Introduction to Astrophysics and Cosmology

The Thermal history of the Universe

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Timescales

The present uniform expansion of the Universe suggests that there was an epoch in the past when the Universe was in a singular state with infinite density. Since most of the known laws of physics become inapplicable to such a singular state, **we cannot extrapolate to earlier times before this epoch of singularity.** We therefore concern ourselves only with what happened after this **epoch of singularity, which is called the *Big Bang***. In the previous equations the time t was measured from the Big Bang.

How the temperature of the early Universe varied with time is given by (10.67) and (10.69).

$$T \text{ (in K)} = \frac{1.52 \times 10^{10}}{\sqrt{t}}. \tag{10.67}$$

$$\boxed{T \text{ (in MeV)} = \frac{1.31}{\sqrt{t}}. \qquad (10.69)}$$

At **times earlier than 1 s after the Big Bang**, typical photons had energies somewhat **larger than 1MeV.** Since such photons are known to **produce electron-positron pairs**, the Universe at these early times must have been **full of electrons and positrons** which would have been approximately as abundant as photons.

When photons had **energies larger than 2 GeV at still earlier times**, they would have given rise to **proton-antiproton pairs and neutron-antineutron pairs** along with pairs of many other particles.

Timescales

At even higher energies existing at still earlier times, **the quarks making up the elementary particles would have been free.** The Universe at this very early time **consisted of quarks and leptons along with their antiparticles and the bosons** which mediate the various fundamental interactions.

Timescales

Figure 11.1 shows a timetable with time plotted in a logarithmic scale along with temperature in K and in eV.

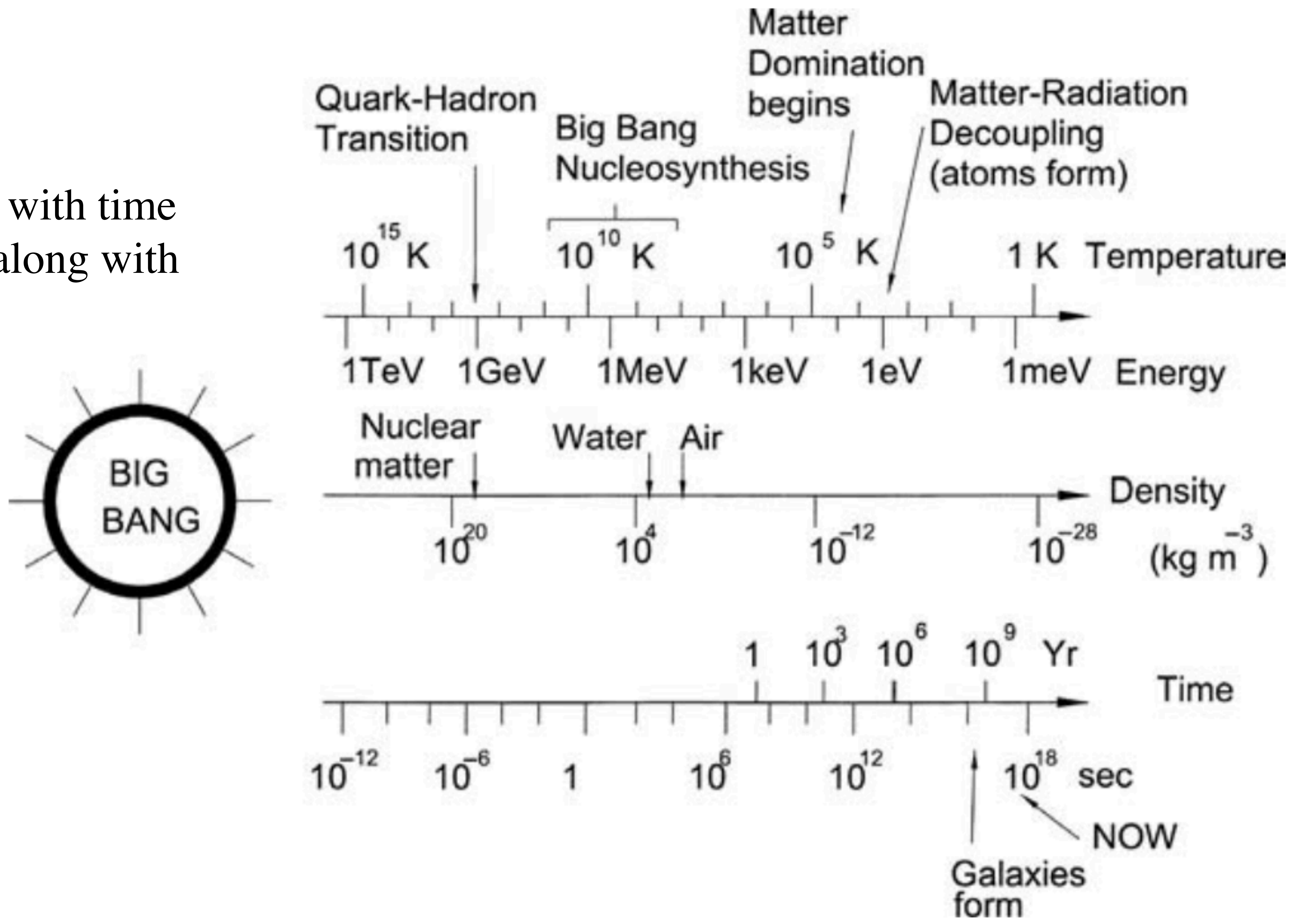


Fig. 11.1 Landmarks in the thermal history of the Universe. Adapted from Kolb and Turner (1990, p. 73).

Timescales

When the **temperatures were higher than 1GeV, we enter the particle physics era**. Since theoretical considerations of these very early times do not have much direct connections with observational data, we shall not discuss much about times earlier than the particle physics era.

Typical nuclear reactions involve **energies of order MeV.** During the epoch from **about $t = 1$ s to $t = 10^2$ s**, particles in the Universe had energies **appropriate for nuclear reactions** and various kinds of nuclear reactions must have taken place.

It follows from (10.64) that **the Universe was a few thousand years old when it changed from being radiation-dominated to being matter-dominated.**

Another landmark epoch came a little bit later when T fell to about **1 eV** and **formation of atoms took place.** Since typical ionization energies are of order eV, we would not expect atoms when T was larger than this and matter must have existed in the form of free electrons and bare nuclei. Although (10.69) is no longer valid at the epoch of atom formation when the Universe is already matter-dominated, we can get an **approximate value of time t for this epoch** by setting $T = 1$ eV in (10.69), which gives $t \approx$ **5×10^4 yr** as the **epoch of atom formation.**

Timescales

There was a very important consequence of the formation of atoms. **Radiation interacted with matter before this epoch primarily through Thomson scattering.** When the atoms formed, all the electrons got locked inside them and there ceased to be any free electrons available to produce Thomson scattering. So **radiation got decoupled from matter and the Universe suddenly became transparent to radiation.**

Thermodynamic equilibrium

Let us consider a reaction in which A, B, C, ...combine to produce L, M, N, ...:

$$A + B + C + \cdots \longleftrightarrow L + M + N + \cdots \,, \tag{11.1}$$

which **can proceed in both directions.**

Under normal circumstances, we expect this reaction to reach a ***chemical equilibrium*** **when the concentrations of A, B, C, ..., L, M, N, … are such that the backward rate balances the forward rate and the concentrations do not change any more.**

It may be noted that we shall be using the term chemical equilibrium even if (11.1) is strictly not a 'chemical' reaction, i.e. it can be a nuclear reaction involving nuclei.

