Introduction to Astrophysics and Cosmology

Our Galaxy and the Interstellar Medium

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Our Galaxy

We are in a disk-like stellar system called a **spiral galaxy**

How to get the shape of the Milky Way?

Milky Way over Chimborazo

Our Galaxy - The Milky Way

- **Mapping the distribution of objects** in the Milky Way, for example the distribution of stars
- One method is to **count stars of a particular spectral type** which have the absolute magnitudes lying in a narrow range.
    - By **obtaining the distribution function *N*(*m*) for these stars** in different directions and by comparing it with a uniform distribution, it is possible to determine the distances in different directions where these stars are under-abundant or over-abundant, thereby generating a **map of the density distribution of these stars.**
    - Usually a particular telescope has a **limit of apparent magnitude *m*** to which it can go.
    - Intrinsically **faint stars** (with large M) reach the apparent magnitude m at a relatively **short distance**, whereas intrinsically **bright stars** (with smaller M) have this magnitude at a **larger distance**. Hence the telescope will show intrinsically bright stars at large distances where intrinsically faint stars are no longer visible. This is called the ***Malmquist bias***. It is important to statistically correct for this bias in data analysis.

The Shapley model

- Shapley noted that most of the **globular clusters are found around the constellation Sagittarius** in the sky and suggested that the **centre of our Galaxy** must be in the direction of this constellation and the globular clusters must be distributed symmetrically around this centre.
- The Figure shows an edge-on view based on our modern perception of what the Galaxy would look like. The **Galaxy has a thin disk with a spheroidal bulge around its centre**.
- The Sun is located in an outlying region of this disk indicated by × in Figure far away from the centre.
- About 200 **globular clusters make up a roughly spherical halo around the galactic centre**.

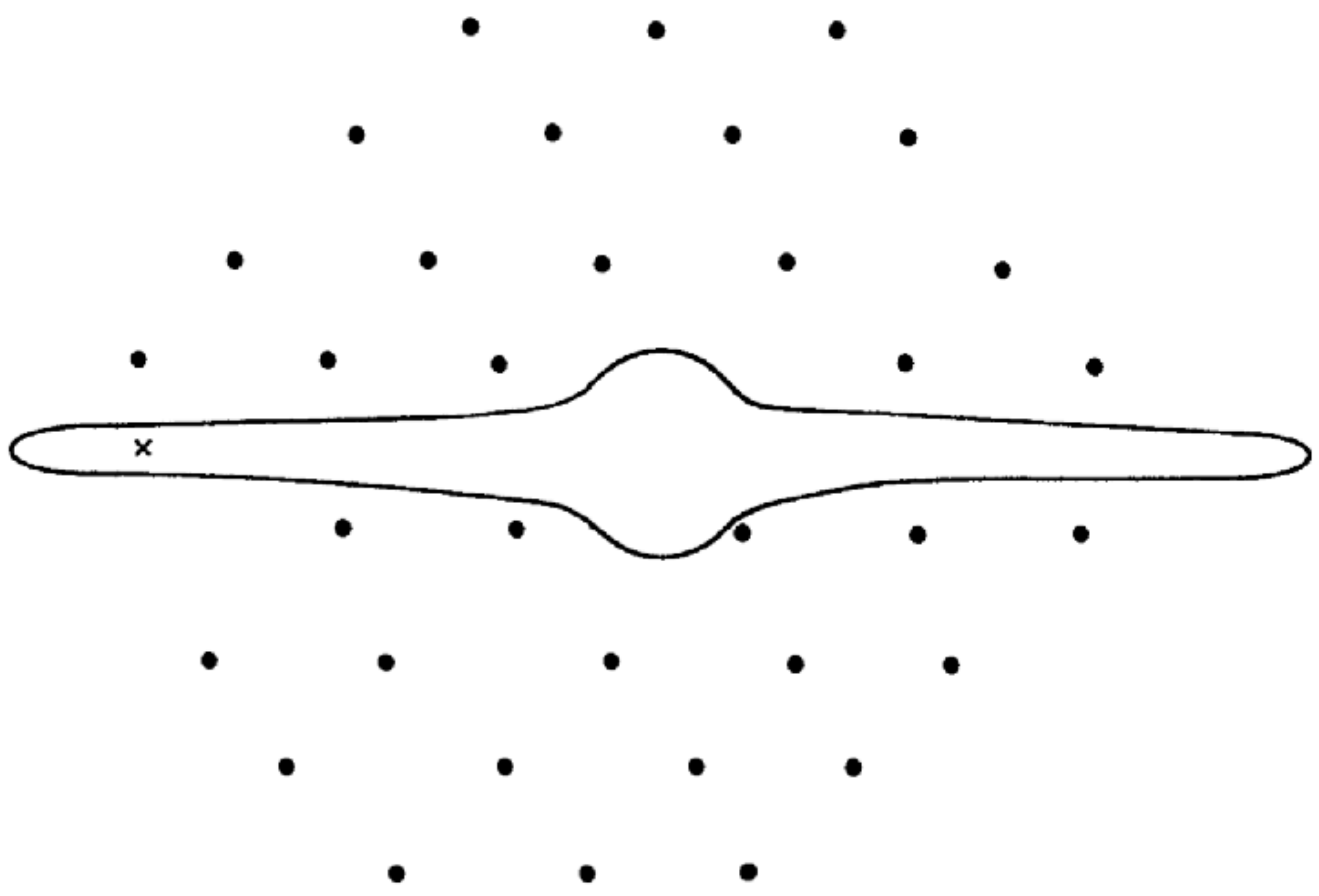


Fig. 6.1 A schematic edge-on view of our Galaxy. The position of the Sun is indicated by ×.

Distance measurement

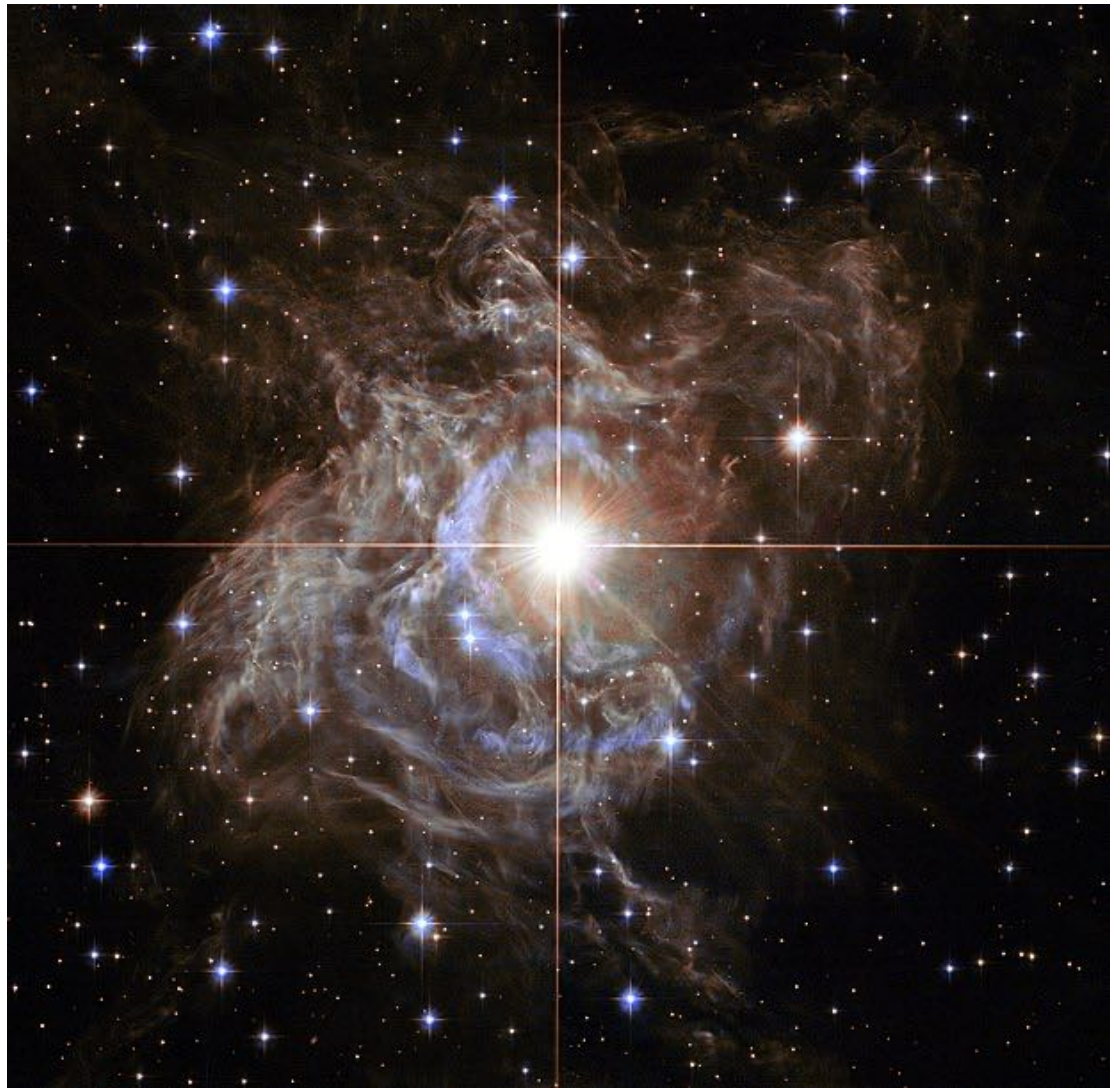
To establish the size of the Galaxy, we need to know the distances of the globular clusters from us.

For measuring distances of reasonably faraway stellar systems, two kinds of stars with periodically varying luminosity can be used **Cepheid variables and RR Lyrae stars**.

Cepheid variable is a type of **star that pulsates radially, varying in both diameter and temperature** and producing **changes in brightness with a well-defined stable period and amplitude**.

The **period–luminosity relation of Cepheid variables** was fully established when the distances based on globular clusters (and hence absolute luminosities) were determined.

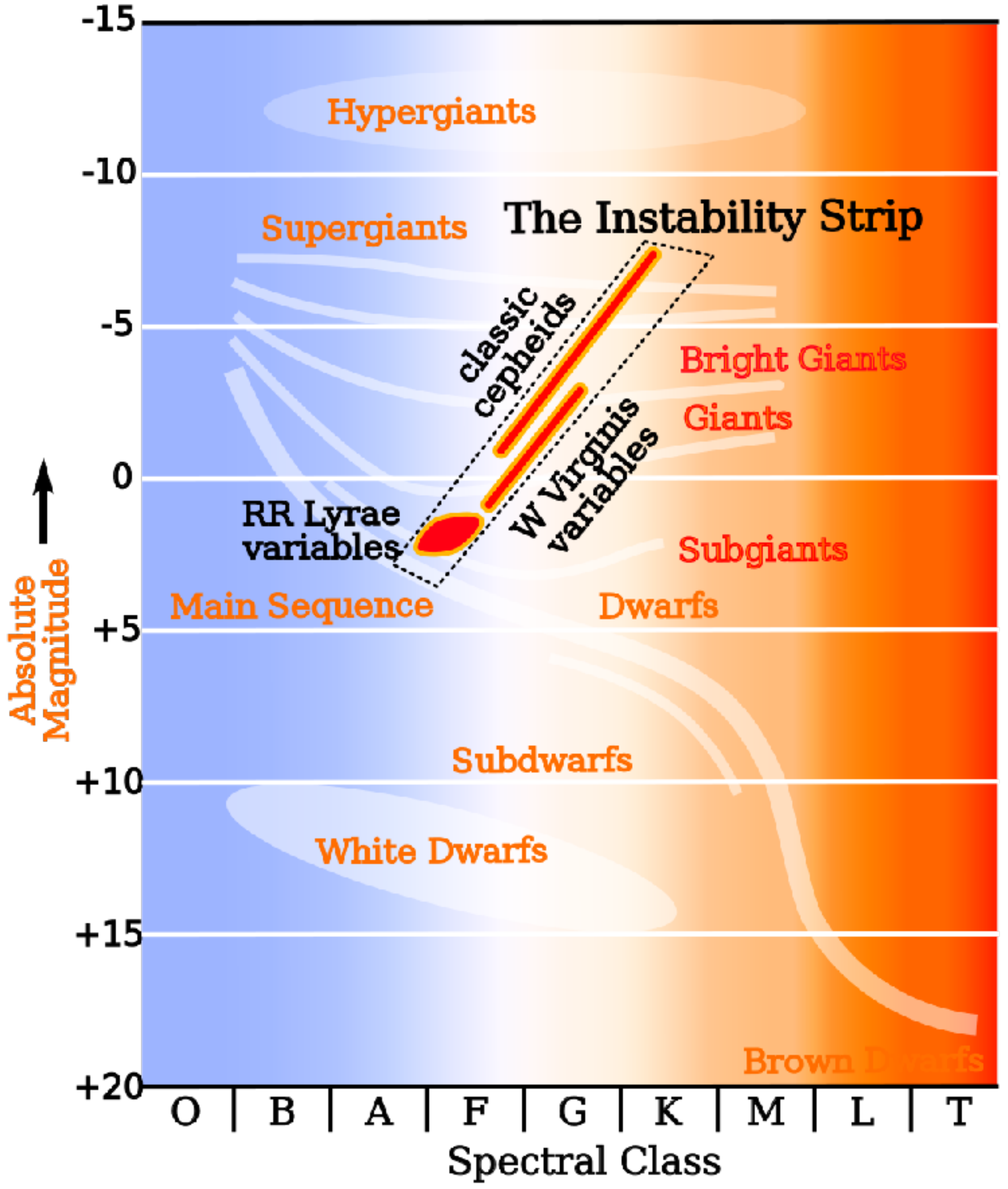
If by measuring the period of a Cepheid variables, we can infer its absolute luminosity and, by comparing with the apparent luminosity, we can then find the **distance.**



RS Puppis, one of the brightest known Cepheid variable stars in the Milky Way galaxy

