Introduction to Astrophysics and Cosmology

Space-time dynamics of the Universe

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Content of the Universe, the CMB

As pointed out in §10.4, we need to specify **how the density ρ of the Universe varies with the scale factor a** in order to solve for a as a function of time. We **have to consider the contents of the Universe** for this purpose.

Before we get into the discussion of the specific contents, let us consider a fluid filling the Universe with density ρ and equivalent energy density ρc^2. Since ρc^2 has the dimension of pressure, we can write the pressure due to this fluid as

$$P = w\rho c^2. \tag{10.36}$$

We consider a volume a^3 of the Universe which is increasing with time. The total internal energy inside this volume is $\rho c^2 a^3$. Assuming the expansion to be adiabatic, the first law of thermodynamics $dQ = dU + P\,dV$ leads to

$$d(\rho c^2 a^3) + w\rho c^2\, d(a^3) = 0,$$

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from which it follows that

$$\rho \propto \frac{1}{a^{3(1+w)}}. \tag{10.37}$$

If we know the appropriate value of w appearing in (10.36) for a particular component of the Universe, (10.37) tells us **how the density of that component will vary with *a*.** One standard result of the kinetic theory of gases is that the **pressure of a gas** is given by

$$P = \frac{1}{3}\rho\overline{v^2},$$

where v is the molecular velocity. Comparing with (10.36), we find that

$$w = \frac{1}{3}\frac{\overline{v^2}}{c^2}.$$

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For a non-relativistic gas, we have

$$w \approx 0, \ \rho \propto \frac{1}{a^3}, \tag{10.39}$$

whereas for a gas of relativistic particles all moving around with speed c,

$$w \approx \frac{1}{3}, \ \rho \propto \frac{1}{a^4}. \tag{10.40}$$

Although matter in the Universe is distributed in a **hierarchy of structures**, we pointed out in §9.6 that the **matter distribution starts looking homogeneous when we go to scales larger than about $100h^{-1}$ Mpc**, in accordance with the cosmological principle.

If the luminous stars constituted all the matter in the Universe, then a careful analysis of observational data indicates that the density parameter Ω defined in (10.30) would be of order

$$\Omega_{\rm Lum} \approx 0.01. \tag{10.41}$$

Is this a good estimation?

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However, rotation curves of galaxies suggest a significant amount of dark matter beyond the stellar disks of galaxies. The application of the virial theorem to galaxy clusters suggests even larger amounts of dark matter. **The density parameter estimated from the virial masses of galaxy clusters** turns out to be independent of the uncertainties in the Hubble constant and is of order

$$\Omega_{\mathrm{M},0} \approx 0.3. \tag{10.42}$$

The subscript 0 implies that this is the present value of the density parameter due to matter, which can have different values at other epochs.

We shall discuss other independent arguments in §14.5 that $\Omega_{M,0}$ indeed has this value. From (10.29) and (10.30), the present matter density should be

$$\rho_{\mathrm{M},0} = 1.88 \times 10^{-26} \Omega_{\mathrm{M},0} h^2 \text{ kg m}^{-3}. \tag{10.43}$$

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Using (10.39), we can now write down the matter density at an arbitrary epoch in the form

$$\rho_M = \rho_{M,0} \left(\frac{a_0}{a}\right)^3 , \tag{10.44}$$

where $a_0$is the value of the scale factor at the present epoch and a its value at that arbitrary epoch.

Hubble's law (9.13) implies that during the *Big Bang* the physical parameters of the Universe like its density and its temperature were infinite. According to straightforward considerations, it is not possible to extrapolate our currently understood physical laws to times earlier than the epoch of the Big Bang.

We know that **hot matter emits radiation.** Since the **early Universe** must have been dense and hot, it would have been **filled with radiation** existing in **thermodynamic equilibrium with matter.**

Radiation in equilibrium with matter **has to be blackbody radiation**.

As the **Universe expanded** and its density fell, at some stage the Universe **became transparent to radiation** and the radiation ceased to be in equilibrium with matter.

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As the **Universe kept expanding** after this decoupling, the **radiation would undergo adiabatic expansion** because it ceases to interact with matter any more. One of the important results from the thermodynamics of blackbody radiation is that blackbody radiation **continues to remain blackbody radiation** under adiabatic expansion, **although its temperature keeps decreasing** with expansion.

The general relativistic analysis of light propagation in the expanding Universe also leads to the same conclusion that the radiation continues to remain blackbody radiation even though it is not interacting with matter any more.

We thus expect on theoretical grounds that **the Universe should still be filled with a blackbody radiation background,** which will be **cooling with the expansion of the Universe**.

Alpher and Herman (1948) were the first to point this out and predicted that the present temperature of this blackbody radiation **should be of order 10 K**.

Without being aware of this theoretical prediction, Penzias and Wilson (1965) **accidentally discovered this radiation** which seemed to have a **temperature of 3 K**. Since much of this blackbody radiation at 3 K lies in the microwave range, it is called the ***cosmic microwave background radiation*, abbreviated as *CMBR***.

