Special Topics in Particle Physics

Introduction and the Standard Model

The Standard model

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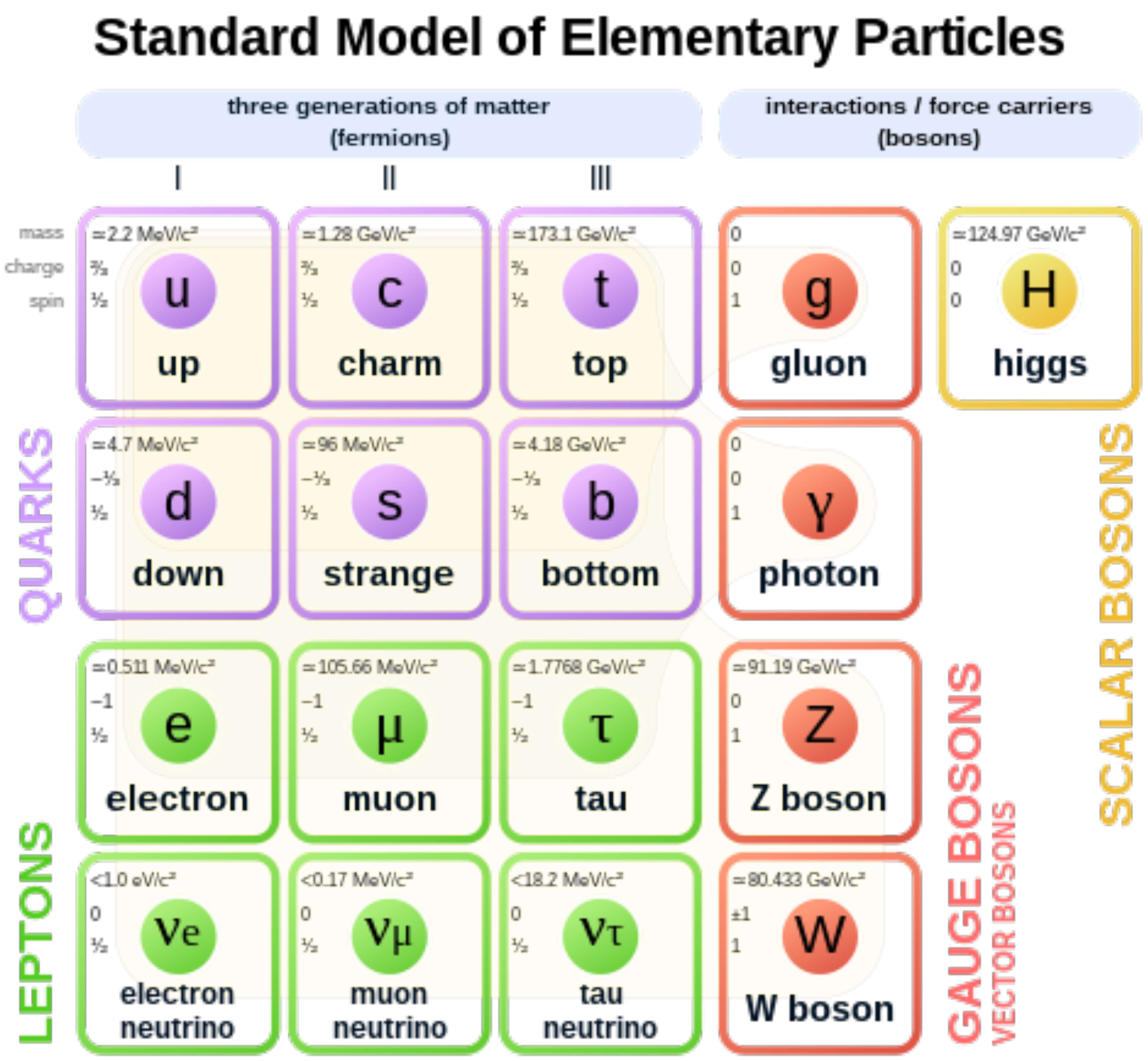
The standard model of elementary particles

- Particles interact via four forces of nature:
  - **strong interaction**, which binds the quarks together into hadrons;
  - **the electromagnetic interaction** between the charged leptons and quarks;
  - **the weak interaction** responsible for β decay; and
  - **gravity**.



The standard model of elementary particles

Quarks are constituents of strongly interacting hadronic matter. The size of quarks is below 10^{-18} m. In addition to quarks, there are leptons that interact weakly and electromagnetically. **With the resolution of the strongest microscopes (accelerators and storage rings), quarks and leptons appear to be point-like particles, having no internal structure.** Three different types of leptons are known: electrons, muons, and taus. Each charged lepton has a separate neutrino: ν_e, ν_μ, ν_τ.

Due to the precise investigations of the ***Z* particle, which is the neutral carrier of weak interactions**, it is known that **there are exactly three particle families** with light neutrinos (Fig. 2.1). This result was obtained from the measurement of the total Z decay width.

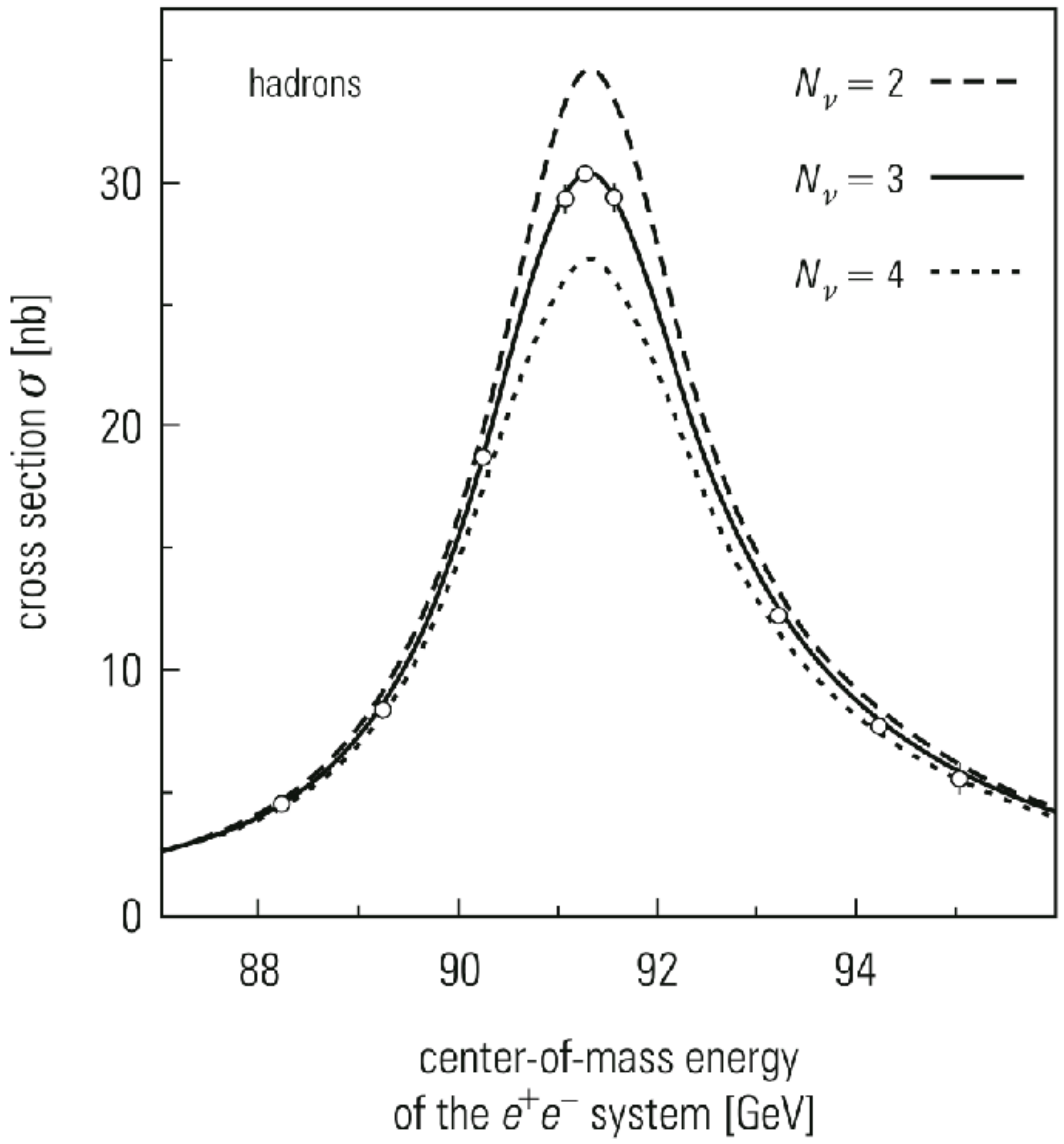


Fig. 2.1 Determination of the number of neutrino generations from Z decay

The standard model - generations

According to Heisenberg's uncertainty principle, the resolution of complementary quantities is intrinsically limited by Planck's constant ($h = 6.626\,0693 \times 10^{-34}$ J s). The relation between the complementary quantities of energy and time is

$$\Delta E\ \Delta t \geq \hbar/2 \ \ (\hbar = h/2\pi)\,.$$

If $\Delta t = \tau$ is the lifetime of the particle, the relation implies that the decay width $\Delta E = \Gamma$ is larger when τ is shorter. **If there are many generations of light neutrinos, the Z particle can decay into all these neutrinos,**

$$Z \to \nu_x + \bar{\nu}_x\,.$$

Remember the rules for decays and half life!

These decays can occur even if the charged leptons l_x associated with the respective generation are too heavy to be produced in Z decay. **A large number of different light neutrinos will consequently reduce the Z lifetime, thereby increasing its decay width**. The exact measurement of the Z decay width took place at the LEP storage ring (Large Electron–Positron Collider) in 1989, enabling the total number of neutrino generations to be determined: there are exactly three lepton generations with light neutrinos.

The standard model - generations

The **measurement of the primordial helium abundance** had already allowed physicists to derive a **limit for the number of neutrino generations**.

The **nucleosynthesis in the early universe was essentially determined by the number of relativistic particles, which were able to cool down** the universe after the Big Bang. At temperatures of $\approx 10^{10}$ K that correspond to energies, where nucleons start to bind in nuclei ($\approx$1 MeV), these relativistic particles would have consisted of **protons, neutrons, electrons, and neutrinos**.

If **many different neutrino flavours existed, a large amount of energy would have escaped from the original fireball**, owing to the low interaction probability of neutrinos. This has the consequence that the **temperature would have decreased quickly**. A rapidly falling temperature means that **the time taken for neutrons to reach nuclear binding energies would have been very short,** and consequently they would have had very **little time to decay** (lifetime τ_n= 885.7 s). If there were **many neutrons** that did not decay, they would have been able to **form helium** together with stable protons.

The primordial helium abundance is therefore an indicator of the number of neutrino generations. In 1990, the experimentally determined primordial helium abundance allowed physicists to conclude that the maximum number of different light neutrinos is at most four.