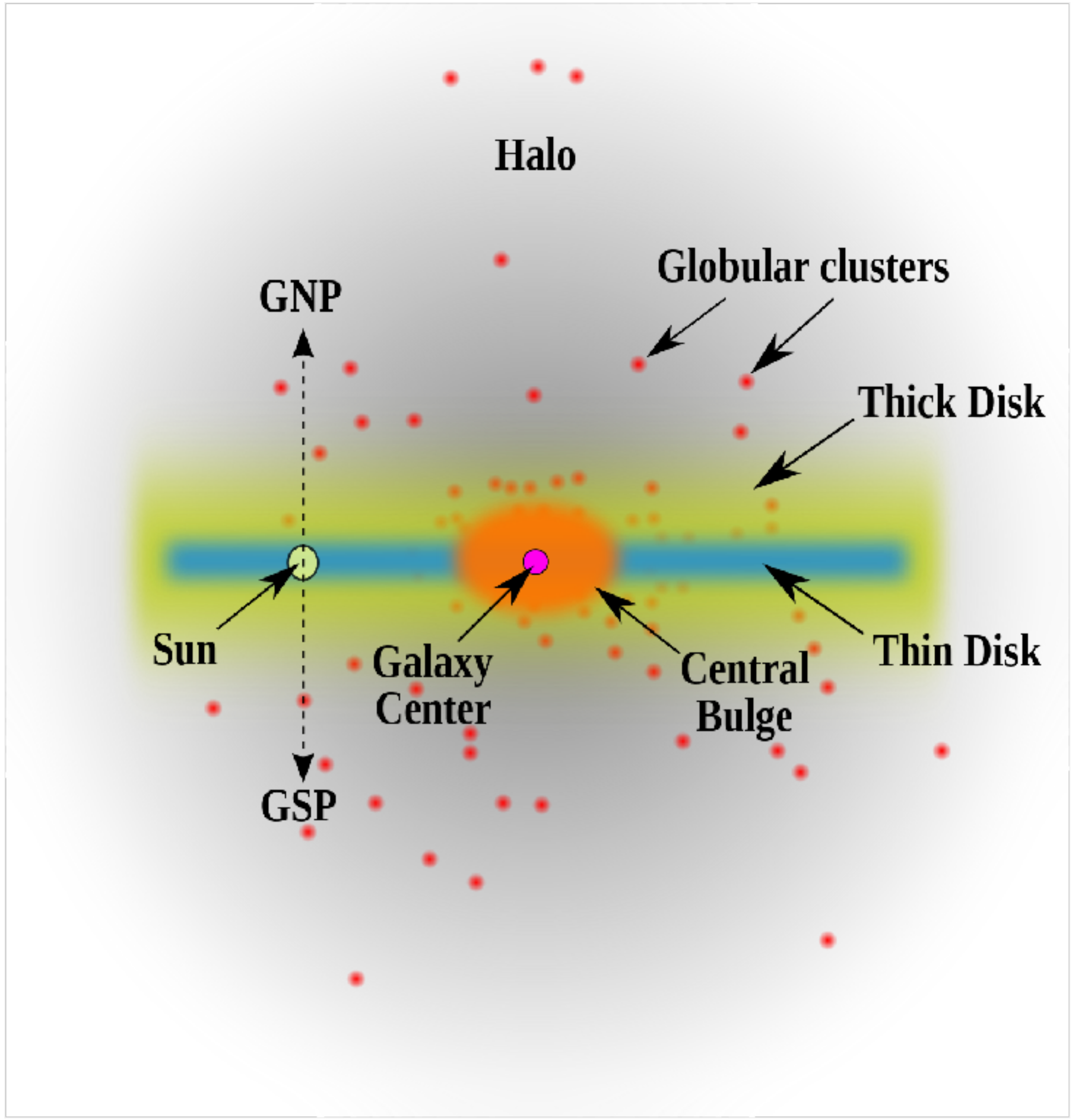
Distance measurement

- **RR Lyrae variables are periodic variable stars**, commonly found **in globular clusters**. They are used as standard candles to **measure (extra) galactic distances.**
- Named after the prototype and brightest example, RR Lyrae.
- They are pulsating horizontal branch stars of spectral class A or F, with a mass of around half the Sun's. They are thought to have shed mass during the red-giant branch phase, and were once stars at around 0.8 solar masses.



The Shapley model

- Shapley (1919) used the RR Lyrae stars in some globular clusters to estimate their distances.
- From these measurements, he concluded that t**he galactic centre is situated at a distance** of 15 kpc from us. The current best estimate for this distance is **about 8 kpc.**
- The disk of the Galaxy has a thickness of the order of 500 pc. The actual estimate of the thickness depends on the kinds of stars we use to find this thickness.
    - The bright **O and B stars are usually found close to the mid-plane of the disk**, such that one gets a lower value of the thickness of the disk (a scale height of about 50 pc from the mid-plane). -> **"thin disk"**

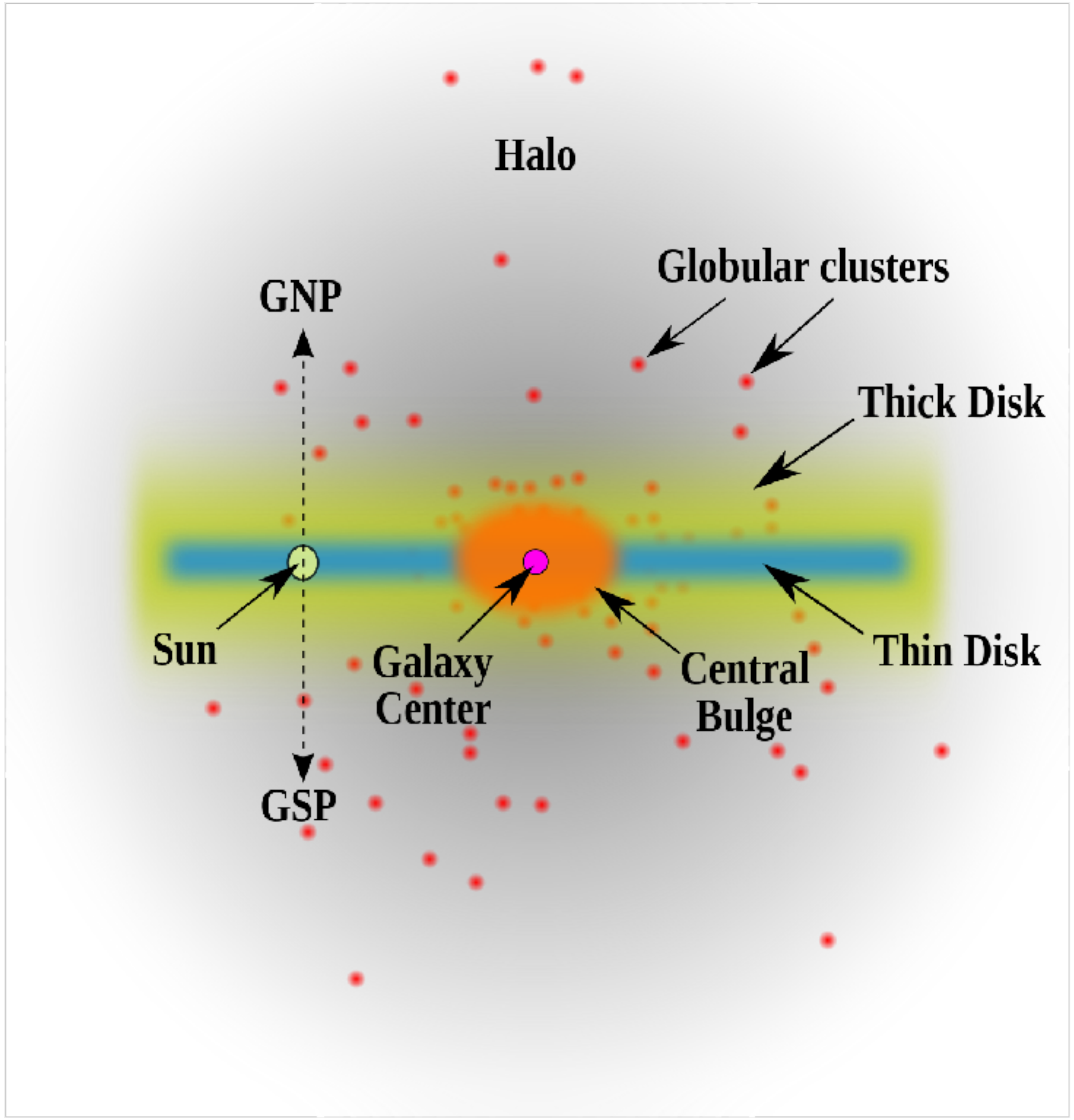


GNP - Galactic North Pole
GSP - Galactic South Pole

The Shapley model

- On the other hand, **stars** of the other types can be found **at greater distances from the mid-plane**, their densities falling with more typical scale heights of order 200 or 300 pc. -> **"thick disk"**
- Since **O and B stars are short-lived,** they are **statistically younger than other stars.** So, presumably, as the stars grow older, they can acquire larger random velocities, enabling them to rise further from the mid-plane against gravity.

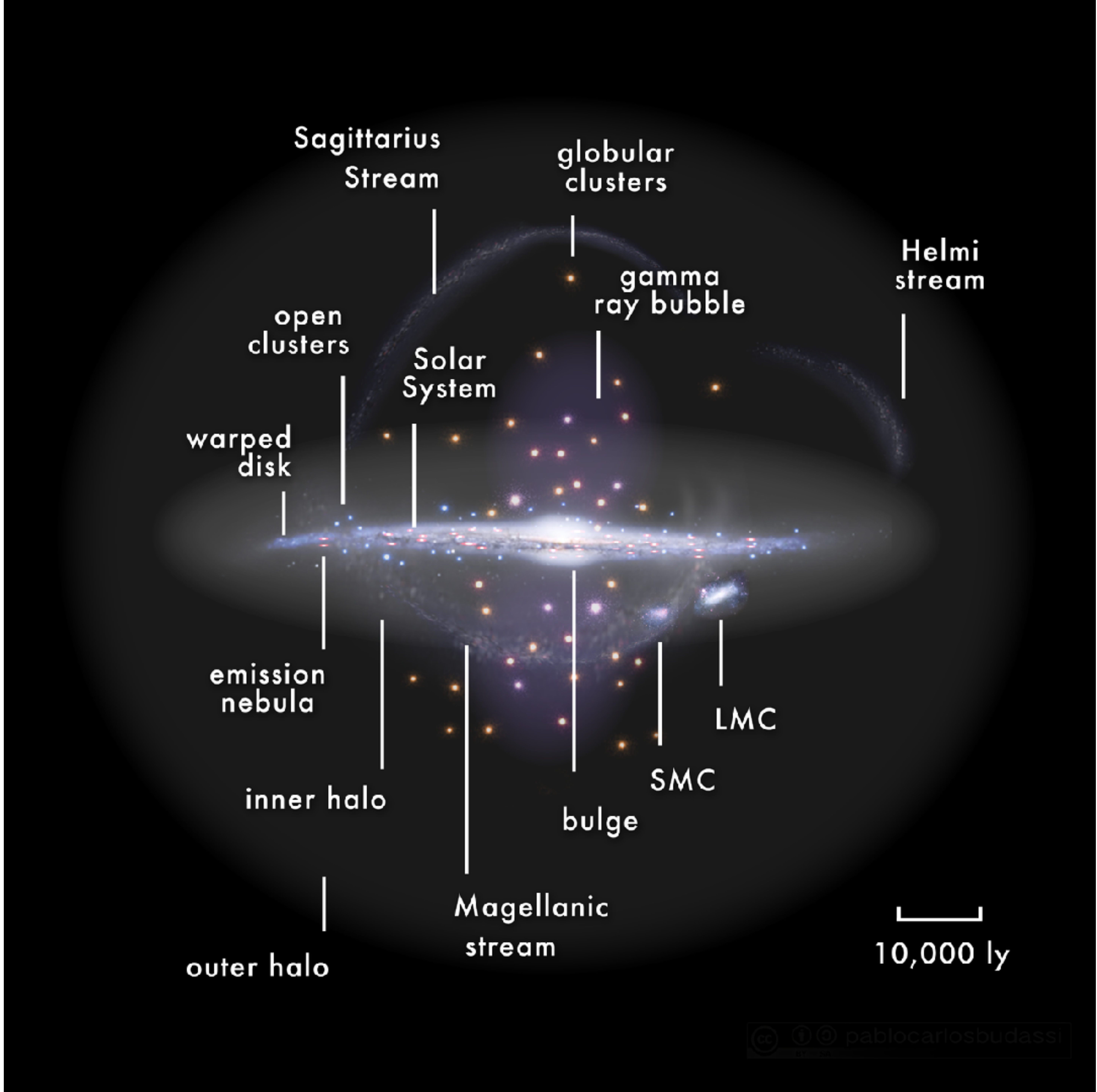
- Although we now know many more details not known in Shapley's time, our **present view of the Galaxy is still essentially what Shapley surmised**.



Our Galaxy

The modern view of our Galaxy:

- Warped stellar disk with spiral arms
- Bulge in the centre
- A bar in the centre
- Open star clusters near the disk
- Globular clusters distributed all over the halo
- Stellar streams in the halo
- A large gamma ray bubble above and below the centre
- Small companion (satellite) galaxies, e.g. the Large and the Small Magellanic Clouds
- The Sun is about halfway between the edge and the centre



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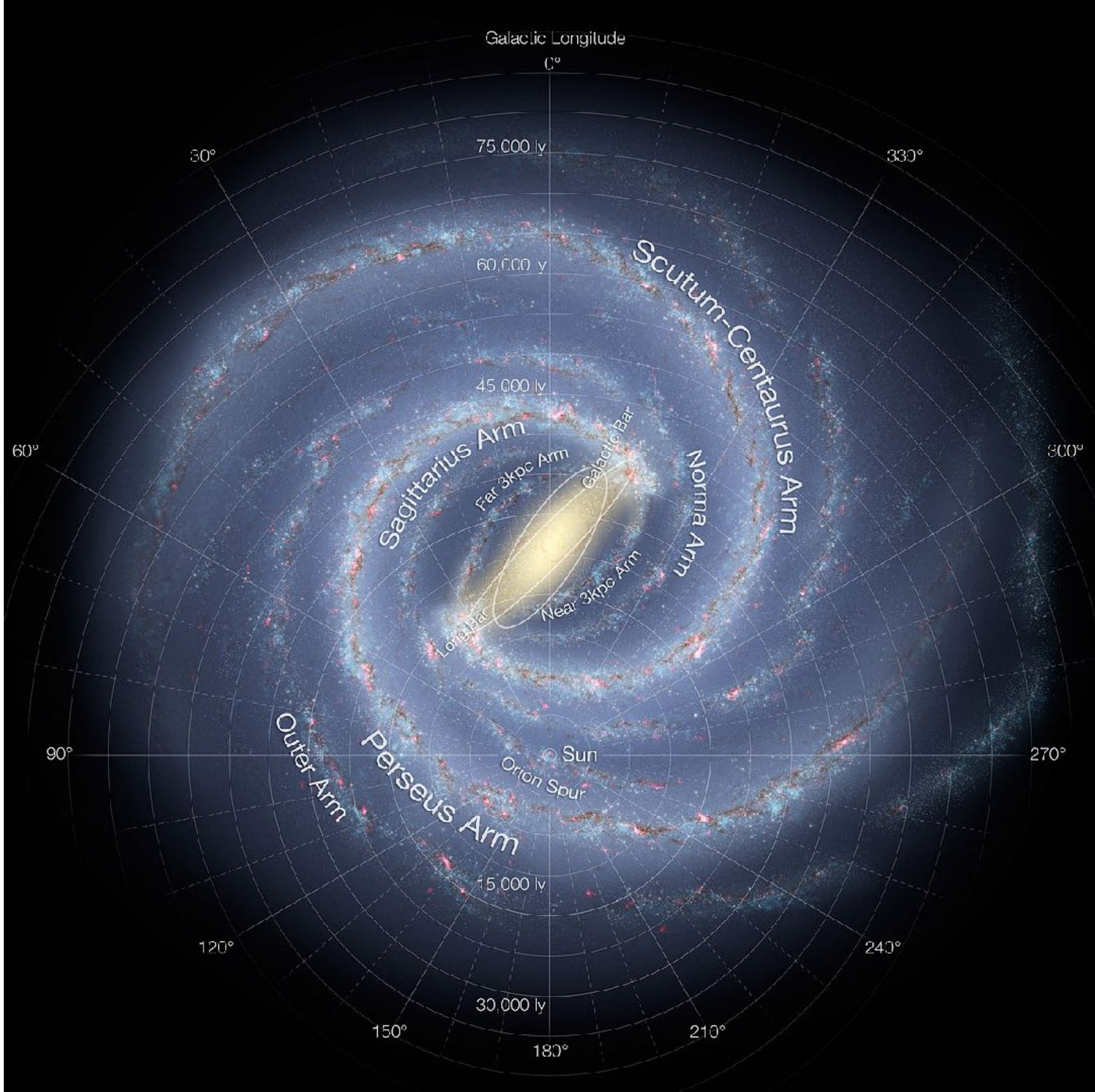


Our Galaxy

What is the galaxy made of?

