Basic Astrophysics - Summary

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This summary is based on the book Chapters 2 from Arnab Rai Choudhuri: Astrophysics for physicists.

1 Radiation Transfer

Radiation Transfer describes how radiation interacts with matter

- Macroscopic: using emission and absorption coefficients Microscopic: calculating the emission and absorption coefficients

Radiation field, simple case: **blackbody radiation** (homogeneous and isotropic inside a container)

Planck's law - specifies energy density

$$U_\nu d\nu = \frac{8\pi h}{c^3} \frac{\nu^3 d\nu}{e^{\frac{h\nu}{\kappa_B T}} - 1}$$

In general the radiation is not isotropic and we need to consider the direction.

$$dE_\nu d\nu = I_\nu cos\Theta dAdtd\Omega d\nu$$

Radiation Flux:

$$F_\nu = \int I_\nu cos\Theta d\Omega$$

$$F = \int F_\nu d\nu$$

Energy density: density of energy in a cylinder filled with radiation

$$U_\nu = \int \frac{I_\nu}{c} d\Omega$$

Radiation Pressure:

$$P_\nu = \frac{1}{3} U_\nu$$

1.1 Radiative transport

The specific intensity due to a source is essentially its intensity divided by the solid angle it subtends, which means that specific intensity is a measure of the surface brightness.

How does radiation propagate trough matter.

- in empty space: radiation does not change
- trough matter: emission and absorption by the matter
- all matter has the following properties:
  - emission coefficient j_ν
  - absorption coefficient α_ν
- The same material can simultaneously emit and absorb radiation.

Radiation transfer equation:

$$\frac{dI_\nu}{ds} = j_\nu - \alpha_\nu I_\nu$$

If the matter only absorbes: $j_\nu = 0$

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu$$

Optical depth:

$$d\tau_\nu = \alpha_\nu ds \rightarrow \tau_\nu = \int \alpha_\nu(s')ds'$$

$$I_\nu \tau_\nu = I_\nu(0)e^{-\tau_\nu}$$

where $I_\nu(0)$ is the initial intensity of the radiation.

If $\tau_\nu \geq 1$ the material is optically thick.
If $\tau_\nu << 1$ the material is optically thin.

Source Function: describes the absorption and emission properties of matter.

$$S_\nu = \frac{j_\nu}{\alpha_\nu}$$

Kirchof's law: In thermodynamical equilibrium:

$$S_\nu = B_\nu(T)$$

$$j_\nu = \alpha_\nu B_\nu(T)$$

both j_ν and α_ν are expected to have peaks at spectral lines.
The radiation coming out of an optically thin source is essentially determined by its emission coefficient. Since the emission coefficient is expected to have peaks at spectral lines, we find the emission from an optically thin system like a hot transparent gas to be mainly in spectral lines.
On the other hand, the specific intensity of radiation coming out of an optically thick source is its source function, which has been shown to be equal to the blackbody function $B_\nu(T)$. Hence we expect an optically thick object like a hot piece of iron to emit roughly like a blackbody.
The nature of radiation from an astrophysical source crucially depends on whether the source is optically thin or optically thick. Often we see a combination of blackbody radiation and spectral lines.

In thermodynamical equilibrium:

- Blackbody radiation
- Maxwell velocity distribution
- Boltzmann distribution law
- Saha equation

Maxwellian velocity distribution: Different particles in a gas move around with different velocities. If the gas is in thermodynamic equilibrium with temperature T , then the number of particles having speeds between v and v + dv

$$dn_v = 4\pi n \left(\frac{m}{2\pi\kappa_B T}\right)^{3/2} v^2 e^{-\frac{mv^2}{2\kappa_B T}} dv$$

If n_0 is the number density of atoms in the ground state, then the number density n_e of atoms in an excited state with energy E above the ground state $\rightarrow$ this is called the Boltzmann distribution law.

$$\frac{n_e}{n_0} = e^{-\frac{E}{\kappa_B T}}$$

The **Saha equation** gives what fraction of a gas will be ionized at a certain temperature T and pressure P.

The condition of validity of the Maxwellian velocity distribution (as well as the Boltzmann equation and the Saha equation) is that the mean free path has to be small enough such that the temperature does not vary much over the mean free path. When collisions are frequent, the **mean free path** turns out to be small, then particles in a gas will interact with each other more effectively and we expect that principles like the Maxwellian velocity distribution, the Boltzmann equation or the Saha equation will hold.
Its inverse α_ν^{-1} gives the distance over which a significant part of a beam of radiation would get absorbed by matter. This is referred to as the **mean free path of photons**. If α_ν^{-1} is sufficiently small such that the temperature can be taken as constant over such distances, then we expect Planck's law of blackbody radiation

to hold. If both α_ν^{-1} and the mean free path of particles are small compared to the length over which the temperature varies significantly than we have local thermodynamic equilibrium (LTE).
Examples of LTE: Inside a star, in the interstellar medium

2 Radiative transport trough stellar atmospheres

Plane parallel atmosphere: When we focus on a local region of a stellar atmosphere, we can neglect the curvature and assume the various thermodynamic quantities like the temperature T to be constant over horizontal planes.

The total flux is the flux which eventually emerges out of the surface and is given by the Stefan-Boltzmann law:

$$F = \sigma T_{eff}^4 \leftarrow \sigma = \frac{ca_B}{4}$$

The radiation field is nearly isotropic in sufficiently deep layers of a stellar atmosphere where the temperature is considerably higher than the surface temperature.

Grey atmosphere: If the **absorption coefficient** α_ν **is constant for all frequencies**, then the atmosphere is called a grey atmosphere. There is no real stellar atmosphere which has this property. The grey atmosphere is an **idealised mathematical model** which is much simpler to treat than a more realistic stellar atmosphere and gives us some insight into the nature of the problem

Limb darkening: the intensity emerging from the surface of a star depends on direction. E.g. the edges (the limb) of the Solar disk appear darker than the centre of the disk.

2.1 Formation of spectral lines

In the case of the general non-grey stellar atmosphere, we have frequency-dependent equations. A constant energy passes through the layers of stellar atmosphere, but the energy continuously gets redistributed amongst different frequencies. There are two components for the absorption:

- true absorption (frequency of γ changes)
- scattering (frequency of γ does not changes)

The spectrum in the continuum region will be like the blackbody spectrum. For frequencies within a spectral line the, the optical depth becomes unity at a shallower depth, where the temperature has a lower value.

Equivalent width: We can estimate the strength of the spectral line by integrating the fractional dip of the line over the spectral line.

$$W_\lambda = \int \frac{I_c - I_\nu}{I_c} d\lambda$$

We can use the equivalent width to determine the column density.

2.2 Radiative energy transport inside stars

We can also describe the absorbing property of material using the **opacity** (κ).

$$\alpha_R = \rho\kappa$$

, where α_R is the average absorption coefficient and ρ is the density.

Kramer's law: Describes how opacity changes inside a star.

$$\kappa \propto \frac{\rho}{T^{3.5}}$$

However, Kramer's law is not valid at low temperatures.

Thompson scattering: Free electrons scattering photons. The electric field of the electromagnetic wave will force the electron to undergo an oscillatory motion. A charge in an oscillatory motion emits electromagnetic waves. The energy of this emitted wave must come from the energy of the incident electromagnetic wave.
Examples:

- When starlight passes through interstellar dust, it also becomes redder due to the selective scattering of blue light by the dust particles.
- Thompson scattering in the Solar corona.

Photon diffusion inside the Sun $\rightarrow$ it takes about 10^4 years for γs to reach the surface of the Sun.