# Special Topics in Particle Physics

## Kinematics and Cross Sections

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# Kinematic and Cross Sections

**Book recommendation for special relativity:**

- Robert Resnick: Introduction to Special Relativity
- Griffiths, David J. Introduction to Elementary Particles - Chapter 3
- Lecture note with basic concepts: https://github.com/helgadenes/Special_relativity_yachay

# Kinematic and Cross Sections

In astroparticle physics the energies of participating particles are generally that high, that ***relativistic kinematics*** **must be used**. **Mass and energy are only different facets of the same thing**. Mass is a particularly compact form of energy, which is related to the total energy of a particle by the famous relation

$$E = mc^2$$

In this equation *m* is the mass of a particle, which moves with the velocity v, and *c* is the velocity of light in vacuum.

The experimental result that **the velocity of light in vacuum is the maximum velocity in all inertial systems** leads to the fact that particles with velocity near the velocity of light do not get much faster when accelerated, but mainly only become heavier,

$$\boxed{m = \frac{m_0}{\sqrt{1-\beta^2}} = \gamma m_0}$$

# Kinematic and Cross Sections

In this equation $m_0$ is the rest mass, $\beta = v/c$ is the particle velocity, normalized to the velocity of light, and

$$\boxed{\gamma = \frac{1}{\sqrt{1-\beta^2}}}$$

is the Lorentz factor. Using this result

$$E = \gamma m_0 c^2$$

where $m_0c^2$ **is the rest energy of a particle**. The **momentum** of a particle can be expressed as

$$p = mv = \gamma m_0 \beta c$$

# Kinematic and Cross Sections

Using, the difference

$$E^2 - p^2c^2 = \gamma^2 m_0^2 c^4 - \gamma^2 m_0^2 \beta^2 c^4$$

Can be written as

$$E^2 - p^2c^2 = \frac{m_0^2 c^4}{1-\beta^2}(1-\beta^2) = m_0^2 c^4 .$$

This result shows that $E^2 - p^2c^2$ is a **Lorentz-invariant quantity called the invariant mass** squared of the particle of mass $m$. This quantity is the same in all systems and it equals the **square of the rest energy**. Consequently, the **total energy of a relativistic particle** can be expressed by

$$\boxed{E = c\sqrt{p^2 + m_0^2 c^2}}$$

# Kinematic and Cross Sections

This equation holds for all particles. For massless particles or, more precisely, particles with rest mass zero, one obtains

$$E = cp$$

Particles of total energy $E$ **without rest mass** are **also subject to gravitation**, because they **acquire a mass according to**

$$m = E/c^2$$

The transition from relativistic kinematics to classical (Newtonian) mechanics ($p \ll m_0 c$) can also be derived by series expansion. The kinetic energy of a particle is obtained to

$$E^{\text{kin}} = E - m_0c^2 = c\sqrt{p^2 + m_0^2c^2} - m_0c^2$$

$$= m_0c^2\sqrt{1 + \left(\frac{p}{m_0c}\right)^2} - m_0c^2$$

$$\longrightarrow$$

$$\approx m_0c^2\left(1 + \frac{1}{2}\left(\frac{p}{m_0c}\right)^2\right) - m_0c^2$$

$$= \frac{p^2}{2m_0} = \frac{1}{2}m_0v^2\,,$$

# Kinematic and Cross Sections

In accordance with classical mechanics, the velocity can be expressed by

$$v = \frac{p}{\gamma m_0} = \frac{c^2 p}{E}$$

$$\beta = \frac{cp}{E} \, .$$

In relativistic kinematics it is usual to set $c = 1$. This simplifies all formulae. If, however, numerical quantities have to be calculated, the actual value of the velocity of light has to be considered.

# Treshold energies

In particle physics, the **threshold energy** for production of a particle is the minimum kinetic energy that must be imparted to one of a pair of particles in order for their collision to produce a given result.

If the desired result is to produce a third particle then the threshold energy is greater than or equal to the rest energy of the desired particle. In most cases, since momentum is also conserved, the threshold energy is significantly greater than the rest energy of the desired particle.

In astroparticle physics frequently the problem occurs to determine the ***threshold energy*** **for a certain process of particle production**. This requires that **in the center-of-mass system of the collision** at least **the masses of all particles in the final state of the reaction have to be provided**. In storage rings the center-of-mass system is frequently identical with the laboratory system so that, for example, the creation of a particle of mass $M$ in an electron–positron head-on collision ($e^+$ and $e^-$ have the same total energy $E$) requires

$$2E \geq M$$

If, on the other hand, a particle of energy $E$ interacts with a target at rest as it is characteristic for processes in cosmic rays, the center-of-mass energy for such a process must first be calculated.