The condition for the chemical equilibrium of reaction (11.1) is

$$\mu_A + \mu_B + \mu_C + \cdots = \mu_L + \mu_M + \mu_N + \cdots, \tag{11.2}$$

where the μ-s are the chemical potentials.

Thermodynamic equilibrium

Various kinds of nuclear and particle reactions were possible in the early Universe.

The fundamental question we want to ask now is **whether these reactions could reach chemical equilibrium**.

As the Universe expanded at the rate $H = \dot{a}/a$, **the condition for chemical equilibrium kept changing.** Only if a **reaction proceeded at a rate faster than this expansion rate**, can we expect the reaction to reach equilibrium.

Suppose Γ **is the interaction rate per particle,** i.e. it is the number of interactions a particle is expected to have in unit time. Within time $\sim 1/\Gamma$, most of the particles will have one interaction and we may expect the system to reach equilibrium if it starts from a state not too far from equilibrium.

If $\Gamma \gg H$, then the reaction would be able to reach chemical equilibrium and, as the Universe expanded, it would evolve through successive states in chemical equilibrium appropriate to the physical conditions at the successive instants of time.

On the other hand, **if $\Gamma \ll H$, then the reaction would not proceed fast enough to change the concentrations of the particles involved** in the reaction.

Thermodynamic equilibrium

Let us write down

$$\boxed{\frac{\Gamma}{H} \gg 1 \qquad (11.3)}$$

as the condition for a reaction to reach chemical equilibrium.

Since the interaction rate Γ depends on the density of the Universe, we expect Γ **to decrease with the expansion of the Universe**.

Although H also decreases with time (for example, H decreases as t^{-1} if a goes as some power t^n of t), the decrease in Γ is typically faster.

We then expect the condition $\Gamma \gg H$ to change over to the condition $\Gamma \ll H$ as the Universe evolves. In such a situation, a reaction which was initially able to remain in chemical equilibrium eventually falls out of equilibrium. **The concentrations of the particles involved in the reaction are not expected to change after the reaction goes out of equilibrium.**

So we can assume the **concentrations to be frozen at the values which they had when $\Gamma \approx H$** and the reaction was just going out of equilibrium.

Thermodynamic equilibrium

It should be emphasized that the ideas presented here should be taken as **rough rules of thumb.** To obtain more accurate results, one has to carry out a detailed calculation of the reaction in the expanding Universe (usually done numerically with computers).

When a reaction is able to reach chemical equilibrium, the concentrations of the various species of particles involved in the reaction will be given by the standard results of thermodynamic equilibrium.

The **number of particles occupying a quantum state** with momentum **p** is given by

$$f(\mathbf{p}) = \left[\exp\left(\frac{E(\mathbf{p}) - \mu}{\kappa_{\mathrm{B}} T}\right) \pm 1\right]^{-1}, \tag{11.4}$$

where we have to use the **plus sign** for fermions obeying **Fermi–Dirac statistics** and the **minus sign** for bosons obeying **Bose–Einstein statistics**.

$E(\mathbf{p}) = \sqrt{p^2c^2 + m_0^2c^4}$ being the energy associated with the momentum **p**.

Thermodynamic equilibrium

To obtain the actual number density from (11.4), we have to keep in mind that the six-dimensional phase space volume element dVd^3p (where dV is the ordinary volume element) has $g\, dV\, d^3\, p/h^3$ quantum states in it. Here g is the degeneracy, which is 2 for both electrons and photons, corresponding to the two spin states and two degrees of polarization respectively.

Hence the **number density per unit volume** must be given by multiplying (11.4) by $g\, d^3\, p/h^3$ and then integrating over all possible momenta, i.e.

$$n = \frac{g}{(2\pi)^3} \int f(\mathbf{p}) \frac{d^3p}{\hbar^3}. \qquad (11.5)$$

The **contribution to density** made by this species of particles is

$$\rho = \frac{g}{(2\pi)^3} \int \frac{E(\mathbf{p})}{c^2} f(\mathbf{p}) \frac{d^3p}{\hbar^3}. \qquad (11.6)$$

Thermodynamic equilibrium

Let us make some comments on the chemical potential μ appearing in (11.2) and (11.4).

It is well known that **$\mu = 0$ for photons which can be created or destroyed easily**.

If photons produce particle-antiparticle pairs, then (11.2) is satisfied only if **the sum of the chemical potentials of the particle and the antiparticle is zero**. In other words, if the chemical potential of the particle is $+\mu$, the chemical potential of the antiparticle has to be $-\mu$. It follows from (11.4) that these **differences in chemical potential would make the number densities of particles and antiparticles different.**

We now consider the case $\kappa_B T \gg mc^2$ such that most particles would have energies much larger than the rest mass energy and would be **relativistic**, i.e. we can write $E \approx pc$. **At such a high temperature, the Universe is expected to be full of these particles and their antiparticles**, which should be present in **comparable numbers.**

This is possible only if μ is much smaller compared to $\kappa_B T$. Writing $d^3p = 4\pi p^2\, dp$, $E = pc$ and $\mu = 0$, we find from (11.4) and (11.5) that

$$n = \frac{g}{2\pi^2\hbar^3}\int_0^\infty \frac{p^2 dp}{e^{pc/\kappa_B T} \pm 1}. \tag{11.7}$$

Thermodynamic equilibrium

It similarly follows from (11.4) and (11.6) that

$$\rho = \frac{g}{2\pi^2 c\hbar^3} \int_0^\infty \frac{p^3 dp}{e^{pc/\kappa_B T} \pm 1}. \tag{11.8}$$

We can find solutions to both equations analytically. For the cases of bosons and fermions, (11.7) can be integrated to give

$$n = \begin{cases} \frac{\zeta(3)}{\pi^2} g \left(\frac{\kappa_B T}{\hbar c}\right)^3 & \text{(boson)}, \\ \frac{3}{4} \frac{\zeta(3)}{\pi^2} g \left(\frac{\kappa_B T}{\hbar c}\right)^3 & \text{(fermion)}, \end{cases} \tag{11.9}$$

where $\zeta(3) = 1.202$ in the Riemann zeta function.

Thermodynamic equilibrium

Similarly from (11.8) we get

$$\rho = \begin{cases} \frac{g}{2c^2} a_{\mathrm{B}} T^4 & \text{(boson)}, \\ \frac{7}{8} \frac{g}{2c^2} a_{\mathrm{B}} T^4 & \text{(fermion)}. \end{cases} \tag{11.10}$$

Where

$$a_{\mathrm{B}} = \frac{\pi^2 \kappa_{\mathrm{B}}^4}{15 \hbar^3 c^3} \tag{11.11}$$

is the Stefan-Boltzmann constant.

If we take $g = 2$ for photons, then the energy density ρc^2 according to (11.10) is $a_B T^4$, which is the standard expression for energy density of **blackbody radiation at temperature T**.