Our Galaxy

The Galaxy is composed of:

- Stars
- Dust
- Gas
- And Dark matter

Interstellar extinction

- The inaccuracy of the early measurements of the Galaxy is partially due to **interstellar extinctions**. The interstellar medium contains particles of dust mixed with gas. It is the **dust particles** which are responsible for the **absorption of starlight.**
- In the presence of **interstellar absorption the magnitude relation** changes to

$$\boxed{m = M + 5\log_{10} d - 5 + A_\lambda}$$

where A_λ gives the dimming caused by the interstellar dust.

- Since dimming implies an increase of the apparent magnitude m, it should be clear that A_λ has to be positive. For visible light coming from stars in the galactic plane, a rough rule of thumb for the dimming term is

$$A_V \approx 1.5\, d$$

if d is measured in kpc. The amount of dimming of visible light with distance is approximately equal to 1.5 magnitude kpc^{-1} in the galactic plane. The subscript V in implies A_λ in the V band.

Interstellar extinction

- Since the **dust particles absorb more light at the shorter wavelengths** (on the bluer side), distant stars appear redder.
- The redness of a star is given by $(B - V)$.
- As starlight passes through interstellar matter, its **redness measure $(B - V)$ keeps increasing**. The change in it is denoted by $E(B - V)$ and the rule of thumb for this in the galactic plane is

$$E(B - V) \approx 0.5\, d$$

- Again d has to be in kpc.
- Since both A_λ and $E(B - V)$ depend linearly on the distance d, their ratio $A_\lambda/E(B - V)$ **is independent of d and is a measure of interstellar extinction as a function of wavelength λ.**

Interstellar extinction

- The Figure plots the related quantity ***E* (λ – *V*)/ *E* (*B* – *V*), which is also a measure of interstellar extinction**, as a function of inverse wavelength in the directions of a few stars.
- It is seen that there is an extinction peak around 2200 Å , which is usually interpreted to be due to graphite present in the dust.
- Apart from this peak, a straight line would not be a too bad fit for the absorption curve. This implies that **interstellar absorption roughly goes as** λ^{-1}, which is a much weaker dependence than the dependence λ^{-4}expected from Rayleigh scattering by molecules.

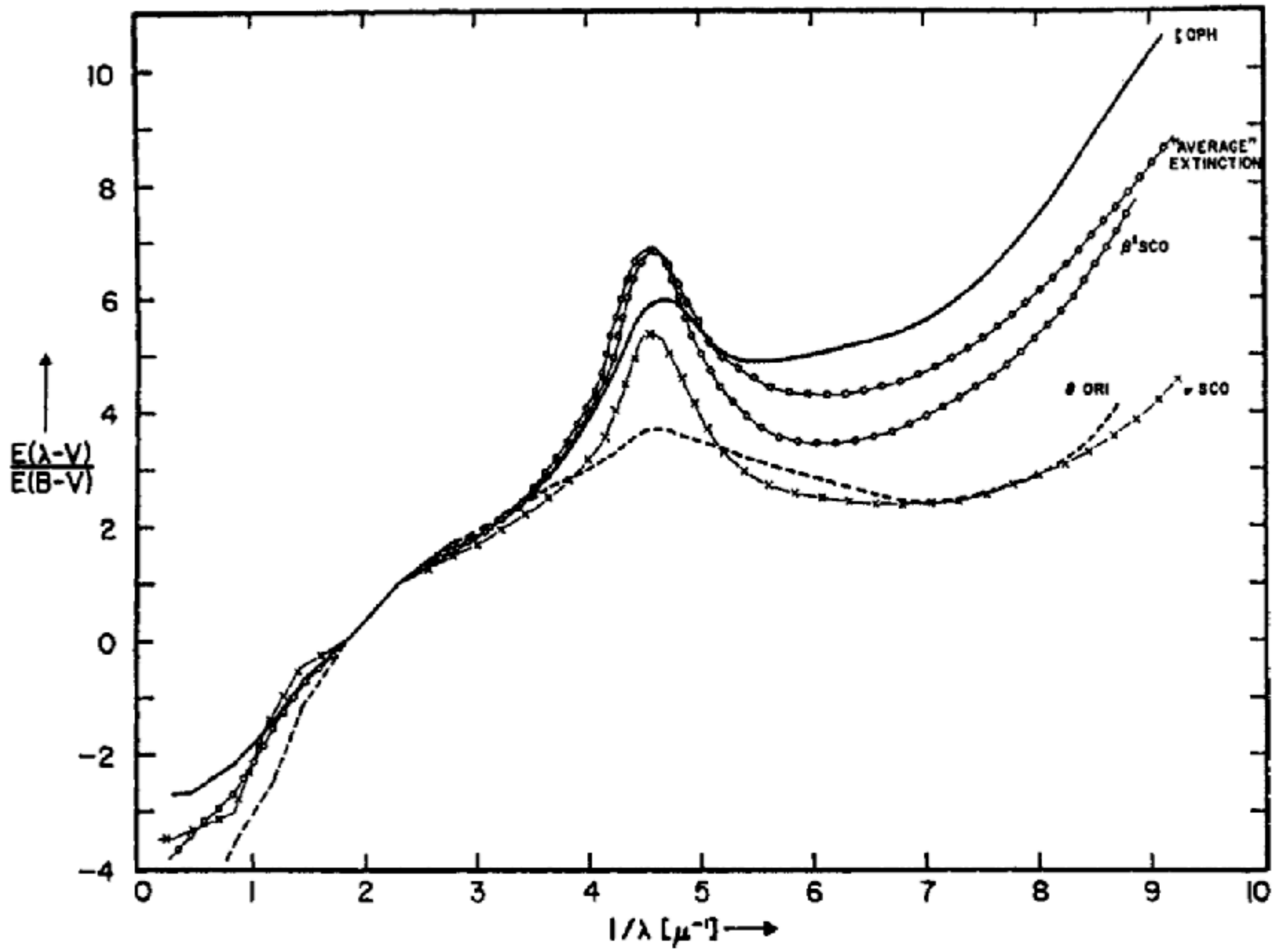
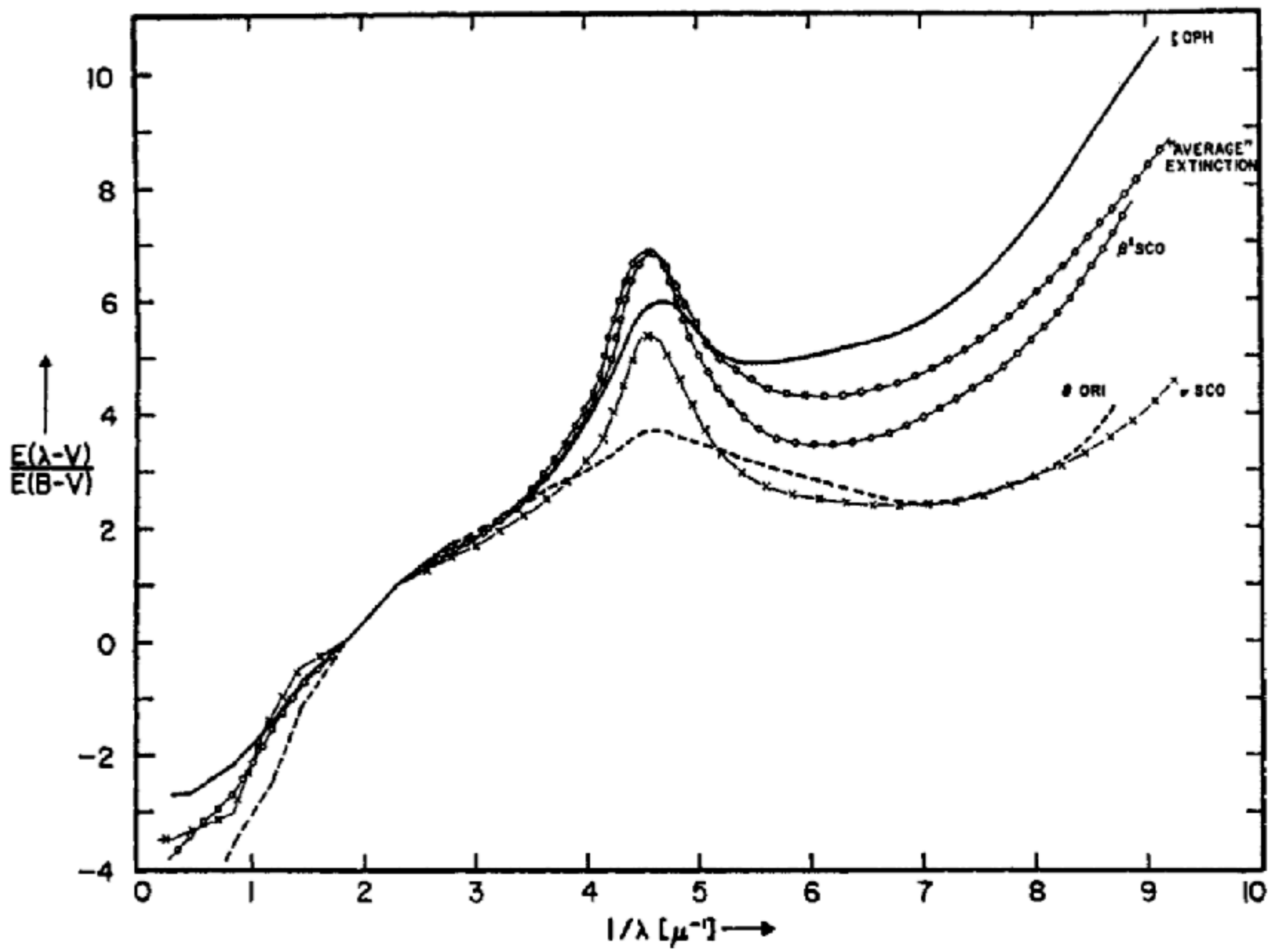


Fig. 6.3 A plot of $E(\lambda - V)/E(B - V)$, which is a measure of light extinction by interstellar dust, as a function of λ^{-1} in the directions of a few stars. An 'average' extinction curve is also indicated. From Bless and Savage (1972). (©American Astronomical Society. Reproduced with permission from *Astrophysical Journal*.)

Interstellar extinction

- The existence of interstellar extinction and reddening makes the star count analysis more complicated than what it would have been in the absence of interstellar matter.
- Luckily interstellar **dust is confined in a layer of thickness of about ±150 pc around the mid-plane** of the Galaxy, close to which we lie.
- When we look in directions away from the galactic plane, our view is not impaired by interstellar extinction or reddening.
- It was known for a long time that **external galaxies could not be seen in a narrow zone near the galactic plane.** This is known as the ***zone of avoidance*****.**



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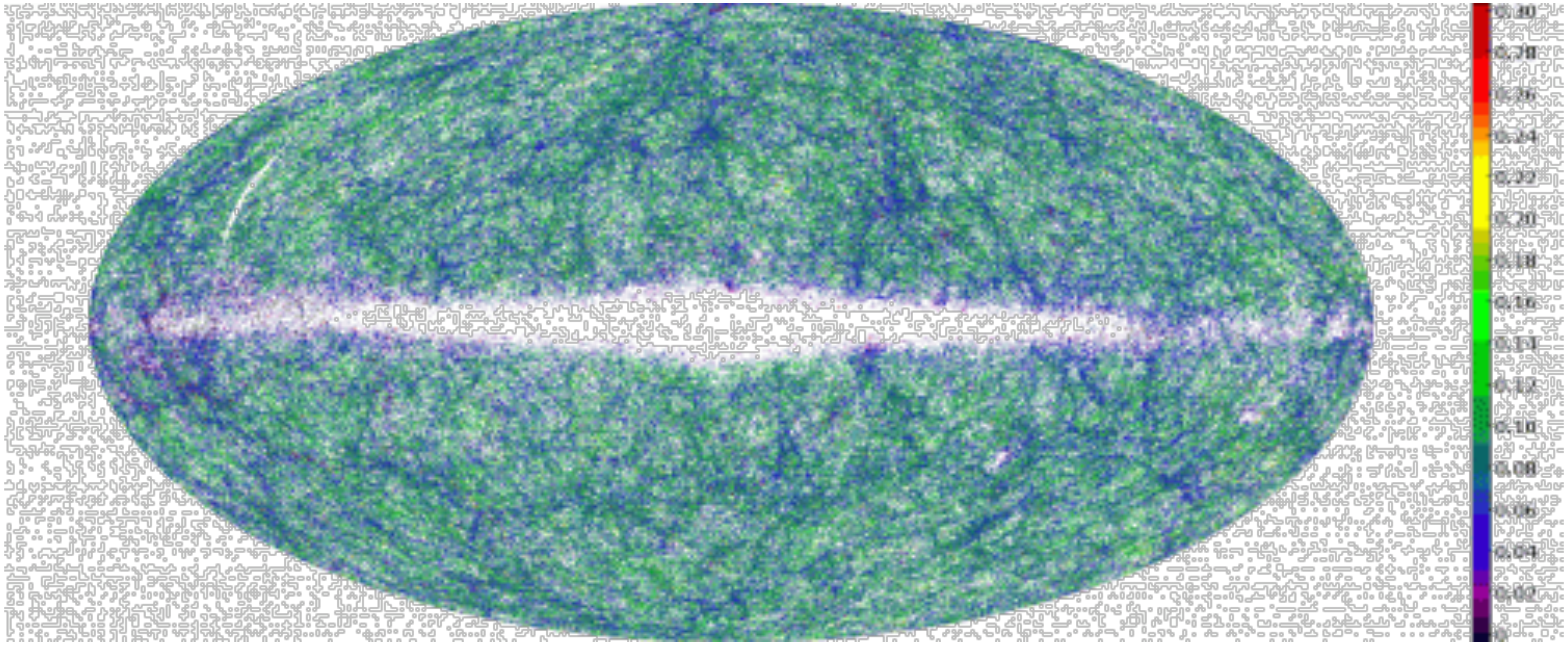
Interstellar extinction