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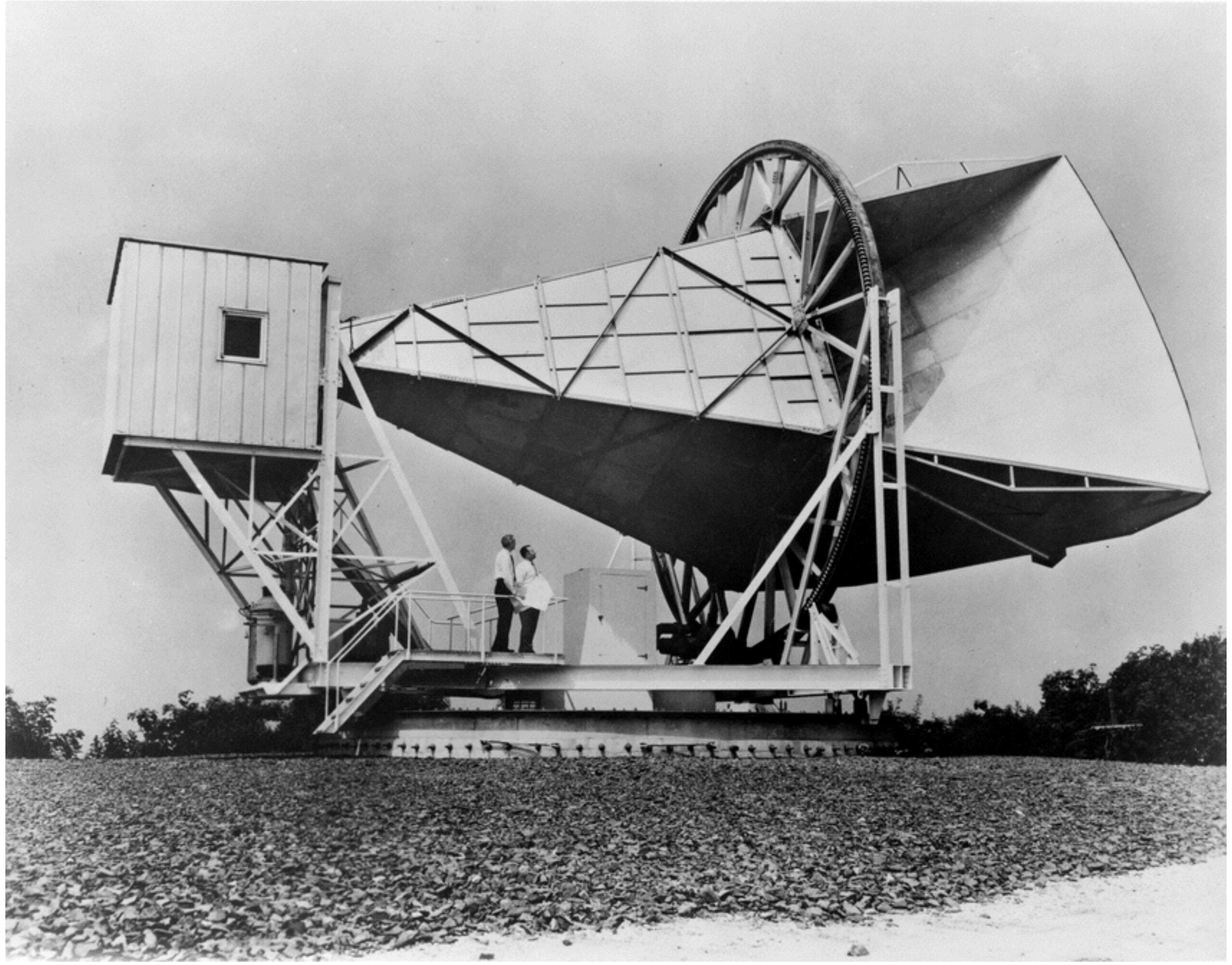
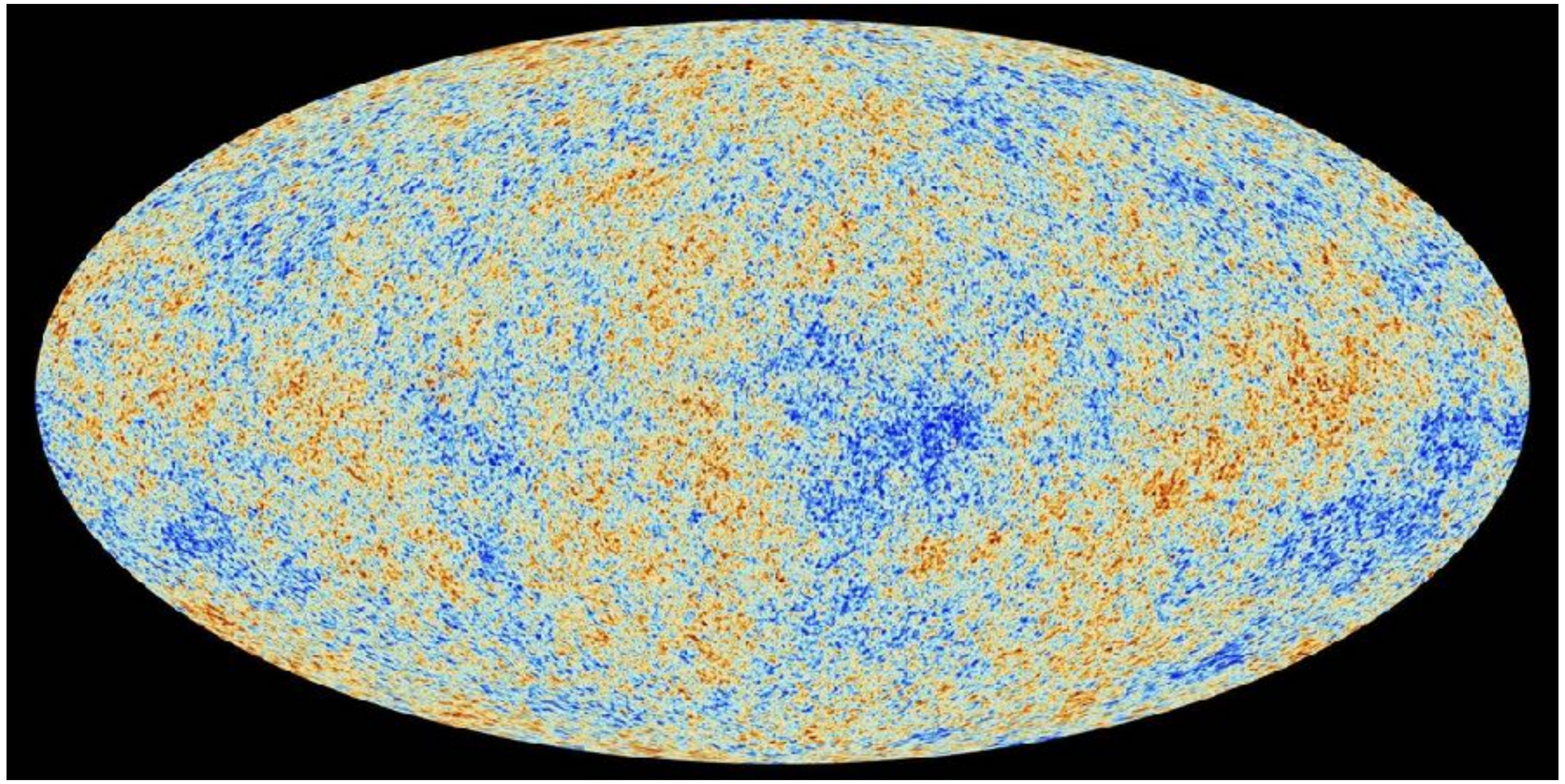


Horn antenna used in 1964 by Penzias and Wilson to discover the CMB. (Credit: NASA image)

The Planck satellite



CMB measurement from the Planck satellite

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Lemaitre (1927) was the first person to argue that the Universe must have begun from a hot Big Bang.

The CMBR is a remnant of this Big Bang and its existence provided reasonably compelling proof (at least compelling enough to most astrophysicists) that there was really a hot Big Bang from which the Universe was born.