Thermodynamic equilibrium

Now we determine the **entropy density** of this gas of relativistic bosons or fermions. For this purpose, we begin with the standard thermodynamic relation

$$T\,dS = dU + P\,dV, \tag{11.12}$$

where the internal energy U inside a volume V is given by $\rho c^2 V$. If **s is the entropy density**, then we can write $S = sV$. On making these substitutions, (11.12) gives

$$TV\,ds = Vc^2 d\rho + [(\rho c^2 + P) - Ts]dV. \tag{11.13}$$

We see in (11.10) that **ρ is a function of T alone.** We expect the entropy density s also to be a function of T alone. If **both s and ρ are independent of V,** then consistency requires that the coefficient of dV in (11.13) must be zero. This means

$$s = \frac{\rho c^2 + P}{T}. \tag{11.14}$$

Thermodynamic equilibrium

On making use of the fact that $P = (1/3)\rho c^2$ for a relativistic gas and substituting for ρ from (11.10), we get

$$s = \begin{cases} \frac{2g}{3} a_{\mathrm{B}} T^3 & \text{(boson)}, \\ \frac{7}{8}\frac{2g}{3} a_{\mathrm{B}} T^3 & \text{(fermion)}. \end{cases} \tag{11.15}$$

We now take a brief look at the opposite case $\kappa_B T \ll mc^2$ (non-relativistic case). We can take

$$E(\mathbf{p}) \approx mc^2 + \frac{p^2}{2m}$$

in (11.4) in this case. Substituting (11.4) in (11.5), the **particle density in the non-relativistic limit** is found to be

$$n = \frac{g}{\hbar^3}\left(\frac{m\kappa_{\mathrm{B}}T}{2\pi}\right)^{3/2} \exp\left(-\frac{mc^2-\mu}{\kappa_{\mathrm{B}}T}\right). \tag{11.16}$$

Thermodynamic equilibrium

Note that the exponential factor is supposed to be quite small in the non-relativistic limit. So **the number density of non-relativistic particles under thermodynamic equilibrium would be negligible** compared to the number density of relativistic particles.

We already pointed out that the CMBR still has a spectral distribution appropriate for thermodynamic equilibrium even though the photons in the CMBR are no longer in equilibrium with matter. So we can still use (11.9) to calculate the number density of photons in the CMBR at present. Using T as given by (10.45) and taking $g = 2$, we find from (11.9) that the **present photon number density is**

$$\boxed{n_{\gamma,0} = 4.14 \times 10^8 \ \mathrm{m}^{-3}. \qquad (11.17)}$$

It is instructive to compare this with the **present number density of baryons.**

If $\Omega_{B,0}$ is the contribution of baryons to the density parameter, then the number density $n_{\mathrm{B},0}$ of baryons should clearly be $\Omega_{B,0}\rho_{c,0}/m_p$, where m_p is the mass of a proton. On using (10.29), we get

$$n_{\mathrm{B},0} = 11.3\, \Omega_{\mathrm{B},0} h^2 \ \mathrm{m}^{-3}. \qquad (11.18)$$

Thermodynamic equilibrium

The **ratio of baryon to photon number densities** is an interesting dimensionless quantity which has the value

$$\boxed{\eta = \frac{n_{\mathrm{B},0}}{n_{\gamma,0}} = 2.73 \times 10^{-8}\Omega_{\mathrm{B},0}h^2. \qquad (11.19)}$$

As the **Universe expands, the photon number density** n_γ **falls** as T^3, which, in combination with (10.49), suggests that n_γ falls as a^{-3}.

The **baryon number density also is obviously falling** as a^{-3}.

Hence **the ratio of these number densities does not change with time**. In other words, the value of this ratio given by (11.19) is not only its present value, but also its value at earlier or later times (as long as there are no reactions to change n_γ or n_B suddenly). Thus **η is an important quantity in cosmology.**

Primordial nucleosynthesis

The epoch from **1 s to 10^2 s after the Big Bang** was **suitable for nuclear reactions**.

In early work, Gamow suggested that most of the heavy nuclei were synthesized during this short epoch just after the Big Bang. However, there is no stable nucleus of mass 5 or 8 and hence it is **not easy for nuclear reactions to synthesize nuclei heavier than helium**. In the interiors of **heavy stars,** this mass gap is superseded by the **triple alpha reaction**. This reaction, however cannot take place unless the number density of **helium nuclei is sufficiently high**. Detailed calculations show that this reaction was very **unlikely under the conditions of the early Universe** and heavier nuclei could not be synthesized there.

We now believe that **all heavy nuclei starting from ^{12}C are produced inside stars.**

Realistic calculations of nucleosynthesis in the early Universe have to be done numerically. The first codes for this were developed in the 1960s soon after the discovery of the CMBR, which established that the Universe really began with a hot Big Bang.

Primordial nucleosynthesis

$$\eta = \frac{n_{B,0}}{n_{\gamma,0}} = 2.73 \times 10^{-8} \Omega_{B,0} h^2.$$

The code perfected by Wagoner (1973) came to be regarded as the 'standard code' on which most of the later computations are based.

The **results depend on η** introduced in (11.19). A **higher η implies** that there were **more baryons per unit volume** and **nuclear reactions were more likely** to take place.

Figure 11.2 shows the **mass fractions of various nuclei produced in the early Universe as a function of η**.

Although this figure is based on results obtained by numerical simulation, at least some of the results can be understood from general arguments.