The standard model - generations

In addition, there are also **three quark generations,** which have a one-to-one cor- respondence with the three lepton generations:

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix} \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix} \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}$$

$$\begin{pmatrix} u \\ d \end{pmatrix} \quad \begin{pmatrix} c \\ s \end{pmatrix} \quad \begin{pmatrix} t \\ b \end{pmatrix}$$

Quarks have fractional electric charges (in units of the elementary charge). The different kinds of quarks (u, d; c, s; t, b) in the three respective generations (families) are characterized by a different **flavour.**
We only have upper limits on the masses of neutrinos, the limits obtained from direct measurements are given in Table 2.1. Neutrino oscillations, however, have shown that neutrinos actually do have a small mass.

Actually, in grand unified theories (GUTs) unifying electroweak and strong interactions, neutrinos are predicted to have small but non-zero masses.

Table 2.1 Periodic table of elementary particles: matter particles (fermions)

LEPTONs ℓ, spin $\frac{1}{2}\hbar$ (antileptons $\bar{\ell}$)						
Electr. charge (e)	1st generation		2nd generation		3rd generation	
	Flavour	Mass (GeV/c^2)	Flavour	Mass (GeV/c^2)	Flavour	Mass (GeV/c^2)
0	ν_e electron neutrino	$<2 \times 10^{-9}$	ν_μ muon neutrino	$<1.7 \times 10^{-4}$	ν_τ tau neutrino	<0.018
−1	e electron	5.11×10^{-4}	μ muon	0.106	τ tau	1.777
QUARKs q, spin $\frac{1}{2}\hbar$ (antiquarks $\bar{q}$)						
Electr. charge (e)	Flavour	$\simeq$ mass (GeV/c^2)	Flavour	$\simeq$ mass (GeV/c^2)	Flavour	$\simeq$ mass (GeV/c^2)
+2/3	u up	2×10^{-3} up to 8×10^{-3}	c charm	1.1 up to 1.7	t top	173
−1/3	d down	5×10^{-3} up to 15×10^{-3}	s strange	0.1 up to 0.3	b bottom	4.3

The standard model

Neutrino oscillation measurements, however, provide only values for the difference of squared masses like $\delta m^2 = m^2\nu_1 - m^2\nu_2$ for two neutrinos, respectively, with the masses $m\nu_1$ and $m\nu_2$.

Under plausible assumptions and the known limit for the electron neutrino mass one can suppose $m\nu <$ $2\text{eV}/c^2$ to be valid for all neutrino flavours. If the mass hierarchy in the neutrino sector were the same as for the charged leptons, the neutrino masses could be around 50 meV/c^2. Cosmological arguments based on the data from the Planck experiment suggest $\Sigma m\nu < 0.23\ \text{eV}/c^2$ for the sum of the three light neutrinos.

Only approximate values of masses for quarks can be given, because **free quarks do not exist** and the binding energies of quarks in hadrons can only be estimated roughly.

For each particle listed in Table 2.1 there exists an **antiparticle,** which is in all cases different from the original particle. This means that there are actually **12 fundamental leptons and an equal number of quarks**.

Bosons also have anti particles. The **neutral bosons are their own anti particles** e.g. γ, Z, Higgs etc.

The W^+ is the anti particle of the W^-.

The standard model

The interactions between elementary particles are governed by different forces. **There are four forces in total**, distinguished by **strong, electromagnetic, weak, and gravitational** interactions.

The electromagnetic and weak interactions are united in the **electroweak theory**.

The carriers of all the interactions are particles with integer spin (*bosons*), in contrast to the matter particles that all have half-integer spin (*fermions*).

The properties of these bosons are compiled in Table 2.2.

Table 2.2 Periodic table of elementary particles: carriers of the forces (bosons)

Electroweak interaction	γ	W^-	W^+	Z
Spin ($\hbar$)	1	1	1	1
Electr. charge (e)	0	-1	$+1$	0
Mass (GeV/c^2)	0	80.3	80.3	91.2

Strong interaction	Gluon g
Spin ($\hbar$)	1
Electr. charge (e)	0
Mass (GeV/c^2)	0

Gravitational interaction	Graviton G
Spin ($\hbar$)	2
Electr. charge (e)	0
Mass (GeV/c^2)	0

Note, there is no evidence for the graviton yet!

The standard model

Table 2.3 Properties of interactions

Interaction →	Gravitation	Electroweak interaction		Strong
Property ↓		Weak	Electromagnetic	
Acts on	Mass–energy	Flavour	Electric charge	Colour charge
Particles concerned	All	Quarks, leptons	All charged particles	Quarks, Gluons
Exchange particle	Graviton G	W^+, W^-, Z	γ	Gluons g
Relative strength	$\approx 10^{-40}$	10^{-5}	10^{-2}	1
Range	∞	$\approx 10^{-3}$ fm	∞	≈ 1 fm
Example	System Earth–Moon	β decay	Atomic binding	Nuclear binding

The standard model

While the existence of **the gauge bosons of electroweak interactions and the gluon of strong interactions are well established,** the *graviton*, the carrier of the gravitational force, has not yet been discovered.

The properties of interactions are compared in Table 2.3. It is apparent that **gravitation can be completely neglected in the microscopic domain,** because its strength in relation to strong interactions is only 10^{-40}.

In the primitive quark model, all strongly interacting particles **(hadrons) are composed of valence quarks.**

A baryon is a three-quark system, whereas a meson consists of a quark and an antiquark.

Examples of baryons include the **proton, which is an *uud*** system, and the **neutron is an *udd*** composite. Correspondingly, an example of a meson is the positively charged pion, which is an $u\bar{d}$ system.

The standard model

The existence of **baryons consisting of three identical quarks with parallel spin** ($\Omega^- = (sss)$, spin $\frac{2}{3}\hbar$) indicates that **quarks must have an additional quantum number**, otherwise the Pauli exclusion principle would be violated. This hidden quantum number is **called *colour***.

Electron–positron interactions show that there are exactly **three different colours**.

Each quark therefore comes in three colours, however, **the observed hadrons are colour-neutral**, i.e., they are colourless.

If the three degrees of freedom in colour are denoted by **red (*r*), green (*g*), and blue (*b*)**, the proton is a composite object made up from $u_{\text{red}}\, u_{\text{green}}\, d_{\text{blue}}$.

In addition to valence quarks, there is also a sea of virtual quark–antiquark pairs in hadrons.

The standard model

The quarks that form hadrons are held together by the **exchange of gluons**.

Since gluons mediate the interactions between quarks, they must **possess two colours: they carry a colour and an anticolour.**

Since **there are three colours and anticolours each**, one would expect that $3 \times 3 = 9$ gluons exist. Quantum chromodynamics, however, is only mediated by by **eight gluons**. A singlet consisting of a colour-neutral mixed state of *all* colours and anticolours, $\frac{1}{\sqrt{3}}(r\bar{r} + g\bar{g} + b\bar{b})$, does not carry colours and does not exist. The eight states of gluons are: $r\bar{g}$, $r\bar{b}$, $g\bar{b}$, $g\bar{r}$, $b\bar{r}$, $b\bar{g}$, $\frac{1}{\sqrt{2}}(r\bar{r} - g\bar{g})$.

Nucleons in a nucleus are bound together by the residual interaction of gluons, in very much the same way as molecular binding is a result of the residual interactions of electric forces.

Remember the **Feynman diagrams**

The standard model

In a very simplified picture, the gluon radiation of a quark can be illustrated by the diagram shown in Fig. 2.2.

Note this is also the fundamental vertex for the strong interaction

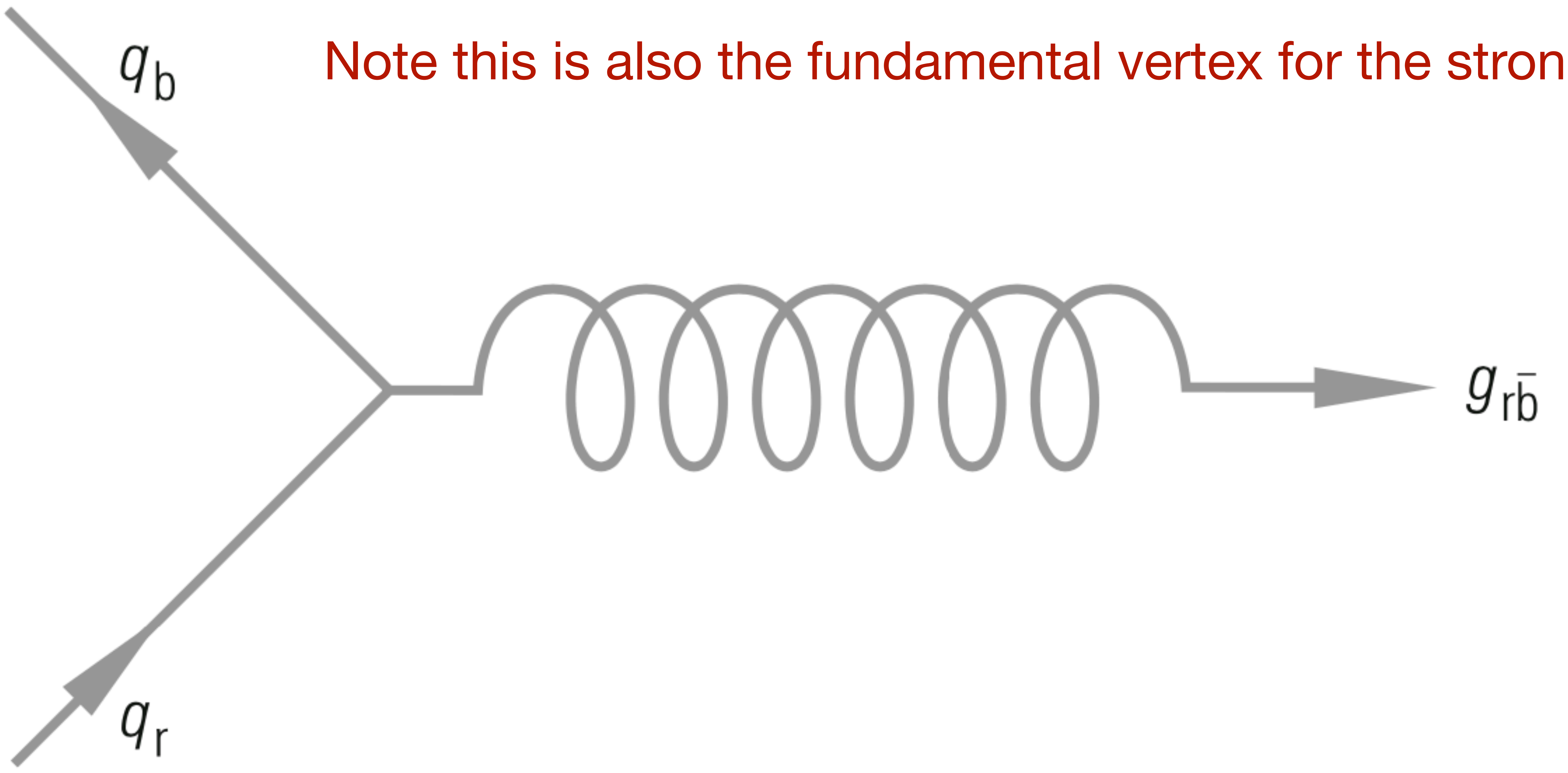


Fig. 2.2 Creation of coloured gluons by quarks

The standard model

The Standard Model of elementary particles has been supplemented by the **discovery of the Higgs particle** in 2012. The mass of the Higgs boson is around 125GeV/c^2.