Penzias and Wilson (1965) were able to measure only a small part of the CMBR spectrum.

The satellite COBE (Cosmic Background Explorer) was launched in 1989 to study the CMBR in detail. Figure 10.4 shows the spectrum of CMBR measured by COBE.

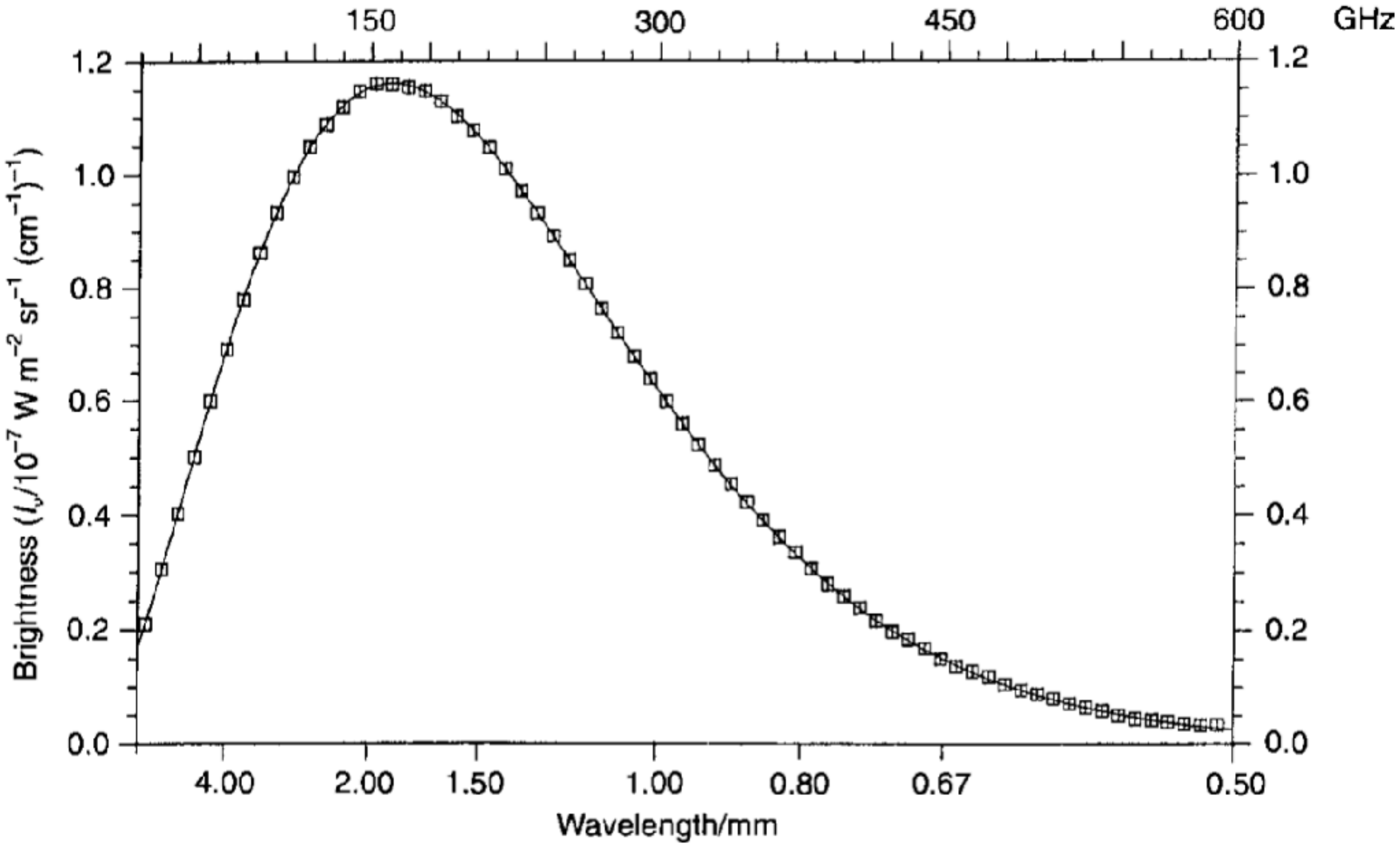

Fig. 10.4 The spectrum of cosmic microwave background radiation (CMBR) as obtained by COBE. From Mather *et al.* (1990). (©American Astronomical Society. Reproduced with permission from *Astrophysical Journal*.)

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It is a fantastic fit to the Planck spectrum (2.1) for blackbody radiation at temperature

$$T_0 = 2.735 \pm 0.06 \text{ K}. \qquad (10.45)$$

We know that the energy density of blackbody radiation at temperature T is given by $a_B T_0^4$, where a_B is Stefan's constant. Hence the contribution of CMBR to the density of the Universe is

