

'Astro summary:

Stellar models - observed properties of stars, models vs. observations
Stellar evolution

Stellar models:

What is a star: self-gravitating bodies of dynamical equilibrium
internal pressure \leftrightarrow gravity
hydrostatic equilibrium

- lose energy by radiation \rightarrow need energy production: nuclear energy, gravitational energy
- temperature structure \rightarrow influences energy transport: radiation, conduction

Basic equations of stellar structure:

① mass conservation $\frac{dm_r}{dr} = 4\pi r^2 \rho$

② hydrostatic equilibrium: $\frac{dP}{dr} = -\frac{GM_r}{r^2} \rho$

\rightarrow assuming perfect gas: \rightarrow calculate P and T

Virial theorem: thermal energy \leftrightarrow balances gravity
 $2E_T + E_G = 0$

③ ~~Energy transport~~ - Energy conservation

$$\frac{dL}{dr} = 4\pi r^2 \rho \epsilon$$

④ Energy transport

$$\frac{dT}{dr} = -\frac{3}{4a_B C} \frac{\rho P}{T^3} \frac{L_r}{4\pi r^2}$$

radiative E transport

$$\frac{dT}{dr} = \left(1 - \frac{1}{\gamma}\right) \frac{T}{P} \frac{dP}{dr}$$

convective E transport
 γ - adiabatic pressure

\rightarrow Schwarzschild stability condition

$$\left| \frac{dT}{dr} \right| < \left(1 - \frac{1}{\gamma}\right) \frac{T}{P} \left| \frac{dP}{dr} \right|$$

if the temperature gradient is steeper than this then the atmosphere is unstable
 \rightarrow convection

- ②
- composition of the star is important $P, \chi, \varepsilon (p_i, T_i, \chi_i)$
 - adding boundary conditions for the inside and outside of the star
 - solving the equations to get a model

Voigt - Russell theorem of unique solutions → not true, possible degeneracy

⇒ model - simplifications ⇒ relations between different quantities

$$P \propto \frac{M^2}{R^2} \quad P \propto \frac{M}{R^3} T \Rightarrow \boxed{T \propto \frac{M}{R}} \quad \boxed{L \propto M^3}$$

mass-luminosity relation → more massive stars → more luminous
 T_{eff} - surface temp.

$$\boxed{M \propto T^2} \quad \boxed{L \propto T_{\text{eff}}^6}$$

⇓
 L vs. T_{eff} ⇒ Hertzsprung Russell diagram

$\tau \propto \frac{M}{L} \Rightarrow \boxed{\tau \propto M^{-2}}$ lifetime of a star is related to the mass inversely

Surface temperature → spectral lines → composition → spectral classes
 → colour → blackbody

O, B, A, F, G, K, M

↓
 O, B - young massive
 G - Sun

Spectra → doppler shift → motion of the star (binary, planet, etc.)
 → Zeeman effect → magnetic field (e.g. sunspots)

Luminosity - needs distance measurement:

- parallax
- globular clusters
- variable stars

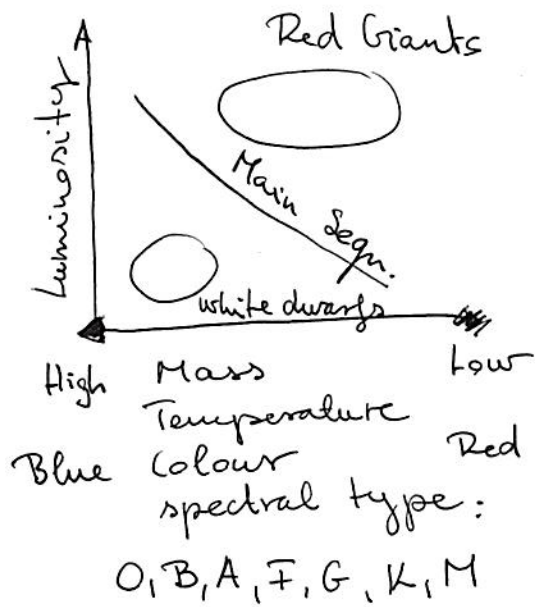
distance → absolute magnitude, abs. bolometric magnitude, luminosity