# Treshold energies

**Example 1:** Let us assume that a high-energy cosmic-ray proton (energy $E_p$, momentum $\boldsymbol{p}$, rest mass $m_p$) produces a proton–antiproton pair on a target proton at rest:

$$p + p \to p + p + p + \bar{p} \, . \tag{3.2.1}$$

# Treshold energies

**Example 2:** For the equivalent process of $e^+e^-$ pair production by an energetic electron on an electron target at rest,

$$e^- + e^- \to e^- + e^- + e^+ + e^- \,, \tag{3.2.5}$$

**Do we expect the same result as for the protons?**

# Treshold energies

**Example 3:** Let us consider the photoproduction of an electron–positron pair on a target electron at rest,

$$\gamma + e^- \to e^- + e^+ + e^- \, ; \tag{3.2.7}$$

# Treshold energies

**Example 4:** Consider the photoproduction of a neutral pion (mass $m_{\pi^0} \approx 135\,\text{MeV}$) on a target proton at rest (mass $m_p$):

$$\gamma + p \rightarrow p + \pi^0 ,$$

# Four vectors

**Example 5:** Photoproduction of an electron–positron pair in the Coulomb field of a nucleus (see Fig. 3.1).
In this example the incoming photon $\gamma$ is real, while the photon $\gamma^*$ exchanged between the electron and the nucleus is virtual (Fig. 3.2)

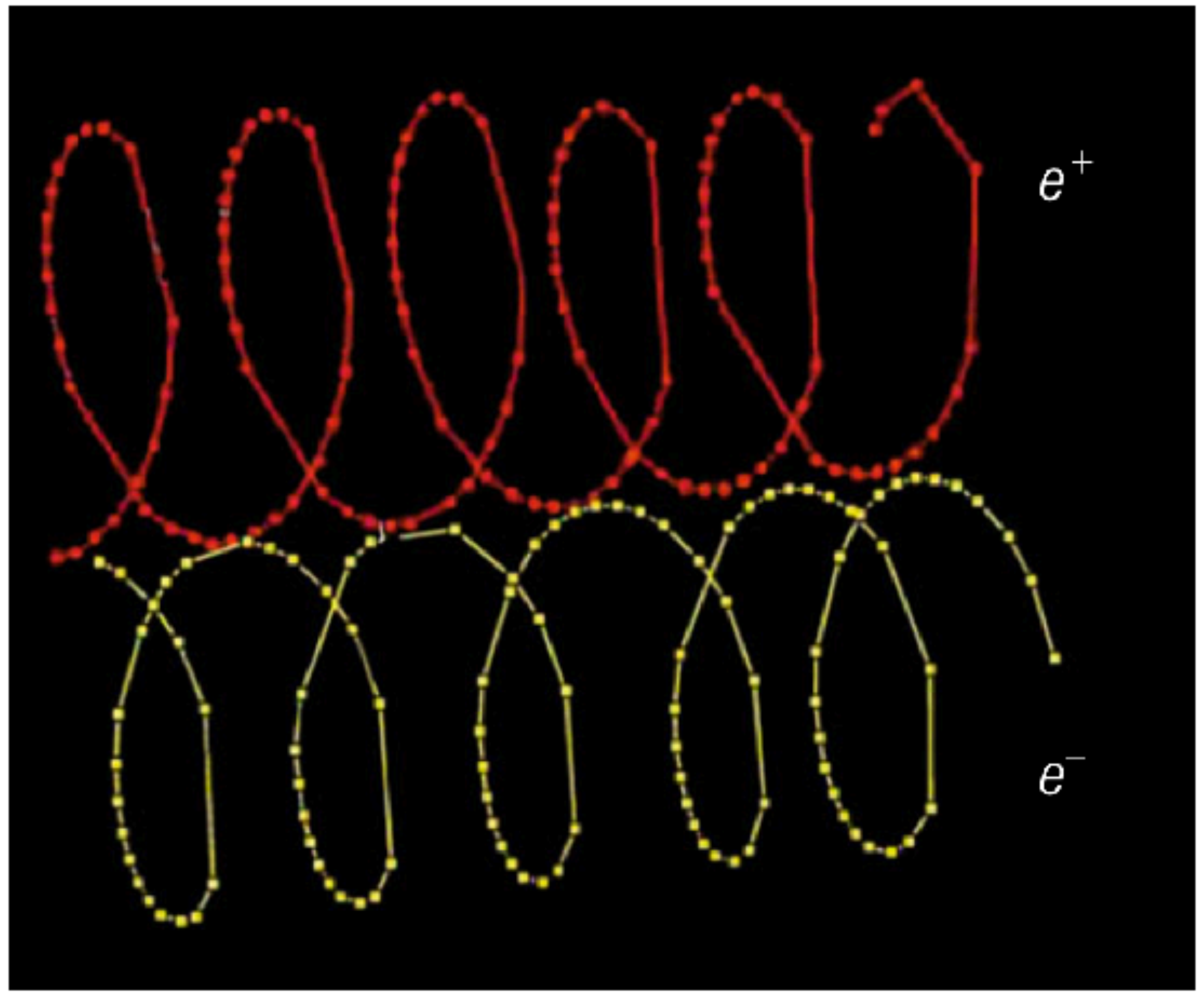



**Fig. 3.1** Electron–positron pair production by a photon (coming from the left) in the time projection chamber of the ALEPH experiment at CERN. The electron and positron tracks are bent in the solenoidal magnetic field producing helical tracks each [26]