$$\rho_\gamma = \frac{a_\mathrm{B}}{c^2} T^4. \qquad (10.46)$$

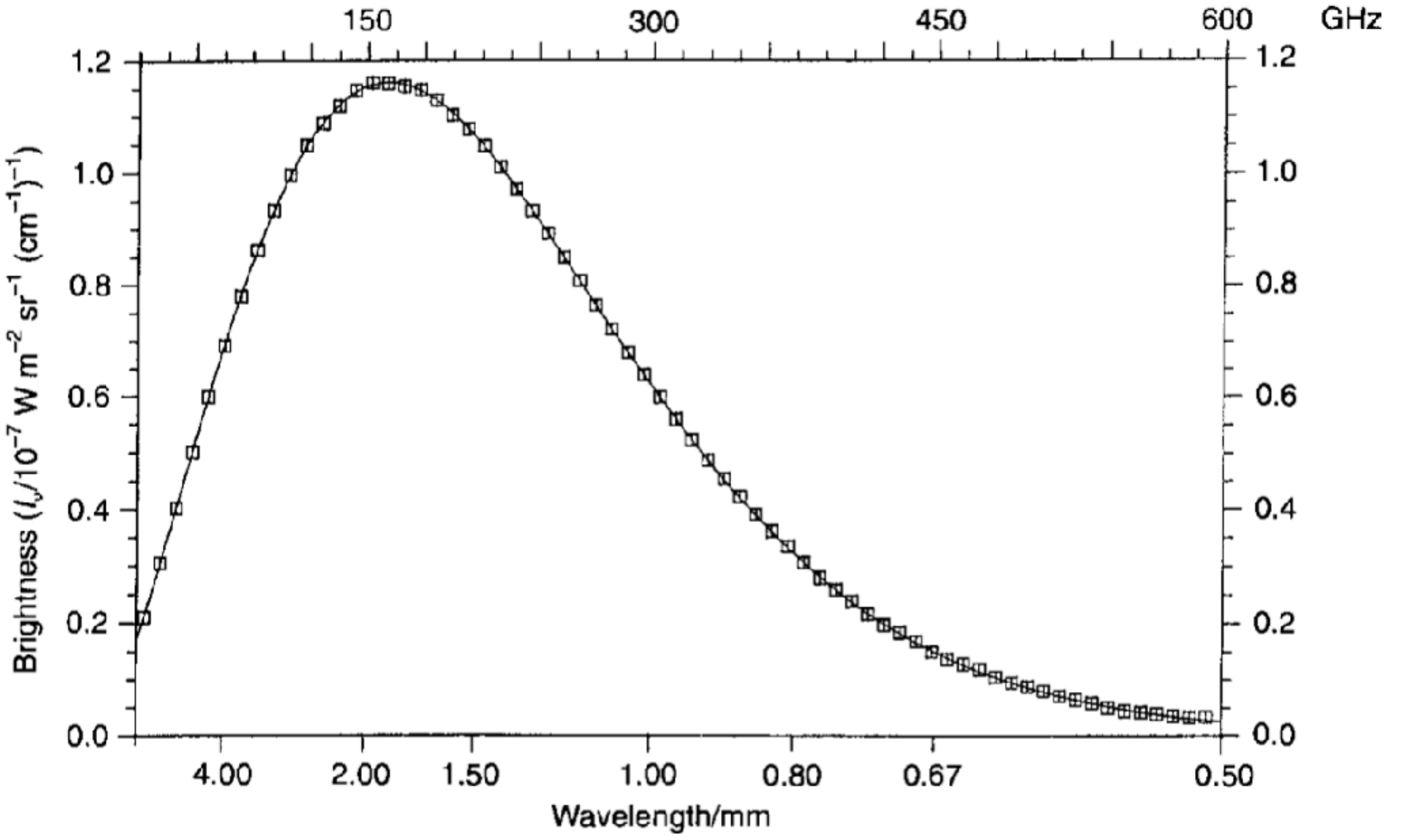

Fig. 10.4 The spectrum of cosmic microwave background radiation (CMBR) as obtained by COBE. From Mather *et al.* (1990). (©American Astronomical Society. Reproduced with permission from *Astrophysical Journal*.)

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The Universe is expected to have a **background of neutrinos** in addition to the background of photons in the CMBR. The detailed properties of the neutrino background will be discussed later. For the time being, let us only mention that the **total density of relativistic background particles** (photons and neutrinos together) at the present epoch will be shown to be

$$\rho_{R,0} = 1.68\rho_{\gamma,0}, \qquad (10.47)$$

where $\rho_{\gamma,0}$ is the present-day contribution of CMBR to the density of the Universe. Since the CMBR photons and the neutrinos are both relativistic gases, we expect (10.40) to hold for both so that we can write

$$\rho_R = \rho_{R,0}\left(\frac{a_0}{a}\right)^4. \qquad (10.48)$$

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Since the CMBR density ρ_γ separately would also fall as a^{-4}, it easily follows from (10.46) that

$$T \propto \frac{1}{a}, \tag{10.49}$$

which tells us **how the temperature of the CMBR must be falling with the expansion of the Universe**. The neutrino temperature also should fall in the same way, although its value will be different from the value of CMBR temperature at the same instant.

By summing up (10.44) and (10.48), the total density of the Universe can be written as

$$\rho = \rho_{\mathrm{M},0}\left(\frac{a_0}{a}\right)^3 + \rho_{\mathrm{R},0}\left(\frac{a_0}{a}\right)^4. \tag{10.50}$$

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In the following discussion, we shall refer to both photons and neutrinos as 'radiation'.

Some obvious conclusions can be drawn from the expression (10.50) for the density. **The radiation density (falling as a^{-4}) falls more rapidly than the matter density (falling as a^{-3}) with the expansion of the Universe.**

Going backwards in time, as we approach the Big Bang closer and closer, the radiation density must be rising faster than the matter density with increasingly smaller a. **At the present time**, we have

$$\boxed{\rho_{\mathrm{M},0} \gg \rho_{\mathrm{R},0}.}$$

However, there must be **a past epoch when the matter and radiation densities were equal,** before which the radiation was dominant. If a_{eq} was the value of the scale factor at that epoch of ***matter-radiation equality***, on equating (10.44) and (10.48) we get

$$\frac{a_0}{a_{\mathrm{eq}}} = \frac{\rho_{\mathrm{M},0}}{\rho_{\mathrm{R},0}} = \frac{\rho_{\mathrm{M},0}c^2}{1.68 a_{\mathrm{B}} T_0^4} \tag{10.51}$$

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where we have made use of (10.46) and (10.47). Substituting from (10.43) and (10.45), we get

$$\frac{a_0}{a_{\mathrm{eq}}} = 2.3 \times 10^4 \Omega_{\mathrm{M},0} h^2. \tag{10.52}$$

We can divide the history of the Universe into **two distinct periods.** At times earlier than the matter-radiation equality when the scale factor ***a* was smaller than** a_{eq} given by (10.52), the Universe is said to be ***radiation-dominated***. On the other hand, after the matter-radiation equality the **Universe has become *matter- dominated*.**

To study the evolution of the Universe, we now need to solve the Friedmann equation (10.27) after substituting for ρ from (10.50).

The **calculations,** however, **become much simpler** without introducing any significant error **if we assume** ρ **=** ρ_M given by (10.44) when we study the **matter-dominated period** of the Universe **and assume** ρ **=** ρ_R given by (10.48) when we study the **radiation-dominated period** of the Universe. It is possible to get analytical solutions in these simpler cases.

The matter-dominated Universe

We shall now solve the Friedmann equation (10.27) by taking $\rho = \rho_M$ given by (10.44) appropriate for the **matter-dominated Universe.** We need to consider the three cases $\boldsymbol{k = -1, 0, +1}$.

Let us first consider the case $k = 0$, for which the Friedmann equation leads to (10.32). On substituting from (10.44) in (10.32), we have

$$\dot{a} = \sqrt{\frac{8\pi G \rho_{\mathrm{M},0} a_0^3}{3}}\, a^{-1/2},$$

of which the solution is

$$\frac{2}{3} a^{\frac{3}{2}} = \sqrt{\frac{8\pi G \rho_{\mathrm{M},0} a_0^3}{3}}\, t \qquad (10.53)$$

on setting $t = 0$ at the epoch of the Big Bang when $a = 0$. For the $k = 0$ case, the density is equal to the critical density given by (10.28) so that

$$\rho_{\mathrm{M},0} = \frac{3H_0^2}{8\pi G}.$$

The matter-dominated Universe

On substituting this in (10.53), we get

$$\boxed{\frac{a}{a_0} = \left(\frac{3}{2} H_0 t\right)^{2/3}. \qquad (10.54)}$$

We thus reach the very important conclusion that **the size of the Universe increases with time as $t^{2/3}$**. This solution for the **$k = 0$ case** is often called the ***Einstein–de Sitter model***.