Stellar mass - binary systems (dynamic measurement)

- eclipsing binaries
- spectroscopic binaries

- Mass - Luminosity relation

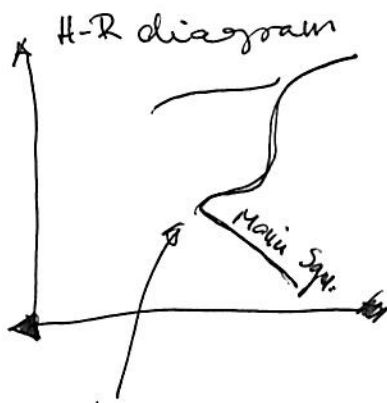
⇒ H-R diagram



\Rightarrow mass limit of stars: $0.1 - 100 M_{\odot}$
 Lower limit: no fusion: brown dwarf
 upper limit: Eddington luminosity limit
 (based on radiation pressure)

Star clusters - globular clusters: old, gravitationally bound
 \downarrow
 - open clusters: young, less bound
 Born together

\downarrow
 same distance \rightarrow distance measurement with the H-R diagram



\Rightarrow match the main sequence to nearby stars \rightarrow absolute magnitude \rightarrow distance

turn off point - related to age of cluster

Stellar nucleosynthesis:

mass-binding energy relation $E_B = [Zm_p + (A-Z)m_n - m_{nuc}]c^2$
 tightest bound nucleus: Fe \rightarrow lighter elements fusion } energy release
 \rightarrow heavier elements fission

Stars: nuclear fusion - quantum mechanics tunneling

reaction rate - related to the cross section $r = n_1 n_2 \langle \sigma v \rangle$

reaction rate \rightarrow energy release ΔE , $rE = r\Delta E = n_1 n_2 \langle \sigma v \rangle \Delta E$

$\hookrightarrow E$ - nuclear energy generation function

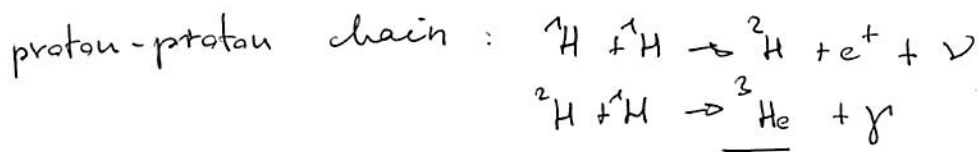
\hookrightarrow increases with temperature sharply

①

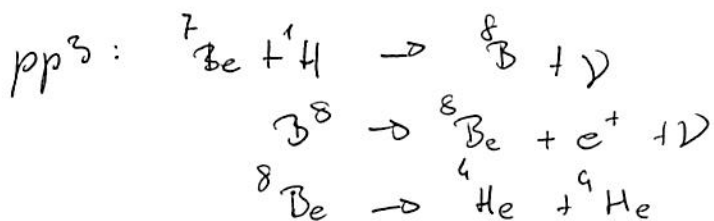
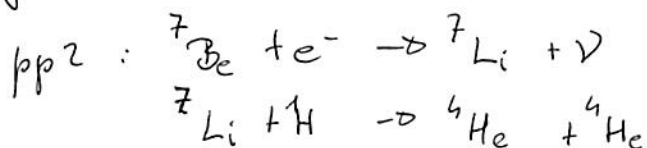
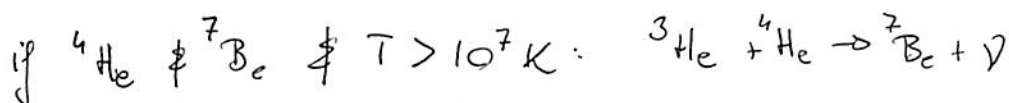
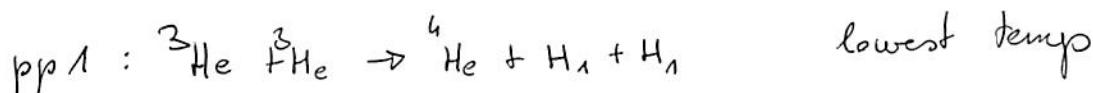
→ reaction of heavier nuclei needs higher temperature