In the framework of the Brout–Englert– Higgs mechanism it is possible to **assign masses to the fundamental fermions** by spontaneous symmetry breaking.

Figure 2.3 shows the production of a Higgs boson by gluon fusion and the decay of the Higgs particle into two energetic photons. This production and decay mechanism played a leading role in the discovery of the Higgs.

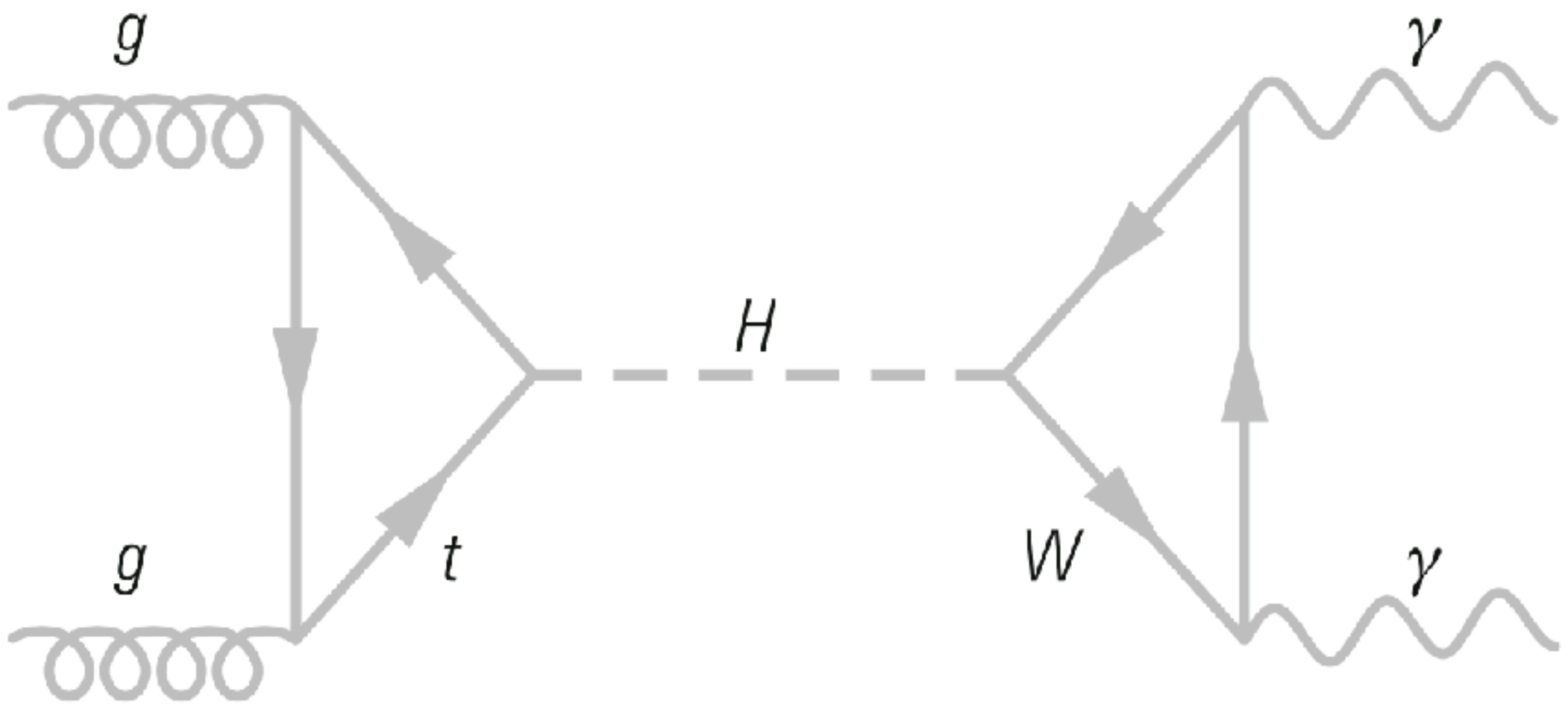


Fig. 2.3 Production of a Higgs particle by gluon fusion and subsequent decay into two photons. The gluons are provided by the colliding beam protons

The standard model

Our understanding of the Standard Model is still not complete.

This model still has **25 free parameters**, which need to be adjusted by experiments.

These parameters are:

- 12 values for the fermion masses (6 quarks, 6 leptons),
- 3 mixing angles and
- a phase of the Cabibbo–Kobayashi–Maskawa matrix,
- 3 mixing angles and
- a phase (if the neutrinos are Dirac particles) of the Pontecorvo–Maki–Nakagawa–Sakata matrix,
- 2 parameters for the Higgs mass and the vacuum expectation value of the Higgs field, and, finally,

3 couplings for the interactions:

- the fine-structure constant α,
- the coupling of the strong interaction α_s, and
- the coupling constant of the electroweak interaction.

The standard model

The **Cabibbo–Kobayashi–Maskawa matrix,** (CKM matrix, quark mixing matrix, or KM matrix) is a unitary matrix which contains information on the s**trength of the flavour-changing weak interaction**.

The **Pontecorvo–Maki–Nakagawa–Sakata matrix** (PMNS matrix, Maki–Nakagawa–Sakata matrix (MNS matrix), lepton mixing matrix, or **neutrino mixing matrix)** is a unitary mixing matrix which contains information on the mismatch of quantum states of neutrinos when they propagate freely and when they take part in weak interactions. It is a **model of neutrino oscillation.**

A **Dirac fermion** (Dirac particle) is a spin-½ particle (a fermion) which is different from its antiparticle. A vast majority of fermions fall under this category. The counterpart to a Dirac fermion is a **Majorana fermion,** a particle that must be its own antiparticle. We do not know if the neutrinos are Dirac or Majorana particles.

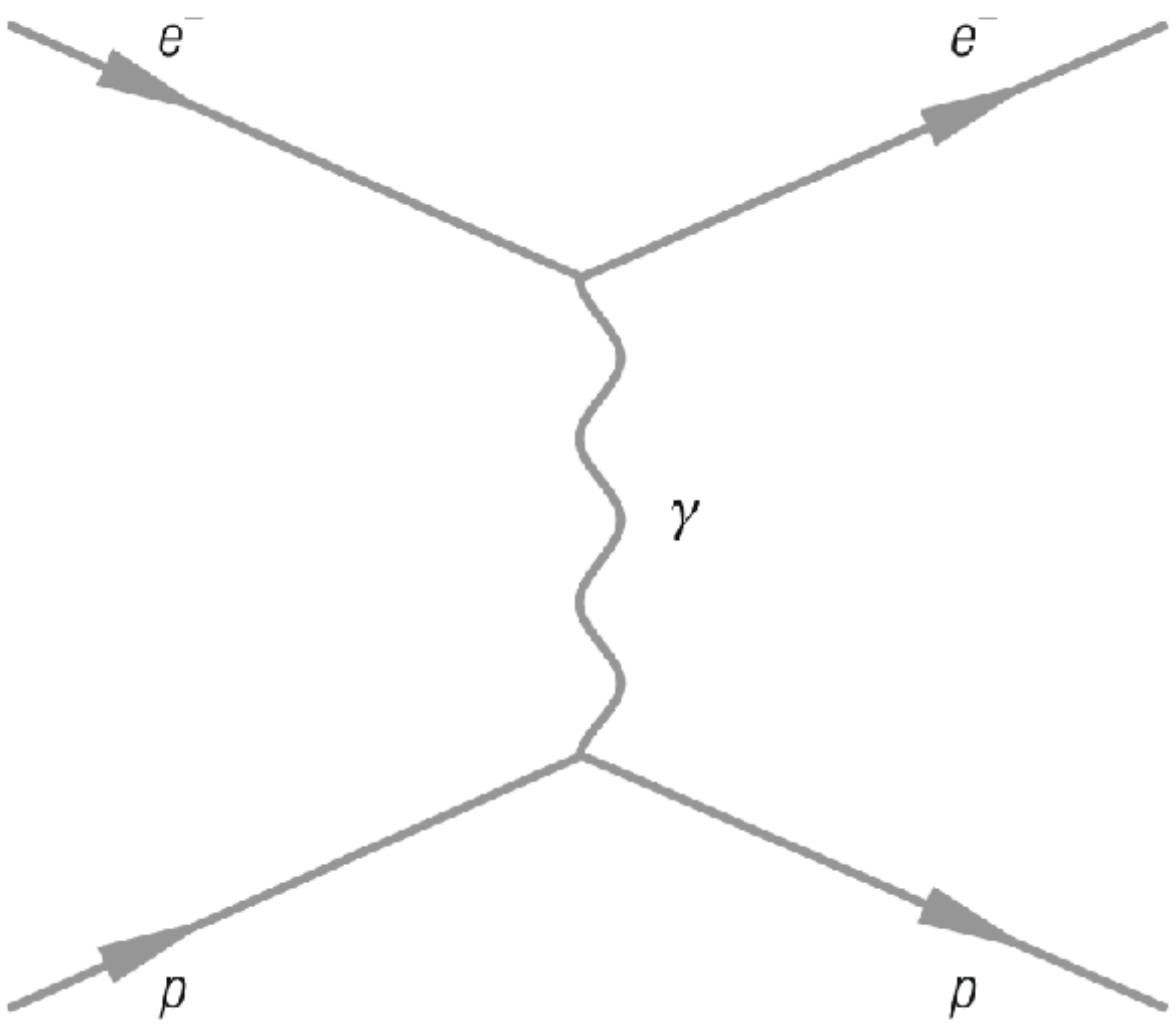
Note: The photons and other neutral bosons are not Majorana particles.

Examples of interaction processes

Interactions of elementary particles can be **graphically represented by Feynman diagrams, which present a short-hand for the determination of cross sections.**

Rutherford scattering of electrons on protons is mediated by photons (Fig. 2.4).