For the **cases $k = \pm 1$**, we use the quadrature formula (10.35) obtained from the Friedmann equation (10.27). On substituting (10.44) into (10.35), we get

$$\eta = \pm \int \frac{da}{\sqrt{\frac{8\pi G \rho_{M,0} a_0^3}{3c^2} a - k a^2}}. \qquad (10.55)$$

The closed solution k=+1

As already discussed in §10.4 and as should be clear from (10.31), this is the case **where the density is larger than the critical density**, i.e. $\Omega_{M,0} > 1$. **When k = +1**, the quadrature (10.55) can be worked out to give

$$a = \frac{4\pi G}{3c^2}\rho_{\mathrm{M},0}a_0^3(1 - \cos\eta).$$

This can be written as

$$\frac{a}{a_0} = \frac{1}{2}\left(\frac{8\pi G\rho_{\mathrm{M},0}}{3H_0^2}\right)\frac{a_0^2H_0^2}{c^2}(1 - \cos\eta).$$

On making use of (10.28), (10.30) and (10.31), it becomes

$$\frac{a}{a_0} = \frac{\Omega_{\mathrm{M},0}}{2(\Omega_{\mathrm{M},0} - 1)}(1 - \cos\eta), \qquad (10.56)$$

where $\Omega_{M,0} = \rho_{M,0} / \rho_{c,0}$ is the density parameter at the present epoch due to matter. The evolution of a as a function of the time-like variable η is given by (10.56).

The closed solution k=+1

In order to bring in t, we use (10.33) which gives

$$t = \frac{1}{c}\int a\, d\eta = \frac{\Omega_{M,0} a_0}{2c(\Omega_{M,0} - 1)}(\eta - \sin\eta)$$

on substituting from (10.56) for a. Multiplying by H_0 and making use of (10.31), we get

$$H_0 t = \frac{\Omega_{M,0}}{2(\Omega_{M,0} - 1)^{3/2}}(\eta - \sin\eta). \qquad (10.57)$$

The two equations (10.56) and (10.57) together give an implicit solution of a as a function of t. It follows from (10.56) that ***a* goes to zero when η increases to 2π**.

In other words, this is a solution which corresponds to **a Universe eventually ending up in a big crunch.**

We have already used arguments based on simple Newtonian mechanics in §10.4 to conclude that a Universe with $k = +1$, which has finite volume and has density more than the critical density, should eventually collapse. This is now explicitly seen in the solution.

The closed solution k=+1

Figure 10.5 shows a/a_0 as a function of t.

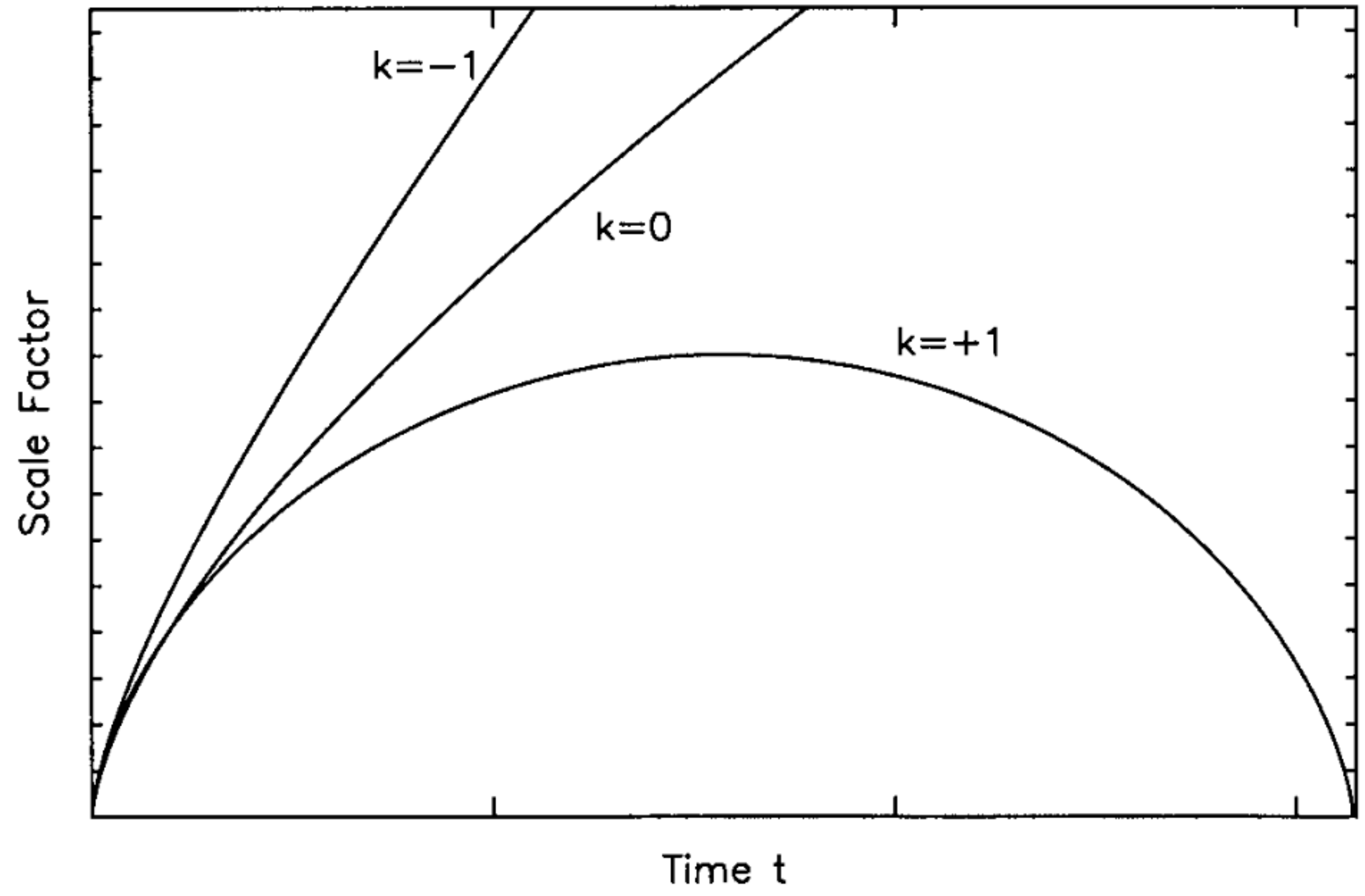

Fig. 10.5 The evolution of the scale factor a/a_0 as a function of the time t for the cases $k = -1, 0, +1$.