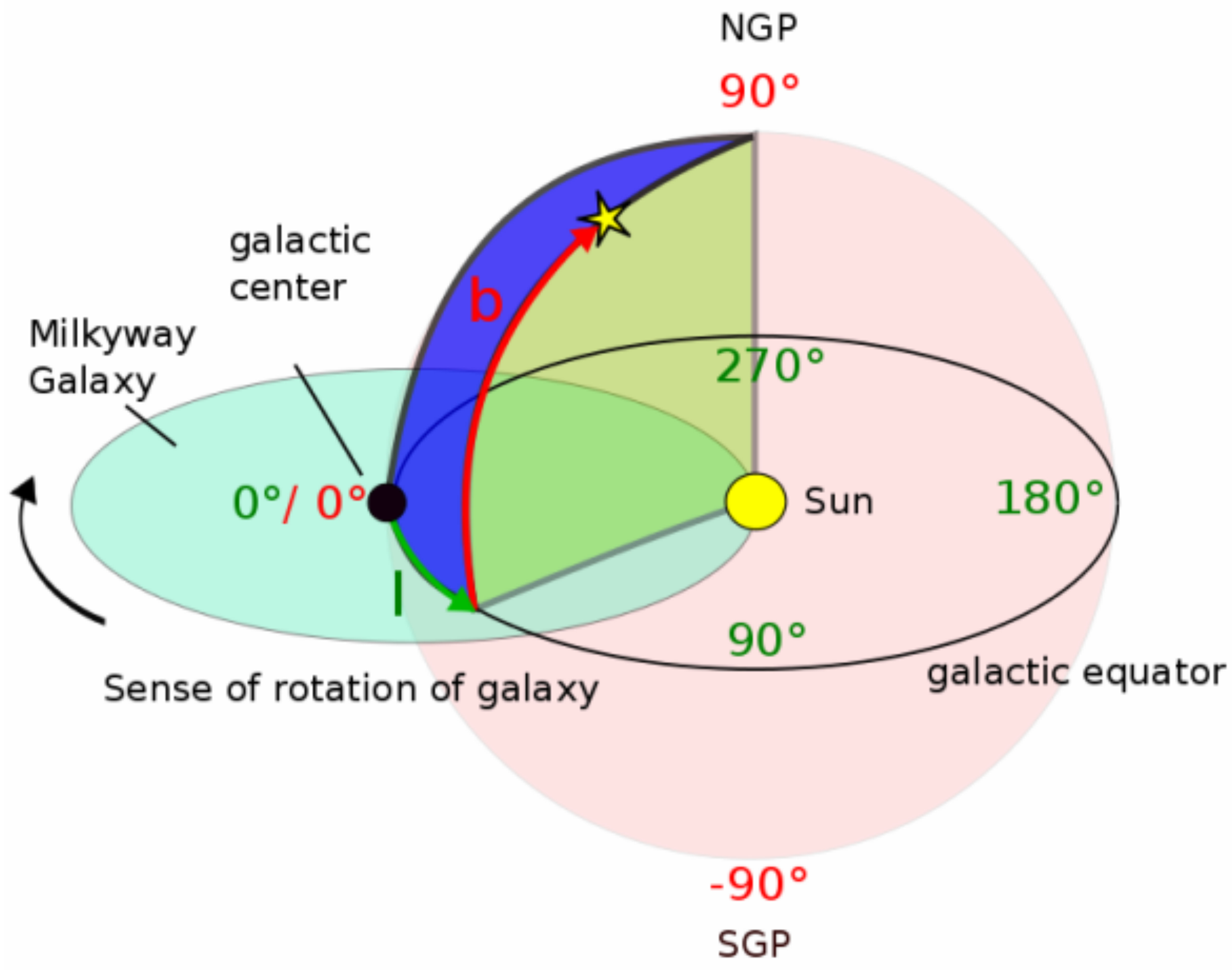
Projection of all-sky distribution of galaxies in the 2MASS Photometric Redshift catalogue. Each dot is a galaxy, colour-coded by its redshift (right-hand scale), and the plot includes almost 1 million of them. The **empty space in the middle is the area obscured by our Galaxy.** Image credit: M. Bilicki



https://science.uct.ac.za/department-astronomy/research/large-scale-structures-galaxies-zone-avoidance

Galactic coordinates

- For presenting Galactic observations, it is often useful to introduce galactic coordinates.
- The **galactic latitude *b*** of an object is its angular distance from the galactic plane, which is taken as the equator in this system.
- The **galactic longitude *l*** is measured from the direction of the galactic centre, which is taken to be at $l = 0^\circ$, $b = 0^\circ$.



https://auger.org/education/Auger_Education/galacticcoordinates.html

Galactic rotation

- The gravitational field at a point inside or near the Galaxy is expected to be directed towards the galactic centre.
- **How is this gravitational field balanced, to ensure that there is not a general fall of everything towards the galactic centre?**

Galactic rotation

- The gravitational field at a point inside or near the Galaxy is expected to be directed towards the galactic centre.
- **How is this gravitational field balanced, to ensure that there is not a general fall of everything towards the galactic centre?**
- There are basically two ways of balancing gravity.
  - A star may move in a circular orbit such that the **centrifugal force** balances gravity (as in the case of planets in the solar system).
  - The other way of balancing gravity is through **random motions.**
- Lindblad (1927) was the first to recognize that our Galaxy must be having two subsystems.
  - Most of the **stars in the disk move in roughly circular orbits** around the galactic centre and constitute one subsystem.
  - From the **nearly spherical shape of the halo of globular clusters**, Lindblad guessed that this must be a **non-rotating subsystem in which gravity is balanced by random motions.** At any particular instant of time, a globular cluster may be falling towards the galactic centre. Eventually, however, this globular cluster will come out on the other side of the Galaxy because of the kinetic energy it gains in falling towards the galactic centre.

Galactic rotation

- The Sun, in its orbit around the galactic centre, would circle around the non-rotating subsystem of globular clusters.
- The line of sight component of the **relative velocity of the Sun with respect to a globular cluster** can be determined by measuring the Doppler shifts of lines in the spectra of stars in this cluster. From the statistical analysis of such measurements for many globular clusters, it is possible to **estimate the speed with which the Sun is going around the galactic centre**, if we assume that the system of globular clusters has zero net rotation around the galactic centre.
- The best value for the **speed of the Sun around the galactic centre**, usually denoted by Θ_0,
  - is about Θ_0**= 220 km/s.**
- If the **Sun is located at a distance of $R_\odot$= 8 kpc from the galactic centre,** then the period of revolution of the Sun around the galactic centre is

$$\boxed{P_{\rm rev} = \frac{2\pi R_0}{\Theta_0} \approx 2 \times 10^8 \,\mathrm{yr}.}$$

The age of the Galaxy is believed to be of order 10^{10} yr, the Sun had time to make not more than 50 rounds about the galactic centre.

Galactic rotation

- The **approximate mass M of the Galaxy inside the solar orbit** can be estimated by balancing the gravitational and centrifugal forces:

$$\frac{GM}{R_0^2} \approx \frac{\Theta_0^2}{R_0}.$$

The gravitational field would have been given by GM/R_0^2 exactly if the mass inside the solar orbit were distributed in a spherically symmetric manner. On substituting the estimated values of R_0 and Θ_0 in the above approximate equation, we find **M to be of order 10^{11} $M_\odot$.**

As the gravitational field of the Galaxy is expected to fall off with distance, stars further out in the disk will have to move around the galactic centre with slower speeds. In other words, **the disk of the Galaxy should have *differential rotation*.** - **However, is this actually the case?**

Galactic rotation

- Oort (1927) carried out a classic analysis to show how this can be demonstrated by **studying the motions of stars in the solar neighbourhood.**
- We now present this analysis, based on the **simplifying assumption that all stars move in exactly circular orbits**. This assumption, of course, is not strictly true.
- Figure 6.4 shows the Sun at a distance R_0 from the galactic centre moving with speed Θ_0 in a circular orbit.
- We consider a star at a distance d from the Sun at galactic longitude l. This star is at a distance R from the galactic centre moving with circular speed Θ.

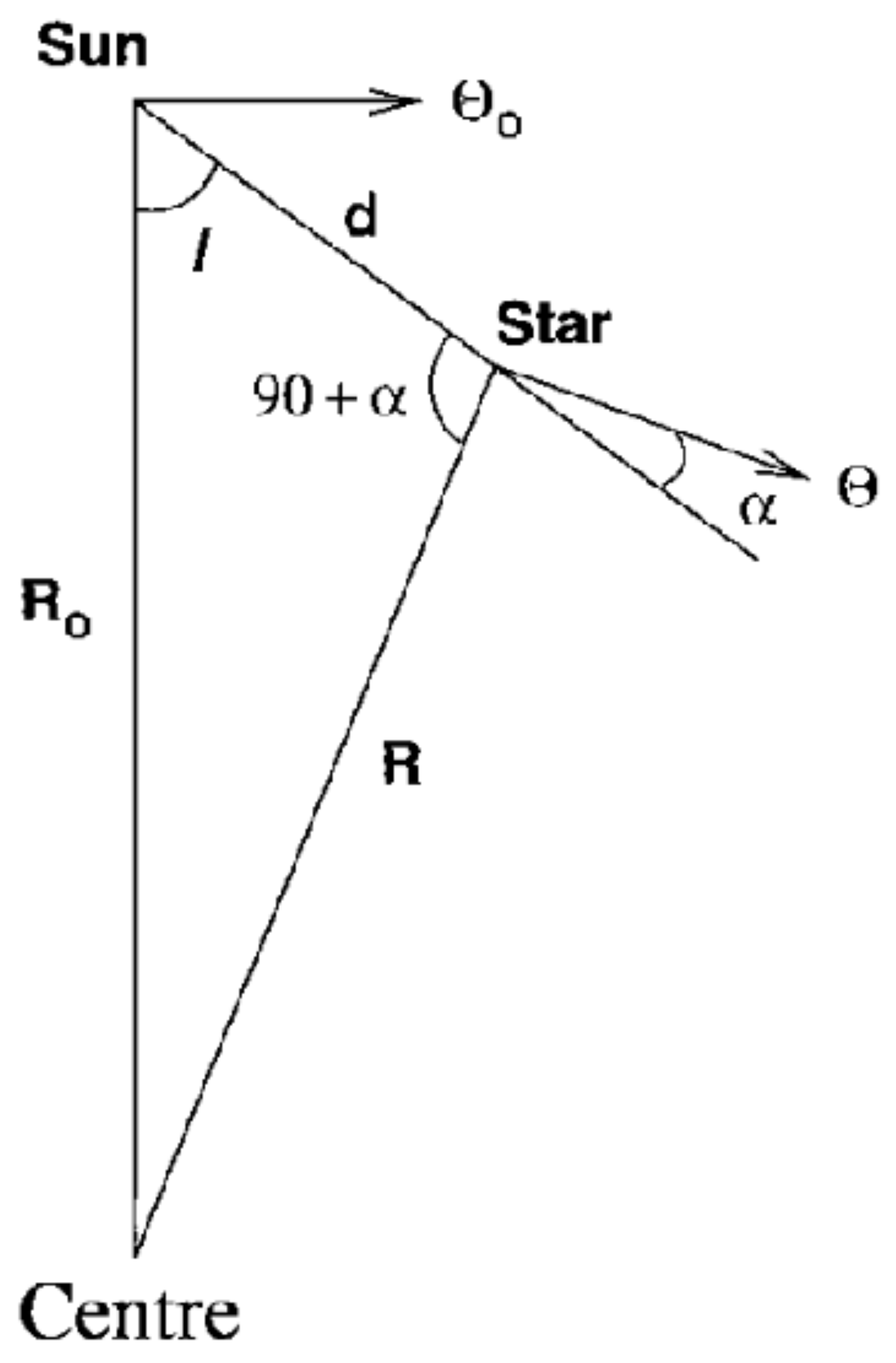


Fig. 6.4 A sketch indicating the Sun and a star going around the galactic centre.

Galactic rotation

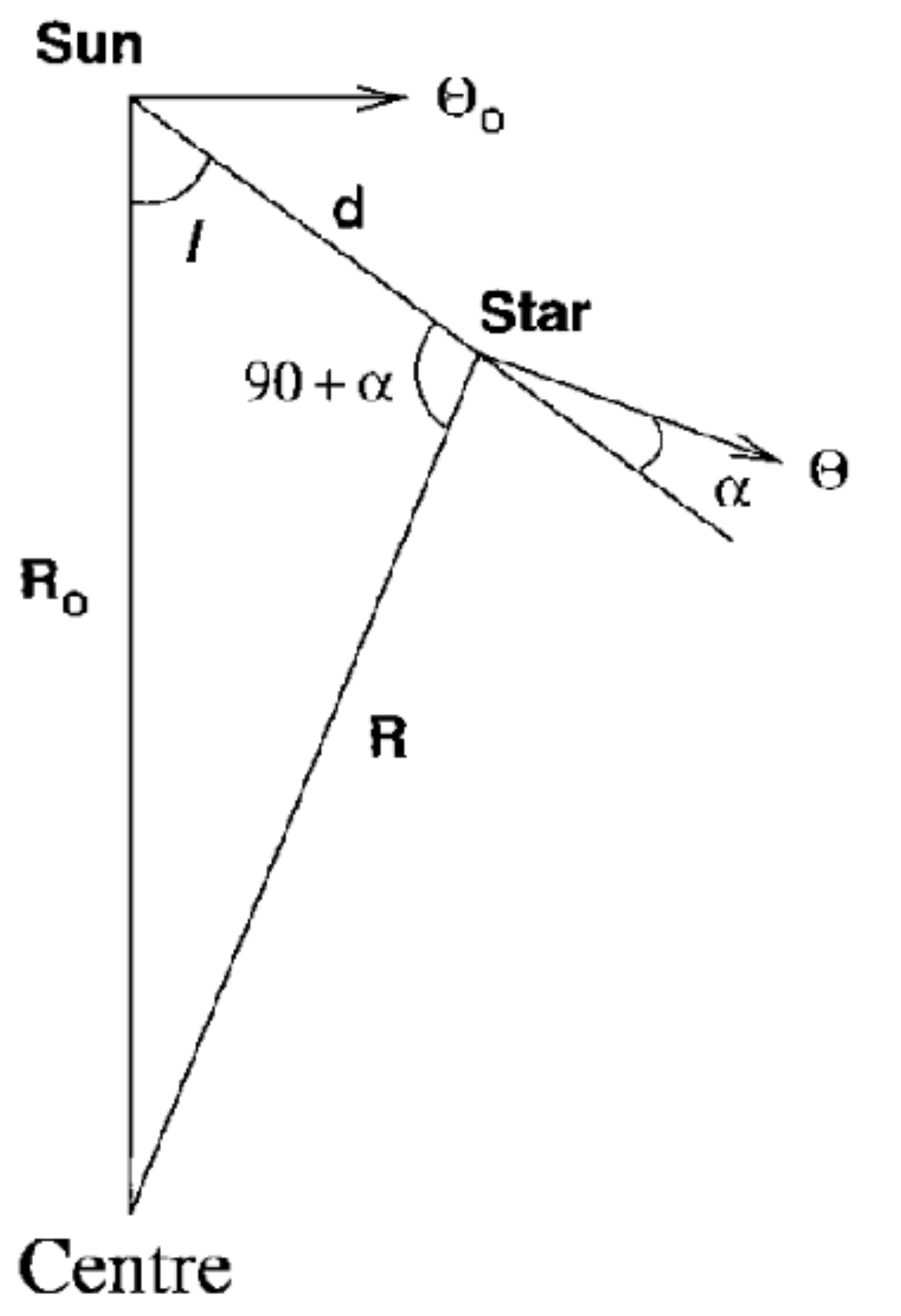
- Let us consider the triangle made up by the lines R_0, R and d. If α is the angle made by the direction of the star's velocity Θ with d, then it follows from Figure 6.4 that the angle opposite to R_0 in our triangle is $90^\circ + \alpha$. From the standard trigonometric properties of a triangle, we have

$$\frac{R}{\sin l} = \frac{R_0}{\cos \alpha}$$

$$R_0 \cos l = d + R \sin \alpha.$$

The **relative radial velocity of the star** (along the line of sight) **with respect to the Sun** is

$$v_R = \Theta \cos \alpha - \Theta_0 \sin l = \left(\frac{\Theta}{R} R_0 - \Theta_0 \right) \sin l$$



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Galactic rotation

Writing the **angular velocities** of the star and the Sun as

$$\omega = \frac{\Theta}{R}, \quad \omega_0 = \frac{\Theta_0}{R_0};$$

$$\boxed{v_R = (\omega - \omega_0) R_0 \sin l}$$

The **tangential velocity of the star with respect to the Sun is**

$$v_T = \Theta \sin \alpha - \Theta_0 \cos l = \Theta \frac{R_0 \cos l - d}{R} - \Theta_0 \cos l$$

$$\boxed{v_T = (\omega - \omega_0) R_0 \cos l - \omega d}$$

These expressions give general expressions of radial and tangential velocities of stars in the galactic disk moving in circular orbits around the galactic centre.