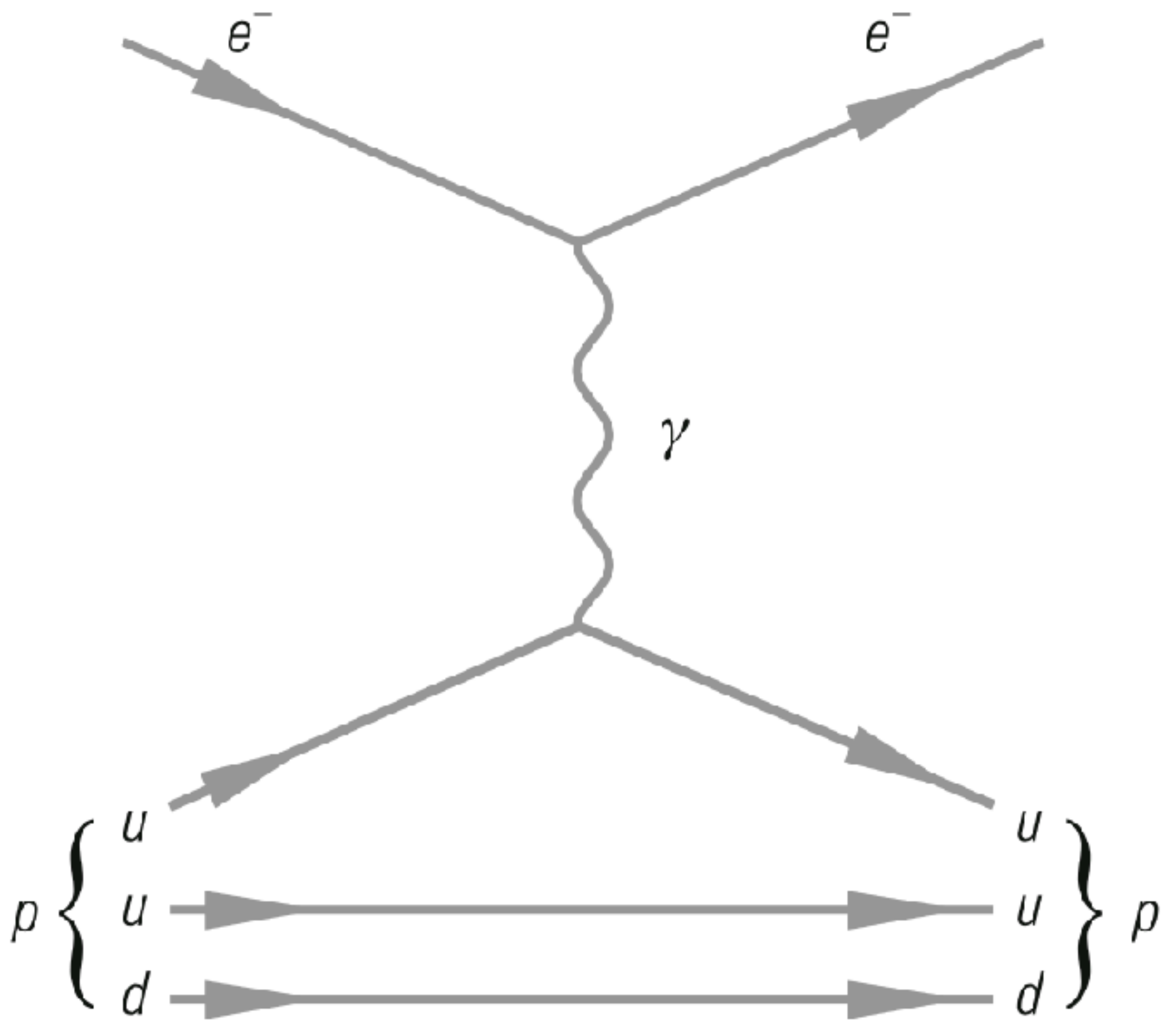
Fig. 2.4 Rutherford scattering of electrons on protons



Examples of interaction processes

At high energies, however, the photon does not interact with the proton as a whole, but rather only with one of its constituent quarks (Fig. 2.5). The other quarks of the nucleon participate in the interaction only as spectators. As **photons are electrically neutral particles, they cannot change the nature of a target particle in an interaction. -> flavour does not change**

Fig. 2.5 Rutherford scattering as photon–quark scattering process



Examples of interaction processes

In **weak interactions, however, there are charged bosons that can cause an interchange between particles within a family.** As an example, Fig. 2.6 shows the scattering of an electron neutrino on a neutron via a charged-current (W^+, W^- exchange) reaction.

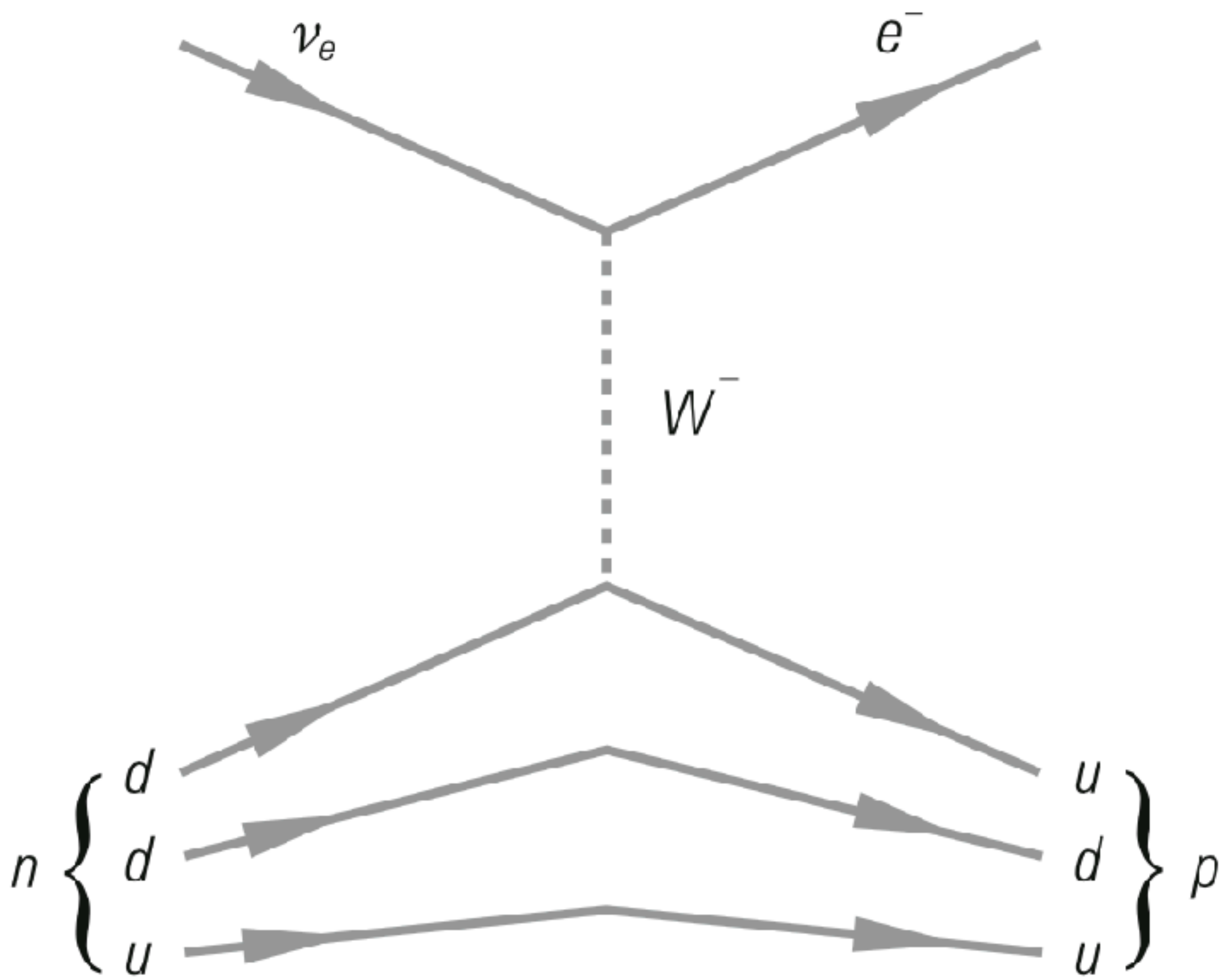


Fig. 2.6 Neutrino–nucleon scattering via a charged current

Which decay is this?

Examples of interaction processes

In a neutral-current (neutral-weak) interaction (*Z* exchange), the neutrino would not alter its nature when scattered off the neutron. If electron neutrinos are scattered on electrons, charged and neutral currents can contribute. For the scattering of muon or tau neutrinos on electrons only neutral currents contribute, because ν_μ and ν_τ are not members of the electron family (Fig. 2.7).