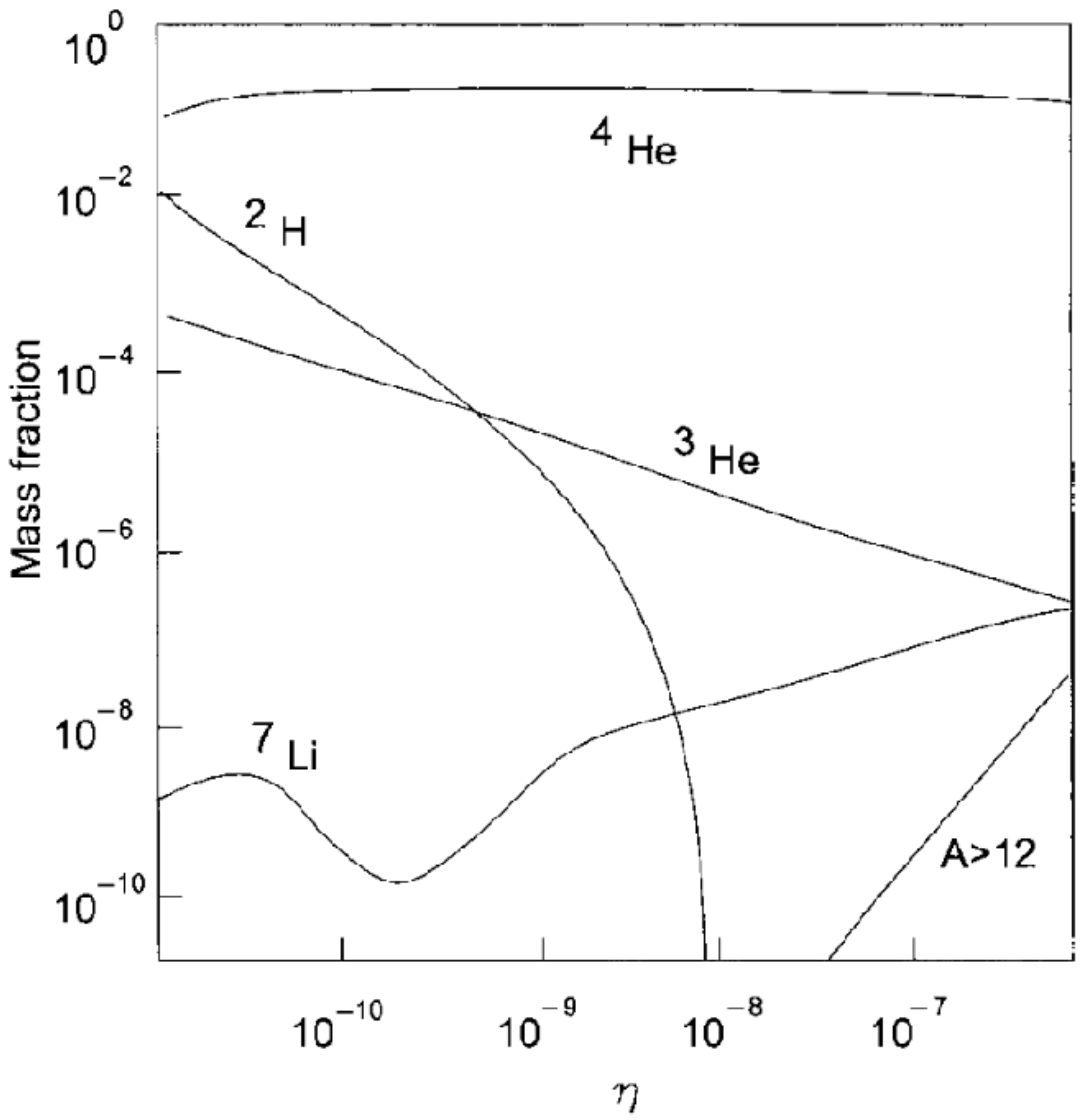


Fig. 11.2 Theoretically calculated primordial abundances of various light nuclei as a function of η. Adapted from Wagoner (1973).

Primordial nucleosynthesis

Let us first consider the **production of helium**.

Two protons and two neutrons have to combine to produce a helium nucleus. During the epoch of nuclear reactions, the protons and neutrons should take part in reactions like

$$n \to p + e + \bar{\nu} \qquad\qquad p + e \to n + \nu.$$

which were listed as **possible reactions inside neutron stars.**

These reactions are **mediated by the weak interaction.** The crucial question is **whether these reactions would have been able to reach chemical equilibrium?**

To answer this question, we need to find out the reaction rate Γ, which **depends on the coupling constant of the weak interaction** and can be calculated from a knowledge of the half-life of a free neutron. We know that free neutrons decay with a half-life of 10.5 minutes. The result is

$$\frac{\Gamma}{H} \approx \left[\frac{T\ (\text{in MeV})}{0.8\,\text{MeV}}\right]^3. \qquad (11.20)$$

Primordial nucleosynthesis

It follows from this that the condition of chemical equilibrium would be satisfied **when T (in MeV) $\gg$ 0.8 MeV.** Under this situation, we can calculate the **number densities of protons or neutrons**. Since chemical potentials are expected to be small compared to the rest masses, it follows from (11.16) that the ratio of these number densities will be

$$\frac{n_{\rm n}}{n_{\rm p}} = \exp\left[-\frac{(m_{\rm n} - m_{\rm p})c^2}{\kappa_{\rm B} T}\right], \tag{11.21}$$

There is a very simple way of understanding (11.21). **We can regard the proton as the ground state for a baryon and the neutron as the first excited state.** Then (11.21) can be regarded merely as the **Boltzmann distribution law** (2.28) applied to this situation.

As the temperature keeps decreasing, we expect (11.21) to be **valid till $T \approx$ 0.8 MeV, after which** the reactions will no longer be able to maintain thermodynamic equilibrium and **the ratio $n_{\rm n}/n_{\rm p}$ will be approximately frozen.** Since $(m_{\rm n} - m_{\rm p})c^2 = 1.29$ MeV, this frozen ratio is given by

$$\boxed{\frac{n_{\rm n}}{n_{\rm p}} \approx e^{-\frac{1.29}{0.8}} \approx 0.20. \qquad (11.22)}$$

Primordial nucleosynthesis

After the neutron number is frozen, these **neutrons may eventually be used up to synthesize helium.**

Let us assume that n_n neutrons in the unit volume combine with $\boldsymbol{n_n}$ **protons to synthesize helium nuclei and the other** $\boldsymbol{n_p - n_n}$ **protons remain as protons.** Then the **helium mass fraction** should be given by

$$\frac{2n_\mathrm{n}}{n_\mathrm{n} + n_\mathrm{p}} = \frac{2(n_\mathrm{n}/n_\mathrm{p})}{1 + (n_\mathrm{n}/n_\mathrm{p})} = 0.33 \tag{11.23}$$

It may be noted that the ratio $\boldsymbol{n_n/n_p}$ **does not remain completely frozen during the time of order 100 s** when nucleosynthesis takes place, but **decreases due to the decay of neutrons** which is still possible even when thermodynamic equilibrium no longer prevails.

So the ratio becomes somewhat less than 0.20, **leading to a smaller helium fraction** compared to what is given in (11.23).

Careful **numerical simulations suggest a value of about 0.25 for the helium mass fraction.**

Primordial nucleosynthesis

We see in Figure 11.2 that the **helium mass fraction is nearly independent of η**, although it becomes slightly smaller for low η.

A low η implies a low baryon number density, which means that the nuclear reactions which build up helium nuclei would proceed at a slower rate and **more neutrons would decay** before being bound up inside helium nuclei, thereby leading to a lower helium mass fraction.