The open solution k=-1

This is the case corresponding to $\Omega_{M,0} < 1$. With $\boldsymbol{k = -1}$, instead of (10.56) and (10.57), we have

$$\frac{a}{a_0} = \frac{\Omega_{M,0}}{2(1-\Omega_{M,0})}(\cosh\eta - 1), \tag{10.58}$$

$$H_0 t = \frac{\Omega_{M,0}}{2(1-\Omega_{M,0})^{3/2}}(\sinh\eta - \eta). \tag{10.59}$$

Again these two equations together give an implicit solution of a as a function of t. This solution of a/a_0 as a function of t is also plotted in Figure 10.5, which shows the solution for the $k = 0$ case as well. It is seen that the solution for **$k = -1$ increases forever at a rate faster than the rate of increase of the critical solution $k = 0$.**

We had already anticipated in §10.4 that the $k = -1$ solution, which corresponds to an infinite Universe with density less than the critical density, will expand forever. The explicit solution now confirms this.

The open solution k=-1

It appears that $\Omega_{M,0}$ **is about 0.3.** It would then seem that the open solution is the appropriate solution for our Universe.

However, so far in our discussion, **we have not included the cosmological constant which would contribute another term to the Friedmann equation** (10.27).

In §14.5 we shall discuss the **observational evidence that this cosmological constant may actually be non-zero, leading to an acceleration of the Universe.**

The cosmological constant term, however, **becomes more important with time**. **At the present epoch, this term is comparable to the matter density term.** At earlier epochs, the cosmological term was less important.

So the open solution is presumably the appropriate solution for the evolution of the Universe at early epochs and can be used up to the present epoch in many calculations without introducing too much error.

Current measurement results

$\Omega_{mass} \approx 0.315 \pm 0.018$
$\Omega_{relativistic} \approx 9.24 \times 10^{-5}$
$\Omega_{\Lambda} \approx 0.6817 \pm 0.0018$
$\Omega_{total} = \Omega_{mass} + \Omega_{relativistic} + \Omega_{\Lambda} = 1.00 \pm 0.02$

The actual value for critical density value is measured as $\rho_c = 9.47 \times 10^{-27}$ kg m^{-3}. From these values, within experimental error, **the universe seems to be spatially flat**.

https://en.wikipedia.org/wiki/Shape_of_the_universe

Approximate solution for early epochs

We see in Figure 10.5 that **the solutions for all the three values of k behave very similarly at sufficiently early times.** We can simplify the solutions for $k = \pm 1$ when $\eta \ll 1$. Both (10.56) and (10.58) reduce to

$$\frac{a}{a_0} \approx \frac{\Omega_{M,0}}{2|1 - \Omega_{M,0}|} \frac{\eta^2}{2}$$

when η is small. Similarly both (10.57) and (10.59) reduce to

$$H_0 t \approx \frac{\Omega_{M,0}}{2|1 - \Omega_{M,0}|^{3/2}} \frac{\eta^3}{6}.$$

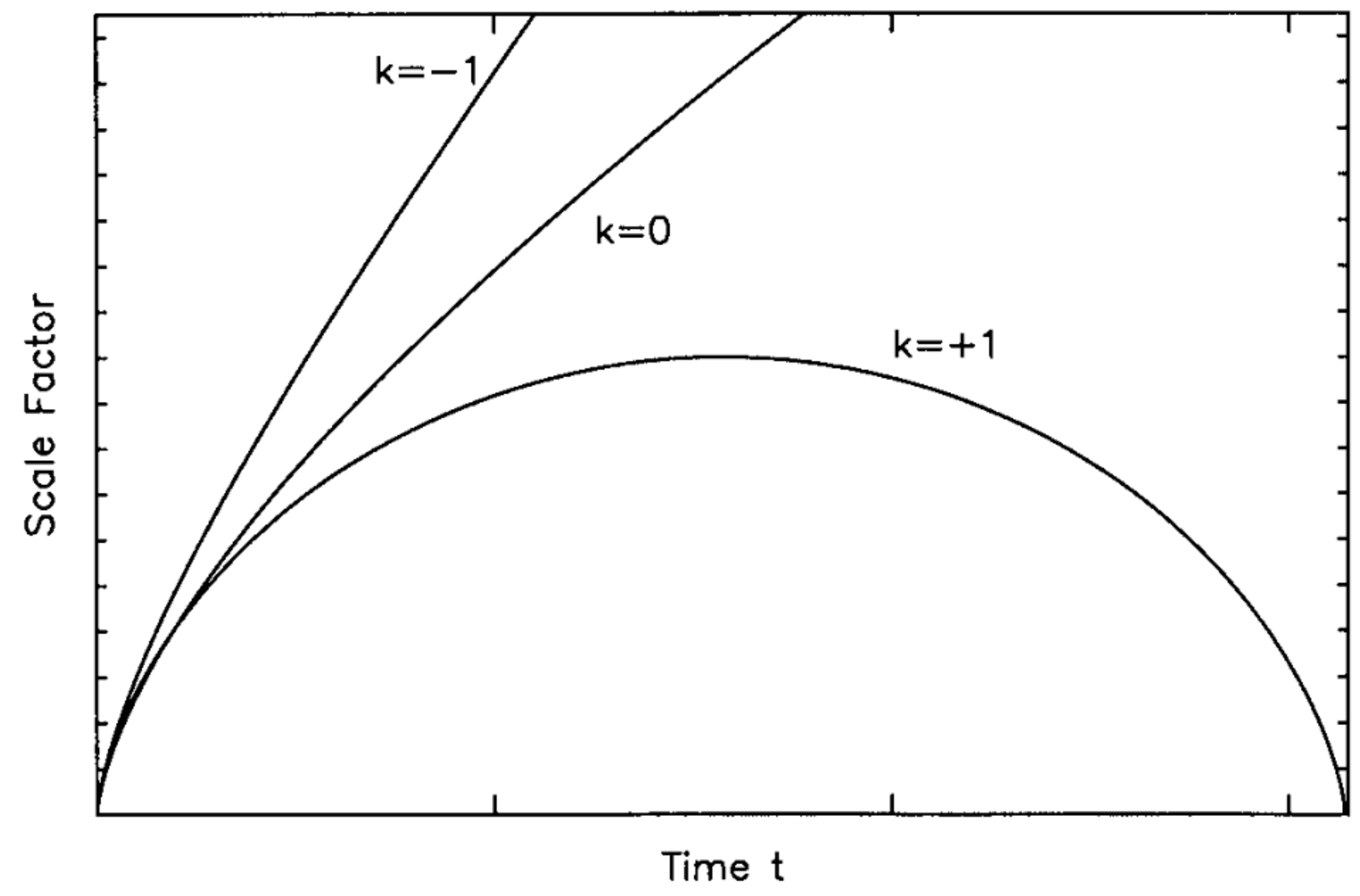

Fig. 10.5 The evolution of the scale factor a/a_0 as a function of the time t for the cases $k = -1, 0, +1$.