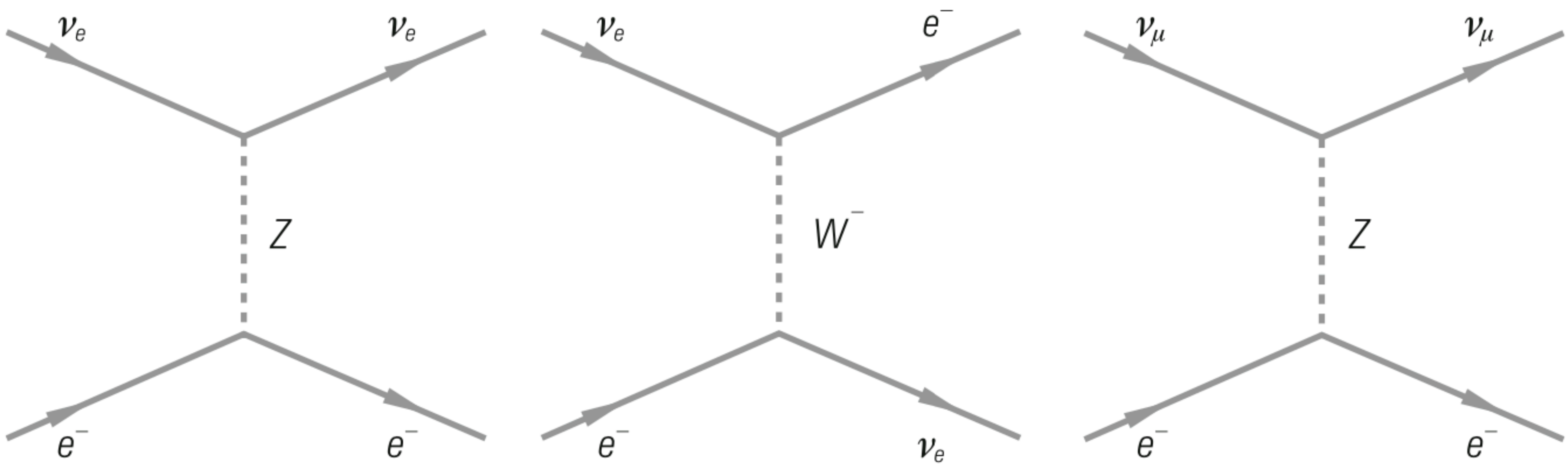


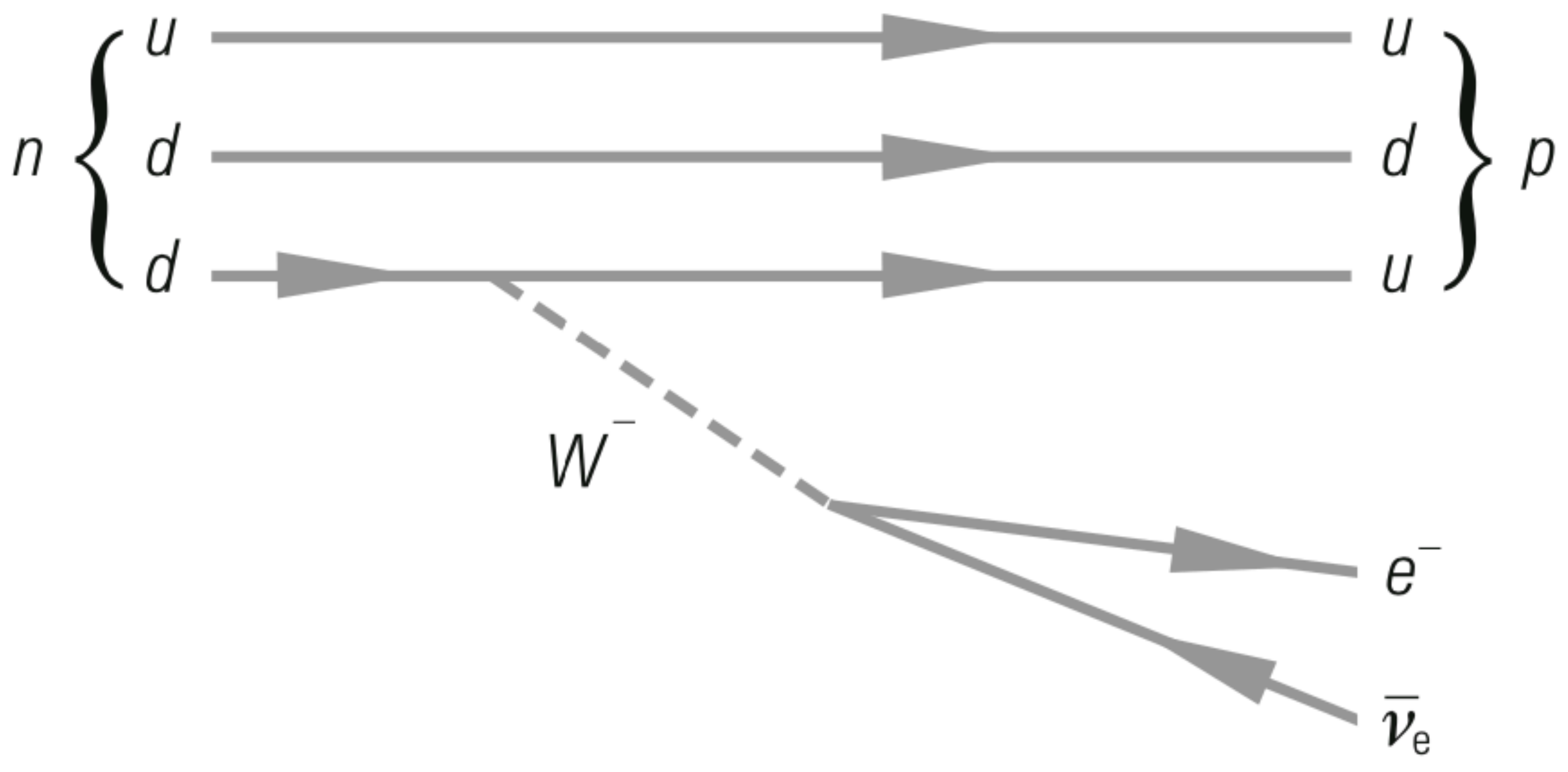
Fig. 2.7 Scattering possibilities of neutrinos on electrons

Examples of interaction processes

Decays of elementary particles can be described in a similar way. Nuclear beta decay of the neutron $n \to p + e^- + \bar{\nu}_e$ is mediated by a weak charged current (Fig. 2.8), where a d quark in the neutron is transformed into a u quark by the emission of a virtual W^-. The W^- immediately decays into members of the first lepton family ($W^- \to e^- \bar{\nu}_e$).

In principle, the W^- could also decay according to $W^- \to \mu^- \bar{\nu}_\mu$ or $W^- \to u\bar{d}$, but this is kinematically not allowed.

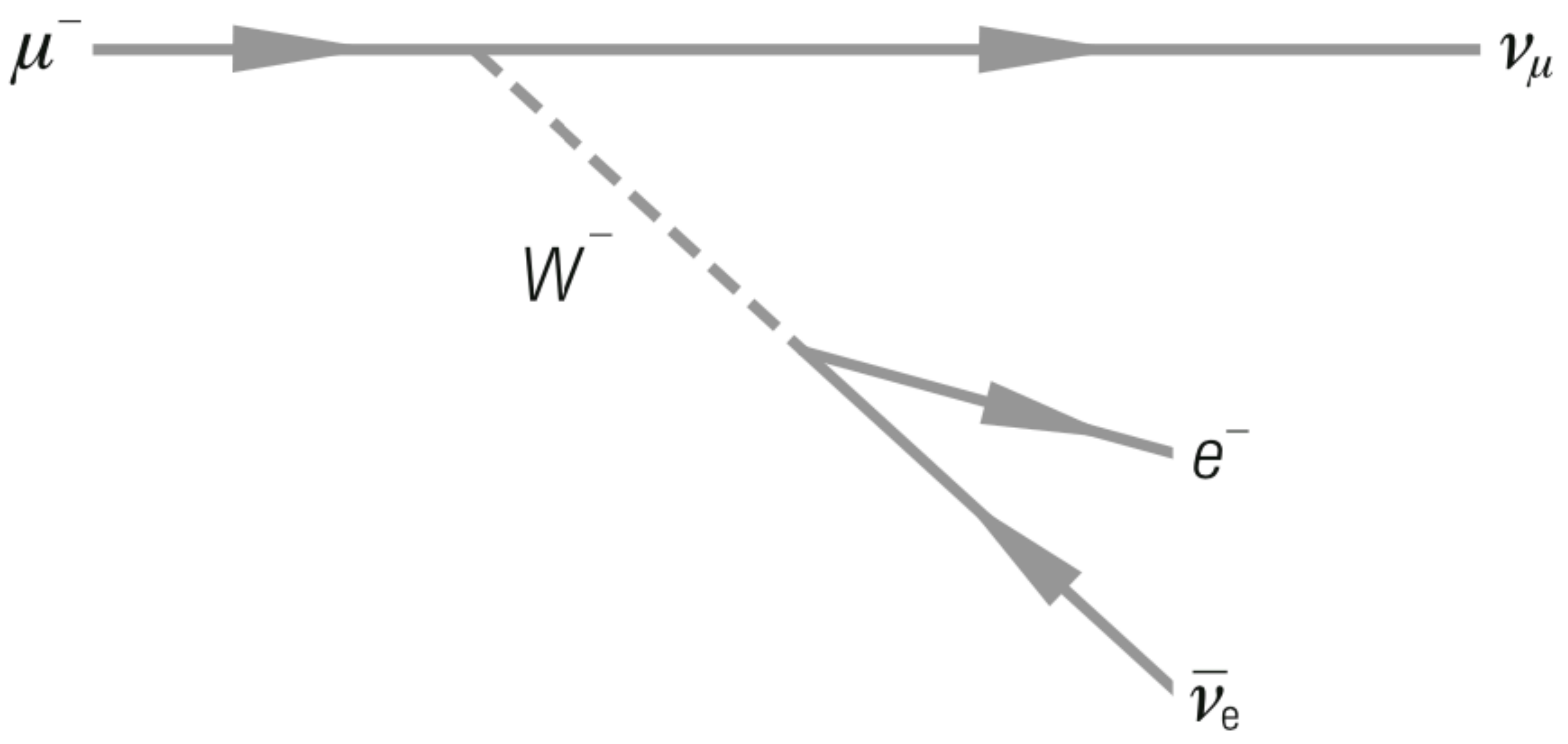
Fig. 2.8 Neutron decay



Examples of interaction processes

Muon decay can be described in a similar fashion (Fig. 2.9). The muon transfers its charge to a W^-, thereby transforming itself into the neutral lepton of the second family, the ν_μ. The W^- in turn decays again into in $e^- \bar{\nu}_e$.

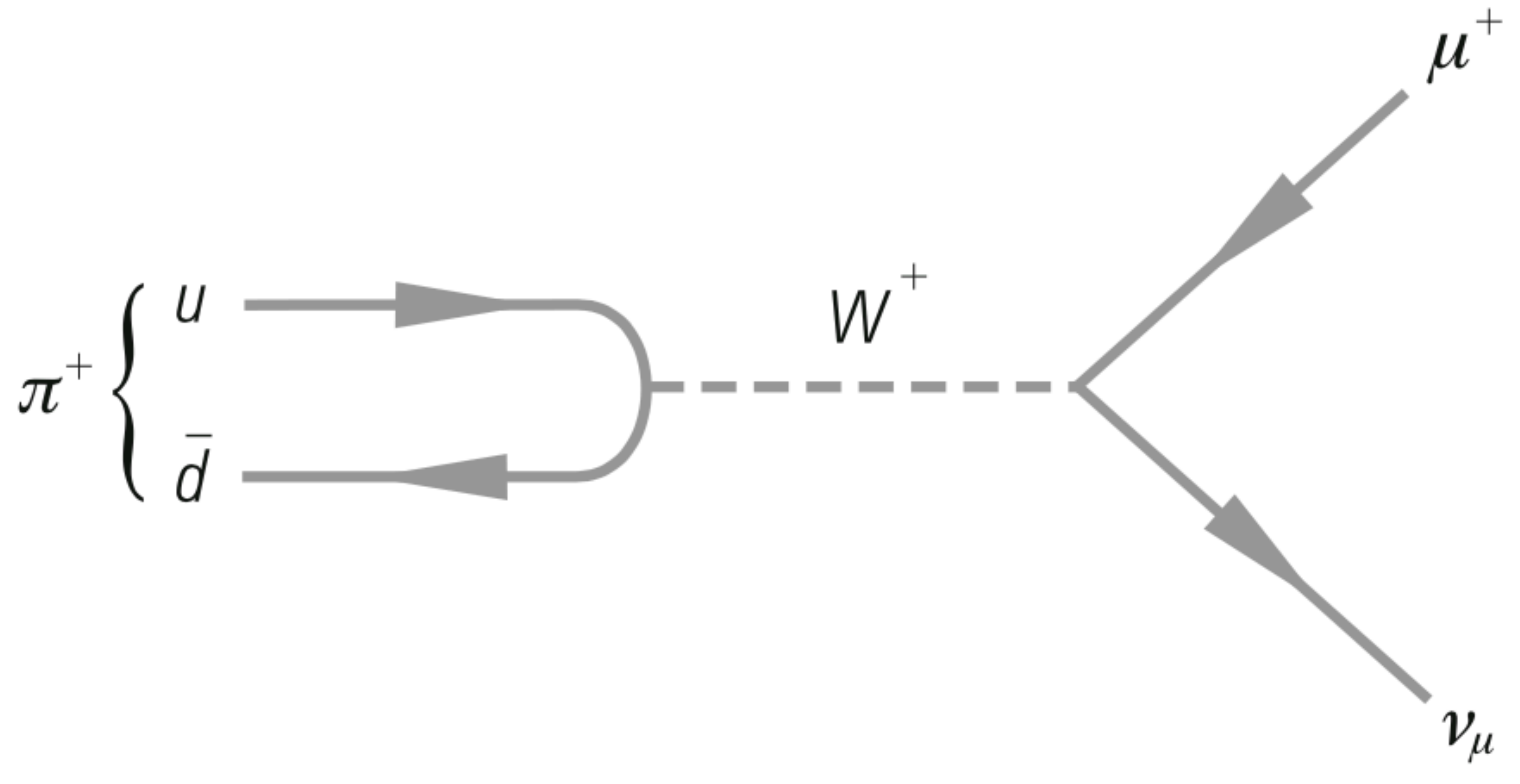
Fig. 2.9 Muon decay



Examples of interaction processes

Finally, pion decay will be discussed (Fig. 2.10). In principle, the W^+ can also decay in this case, into an e^+ ν_e state. Helicity reasons, however, strongly suppress this decay: **as a spin-0 particle, the pion decays into two leptons that must have antiparallel spins** due to **angular-momentum conservation.**