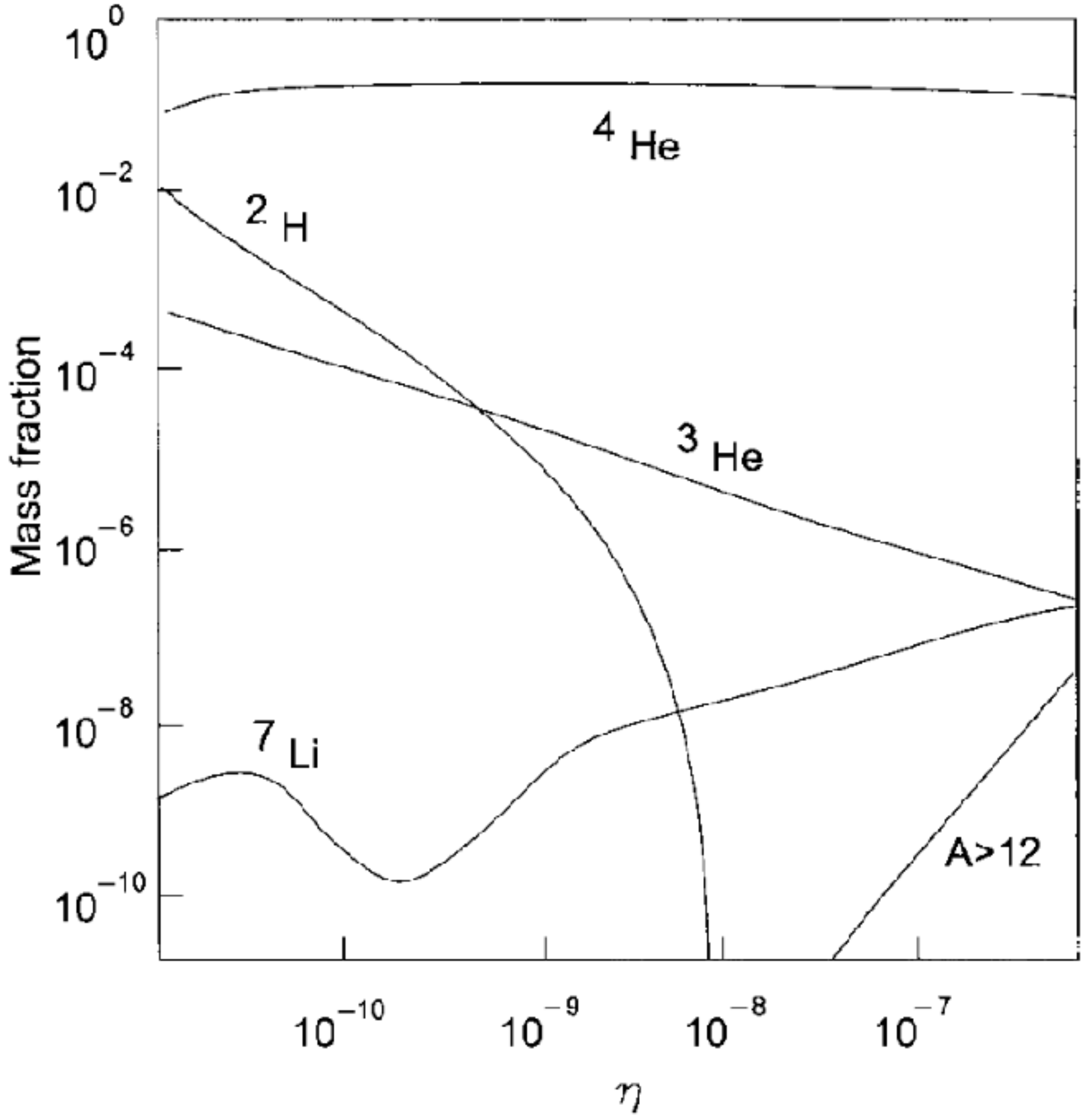


Fig. 11.2 Theoretically calculated primordial abundances of various light nuclei as a function of η. Adapted from Wagoner (1973).

Primordial nucleosynthesis

Let us now point out what the observational data tell us. Although the mass fractions of higher elements are found to vary considerably in different astrophysical sources, **the helium fraction is found to be very similar in widely different astrophysical systems, lying in the range 0.23–0.27.**

The Big Bang cosmology can explain this observation very naturally if we assume that most of the helium was produced by primordial nucleosynthesis, with nuclear reactions in the interiors of **stars contributing a little bit afterwards.** The theoretically calculated value of the helium fraction is completely in agreement with observational data.

The fact that **the abundances of higher elements vary considerably in different astrophysical sources suggests that they were not produced in the early Universe**, because such a production would imply a more uniform distribution today.

Primordial nucleosynthesis

We now consider the abundance of **deuterium ^{2}H**.

It is an **intermediate product in the synthesis of helium**, as can be seen from the *pp* chain reactions. **If η is high and the nuclear reaction rate is fast**, then **most of the deuterium would get converted into helium** during the primordial nucleosynthesis era.

On the other hand, a **lower η and a slower reaction rate** would imply that the synthesis of helium would not be so efficient in the nucleosynthesis era and some **deuterium would be left over.** We therefore see in Figure 11.2 that the **deuterium fraction falls sharply with the increase in η.**

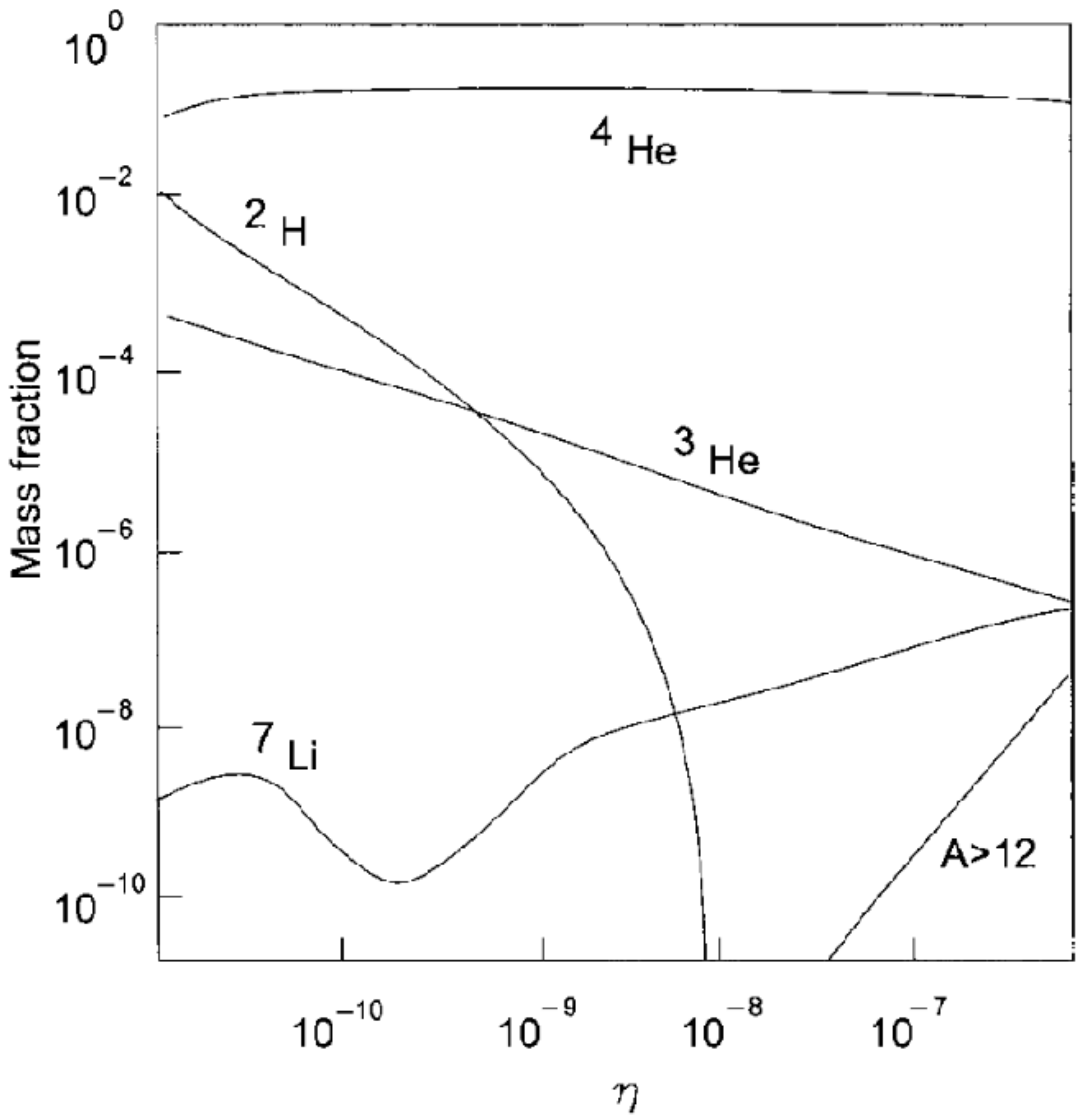


Fig. 11.2 Theoretically calculated primordial abundances of various light nuclei as a function of η. Adapted from Wagoner (1973).

Primordial nucleosynthesis

From observations, it is found that **the cosmic deuterium abundance is not less than 10^{-5}.**

This puts an **upper bound on η,** which is $\eta < 10^{-9}$. A larger value of η would not allow the observed deuterium to be left over after the primordial nucleosynthesis.