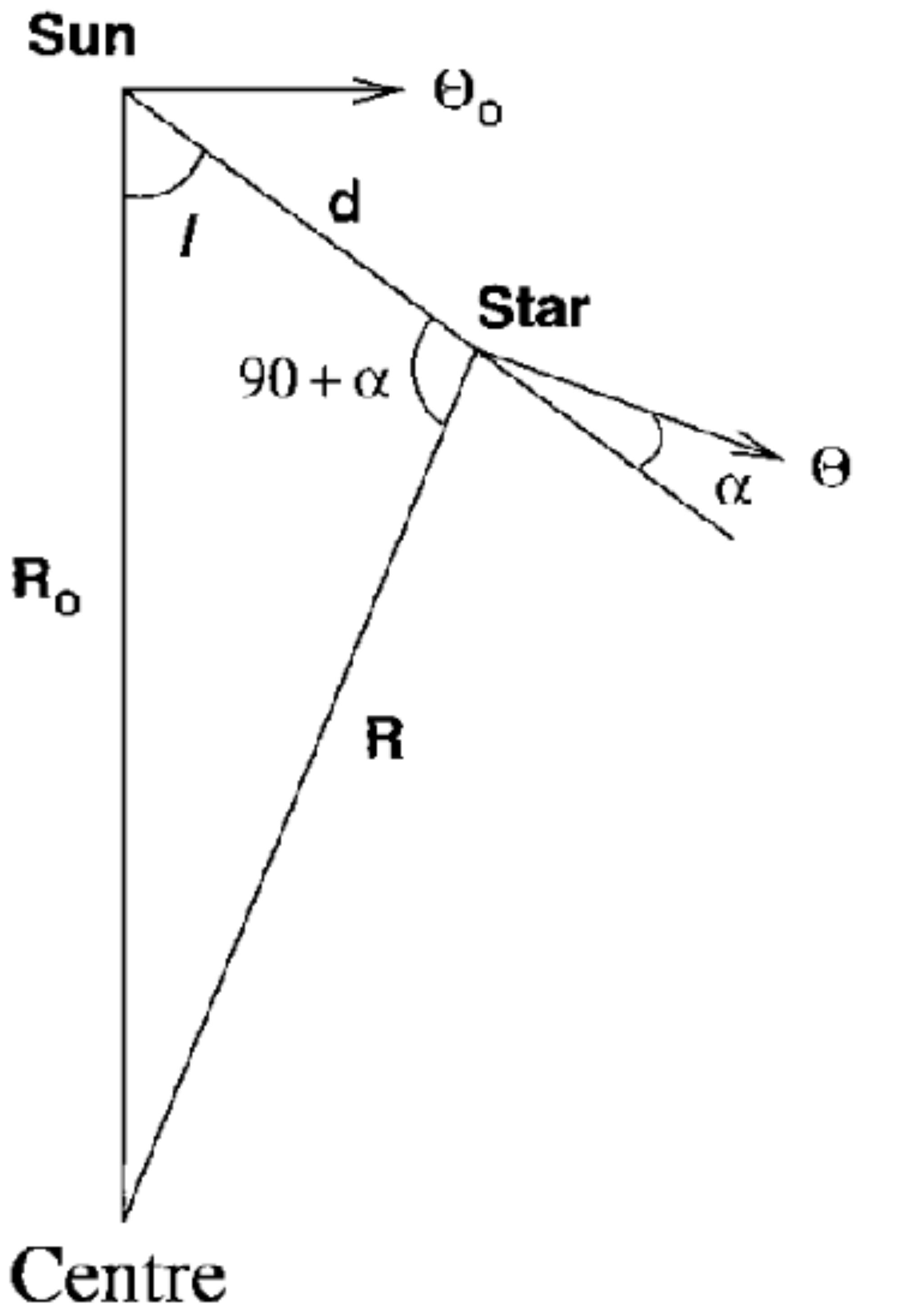


Fig. 6.4 A sketch indicating the Sun and a star going around the galactic centre.

Galactic rotation

We now consider stars in the solar neighbourhood for which $d \ll R_0$. For such stars, we approximately have

$$R_0 - R = d\cos l.$$

$$(\omega - \omega_0) = \left(\frac{d\omega}{dR}\right)_{R_0}(R - R_0) = \left[\frac{1}{R_0}\left(\frac{d\Theta}{dR}\right)_{R_0} - \frac{\Theta_0}{R_0^2}\right](R - R_0) \qquad \omega = \frac{\Theta}{R}, \quad \omega_0 = \frac{\Theta_0}{R_0},$$

$$\downarrow \qquad\qquad \downarrow$$

$$v_\mathrm{R} = \frac{1}{2}\left[\frac{\Theta_0}{R_0} - \left(\frac{d\Theta}{dR}\right)_{R_0}\right] d\sin 2l, \qquad v_\mathrm{T} = \left[\frac{\Theta_0}{R_0} - \left(\frac{d\Theta}{dR}\right)_{R_0}\right] d\cos^2 l - \frac{\Theta}{R} d.$$

Since $cos^2 l = 1/2(cos2l + 1)$, we get:

$$\downarrow$$

$$v_\mathrm{T} = \frac{1}{2}\left[\frac{\Theta_0}{R_0} - \left(\frac{d\Theta}{dR}\right)_{R_0}\right] d\cos 2l - \frac{1}{2}\left[\frac{\Theta_0}{R_0} + \left(\frac{d\Theta}{dR}\right)_{R_0}\right] d$$

Galactic rotation

We can finally write:

$$\boxed{v_R = Ad\sin 2l}$$

$$\boxed{v_T = Ad\cos 2l + B\,d}$$

Oort constants:

$$\boxed{A = \frac{1}{2}\left[\frac{\Theta_0}{R_0} - \left(\frac{d\Theta}{dR}\right)_{R_0}\right] = -\frac{1}{2}R_0\left(\frac{d\omega}{dR}\right)_{R_0}}$$

$$\boxed{B = -\frac{1}{2}\left[\frac{\Theta_0}{R_0} + \left(\frac{d\Theta}{dR}\right)_{R_0}\right]}$$

The Oort constants can tell us how the Galaxy rotates.

Galactic rotation

- The **radial velocity** v_R of a star can be easily **determined from the Doppler shifts** of spectral lines.
- Suppose we measure v_R of many stars located at approximately the same distance d in the galactic plane. **We expect v_R to vary as $\sin(2l)$ with the galactic longitude of the star.**
- The Figure shows one of the first such measurements: radial velocities of four groups of Cepheid variables of which distances could be estimated from periods, the members of each group lying at a fixed distance d.
- We clearly see a **sinusoidal variation in v_R** with the galactic longitude.
- Data points not lying exactly on the fitted curves indicate that **stars do not move in precise circular orbits.**

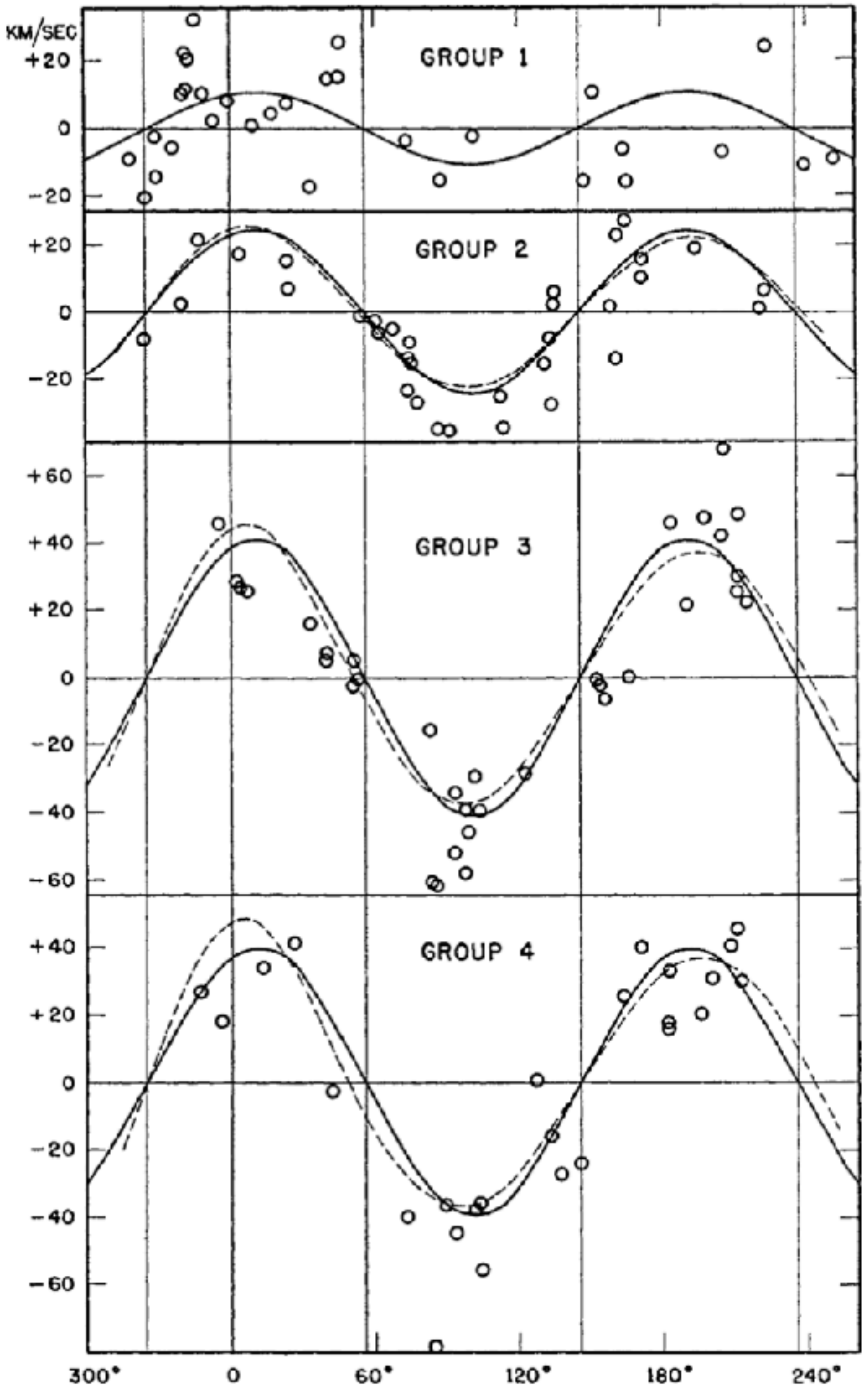
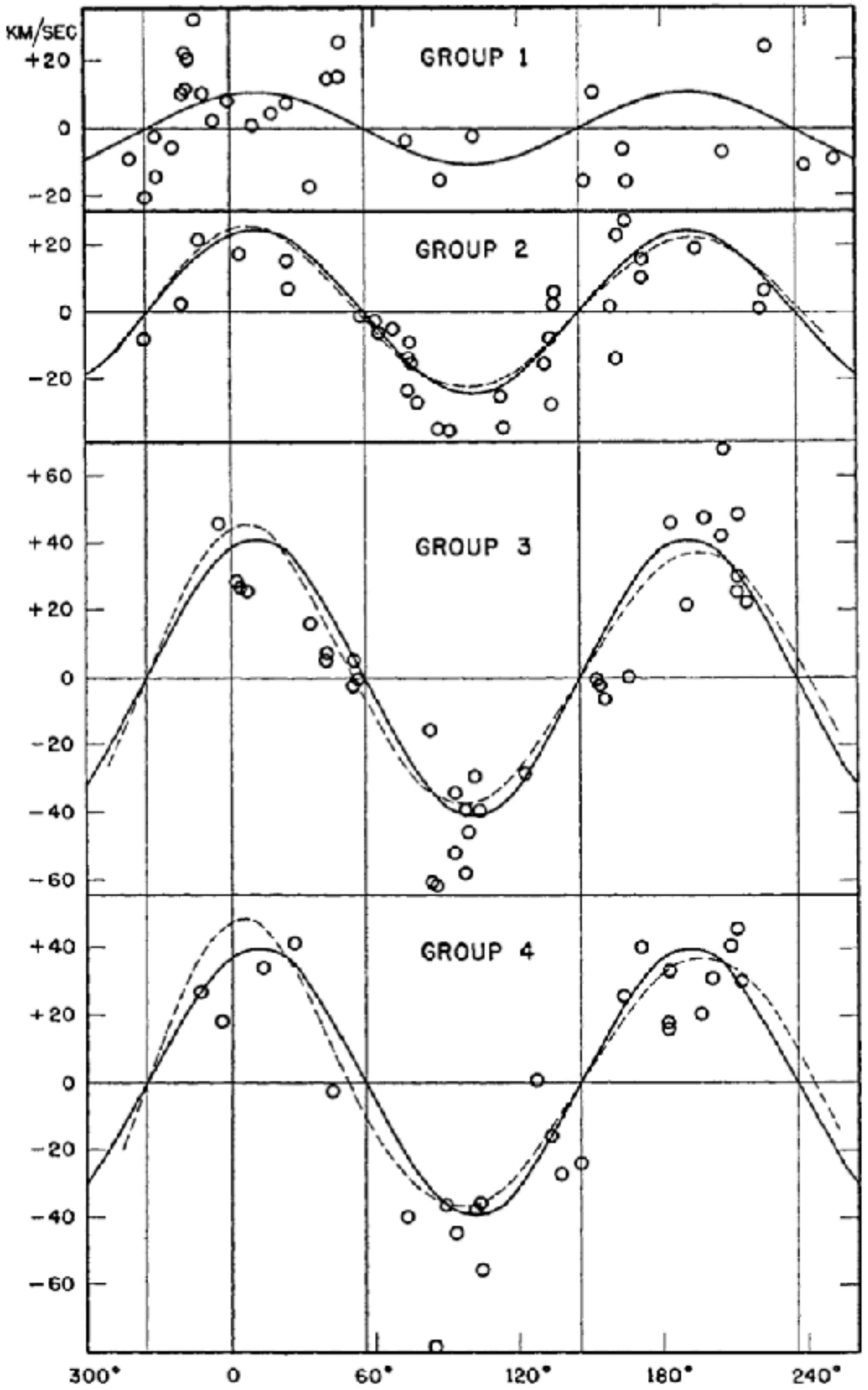


Fig. 6.5 Radial velocities of four groups of Cepheid variables located at four different distances. Note that the galactic coordinates indicated refer to the old system and not the presently used system (in which the galactic centre is taken as zero). From Joy (1939). (©American Astronomical Society. Reproduced with permission from *Astrophysical Journal*.)