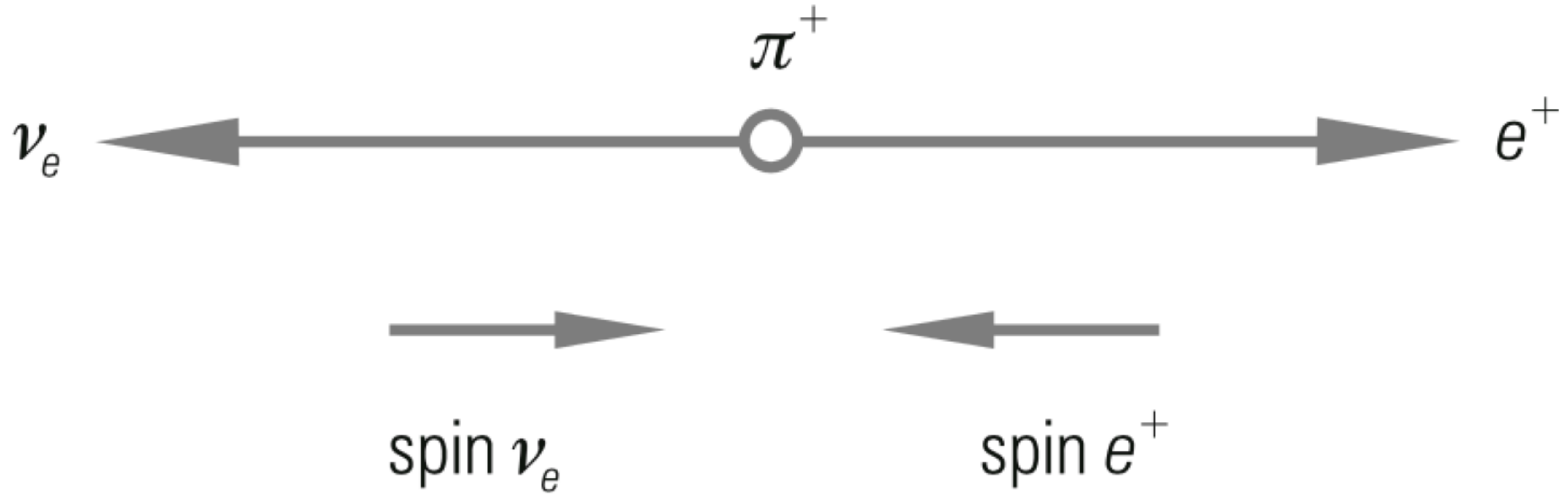
Fig. 2.10 Pion decay



Examples of interaction processes

The *helicity* is the projection of the spin onto the momentum vector, and it is fixed for the neutrino (for massless particles the spin is either parallel or antiparallel to the momentum). Particles normally carry negative helicity (spin $\parallel -\boldsymbol{p}$, left-handed) so that the positron, as an antiparticle (spin $\parallel \boldsymbol{p}$, right-handed), must take on an unnatural helicity (Fig. 2.11). The probability of carrying an abnormal helicity is proportional to $1 - \frac{v}{c}$ (where v is velocity of the charged lepton). Owing to the relatively high mass of the muon ($m_\mu \gg m_e$), it takes on a much smaller velocity compared to the electron in pion decay, i.e., $v(\mu) \ll v(e)$. The consequence of this is that the probability for the decay muon to take on an unnatural helicity is much larger compared to the positron. For this reason, the $\pi^+ \to e^+ \nu_e$ decay is strongly suppressed compared to the $\pi^+ \to \mu^+ \nu_\mu$ decay (the suppression factor is 1.23×10^{-4}).

Fig. 2.11 Helicity conservation in π^+ decay



Examples of interaction processes

The neutrino is always left handed and the anti neutrinos are always right handed. -> violates parity symmetry.

Example: $\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$

The π^- has spin 0, which means that the spin of the μ and the $\bar{\nu}_\mu$ must be oppositely aligned, which means that if the anti neutrino is right handed then the muon must be right handed to (in the pion rest frame). This is also supported by experiments.

Example: $\pi^0 \rightarrow \gamma + \gamma$ photons can be both left handed and right handed. In an individual decay the photons are either left or right handed, but this is an electromagnetic decay, which does not break parity symmetry.

Quantum numbers and symmetries

The various elementary particles are characterized by quantum numbers.

In addition to the **electric charge**, the membership of a quark generation (**quark flavour**) or lepton generation (**lepton number**) is introduced as a quantum number.

Leptons are assigned the **lepton number +1** in their respective generation, whereas **antileptons are given the lepton number –1**. Lepton numbers for the different lepton families (Le, $L\mu$, $L\tau$) are separately conserved, as is shown in the example of the muon decay:

$$
\begin{array}{lccccccc}
 & \mu^- & \rightarrow & \nu_\mu & + & e^- & + & \bar{\nu}_e \\
L_\mu & 1 & & 1 & & 0 & & 0 \\
L_e & 0 & & 0 & & 1 & & -1
\end{array}
$$

Quantum numbers and symmetries

Some particles, like **kaons**, exhibit a very strange behaviour. They are produced copiously, but decay relatively slowly. These particles are **produced in strong interactions, but they decay via weak interactions.**

This property is accounted for by introducing the **quantum number *strangeness*,** which is **conserved in strong interactions, but violated in weak decays**. Owing to the conservation of strangeness in strong interactions, **only the combined production of** hadrons, one of which contains **a strange and** the other an **anti-strange quark, is possible,** such as

$$\pi^- + p \to K^+ + \Sigma^-$$

In this process, the $\bar{s}$ quark in the $K+$ (= $u\bar{s}$) receives the strangeness +1, whilst the s quark in the Σ^- (= dds) is assigned the strangeness −1.

In the weak decay of the $K+ \to \pi^+\pi^0$, the strangeness is violated, since pions do not contain strange quarks (s).

Quantum numbers and symmetries

The parity transformation *P* is the space inversion of a physical state. Parity is conserved in strong and electromagnetic interactions, however, **in weak interactions it is maximally violated**. This means that the **mirror state of a weak process does not correspond to a physical reality**. Nature distinguishes between right and left in weak interactions.

The operation of **charge conjugation *C*** applied to a physical state **changes all the charges**, meaning that **particles and antiparticles are interchanged**, whilst leaving quantities like momentum or spin untouched. **Charge conjugation is also violated in weak interactions.**

In **β decay**, for example, left-handed electrons (negative helicity) and right-handed positrons (positive helicity) are favoured. Even though **the symmetry operations *P* and *C* are not conserved individually, their combination *CP***, which is the application of space inversion (parity operation *P*) with subsequent interchange of particles and antiparticles (charge conjugation *C*) **is a well-respected symmetry**. This symmetry, however, is still **broken in certain decays (K^0 and B^0 decays)**, but it is a common belief that the ***CPT* symmetry** (*CP* symmetry with additional time reversal) **is conserved under all circumstances.**

Why are symmetries important?

CP-symmetry

The parity symmetry -> the equations of particle physics are invariant under mirror inversion. The mirror image of a reaction occurs at the same rate as the original reaction. (Not true for the weak interactions)

CP violation is a violation of CP-symmetry (or charge conjugation parity symmetry): the combination of C-symmetry (charge conjugation symmetry) and P-symmetry (parity symmetry).

CP-symmetry states that the laws of physics should be the same if a particle is interchanged with its antiparticle (C-symmetry) while its spatial coordinates are inverted ("mirror" or P-symmetry).

CP violation meant researchers could distinguish matter from antimatter and create a solution that would explain the existence of the universe as one that is filled with matter. The lack of symmetry gave the possibility of matter-antimatter imbalance.

Examples: C symmetry is violated by the neutrinos, because there are no left handed anti neutrinos.

K^0 spontaneously turning into $\bar{K}^0$ with different decay products and different mean lifetimes.

$K_L \rightarrow \pi^+ + e^- + \bar{\nu}_e$

$K_L \rightarrow \pi^- + e^+ + \bar{\nu}_e$ these two should occur at the same rate but the second one is slightly more common.

The tau-theta puzzle

The **tau-theta puzzle**. The τ and the θ mesons are identical in all their properties, mass, spin, charge etc… However, they decayed into different particles

$\theta^+ \rightarrow \pi^+ + \pi^0$ (P = +1)

$\tau^+ \rightarrow \pi^+ + \pi^0 + \pi^0$ (P = -1)

$\tau^+ \rightarrow \pi^+ + \pi^+ + \pi^-$ (P = -1)

One of these decays conserves the parity and the other does not.

Turns out that the τ and the θ mesons are the same particle the K^+ and the weak interaction does not conserve parity.

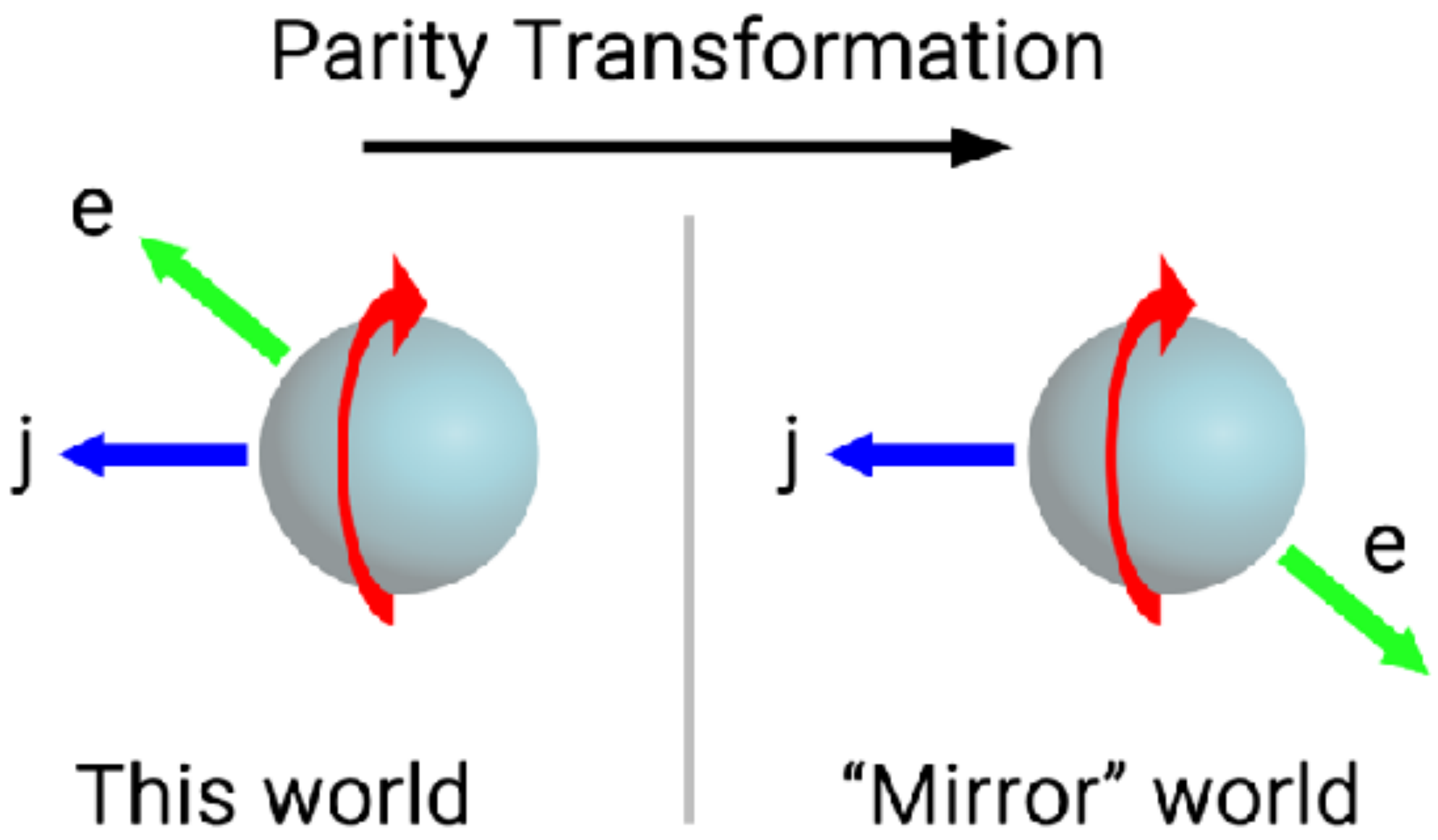
The different behaviour is because the K^0 mixing with $\bar{K}^0$ which results in 2 states K_1 ($K_S \sim \theta^+$) and K_2 ($K_L \sim \tau^+$) which decay differently.