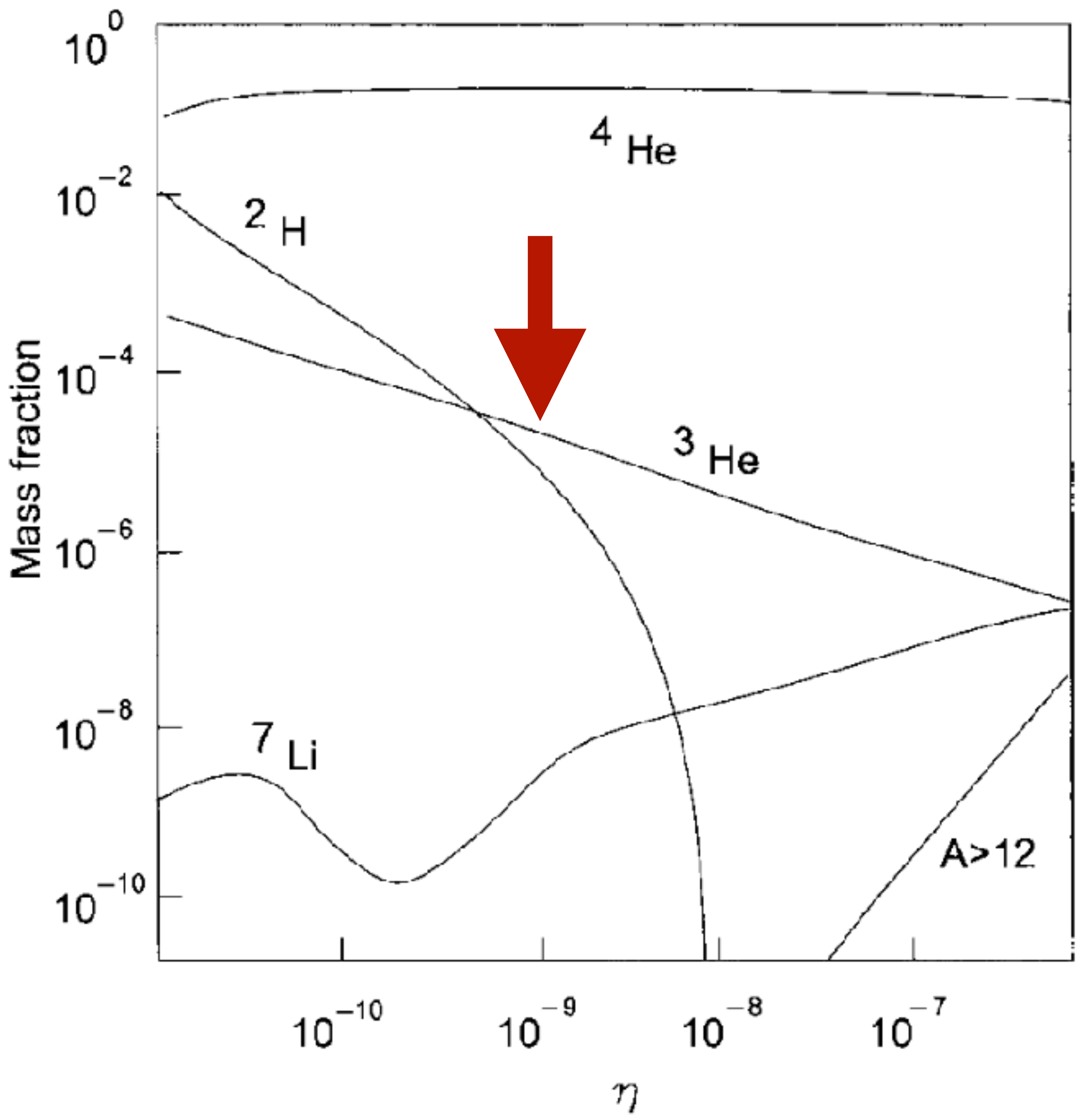


Fig. 11.2 Theoretically calculated primordial abundances of various light nuclei as a function of η. Adapted from Wagoner (1973).

Primordial nucleosynthesis

This upper bound on η has a tremendously important significance. It follows from (11.19) that this **upper bound on η translates into an upper bound on $\Omega_{B,0}$** which is $\Omega_{B,0} < 0.037h^{-2}$. Even if we take 0.64 as the lowest possible value of h in accordance with (9.20), we are still bound by the limit $\Omega_{B,0}$ **< 0.09**.

On the other hand, the value of $\Omega_{M,0}$ based on virial masses of clusters of galaxies is about 0.3. There is other evidence suggesting that $\Omega_{M,0}$ indeed has a value close to 0.3.

How do we reconcile this result with the limit $\Omega_{B,0} < 0.09$?

The only way of reconciling these results is to conclude that **a large part of the matter in the Universe is non-baryonic in nature** and hence is not included when we estimate $\Omega_{B,0}$ or η.

The different objects around us and we ourselves are made up of atoms with nuclei which consist of baryonic matter. If much of the matter in the Universe is non-baryonic, it means that **this matter is not made up of ordinary atoms**. We believe that most of the matter in the Universe does not emit light and hence is dark (**dark matter**).

The cosmic neutrino background

$$p + e \rightarrow n + \nu.$$

$$n \rightarrow p + e + \bar{\nu}$$

Various thermal processes in the early Universe created photons and that is why we still have a cosmic background of blackbody radiation. Exactly similarly **we would expect a background of neutrinos because neutrinos were created in the early Universe** which involved weak interactions and these neutrinos **must be present today.**

When the weak interaction rate was faster than the rate of expansion of the Universe, these **neutrinos must have been in thermodynamic equilibrium with matter** and would have satisfied relations like (11.9), (11.10) and (11.15) for fermions. After the weak interaction rate became slower, the **neutrinos would get decoupled from matter and thereafter must have evolved adiabatically**. We know that photons in thermodynamic equilibrium continue to remain in thermodynamic equilibrium under adiabatic expansion. If neutrinos are either massless or continue to remain relativistic (i.e. the thermal energy continues to remain higher than rest mass energy $m_\nu c^2$), then exactly the same considerations should hold for the background neutrinos as well. So they should continue to have a **distribution appropriate for thermodynamic equilibrium**, with the temperature T falling as a^{-1}.

In the early Universe, photons, neutrinos and matter particles must all have been in thermodynamic equilibrium and must have had the same temperature. Since the temperatures of both photons and neutrinos fall as a^{-1} after decoupling, should we still expect them to have the same temperature?

The cosmic neutrino background

The background neutrinos would have the same temperature as the CMBR photons only if nothing had happened to change the temperature of photons after the neutrinos decoupled from the other particles.

We believe that one important phenomenon had **changed the temperature of the photons. When the neutrinos decoupled,** the **electrons were still relativistic**.

So the Universe must have been filled with electrons and positrons with number densities given by (11.9). **When the temperature fell, the electrons and positrons would have annihilated each other creating photons,** thereby putting **more energy in the photon background** and increasing its temperature.

Let $\boldsymbol{T_i}$ be the temperature **before the electron-positron annihilation** (i.e. it would have been the temperature of the electrons and positrons as well as photons) and let $\boldsymbol{T_f}$ **be the enhanced temperature** of the photons after this annihilation. Since this is an adiabatic process, we expect the entropy to remain conserved. Equating the final entropy with the initial entropy, we shall now obtain a relation between Ti and Tf .