Approximate solution for early epochs

Eliminating η between these two equations, we get

$$\frac{a}{a_0} \approx \left(\frac{3}{2}\Omega_{\mathrm{M},0}^{1/2} H_0 t\right)^{2/3}. \qquad (10.60)$$

This reduces to the critical solution (10.54) when the density parameter $\Omega_{M,0}$ is 1.

Although (10.60) may not be strictly true at the present epoch, it is still very useful in many quick calculations. Since $a = a_0$ at the present epoch $t = t_0$, it follows from (10.60) that

$$t_0 \approx \frac{2}{3} H_0^{-1} \Omega_{\mathrm{M},0}^{-1/2}. \qquad (10.61)$$

Then (10.60) can be written as

$$\frac{a}{a_0} \approx \left(\frac{t}{t_0}\right)^{2/3}. \qquad (10.62)$$

Approximate solution for early epochs

$$\boxed{1 + z = \frac{\lambda_{\mathrm{obs}}}{\lambda_{\mathrm{em}}} = \frac{a_0}{a}.}$$

It may be pointed out that the ratio $\boldsymbol{a/a_0}$ **is a more observationally relevant quantity** than the scale factor a itself, since this ratio is **related to the redshift z** through (10.24).

If a source is at redshift z, then light from it started at time t to reach us at the present epoch t_0. The **relation between z and t** can be obtained by combining (10.24) with (10.62):

$$\frac{t}{t_0} \approx (1 + z)^{-3/2}. \tag{10.63}$$

When we look at a galaxy at redshift $z = 1$, we essentially see the galaxy as it existed when the age of the Universe was $2^{-3/2}$ times its present age.

Approximate solution for early epochs

As we already discussed, the very **early Universe was radiation-dominated.**

So (10.60), based on the assumption that the Universe is matter-dominated, should not hold at those early times. However, the **Universe became matter-dominated fairly early**. From (10.52) and (10.62), the epoch t_{eq} of matter-radiation equality is given by

$$t_{eq} = 2.9 \times 10^{-7} \Omega_{M,0}^{-3/2} h^{-3} t_0. \qquad (10.64)$$

Since t_0 is believed to be of the order of 10^{10} years, **the Universe would have become matter-dominated a few thousand years after the Big Bang**. From then onwards, equations like (10.60) and (10.62) should hold till the present time.

The Age of the Universe

$$t_0 \approx \frac{2}{3} H_0^{-1} \Omega_{\mathrm{M},0}^{-1/2}. \tag{10.61}$$

An approximate expression for the age of the Universe is given in (10.61), where we see that **the age is shorter if $\Omega_{M,0}$ is larger.**

One can understand this result quite easily. A **larger $\Omega_{M,0}$ means a stronger deceleration**, which implies that the **expansion rate of the early Universe would have been faster** if $\Omega_{M,0}$ was larger. This faster expansion **leads to a shorter age**.

The Age of the Universe

$$\frac{a}{a_0} = \frac{\Omega_{\mathrm{M},0}}{2(\Omega_{\mathrm{M},0} - 1)}(1 - \cos\eta), \tag{10.56}$$

$$H_0 t = \frac{\Omega_{\mathrm{M},0}}{2(\Omega_{\mathrm{M},0} - 1)^{3/2}}(\eta - \sin\eta). \tag{10.57}$$

Instead of using the approximate expression (10.61), one can easily find out the exact value of the age t_0 for a given $\Omega_{M,0}$. Since $a = a_0$ at the present epoch, the age t_0 of the Universe is given by the value of t which makes $a = a_0$.

If $\Omega_{M,0} > 1$, then the numerical value of $H_0 t_0$ can be found from (10.56) and (10.57).

On the other hand, we have to use (10.58) and (10.59) if $\Omega_{M,0} < 1$.

$$\frac{a}{a_0} = \frac{\Omega_{\mathrm{M},0}}{2(1 - \Omega_{\mathrm{M},0})}(\cosh\eta - 1), \tag{10.58}$$

$$H_0 t = \frac{\Omega_{\mathrm{M},0}}{2(1 - \Omega_{\mathrm{M},0})^{3/2}}(\sinh\eta - \eta). \tag{10.59}$$

The Age of the Universe

Figure 10.6 plots the numerical values of $H_0 t_0$ as a function of $\Omega_{M,0}$.

The expansion rate of the Universe would have been unchanged if $\Omega_{M,0}$ were equal to zero, making t_0 equal to the Hubble time H_0^{-1}.

We see in Figure 10.6 that t_0 becomes a smaller and smaller fraction of the Hubble time as $\Omega_{M,0}$ is increased, giving the value $(2/3)H_0^{-1}$ when $\Omega_{M,0} = 1$, in accordance with (10.61).