- nuclear burning
- nuclear fuel

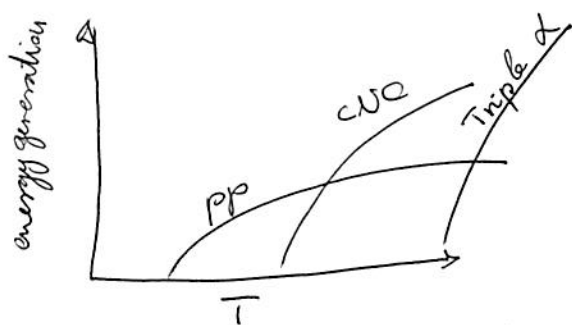
} main sequence burn $H \rightarrow He$



${}^3_2He \rightarrow pp1, pp2, pp3$ chains

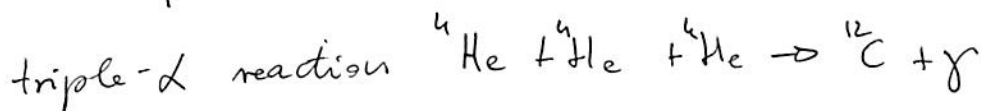


if C, N, O present CNO-cycle: higher temperature



low temp: pp
low mass

high temp: CNO → more energy
high mass



⇒ heavy elements are synthesised in stars

→ after H finished → He can burn in high mass stars

→ up to Si burning into Fe

heavier elements than Fe → in Supernovas

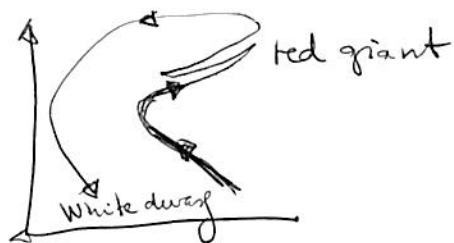
- ⑤ → composition of stars change with time
 → more massive stars hotter → different temp gradient
 → massive stars: convective core
 - low mass stars: convective envelop

testing the inside of stars:

stellar oscillations → sound speed inside → pressure and density
 - helio seismology } test models
 - asteroseismology }

- neutrino production in stars (nuclear fusion)
 ↳ test with observations → neutrino oscillation
⁸B neutrinos depend on solar models

Stellar evolution on the H-R diagram



Binary systems: equipotential surface → Roche lobe → mass transfer

Mass loss → stellar winds (solar wind)

- thermal driven wind
- radiative driven wind
- centrifugally driven winds

Extreme case: planetary nebula → white dwarf

Supernova explosion → neutron star or black hole
 ↳ Type I and Type II.

Type Ia supernova → mass transfer onto degenerate star
 ↳ identical mass → distance measurements

→ heavier elements than iron in supernova explosions (Type II)
 ↳ neutron capture + radioactive decay

⑥ Solar properties

- differential rotation
- magnetic field - sunspots - magnetogram
- solar cycle: 22 years → min, max sunspot
+ magnetic field flips polarity

↳ related: solar flares, CMEs ⇒ space weather

Exoplanets: 5 methods to detect:

- astrometry
- transit
- radial velocity
- direct imaging
- gravitational microlensing

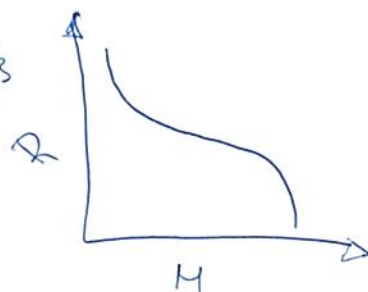
End state of stars

- white dwarf
 - neutron star
 - black hole
- } degenerate stars
degenerate matter

WD: degenerate electron gas $R \propto M^{-1/3}$

higher mass → smaller radius
planet sized star

Chandrasekhar mass limit



Neutron star: degenerate neutron gas

to high pressure → neutron drip

star of M_{\odot} → 12km city sized star

- pulsars
- rotation → rotation period + mass ⇒ density
- associated with supernova remnants
- pulsar glitches
- millisecond pulsars (double system, mass accretion)
↳ X-ray sources