The Wu experiment

Observing **beta decay** and the direction at which e^- exit the sample. Conclusion: the electrons prefer one direction over the other! This means that in a “mirror world” the emission direction would be opposite and the two worlds would be distinguishable. This breaks parity symmetry.

The quark has a left part and a right part. As it walks across the spacetime, it oscillates back and forth from right part to left part and from left part to right part. From analyzing the Wu experiment's demonstration of parity violation, it can be deduced that **only the left part of down quarks decay and the weak interaction involves only the left part of quarks and leptons** (or the right part of antiquarks and antileptons). **The right part of the particle simply does not feel the weak interaction.** If the down quark did not have mass it would not oscillate, and its right part would be quite stable by itself. Yet, **because the down quark is massive, it oscillates and decays.**

Chien-Shiung Wu, after whom the Wu experiment is named, designed the experiment and led the team that carried out the test of the conservation of parity in 1956.



CPT-symmetry

CPT symmetry: Besides C and P, there is a third operation, **time reversal T, which corresponds to reversal of motion.** Invariance under time reversal implies that whenever a motion is allowed by the laws of physics, the reversed motion is also an allowed one and occurs at the same rate forwards and backwards.

The combination of CPT is thought to constitute an exact symmetry of all types of fundamental interactions. Because of the long-held CPT symmetry theorem, provided that it is valid, **a violation of the CP-symmetry is equivalent to a violation of the T-symmetry**. In this theorem, regarded as one of the basic principles of quantum field theory, charge conjugation, parity, and time reversal are applied together.

If CP is violated, then time reversal should be violated to.

This means testing if reactions could occur backwards at the same rate as the forward reactions. To date, experiments confirm T reversal works for electromagnetic and weak interactions and there is no experimental evidence for violation for the weak interactions. However, testing reverse weak interaction is extremely difficult.

In the macroscopic world time reversal is often broken. E.g. 2nd law of thermodynamics, black holes etc.

More on symmetries: Griffiths: Introduction to Elementary Particles Chapter 4

Quantum numbers and symmetries

In nuclear physics and particle physics, **isospin (I) is a quantum number related to the up- and down quark content of the particle.** More specifically, isospin symmetry is a **subset of the flavour symmetry** seen more broadly in the interactions of baryons and mesons.

The name of the concept contains the term spin because its quantum mechanical description is mathematically similar to that of angular momentum (in particular, in the way it couples; for example, a proton–neutron pair can be coupled either in a state of total isospin 1 or in one of 0). But unlike angular momentum, **it is a dimensionless quantity and is not actually any type of spin.**

Flavour symmetry: If there are two or more particles which have identical interactions, then they may be interchanged without affecting the physics. In quantum chromodynamics, flavour is a conserved global symmetry. In the electroweak theory, on the other hand, this symmetry is broken, and flavour changing processes exist, such as quark decay or neutrino oscillations.

Quantum numbers and symmetries

Isospin (I) is defined as a vector quantity in which up and down quarks have a value of I = 1/2, with the 3rd-component **(I_3) being +1/2 for up quarks, and –1/2 for down quarks**, while all **other quarks have I = 0**. Leptons a bosons also have **I = 0. Isospin is only relevant for the strong interaction.** Therefore, for hadrons in general,

$I_3 = \frac{1}{2}(n_u - n_d)$ where n_u and n_d are the numbers of up and down quarks respectively.

In any combination of quarks **I_3 could either be aligned between a pair of quarks, or face the opposite direction,** giving different possible values for total isospin for any combination of quark flavours. **Hadrons with the same quark content but different total isospin can be distinguished experimentally**, verifying that **flavour is actually a vector quantity, not a scalar** (up vs down simply being a projection in the quantum mechanical z axis of flavour space).

Quantum numbers and symmetries

For example, a strange quark can be combined with an up and a down quark to form a baryon, but there are two different ways the isospin values can combine – either adding (due to being flavour-aligned) or cancelling out (due to being in opposite flavour directions), $\Sigma^0(\mathrm{uds}, \mathrm{I} = 1)$ and $\Lambda^0(\mathrm{uds}, \mathrm{I} = 0)$.

Examples: $n^0(\mathrm{udd}, \mathrm{I} = 1/2)$ is the same as Δ^0 $(\mathrm{I} = 3/2)$, but the Δ^0 is heavier and can decay into different particles. $p^+(\mathrm{uud})$ is the same as Δ^+

The Δ^0 and Δ^+ are **higher-mass spin-excitations** of the n^0 and p^+

Quantum numbers and symmetries

Certain particles that **behave in an identical way under strong interactions**, **but differ in their charge state,** are integrated into **isospin multiplets**.

Protons and neutrons are nucleons that form an **isospin doublet of $I = 1/2$.**

When the nucleon isospin is projected onto the z axis, the state with $I_z = +1/2$ corresponds to a proton, whereas the $I_z = -1/2$ state relates to the neutron.

The three pions (π^+, π^-, π^0) combine to form an **isospin triplet with $I = 1$.**

π^-: $I_z = -1$

π^+: $I_z = +1$

π^0: $I_z = 0$

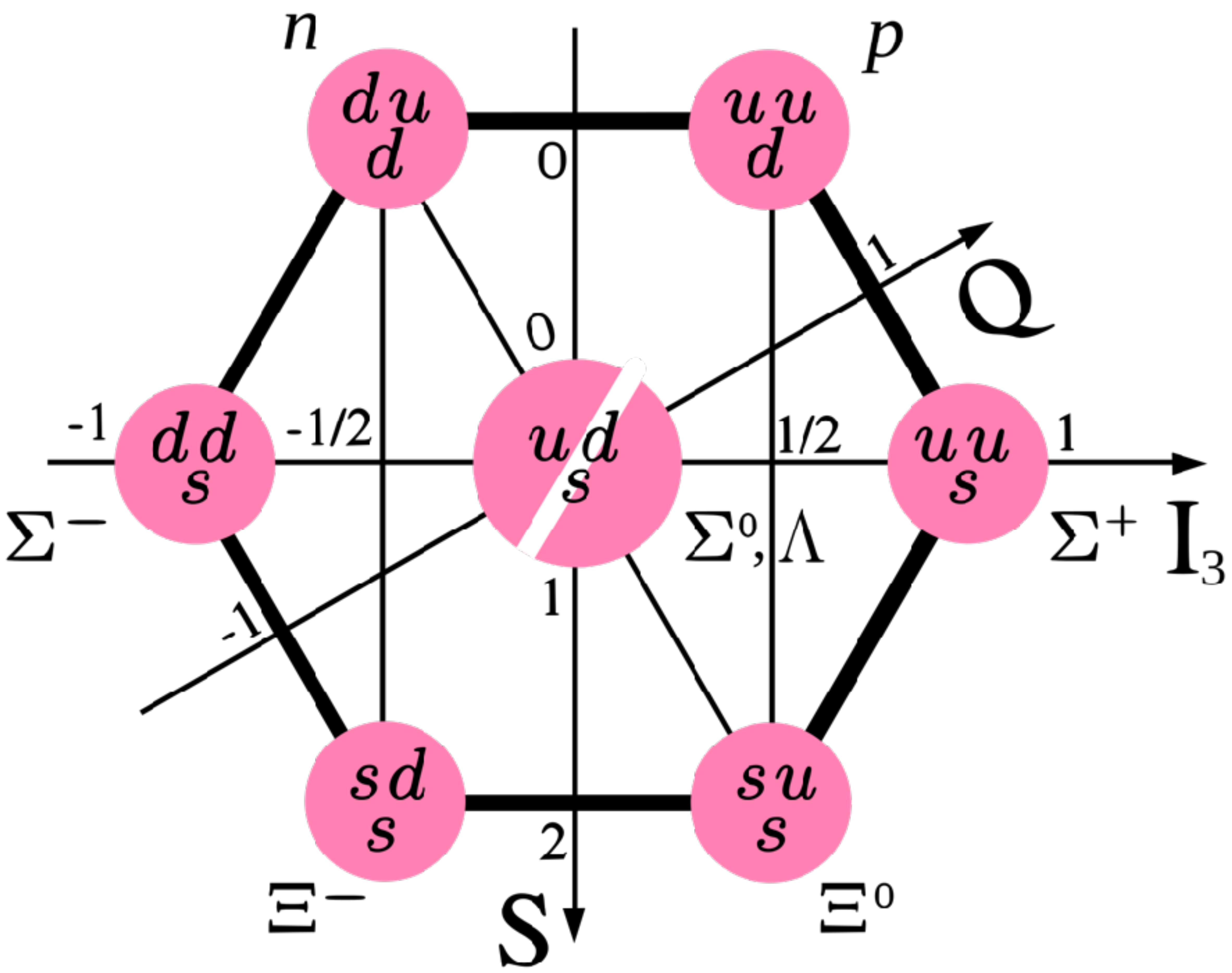
Λ^0 $\mathrm{I} = 0$, $I_z = 0$

The **particle multiplicity *m*** in an isospin multiplet is related to the isospin via the equation