The cosmic neutrino background

To find the **initial entropy density** s_i , we have to add up the contributions of photons, electrons and positrons as given by (11.15). For all these particles, we have $g = 2$ – because of the two polarization states of photons and two spin states of electrons or positrons. Remembering that photons are bosons whereas electrons and positrons are fermions, we get

$$s_i = \left(\frac{4}{3} + \frac{7}{8} \times \frac{4}{3} + \frac{7}{8} \times \frac{4}{3}\right) a_{\mathrm{B}} T_i^3 = \frac{11}{4} \times \frac{4}{3} a_{\mathrm{B}} T_i^3. \qquad (11.24)$$

The final entropy density s_f is due only to the photons and must be

$$s_f = \frac{4}{3} a_{\mathrm{B}} T_f^3. \qquad (11.25)$$

Equating s_i and s_f as given by the above two equations, we get

$$\frac{T_f}{T_i} = \left(\frac{11}{4}\right)^{1/3}. \qquad (11.26)$$

The cosmic neutrino background

In other words, the **photon temperature jumped by this factor** after the electron-positron annihilation, **but the neutrino temperature did not change** because the neutrinos constituted a distinct system decoupled from other matter. Since both the photon temperature and the neutrino temperature afterwards fell as a^{-1}, **the photon temperature would still be larger than the neutrino temperature by this factor (11/4)$^{\mathbf{1/3}}$**.

The present neutrino temperature should be

$$T_{\nu,0} = \left(\frac{4}{11}\right)^{1/3} T_0, \tag{11.27}$$

where T_0 is the present temperature of the CMBR given by (10.45). On taking $T_0 = 2.735$ K, we get

$$T_{\nu,0} = 1.95 \text{ K}. \tag{11.28}$$

The cosmic neutrino background

We can calculate the energy density of the neutrino background by using (11.10). Since there are three types of neutrinos (ν_e, ν_μ, ν_τ) and each neutrino has an antineutrino, we take $g = 6$ so that the present energy density of the neutrino background, according to (11.10), is

$$\rho_{\nu,0} = \frac{7}{8}\frac{3}{c^2}a_{\mathrm{B}}\left[\left(\frac{4}{11}\right)^{1/3} T_0\right]^4 = 0.68\frac{a_{\mathrm{B}}}{c^2}T_0^4 = 0.68\rho_{\gamma,0}, \qquad (11.29)$$

where $\rho_{\gamma,0}$ is the present energy density of the CMBR. We have already made use of this earlier when adding the energy density of neutrinos to the energy density of photons.

We again stress one point: The contribution to density by neutrinos will be given by (11.29) **only if the neutrinos are still relativistic.** If the neutrinos have mass and **if the thermal energies become smaller than $m_\nu c^2$ with the fall in temperature, then it is possible for neutrinos to contribute much more to the density** because there would be a lower bound in the contribution to density made by a neutrino. A neutrino with mass m_ν has to contribute at least m_ν to density.

The nature of dark matter

Much of the matter in our Universe does not emit light.

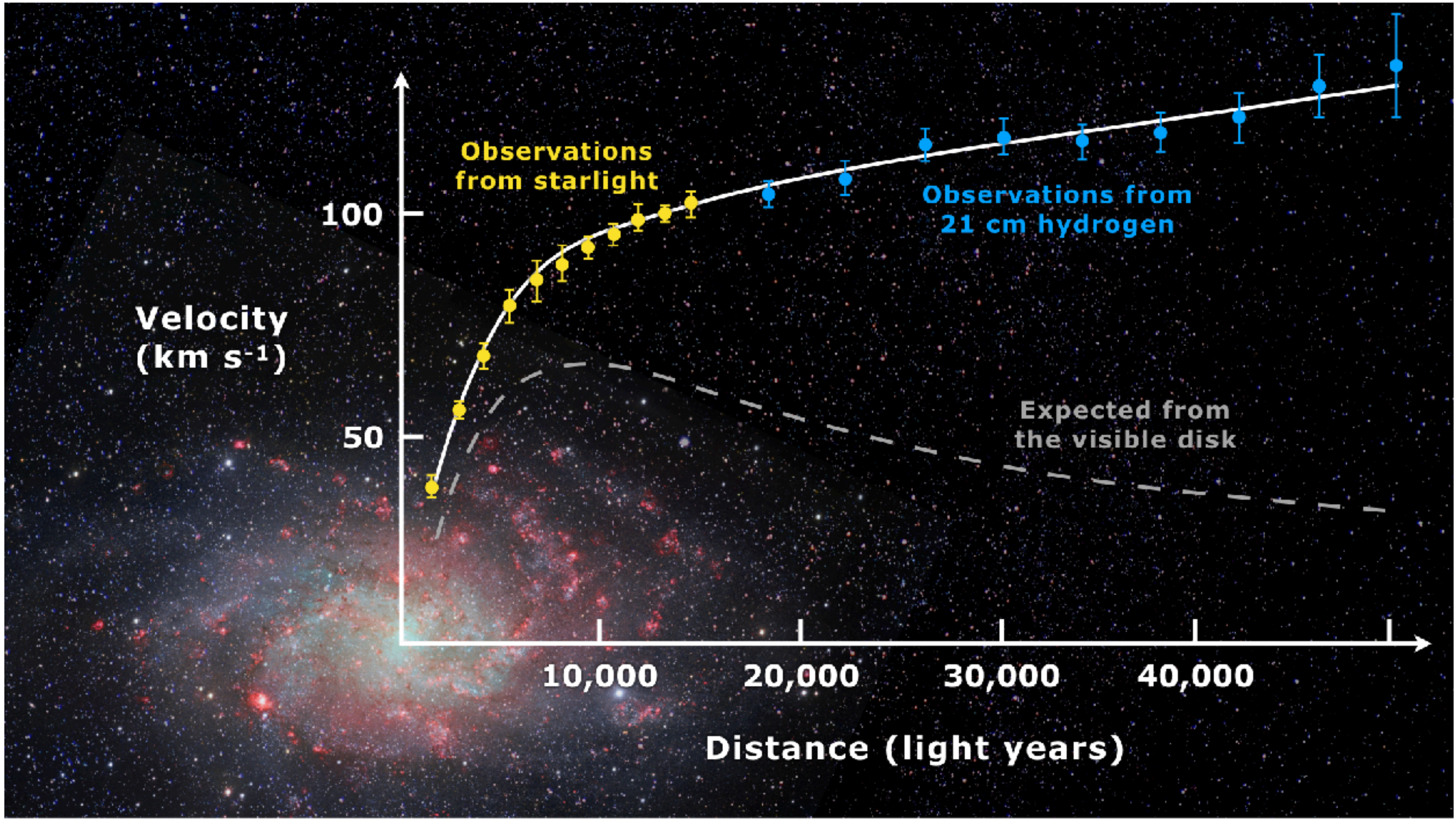
Evidence for dark matter comes from

- **the rotation curves of spiral galaxies**
- **and from the application of the virial theorem to galaxy clusters.**
- **Primordial nucleosynthesis calculations** lead us to the extraordinary conclusion that much of the dark matter is non-baryonic, i.e. not made up of ordinary atoms.

We now briefly discuss the question of what could be the constituents of dark matter.