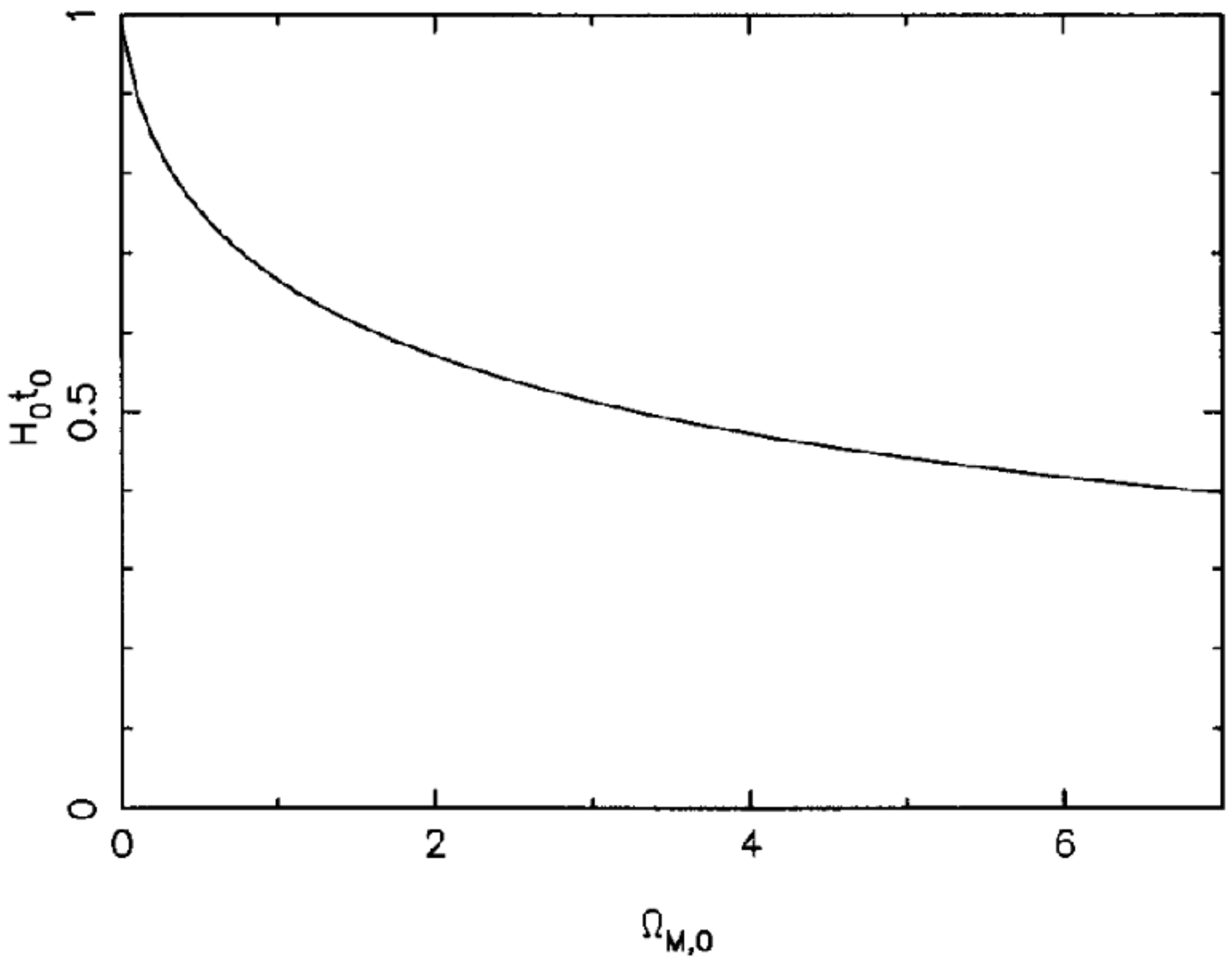

Fig. 10.6 The plot of $H_0 t_0$ against $\Omega_{M,0}$, showing how the age t_0 of the Universe depends on $\Omega_{M,0}$.

The radiation dominated Universe

We shall now discuss how the Universe evolved at the early times before t_{eq} given by (10.64) **when the Universe was dominated by radiation**.

In the case of the matter-dominated Universe, it should be clear from Figure 10.5 that the solutions for $k = -1, 0, +1$ converge at sufficiently early times. The reason for this is not difficult to understand. **It is the curvature term kc^2/a^2 in the Friedmann equation (10.27) which is responsible for making the three solutions different.** At sufficiently **early times, this term becomes negligible** compared to the mass density term which goes as a^{-3} and becomes more dominant when a is sufficiently small.

During the **radiation-dominated epoch,** the radiation density **term going as a^{-4} becomes even more dominant** and we can ignore the curvature term kc^2/a^2, so that (10.32) obtained by putting $k = 0$ in the Friedmann equation (10.27) is applicable.

On substituting $\rho = \rho_R$ as given by (10.48), we obtain from (10.32) that

$$a\dot{a} = \sqrt{\frac{8\pi G\rho_{R,0}}{3}}a_0^2,$$

The radiation dominated Universe

of which the solution is

$$\frac{a}{a_0} = \left(\frac{32\pi G\rho_{\mathrm{R},0}}{3}\right)^{1/4} t^{1/2}. \qquad (10.65)$$

The **radiation-dominated Universe expanded with time as $t^{1/2}$** in contrast to the early **matter-dominated Universe which expanded as $t^{2/3}$** according to (10.62).

We have pointed out in (10.49) that the temperature of the CMBR falls as the inverse of a. So we expect

$$\frac{a}{a_0} = \frac{T_0}{T}.$$

Using this and substituting for $\rho_{R,0}$ from (10.46), we get from (10.65) that

$$T = \left(\frac{3c^2}{32\pi G a_{\mathrm{B}}}\right)^{1/4} t^{-1/2}. \qquad (10.66)$$

The radiation dominated Universe

On substituting the values of c, G and a_{B}, we obtain

$$T \text{ (in K)} = \frac{1.52 \times 10^{10}}{\sqrt{t}}. \tag{10.67}$$

Here T is in kelvin as indicated and t has to be in seconds.

It may be noted that (10.66) and (10.67) are **derived by assuming that photons were the only relativistic particles in the early Universe.** As we shall see in Chapter 11, **there were other relativistic particles and these equations have to be suitably modified in more accurate calculations.**

A typical photon in blackbody radiation at temperature T has energy $E = \kappa_B T$. It can be easily shown that an energy E in eV corresponds to temperature

$$T = 1.16 \times 10^4 E \text{ (in eV)}. \tag{10.68}$$

The radiation dominated Universe

It is sometimes useful to express temperature in units of energy like eV. If we do this, then (10.67) becomes

$$\boxed{T \text{ (in MeV)} = \frac{1.31}{\sqrt{t}}. \qquad (10.69)}$$

This is a very important equation which gives an **indication of the typical energy a photon (or any other kind of particle) would have at time *t* after the Big Bang.** We shall make extensive use of (10.69) in the next chapter.

Since we have neglected the curvature term kc^2/a^2, we now argue that this term really becomes negligible compared to the other terms in the Friedmann equation (10.27) as we go close to the Big Bang. It follows from (10.31) that

$$|\Omega - 1| = \frac{c^2}{a^2 H^2} = \frac{c^2}{\dot{a}^2}$$

The radiation dominated Universe

on using (10.23). Since a goes as $t^{1/2}$, we find from this that

$$|\Omega - 1| \propto t. \tag{10.70}$$

In other words, **as $t \to 0$, the density parameter Ω approaches 1 arbitrarily closely and the curvature term becomes totally insignificant compared to other terms,** in spite of the fact that the curvature term kc^2/a^2 by itself becomes infinite as a goes to zero.