Galactic rotation

- Since the amplitude of the oscillation is Ad, we can **find the Oort constant A if we know d.**
- To determine the other Oort constant B, we need to find the tangential velocity v_T of many nearby stars with respect to some non-rotating frame (such as the frame provided by extragalactic objects). The determination of B is more difficult than the determination of A.
- The unit in which we should be expressing A and B. Both A and B are obtained by dividing velocities by distances. Since stellar velocities are usually expressed in km/s, whereas galactic distances are expressed in kpc, it has been the convention to express A and B in units of **km s^{-1} kpc^{-1}**.
- The newest values of the **Oort constants** are obtained from the proper motion measurements by the Gaia satellite. They are
  - **$A = 15.3 \pm 0.4$ km s^{-1} kpc^{-1}, $B = -11.9 \pm 0.4$ km s^{-1} kpc^{-1}**
  - Values are from 2018



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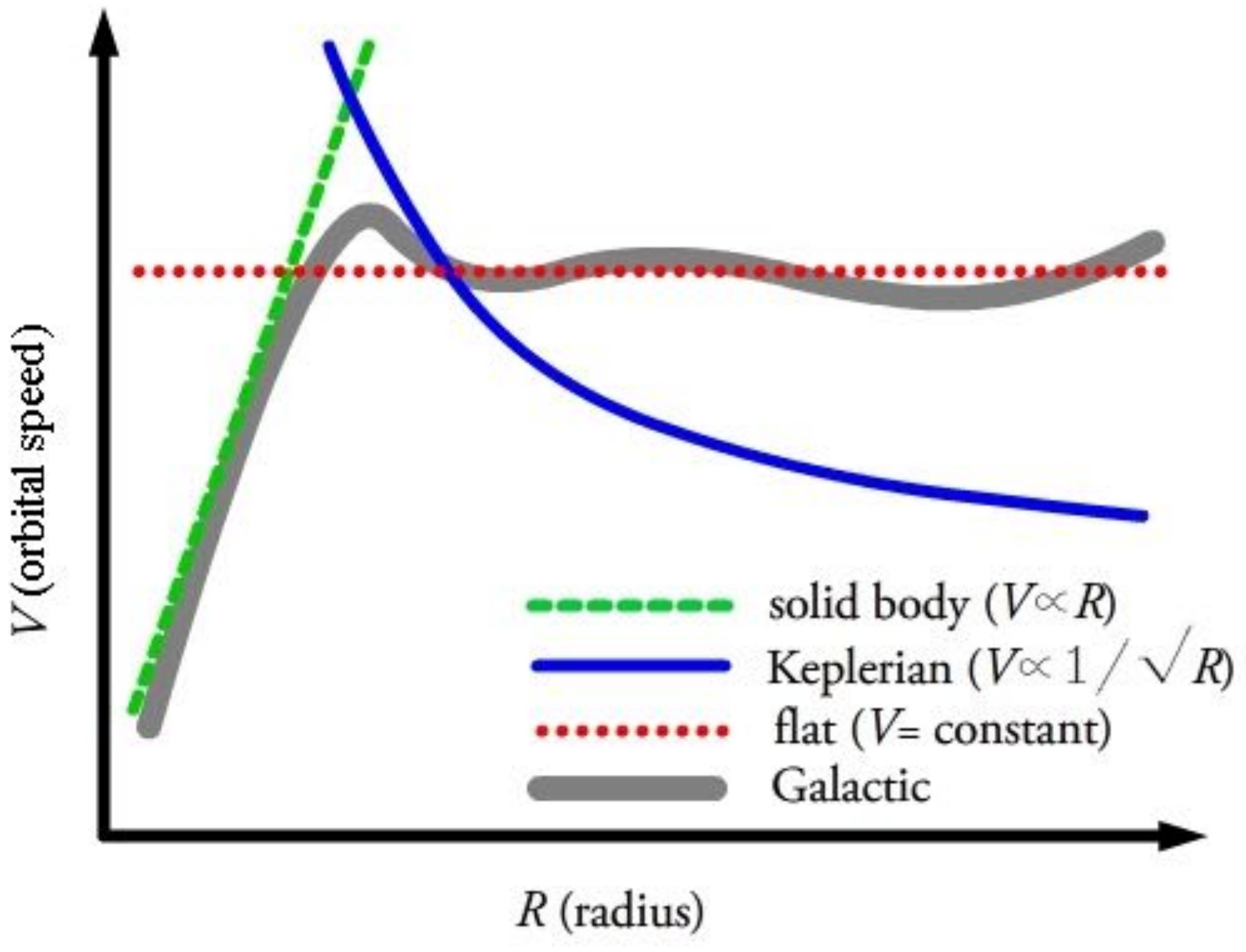
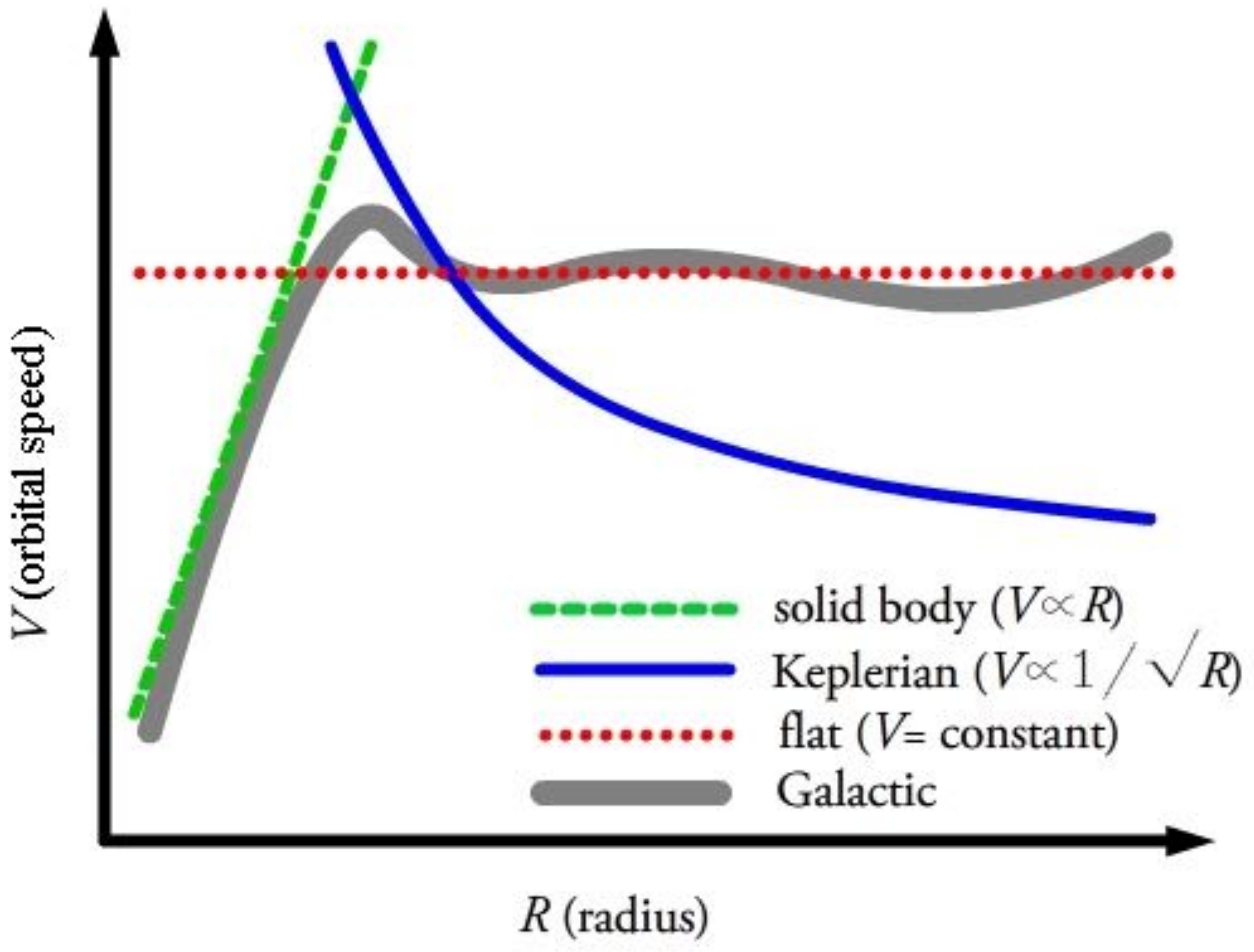
Galactic rotation

- **The Oort constants can tell us how the Galaxy rotates.**
- A and B are both functions of the Sun's orbital velocity as well as the first derivative of the Sun's velocity.
- As a result, **A describes the shearing motion in the disk surrounding the Sun,**
- while **B describes the angular momentum gradient in the solar neighborhood,** also referred to as vorticity.
- To illustrate this, one can look at three examples on how stars and gas orbit within the Galaxy:
    - solid body rotation,
    - Keplerian rotation and
    - constant rotation over different annuli.
- These three types of rotation are plotted as a function of radius (R), and are shown in Figure. The grey curve is approximately the rotation curve of the Milky Way.



Galactic rotation

- The solid body and the Keplerian examples can be interpreted as constraints to the Galactic rotation, for they show the fastest and slowest the Galaxy can rotate at a given radius.
- The **flat rotation curve** serves as an intermediate step between the two rotation curves, and in fact **gives the most reasonable Oort constants as compared to current measurements.**
- Flat rotation curves are also measured in other spiral galaxies - this is a general property of spiral galaxies
- **What does a flat rotation curve indicate?**



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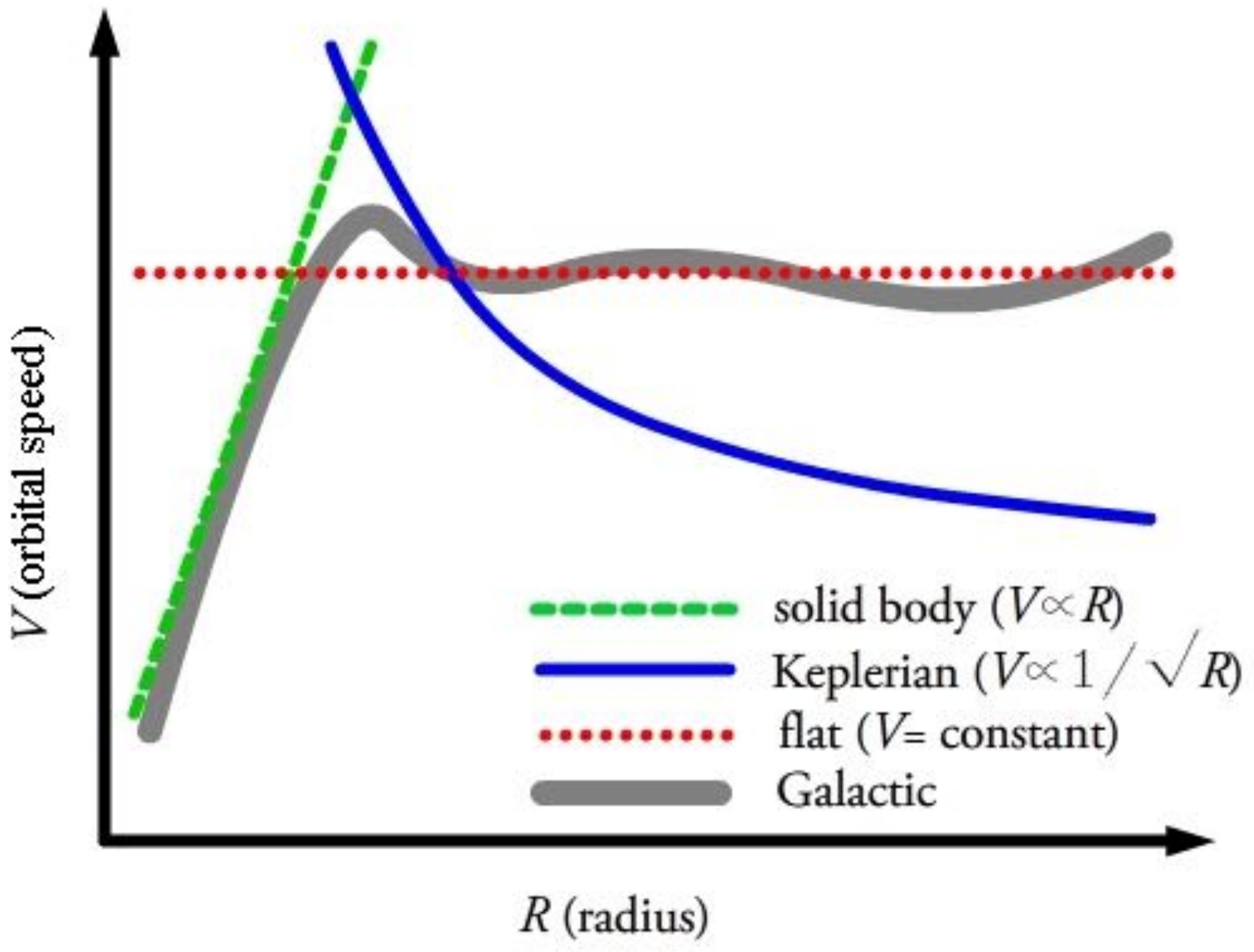
- **What does a flat rotation curve indicate?**
    - **The presence of dark matter (extra mass) in the galaxies**



Near circular orbits of stars

- We assumed that all stars in the galactic disk move exactly in circular orbits.
- In reality, however, **we do not expect most stars to move in exactly circular orbits**, just as planets in the solar system do not move in exactly circular orbits. We know that a planet moves in an ellipse, which is the orbit in a gravitational field falling as the inverse square of distance from the central mass.
- Since the mass of the Galaxy is not concentrated in a central region but distributed all over the Galaxy, we expect that the gravitational field will not follow a simple inverse-square law and the orbits in the galactic disk will not be simple ellipses.
- We now want to **find out the orbit of a star by assuming that the departure from a circular orbit is small**.
- Let $\Theta_{circ}(r)$ be the speed which a star will need to move in a circular orbit at a distance r from the galactic centre. If f_r **is the gravitational force at this distance** $\boldsymbol{r}$, then we must have

$$f_r = -\frac{\Theta_{\rm circ}^2}{r}$$

Near circular orbits of stars

$$f_r = -\frac{\Theta_{\mathrm{circ}}^2}{r}$$

- Let $\Theta_0 = \Theta_{circ}(R_0)$ be the circular speed where the Sun is located, at a distance R_0 from the galactic centre.
- We can think of a frame of reference at the **Sun's position moving with speed Θ_0 in a circular orbit around the galactic centre. This frame of reference is known as the *local standard of rest* (LSR).**
- If a star has a small velocity with respect to the LSR, then the orbit of the star can be found by determining its movements with respect to the LSR by using a perturbation technique.

Near circular orbits of stars

- We consider a star moving with speed Θ_0 in a circular orbit at distance R_0. Suppose the star is suddenly given a small kick in the radial direction. According to classical mechanics, its subsequent motion will be governed by the following equations

$$\ddot{r} - r\dot{\theta}^2 = f_r,$$
$$r^2\dot{\theta} = \text{constant}$$

Since the speed Θ in the θ direction is given by $\Theta = r\dot{\theta}$, the above two equations can be written as

$$\ddot{r} = \frac{\Theta^2}{r} - \frac{\Theta_{\text{circ}}^2}{r},$$
$$r\,\Theta = R_0\Theta_0,$$

Near circular orbits of stars

$$f_r = -\frac{\Theta^2_{\text{circ}}}{r}$$

To substitute for f_r and noting that the angular momentum of the star remains $R_0\Theta_0$, which did not change when we gave the star a radial kick. We now write

$$r = R_0 + \xi = R_0\left(1 + \frac{\xi}{R_0}\right)$$

and assume that $\xi \ll R_0$, since the star will not move too far away from the circular orbit $r = R_0$ after receiving the small radial kick. We shall neglect the quadratic and higher powers of ξ in our discussion.

$$r\,\Theta = R_0\Theta_0, \longrightarrow \frac{\Theta^2}{r} = \frac{R_0^2\Theta_0^2}{r^3} \approx \frac{\Theta_0^2}{R_0}\left(1 - \frac{3\xi}{R_0}\right)$$

Near circular orbits of stars

Keeping only the linear term in ξ . We can write

$$r = R_0 + \xi = R_0\left(1 + \frac{\xi}{R_0}\right) \longrightarrow \Theta_{\rm circ}(r) \approx \Theta_{\rm circ}(R_0) + \left(\frac{d\Theta}{dr}\right)_{R_0}\xi \approx \Theta_0 - (A+B)\xi,$$

where A and B are the Oort constants

$$\frac{\Theta_{\rm circ}^2}{r} = \frac{\Theta_0^2\left[1 - \frac{(A+B)}{\Theta_0}\xi\right]^2}{R_0\left(1 + \frac{\xi}{R_0}\right)} \approx \frac{\Theta_0^2}{R_0}\left[1 - \frac{2(A+B)}{\Theta_0}\xi - \frac{\xi}{R_0}\right]$$

Near circular orbits of stars

$\ddot{r} = \ddot{\xi}$ we can get the following approximate form

$$\ddot{\xi} = 2\frac{\Theta_0}{R_0}(A+B)\xi - 2\frac{\Theta_0^2}{R_0^2}\xi$$

$$\frac{\Theta_0}{R_0} = A - B \quad \longrightarrow \quad \ddot{\xi} = 4B(A-B)\xi$$

Which we can write as:

$$\ddot{\xi} + \kappa^2\xi = 0,$$

$$\kappa = \sqrt{-4B(A-B)}$$

is a real quantity because B is negative.