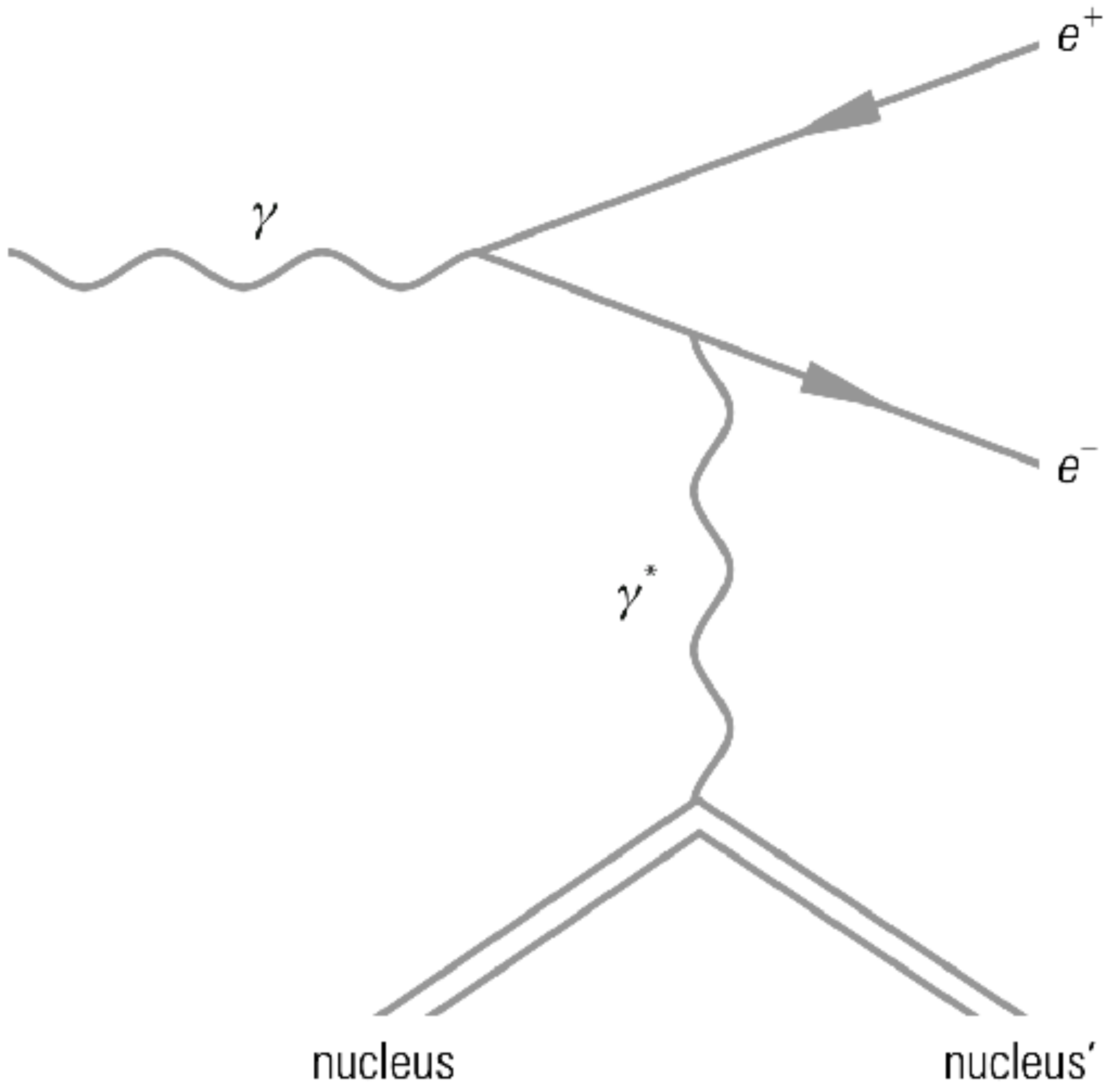



**Fig. 3.2** The process $\gamma + \text{nucleus} \to e^+ + e^- + \text{nucleus}'$

# WIMPs

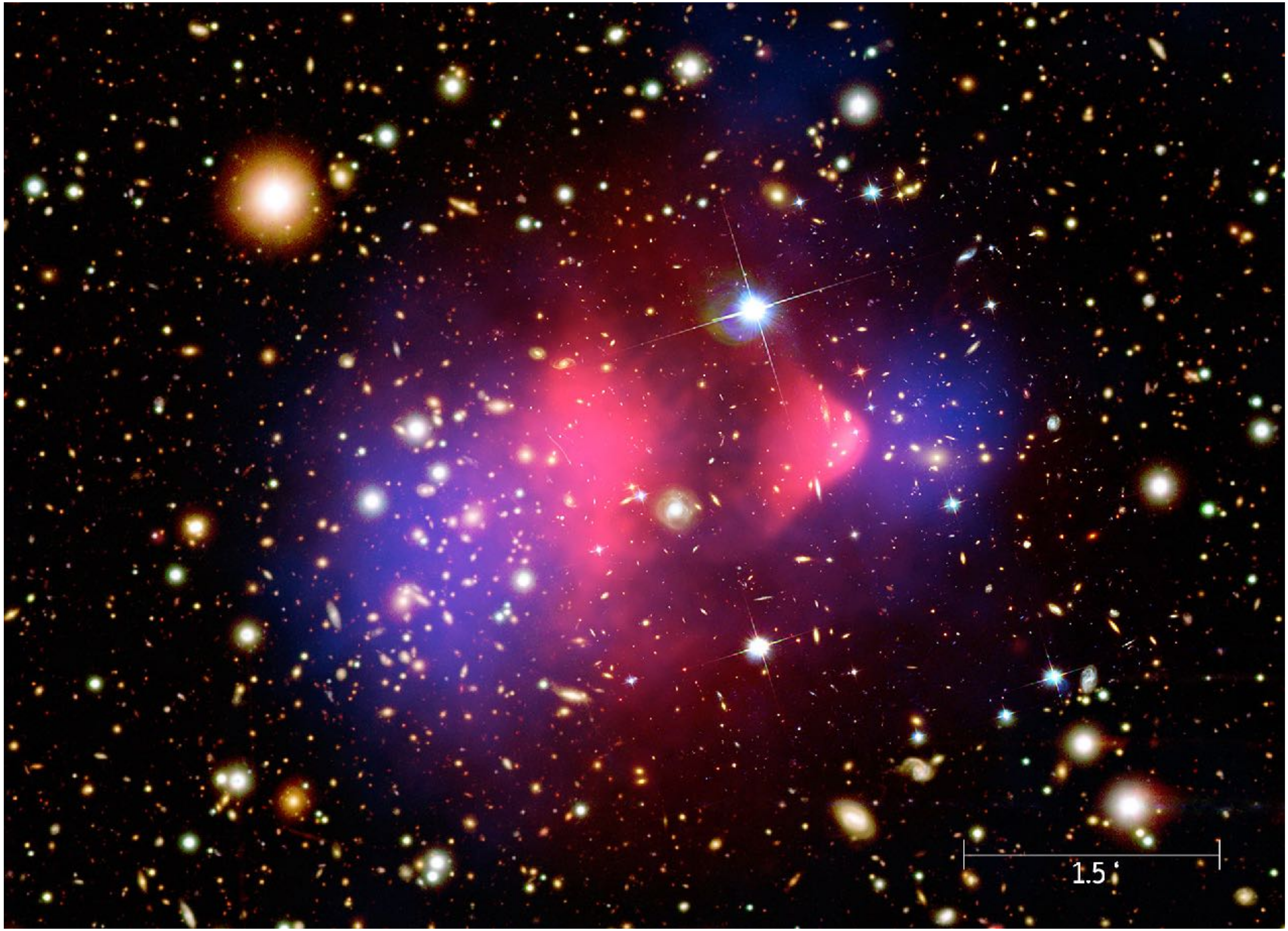
**Weakly interacting massive particles (WIMPs)** are **hypothetical particles** that are one of the proposed **candidates for dark matter.**

There exists no formal definition of a WIMP, but broadly, it is **an elementary particle which interacts via gravity** and any other force (or forces), which are **as weak as or weaker than the weak force**, but also non-vanishing in strength.
Many WIMP candidates are **expected to have been produced thermally in the early Universe**, similarly to the particles of the Standard Model according to Big Bang cosmology, and usually will constitute **cold dark matter**. The expected mass for the WHIMPs is in the 100 GeV mass range.

Experimental efforts to detect WIMPs include the **search for products of WIMP annihilation,** including gamma rays, neutrinos and cosmic rays in nearby galaxies and galaxy clusters; **direct detection experiments** designed to measure the collision of WIMPs with nuclei in the laboratory, as well as attempts to directly produce WIMPs in colliders, such as the Large Hadron Collider at CERN.