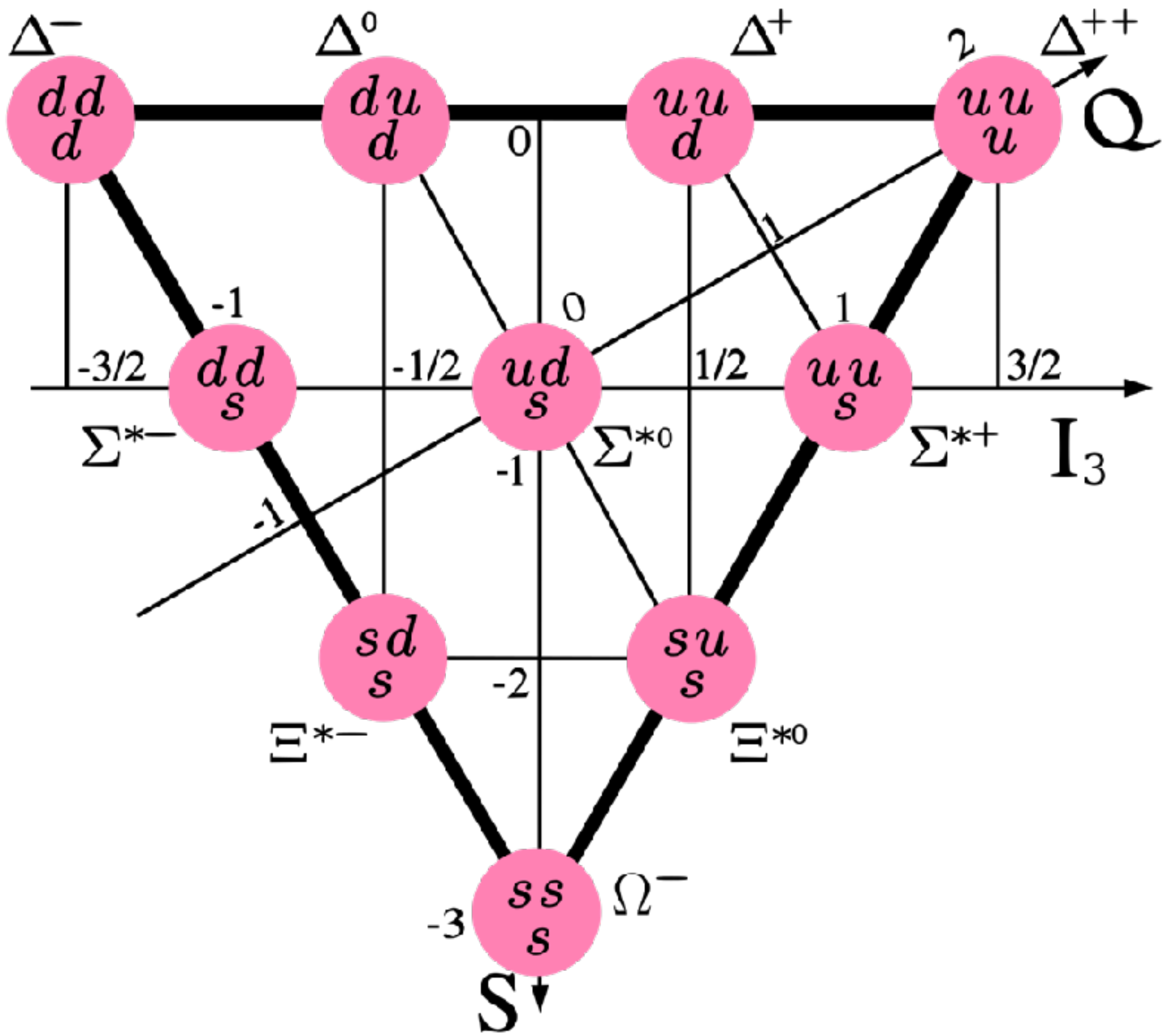
$$m = 2I + 1$$

Quantum numbers and symmetries

Combinations of three u, d or s-quarks forming baryons with spin-1⁄2 form the **baryon octet**



Combinations of three u, d or s-quarks forming baryons with spin-3⁄2 form the **baryon decuplet.**



Quantum numbers and symmetries

Finally, the ***baryon number:*** Quarks are assigned the baryon number $1/3$, and antiquarks are given $-1/3$. All baryons consisting of three quarks are therefore assigned the baryon number 1, whereas all other particles get the baryon number 0.

The properties of the **conservation laws for the different interaction** types in elementary particle physics are compiled in Table 2.4.

Table 2.4 Conservation laws of particle physics (conserved: +; violated: –)

Physics quantity	Interaction		
	Strong	Electromagnetic	Weak
Momentum	+	+	+
Energy (incl. mass)	+	+	+
Angular momentum	+	+	+
Electric charge	+	+	+
Quark flavour	+	+	–
Lepton number	./.	+	+
Parity	+	+	–
Charge conjugation	+	+	–
Strangeness	+	+	–
Isospin	+	–	–
Baryon number	+	+	+

Quantum numbers and symmetries

As can be seen from Table 2.1, **there is a complete symmetry between leptons and quarks. Leptons, however, participate in interactions as free particles, whereas quarks do not.** Due to quark confinement, spectator quarks always participate in the interactions in some way.

For **charged leptons, there is a strict law of lepton-number conservation:** The members of **different generations do not mix** with each other.

For the quarks, it was seen that weak processes can **change the strangeness**. In Λ decay, the s quark belonging to the second generation can transform into a u quark of the first generation. This would otherwise only be allowed to happen to a d quark (Fig. 2.12).

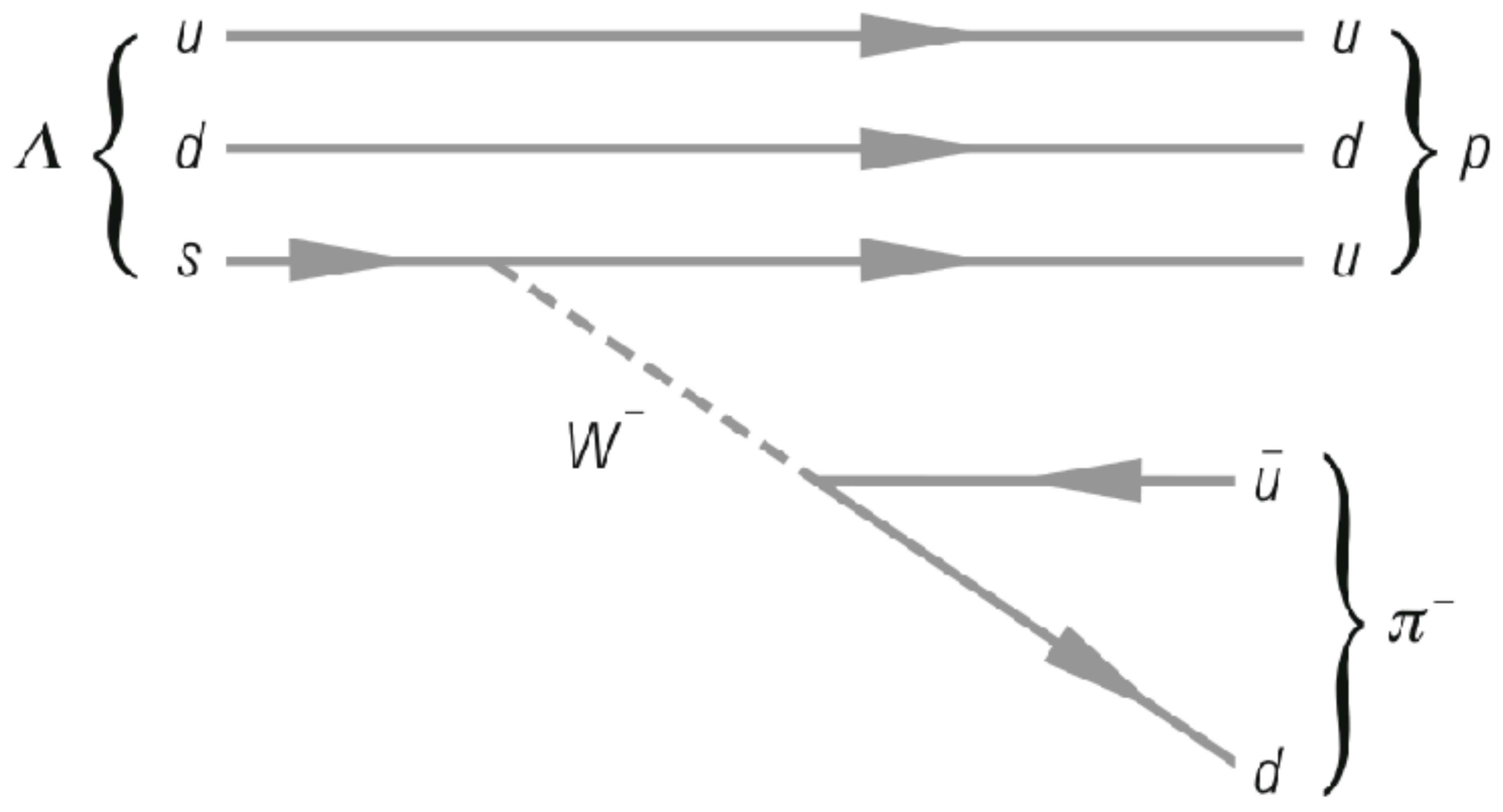


Fig. 2.12 Lambda decay: $\Lambda \to p + \pi^-$

Quantum numbers and symmetries

It appears as if the *s* **quark can sometimes behave like the *d* quark**. It is, in fact, the d' and s' quarks that couple to weak interactions, rather than the d and s quarks. The d' and s' quarks can be described as a rotation with respect to the d and s quarks. This rotation is expressed by

$$\begin{aligned} d' &= d \cos\theta_C + s \sin\theta_C \, , \\ s' &= -d \sin\theta_C + s \cos\theta_C \, , \end{aligned}$$

where θ_C is the **mixing angle** (Cabibbo angle).

The reason that angles are used for weighting is based on the fact that the sum of the squares of the weighting factors, $\cos^2\theta + \sin^2\theta = 1$, automatically guarantees the correct normalization. θ_C has been experimentally obtained to be approximately 13° ($\sin\theta_C \approx 0.2255$). Since $\cos\theta_C \approx 0.9742$, the **$d'$ quark predominantly behaves like the *d* quark, albeit with a small admixture of the *s* quark.**

Quantum numbers and symmetries

The quark mixing originally introduced by Cabibbo was **extended** by Kobayashi and Maskawa **to all three quark families, such that d', s', and b' are obtained from d, s, b by a rotation matrix**. This matrix is called the **Cabibbo–Kobayashi–Maskawa matrix (CKM matrix),**

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = U \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$\begin{bmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{bmatrix} = \begin{bmatrix} 0.97373 \pm 0.00031 & 0.2243 \pm 0.0008 & 0.00382 \pm 0.00020 \\ 0.221 \pm 0.004 & 0.975 \pm 0.006 & 0.0408 \pm 0.0014 \\ 0.0086 \pm 0.0002 & 0.0415 \pm 0.0009 & 1.014 \pm 0.029 \end{bmatrix}$$

The elements on the main diagonal of the (3×3) matrix U are very close to unity. The off-diagonal elements indicate the strength of the quark-flavour violation.

Unified theory of interactions

Can read **chapter 2.3 in the book** (Claus Grupen: Astroparticle Physics (second edition - 2020)).

Problems

1. Which of the following reactions or decays are allowed?

   (a) $\mu^- \to e^- + \gamma$,
   (b) $\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu + e^+ + e^-$,
   (c) $\pi^0 \to \gamma + e^+ + e^-$,
   (d) $\pi^+ \to \mu^+ + e^-$,
   (e) $\Lambda \to p + K^-$,
   (f) $\Sigma^+ \to n + \pi^+$,
   (g) $K^+ \to \pi^+ + \pi^- + \pi^+$,
   (h) $K^+ \to \pi^0 + \pi^0 + e^+ + \nu_e$.

Problems

2. What is the minimum kinetic energy of a cosmic-ray muon to survive to sea level from a production altitude of 20 km ($\tau_\mu = 2.197\,03\,\mu\text{s}$, $m_\mu = 105.658\,37\,\text{MeV}$)? For this problem one should assume that all muons have the given lifetime in their rest frame.

3. Work out the Coulomb force and the gravitational force between two singly charged particles of the Planck mass at a distance of $r = 1\,\text{fm}$!
4. In a fixed-target experiment positrons are fired at a target of electrons at rest. What positron energy is required to produce a Z ($m_Z = 91.188\,\text{GeV}$)?