It is clear that **there will be a simple harmonic motion of the star in the radial direction with respect to the circular orbit $r = R_0$.**

Near circular orbits of stars

The radial velocity $\Pi = \dot{r}$ with respect to the LSR also should vary in a simple harmonic fashion and can be written as

$$\Pi = \Pi_0 \cos \kappa t$$

so that the displacement should be:

$$\xi = \frac{\Pi_0}{\kappa} \sin \kappa t$$

Near circular orbits of stars

The motion in the **θ direction.** From the constancy of the angular momentum $r^2\dot{\theta}$, we have

$$\dot{\theta} = \frac{R_0\Theta_0}{r^2} \approx \frac{\Theta_0}{R_0}\left(1 - \frac{2\xi}{R_0}\right)$$

Since the first term Θ_0/R_0 corresponds to the motion of the LSR, the part corresponding to the motion of the star with respect to the LSR is approximately given by

$$\Delta\dot{\theta} = -\frac{2\Theta_0\xi}{R_0^2}.$$

This translates into a linear velocity which, in the linear order in ξ , is

$$\Delta\Theta = (R_0 + \xi)\Delta\dot{\theta} = -\frac{2\Theta_0\xi}{R_0} = -\frac{2\Pi_0\Theta_0}{\kappa R_0}\sin\kappa t$$

Near circular orbits of stars

The corresponding **displacement** is

$$\eta = \frac{2\Pi_0\Theta_0}{\kappa^2 R_0}\cos\kappa t.$$

$$\frac{\Theta_0}{\kappa^2 R_0} = \frac{(A-B)}{-4B(A-B)} = \frac{1}{-4B} \longrightarrow \eta = \frac{\Pi_0}{-2B}\cos\kappa t.$$

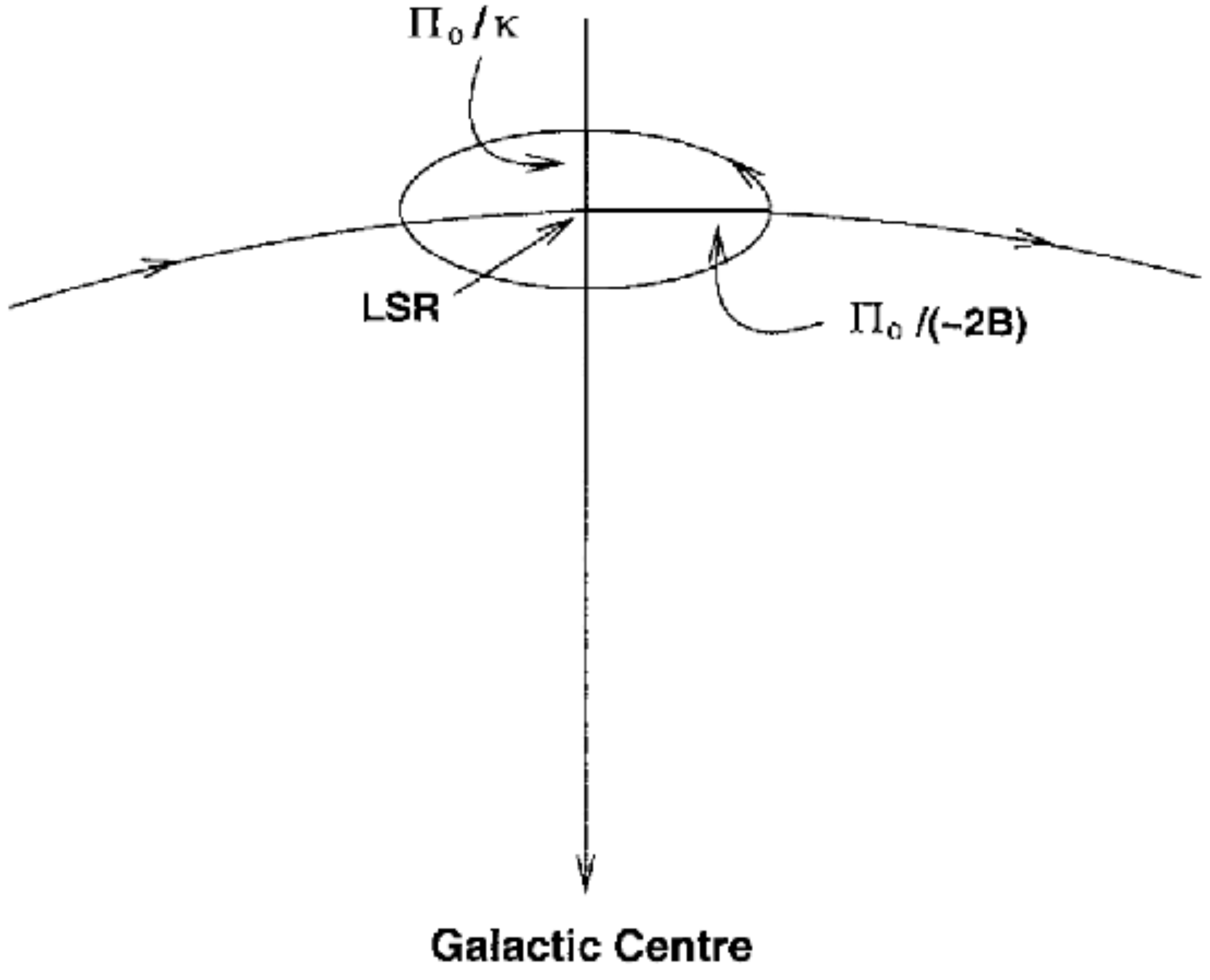


Fig. 6.6 A sketch showing the epicyclic motion of a star around the LSR.

In summary: **the star moves in an ellipse with respect to the LSR, while the LSR is revolving around the galactic centre,** as shown in Figure 6.6.

We call such motions ***epicyclic*.** The elliptical path of the star with respect to the LSR is called an ***epicycle***.

Near circular orbits of stars

The ratio of the semimajor axis (in the θ direction) to the semiminor axis (in the r direction) is

$$\frac{\Pi_0/2|B|}{\Pi_0/\kappa} = \sqrt{\frac{A-B}{|B|}}$$

Substituting for κ and putting values of A and B this ratio turns out to be 1.48.

So the **ellipse is elongated in the tangential direction**. The period of oscillation in the epicycle is related to the revolution period in the following way

$$\frac{P_{\rm osc}}{P_{\rm rev}} = \frac{2\pi/\kappa}{2\pi R_0/\Theta_0} = \frac{A-B}{\sqrt{-4B(A-B)}} = \frac{1}{2}\sqrt{\frac{A-B}{-B}}.$$

On putting the values of A and B, this ratio of periods is found to be 0.74 for stars in the solar neighbourhood. Since this ratio is not in general a rational number for a star at an arbitrary distance from the galactic centre, the **orbit of the star will not close**.

The solar motion

A star in the solar neighbourhood would not in general be at rest in the LSR, but would move in an epicycle with respect to the LSR.

Is the Sun at rest in the LSR?

The solar motion

- A star in the solar neighbourhood would not in general be at rest in the LSR, but would move in an epicycle with respect to the LSR.
- **Is the Sun at rest in the LSR?**
- we expect the answer to be ‘no’.
- **The motion of the Sun with respect to the LSR at the present epoch is called the *solar motion*.**
- This motion can be found out by studying the motions of the stars in the solar neighbourhood and by assuming that these stars do not have any net drift in the radial direction or perpendicular to the galactic plane. This implies

$$\langle \Pi \rangle = 0, \ \langle Z \rangle = 0.$$

- Here Z is the component of velocity perpendicular to the galactic plane and $\langle . \rangle$ implies averaging over stars in the solar neighbourhood.
- Π for a particular star varies sinusoidally and the same is true for Z . So it is no wonder that their averages will be zero.

The solar motion

- Let $(\Pi_\odot, \Theta_\odot - \Theta_0,\ Z_0)$ be the components of solar motion.
- From the Doppler shifts of spectral lines, we can find the line of sight velocity of a star with respect to the Sun, whereas the proper motion gives the velocity perpendicular to the line of sight. Combining these measurements, one can find out the components $\Pi - \Pi_\odot$ and $Z - Z_0$ of relative velocity, and then their averages over the stars in the solar neighbourhood.

$$\langle \Pi \rangle = 0,\ \langle Z \rangle = 0. \longrightarrow \begin{aligned} \langle \Pi - \Pi_\odot \rangle &= \langle \Pi \rangle - \Pi_\odot = -\Pi_\odot \\ \langle Z - Z_\odot \rangle &= -Z_\odot. \end{aligned}$$

- These averages give us the components of solar motion, which are found to be

$$\Pi_\odot = -10.0 \pm 0.4 \text{ km s}^{-1},\quad Z_\odot = 7.2 \pm 0.4 \text{ km s}^{-1}.$$

The solar motion

- Since the LSR itself does not have any Π or Z velocities, it is relatively easy to find the components of solar motion with respect to LSR in these directions. Now let us consider the θ direction. We certainly have

$$\langle \Theta - \Theta_{\odot} \rangle = -(\Theta_{\odot} - \langle \Theta \rangle),$$

- which can be found out from the measurements of stellar velocities with respect to the Sun.
- Now, if $\langle \Theta \rangle$ is equal to the velocity $\Theta_0 = \Theta_{circ}(R_0)$ of the LSR, then $\Theta_{\odot} - \langle \Theta \rangle$ would give the solar motion with respect to the LSR. But is it true that $\langle \Theta \rangle = \Theta_0$?
- From the epicycle theory it would seem that a star would simply oscillate forward and backward with respect to the LSR and $\langle \Theta \rangle$ averaged over many stars in the solar neighbourhood would give Θ_0.
- However, this result is a consequence of the assumption of linearity. If we go beyond the linear theory, then we find that the centre of the epicycle, known as the ***guiding centre***, moves slower than the LSR.

The solar motion

- Figure 6.6. Because of the curvature of the path of the guiding centre, the length of the epicycle path on the outside (i.e. away from the galactic centre) is larger than the length of the epicycle path on the inside.
- The velocity Θ is less when the star is in the outer part of the epicycle. Hence the star covers a longer path with a slower speed and the average Θ of the star should actually be less than Θ_0.

- Things can be even more complicated when we consider the fact that the guiding centres of different stars in the solar neighbourhood may lie at different distances from the galactic centre.
- The solar motion in the θ direction is given by

$$\Theta_\odot - \Theta_0 = -\langle(\Theta - \Theta_\odot)\rangle + \langle\Theta\rangle - \Theta_0$$

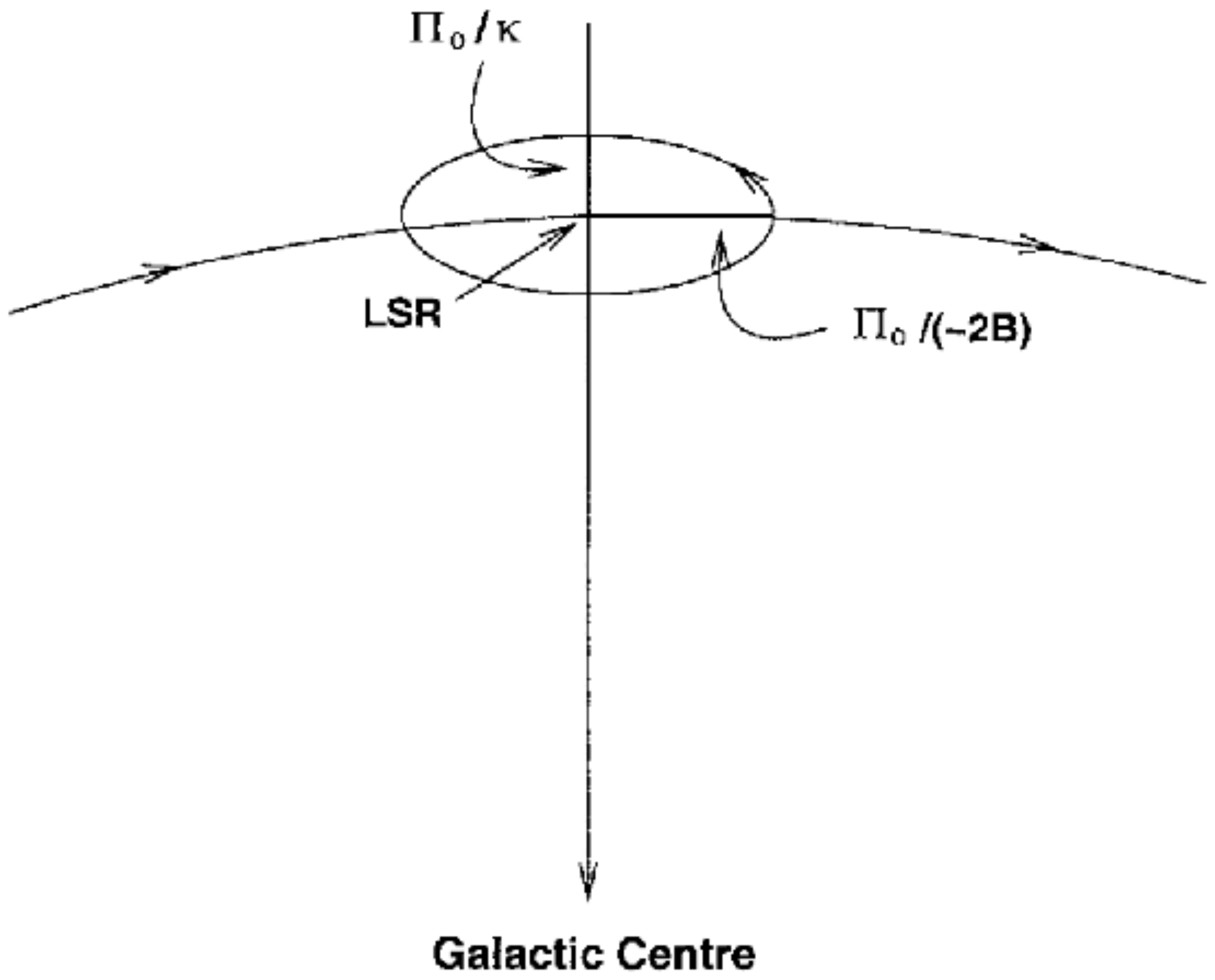


Fig. 6.6 A sketch showing the epicyclic motion of a star around the LSR.

The solar motion

$$\Theta_\odot - \Theta_0 = -\langle(\Theta - \Theta_\odot)\rangle + \langle\Theta\rangle - \Theta_0$$

Apart from $\langle(\Theta - \Theta_\odot)\rangle$, which is found from the observations of stellar motions in the solar neighbourhood, we need to know how $\langle\Theta\rangle$ differs from the velocity of LSR Θ_0 to find out the solar motion in the θ direction.