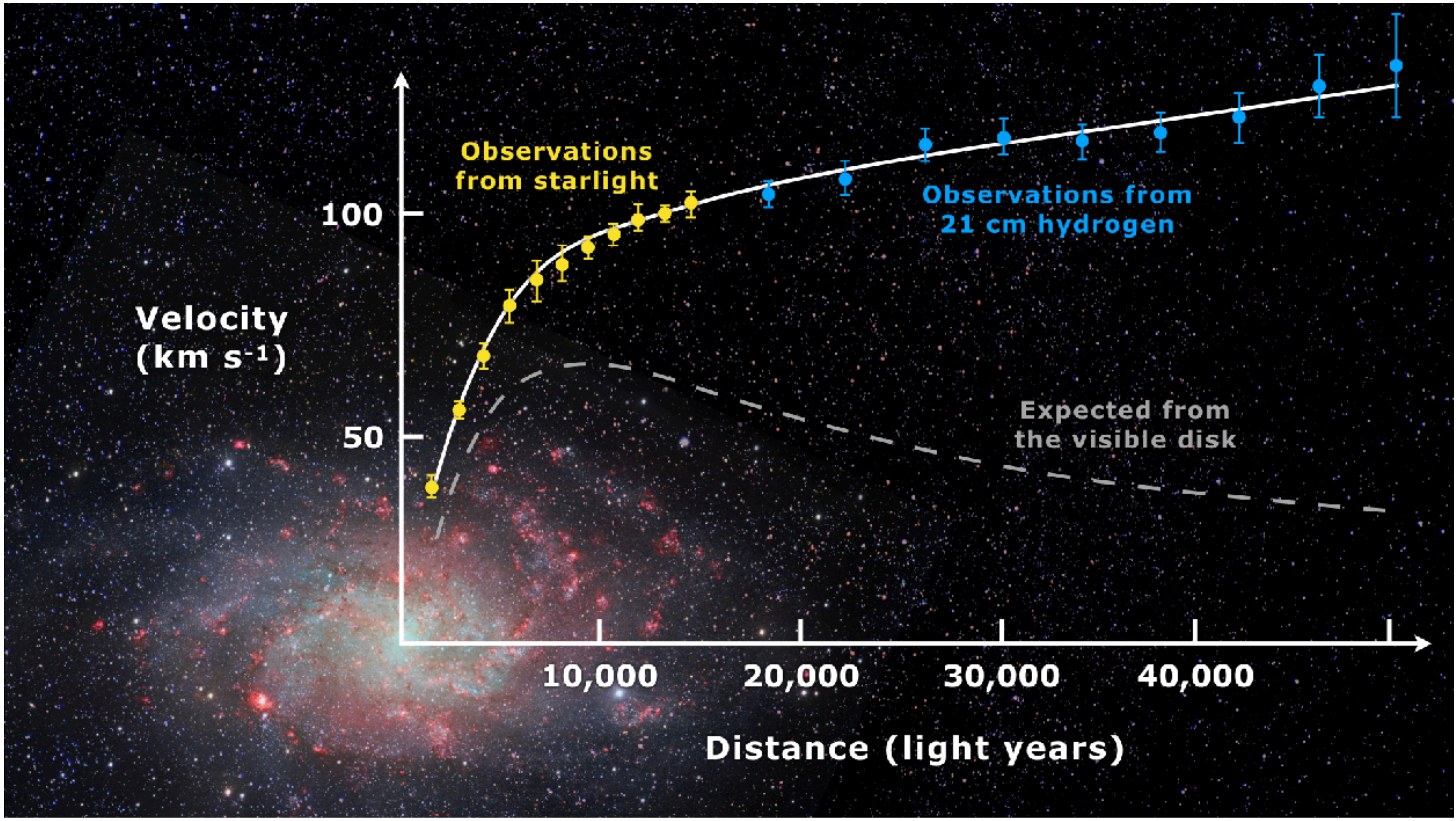
# Dark matter

**Dark matter in the Bullet cluster**
Composite image showing the galaxy cluster known as bullet cluster. The image in **background** showing the **visible spectrum** of light stems from Magellan and Hubble Space Telescope images. The **pink overlay shows the x-ray emission** (recorded by Chandra Telescope) of the **colliding galaxy clusters,** the **blue one represents the mass distribution** of the clusters calculated from gravitational lens effects.



# Dark matter

Rotation curve of spiral galaxy Messier 33 (yellow and blue points with error bars), and a predicted one from distribution of the visible matter (gray line). The data and the model predictions are from Corbelli and Salucci 2000.
The discrepancy between the two curves can be accounted for by adding a **dark matter halo surrounding the galaxy**.



# Supersymmetry

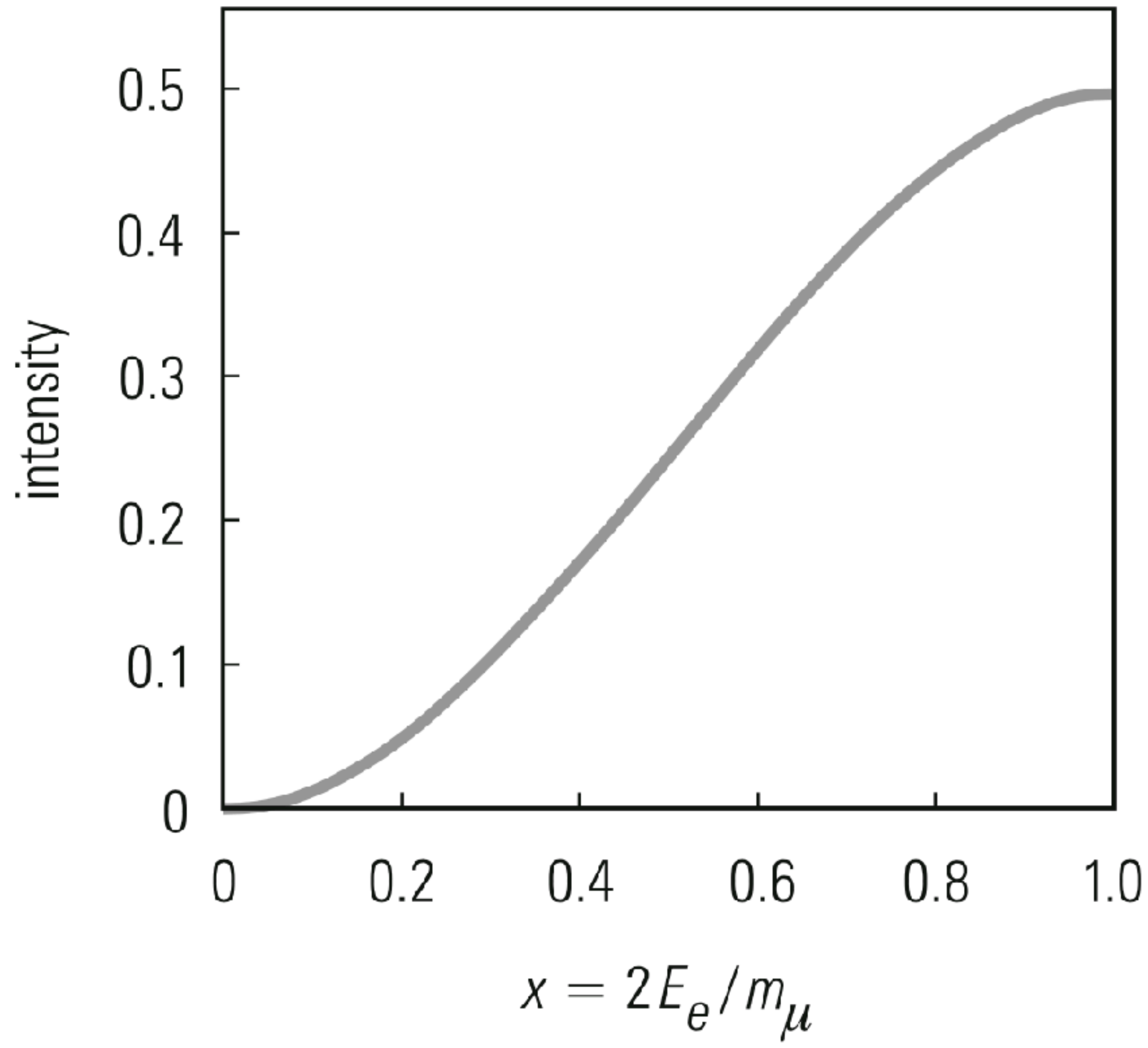
Supersymmetry is a theoretical framework in physics that **suggests the existence of a symmetry between particles with integer spin (bosons) and particles with half-integer spin (fermions)**. It proposes that for every known particle, there exists a partner particle with different spin properties. There have been multiple experiments on supersymmetry that have failed to provide evidence that it exists in nature.

