating GARCH Model: Estimating the GARCH model involves determining the appropriate order of the model (p and q), selecting an appropriate estimation method, and estimating the parameters. Common estimation techniques include maximum likelihood estimation and Bayesian methods.

3. Volatility Forecasting: Once the GARCH model is estimated, it can be used for volatility forecasting. By updating the model with new information, economists and analysts can generate forecasts of future volatility. These forecasts are crucial for risk management, option pricing, and portfolio optimization.

4. ARCH Effect: The GARCH model is an extension of the ARCH (Autoregressive Conditional Heteroskedasticity) model, which assumes only autoregressive conditional heteroskedasticity. The GARCH model captures both autoregressive and moving average components, allowing for a more flexible modeling of the conditional variance.

TAR Model

TAR Model (Threshold Autoregressive Model)

1. Threshold Autoregressive Equation: The TAR model consists of two regimes, each characterized by its own autoregressive relationship. The model switches between regimes based on a threshold variable. The TAR(1) model can be expressed as follows:

$$Y_t = \begin{cases} \alpha_1 + \beta_1 Y_{t-1} + \varepsilon_t, & \text{if } X_{t-1} < \gamma \\ \alpha_2 + \beta_2 Y_{t-1} + \varepsilon_t, & \text{if } X_{t-1} \geq \gamma \end{cases}$$

where: - Y_t is the value of the dependent variable at time 't'. - α_1 and α_2 are the intercepts for each regime. - β_1 and β_2 are the autoregressive coefficients for each regime. - X_{t-1} is the value of the threshold variable at time 't-1'. - γ is the threshold value that determines the regime switch. - ε_t is the error term at time 't'.

2. Estimation and Inference: Estimating the TAR model involves determining the appropriate threshold value, estimating the model parameters, and conducting inference tests. Common estimation techniques include maximum likelihood estimation or non-linear least squares. Hypothesis tests can be performed to examine the presence of regime-switching and the significance of the model parameters.

3. Model Selection: Selecting the appropriate TAR model involves choosing the optimal lag order, threshold value, and the number of regimes. Model selection criteria, such as information criteria (AIC, BIC), can be used to compare different TAR models and select the most suitable one.

4. Forecasting and Applications: Once the TAR model is estimated, it can be used for forecasting future values of the dependent variable. The regime-switching nature of the model allows for capturing different dynamics under different conditions, making it valuable for predicting regime shifts and generating more accurate forecasts.