The final result:

$$\Theta_\odot - \Theta_0 = 5.2 \pm 0.6 \text{ km s}^{-1}$$

- **The amplitude of the solar motion is of order 10 km s^{-1}.**
- The typical random velocity of a star in the solar neighbourhood is also of this order.
- Then the amplitude of oscillation in the radial direction, which is given by Π_0/κ should be of order 1 kpc on taking Π_0 to be of order 10 km s^{-1}.

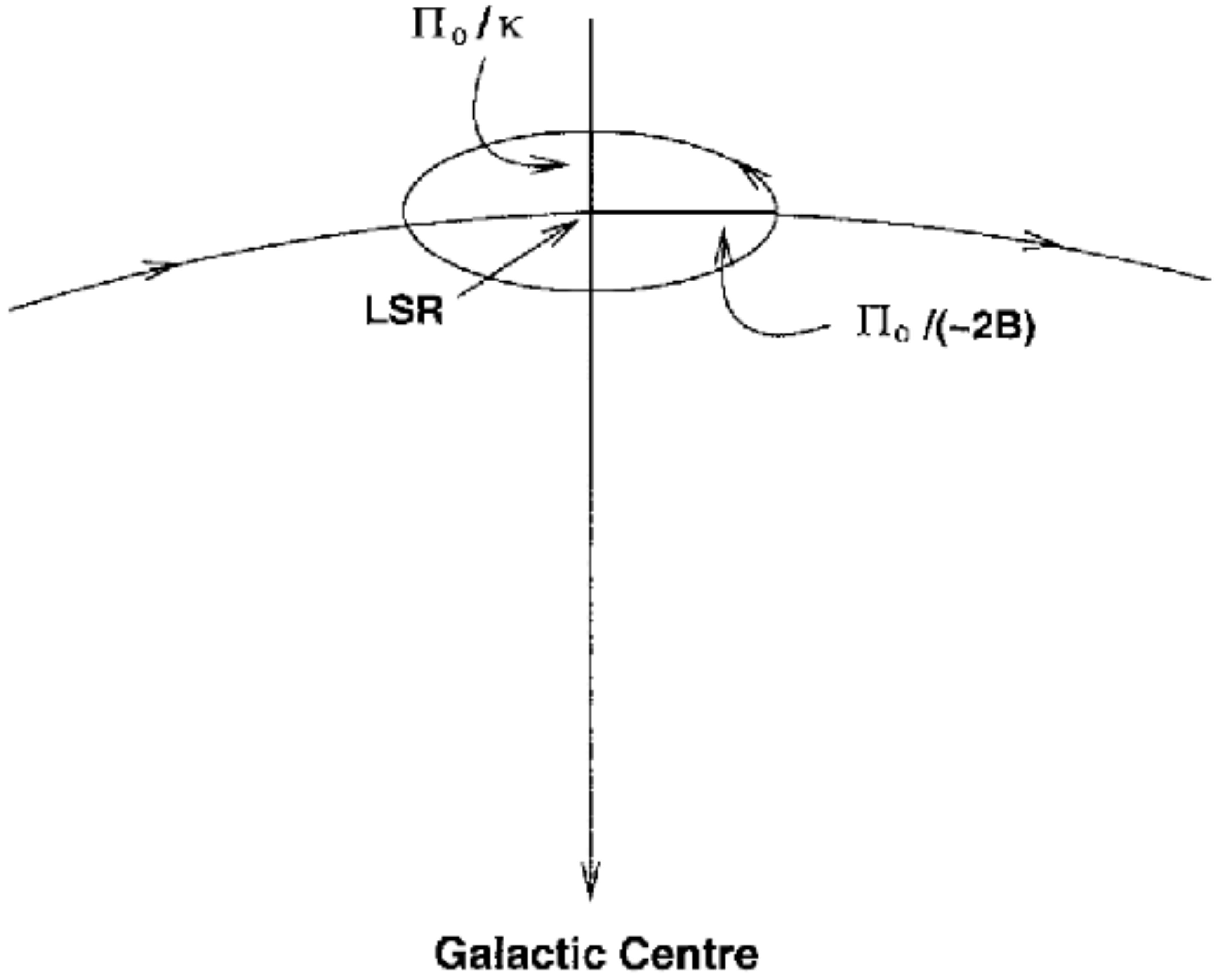
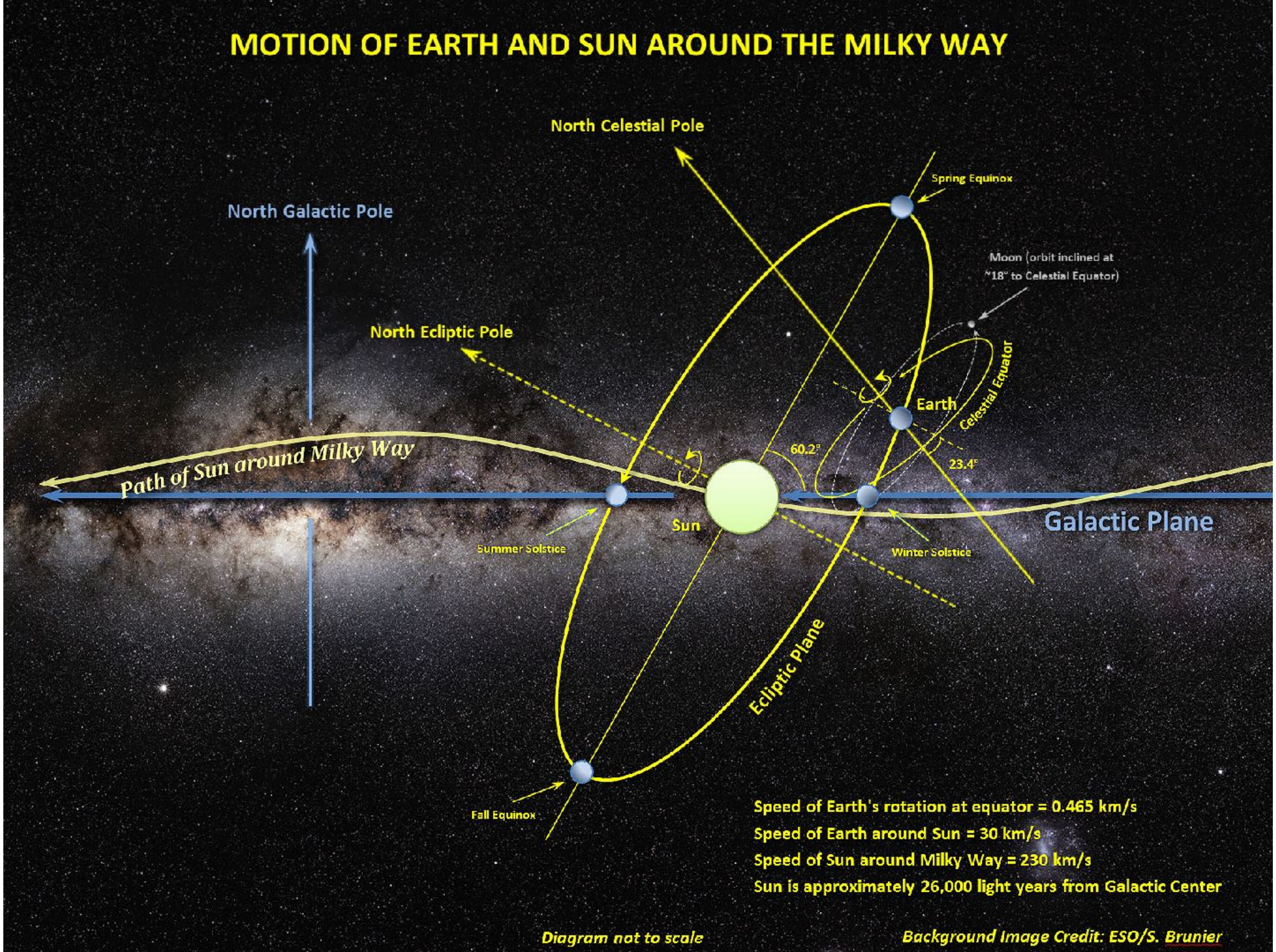


Fig. 6.6 A sketch showing the epicyclic motion of a star around the LSR.


MOTION OF EARTH AND SUN AROUND THE MILKY WAY
North Celestial Pole
Spring Equinox
North Galactic Pole
Moon (orbit inclined at ~18° to Celestial Equator)
North Ecliptic Pole
Celestial Equator
Earth
60.2°
23.4°
Path of Sun around Milky Way
Sun
Galactic Plane
Summer Solstice
Winter Solstice
Ecliptic Plane
Fall Equinox
Speed of Earth's rotation at equator = 0.465 km/s
Speed of Earth around Sun = 30 km/s
Speed of Sun around Milky Way = 230 km/s
Sun is approximately 26,000 light years from Galactic Center
Diagram not to scale
Background Image Credit: ESO/S. Brunier

Stellar populations

- Our Galaxy contains two subsystems.
  - One subsystem consists of **stars in the disk** which revolve around the galactic centre in nearly **circular orbits**. The **interstellar matter** also revolves around the galactic centre with these stars and belongs to this subsystem.
  - The other subsystem contains the **globular clusters** which have no systematic rotation around the galactic centre. The **bulge around the galactic centre** belongs to this subsystem. For stars in the spheroidal component also, gravity is balanced by **random motions**, since these stars have very little systematic rotation. Additionally, the Galaxy has a **non-rotating halo of stars.** Although the density of stars in the halo is much less than the density of stars in the disk, even in the solar neighbourhood we see a handful of stars with high random velocities which presumably belong to the halo.
- There are several distinct differences between the physical characteristics of these two subsystems. The stars in the non-rotating subsystem consisting mainly of the globular clusters and the **spheroidal component are mainly very old stars.**
- The bright O and B stars, which are short-lived, are not found in this subsystem, where formation of new stars does not take place. HR diagrams of globular clusters -> very old systems with no very luminous stars.

Stellar populations

- On the other hand, **star formation out of interstellar matter continuously takes place in the rotating subsystem** comprising stars and interstellar matter in the galactic disk. We see **O and B stars in this subsystem.**
- Finally, the **stars in the non-rotating subsystem are deficient in ‘metals’** (i.e. elements heavier than He such as C, N and O, which are called ‘metals’ by astronomers) compared to stars in the galactic disk belonging to the rotating subsystem.
- Heavier elements are produced inside stars and get strewn in the interstellar matter when massive stars undergo supernova explosions. **With more and more supernova explosions, the interstellar matter of the Galaxy is getting more enriched with these heavier elements.**
- The **old stars of the non-rotating subsystem must have formed from a primordial interstellar matter** which was not yet rich in metals.
- The **stars in the other rotating subsystem are younger** and formed out of interstellar matter after it became enriched with metals.

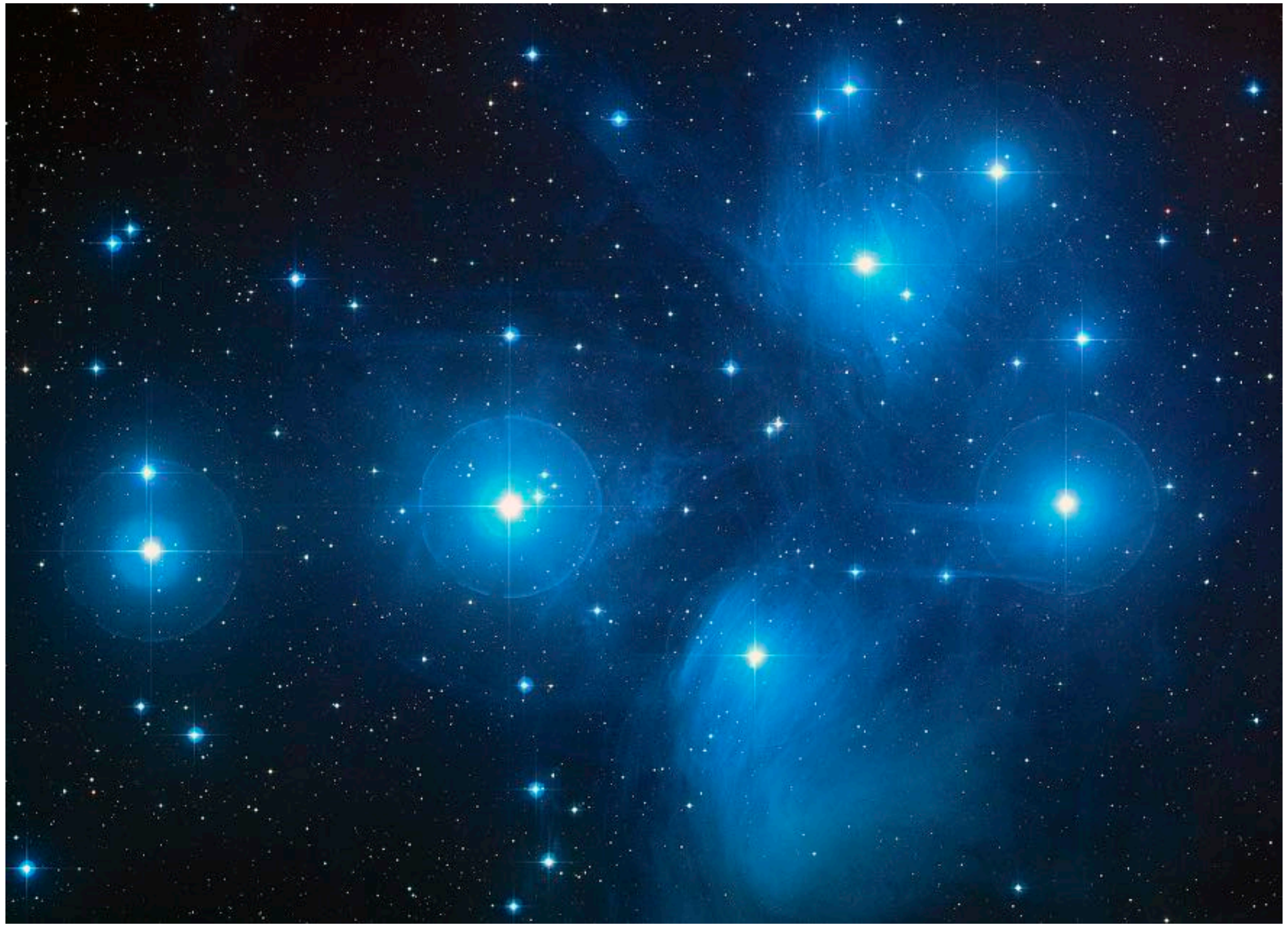
Stellar populations

- Stellar populations are based on these considerations.
- The **Population I** stellar systems are relatively **metal-rich**, contain interstellar matter and very bright O/B stars, and revolve around the galactic centre to balance the pull of gravity. These are relatively **young stars**. The galactic disk is the prime example of a Population I system.
- The **Population II** stellar systems are comparatively **metal-poor,** contain no interstellar matter or O/B stars, and counteract the gravitational field of the Galaxy by having random motions. These are **old stars**. The globular clusters along with the spheroidal bulge and the halo belong to Population II.
- Since we believe that all stars form out of interstellar matter, even Population II systems must have contained interstellar matter at some early stage out of which they formed, even though now they do not have much interstellar matter any more. Presumably, all the interstellar matter has been used up in forming stars.
- **Population III** stars - are the very **first stars in the Universe** and should not contain any metals. However, since they are the oldest stars it is extremely difficult to find such stars. Not yet found with observations.

Stellar populations

Population II stars - metal poor, old

Population I stars - metal rich, young



Pleiades open cluster

M87 globular cluster

Stellar populations

- Population I stars in the disk
- Population II stars in the halo and the bulge