In supersymmetry, each particle from the class of fermions would have an associated particle in the class of bosons, and vice versa, known as a superpartner. The spin of a particle's superpartner is different by a half-integer. For example, if the electron exists in a supersymmetric theory, then there would be a particle called a selectron (superpartner electron), a bosonic partner of the electron. In the simplest supersymmetry theories, with perfectly "unbroken" supersymmetry, each pair of superpartners would share the same mass and internal quantum numbers besides spin. More complex supersymmetry theories have a spontaneously broken symmetry, allowing superpartners to differ in mass.

https://en.wikipedia.org/wiki/Supersymmetry

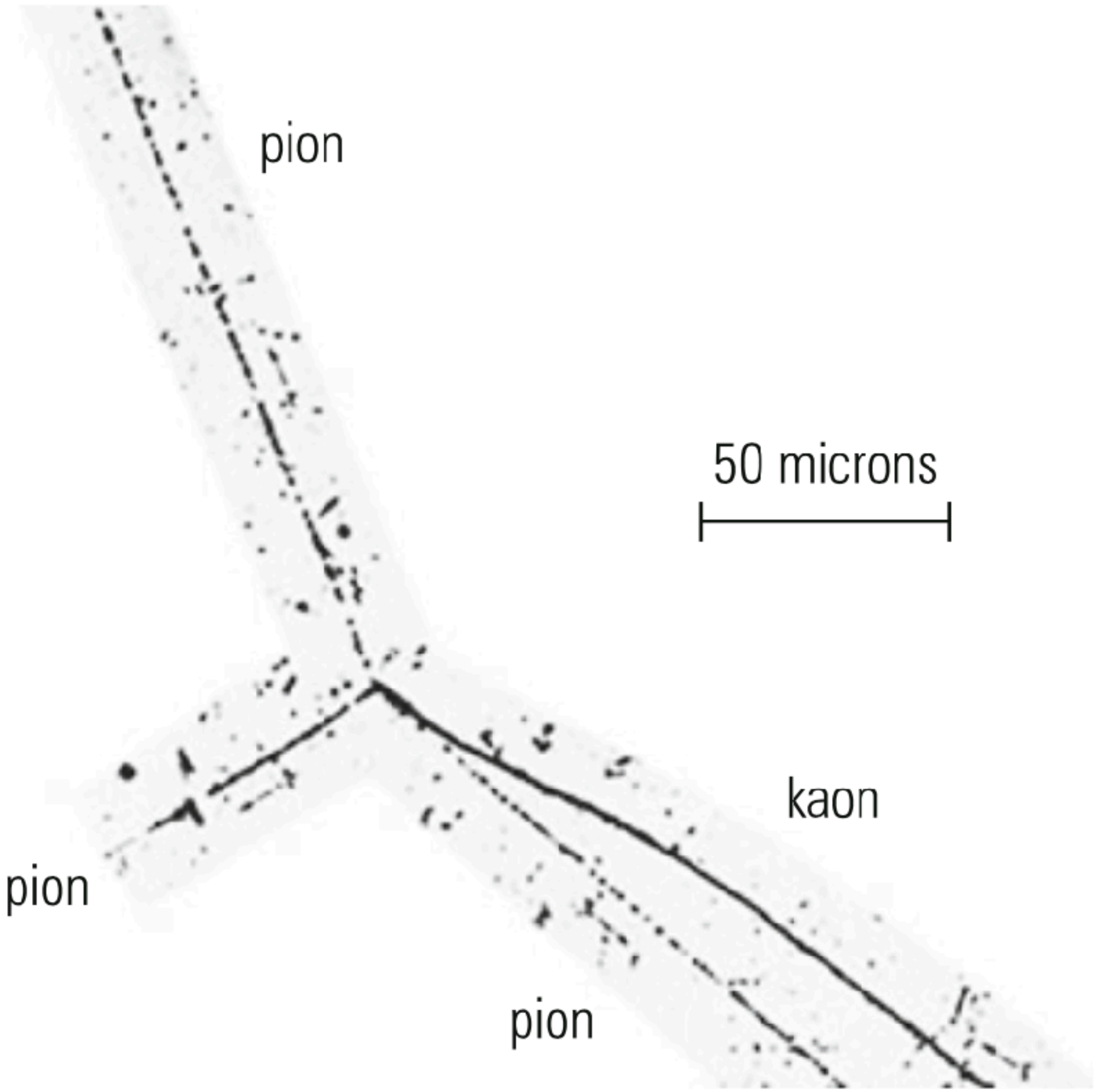
# 3-body decays

**Fig. 3.6** Energy spectrum of electrons from muon decay



# 3-body decays

**Fig. 3.7** Event display of the decay of a charged kaon into three pions in a nuclear emulsion ($K^{\pm} \rightarrow \pi^{\pm} + \pi^{-} + \pi^{+}$; in a nuclear emulsion the charge of a particle cannot be determined) [28]



# Cross sections

To illustrate the idea of differential cross sections the **transverse momentum distributions of pions and kaons in cosmic rays is compared to fixed-target data from accelerators** as an example in Fig. 3.9.

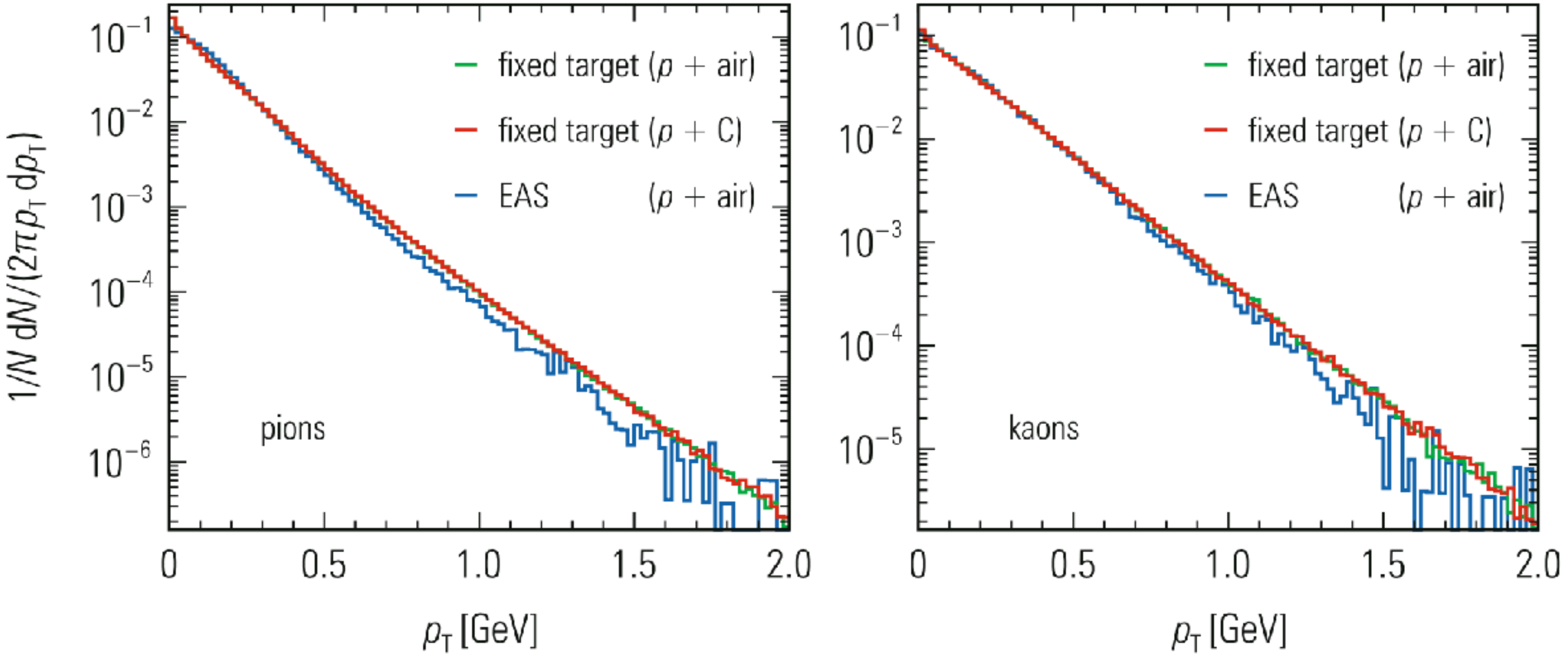



**Fig. 3.9** Transverse momentum distributions of pions and kaons in cosmic rays from the KASCADE experiment (nucleons ≈160 GeV on air) in comparison to fixed-target data from accelerators (protons 160 GeV on air; protons 160 GeV on carbon) [30]

# Cross sections

The double differential cross section for transverse momentum distributions of pions, kaons and protons obtained in **proton–proton collisions** is shown in Fig. 3.10.

Here, as second variable the commonly used **rapidity is introduced, which is interpreted as a measure for the relativistic velocity**

$$y = \frac{1}{2} \cdot ln\left(\frac{E + p_z}{E - p_z}\right).$$

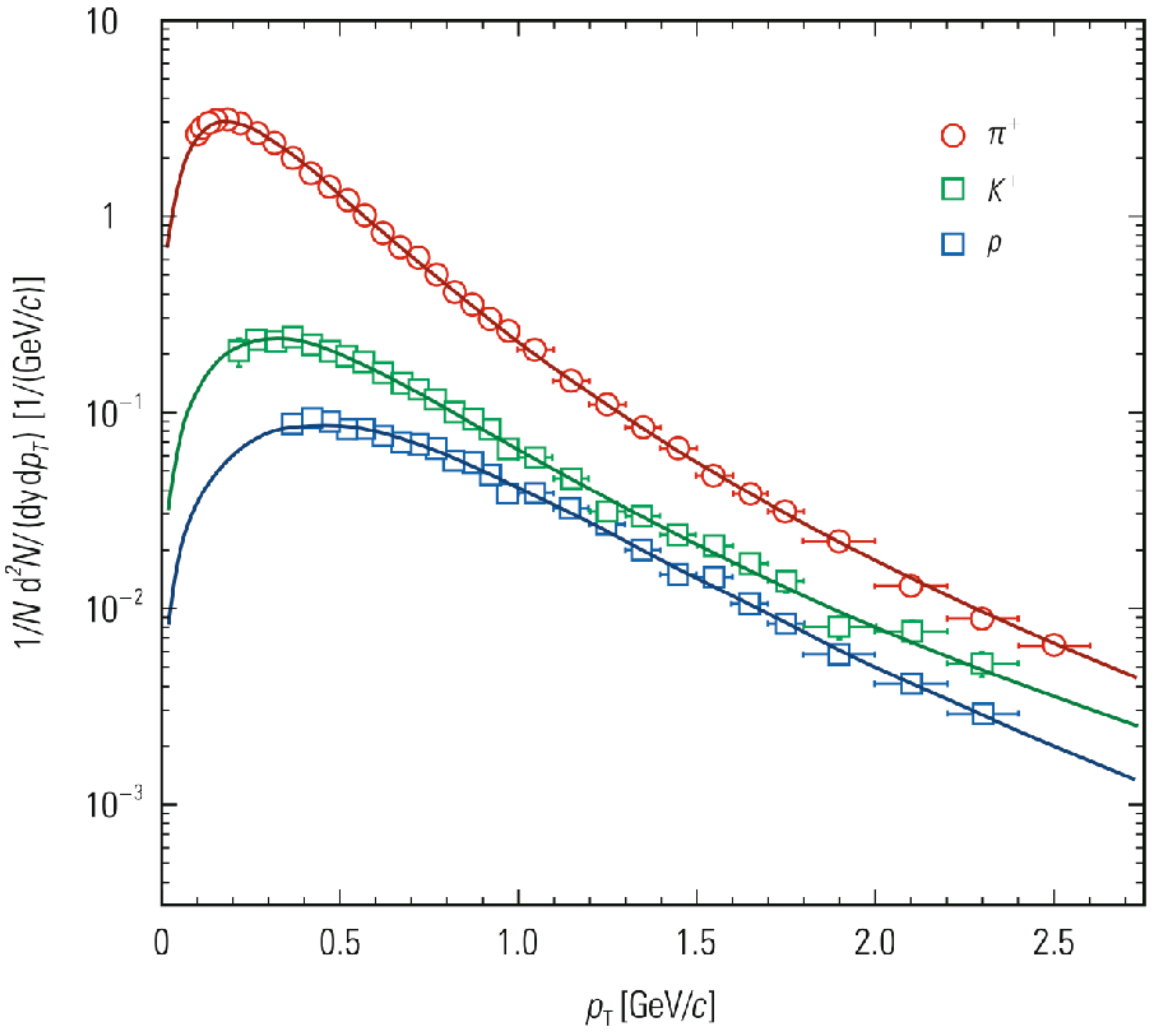



**Fig. 3.10** Double differential transverse momentum distributions of pions, kaons, and protons obtained in proton–proton collisions at a center-of-mass energy of 900 GeV in ALICE at the LHC ($p_T$ is the transverse momentum, $y$ is the rapidity, which is defined as $y = \frac{1}{2} \cdot \ln(\frac{E+p_z}{E-p_z})$, and $z$ is oriented along the beam direction, i.e., longitudinal momentum. Rapidity is commonly used as a measure for relativistic velocity.) [31]

# Problems

1. What is the threshold energy for a photon, $E_{\gamma_1}$, to produce a $\mu^+\mu^-$ pair in a collision with a blackbody photon of energy $1\,\mathrm{meV}$?

# Problems

2. The mean free path $\lambda$ (in $\mathrm{g/cm^2}$) is related to the nuclear cross section $\sigma_{\mathrm{N}}$ (in $\mathrm{cm^2}$) by

$$\lambda = \frac{1}{N_{\mathrm{A}} \sigma_{\mathrm{N}}} ,$$

where $N_{\mathrm{A}}$ is the Avogadro number, i.e., the number of nucleons per g, and $\sigma_{\mathrm{N}}$ is the cross section per nucleon.
The number of particles penetrating a target $x$ unaffected by interactions is

$$N = N_0 \, \mathrm{e}^{-x/\lambda} .$$

How many collisions happen in a thin target of thickness $x$ ($N_{\mathrm{A}} = 6.022 \times 10^{23}\,\mathrm{g}^{-1}$, $\sigma_{\mathrm{N}} = 1\,\mathrm{b}$, $N_0 = 10^8$, $x = 0.1\,\mathrm{g/cm^2}$)?

# Problems

3. The neutrino was discovered in the reaction

$$\bar{\nu}_e + p \rightarrow n + e^+ \,,$$

where the target proton was at rest. What is the minimum neutrino energy to induce this reaction?