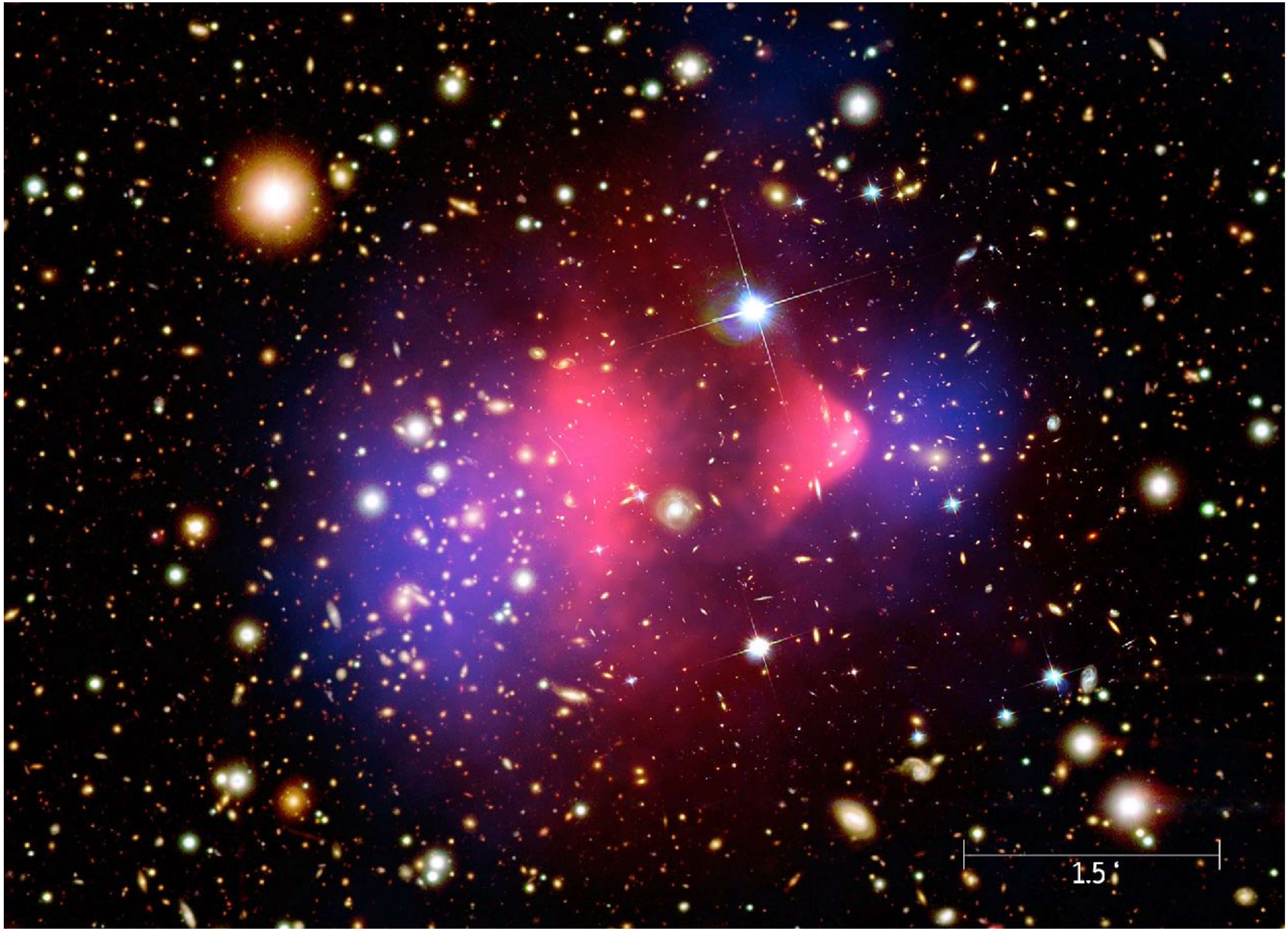
The nature of dark matter

Rotation curve of spiral galaxy Messier 33 (yellow and blue points with error bars), and a predicted one from distribution of the visible matter (gray line). The data and the model predictions are from Corbelli and Salucci 2000.
The discrepancy between the two curves can be accounted for by adding a **dark matter halo surrounding the galaxy**.



Dark matter

Dark matter in the Bullet cluster Composite image showing the galaxy cluster known as bullet cluster. The image in **background** showing the **visible spectrum** of light stems from Magellan and Hubble Space Telescope images. The **pink overlay shows the x-ray emission** (recorded by Chandra Telescope) of the **colliding galaxy clusters,** the **blue one represents the mass distribution** of the clusters calculated from gravitational lens effects.



The nature of dark matter

While neutrinos are very light particles, physicists wondered for a very long time whether they are massless or have very small mass.

If neutrinos have mass and the masses of the three types of neutrinos are different, then **it is possible for neutrinos to have oscillations** in which one type of neutrino gets converted into another.

Since these oscillations **depend on the mass difference between different kinds of neutrinos,** the discovery of neutrino oscillations led to the conclusion that $|\Delta m|^2$ should be of order $5 \times 10^{-5}\ \mathrm{eV}^2$.

While we do not know the individual masses of different types of neutrinos, we now know that neutrinos have mass.

Is it then possible that the background neutrinos are responsible for the non-baryonic mass part of dark matter?

The nature of dark matter

$$n = \begin{cases} \frac{\zeta(3)}{\pi^2} g \left(\frac{\kappa_B T}{\hbar c}\right)^3 & \text{(boson)}, \\ \frac{3}{4} \frac{\zeta(3)}{\pi^2} g \left(\frac{\kappa_B T}{\hbar c}\right)^3 & \text{(fermion)}, \end{cases} \tag{11.9}$$

To answer this question, let us first find out the **number density of neutrinos.**

As long as neutrinos are relativistic, the number density is given by (11.9). It turns out that (11.9) can still be used to calculate the number density of neutrinos even if they had become non-relativistic, provided we take T to be a quantity which falls off as a^{-1} in the expanding Universe.

On taking $g = 6$ and substituting the temperature given by (11.28), we obtain from (11.9) that

$$n_{\nu,0} = 3.36 \times 10^8 \text{ m}^{-3}. \tag{11.30}$$

If m_ν is the average mass of neutrinos, then we have to divide $m_\nu n_{\nu,0}$ by $\rho_{c,0}$ given by (10.29) to get the density parameter $\Omega_{\nu,0}$ due to neutrinos, which turns out to be

$$\Omega_{\nu,0} = 3.18 \times 10^{-2} h^{-2} \left(\frac{m_\nu}{\text{eV}}\right), \tag{11.31}$$

where m_ν has to be in eV.

The nature of dark matter

We have seen that $\Omega_{M,0}$ **should have a value of about 0.3.** Since **the neutrino contribution to the density parameter has to be less than this,** we get the following limit by demanding that $\Omega_{\nu,0}$ given by (11.31) should be less than 0.3:

$$\boxed{m_\nu < 9.4h^2 \text{ eV}. \qquad (11.32)}$$

This is known as the ***Cowsik–McClelland limit***.

If we hypothesize that neutrinos have an average mass close to the Cowsik-McClelland limit, then we can solve the mystery of non-baryonic dark matter. There are, however, **some serious difficulties** with this hypothesis that dark matter is made up of neutrinos with mass just a little less than the limit.

Although we do not have a good idea about the **distribution of dark matter**, observations indicate that **it is not uniformly spread throughout the Universe.** Estimates of the masses of spiral galaxies and galaxy clusters suggest that **much of the dark matter should be gravitationally bound** with these systems.

The nature of dark matter

$$m_\nu < 9.4h^2 \text{ eV}. \tag{11.32}$$

For this to be possible, the typical **kinetic energy of dark matter particles should not exceed the gravitational binding energy** $m_\nu|\Phi|$, where Φ is the gravitational potential associated with structures like galaxies and galaxy clusters.

This condition is hard to satisfy if the limit of m_ν is given by (11.32). **If dark matter particles satisfy (11.32), then that type of dark matter is called *hot dark matter***. Hot dark matter is a theoretical form of dark matter which consists of particles that travel with **ultrarelativistic** velocities.

Such dark matter **would tend to be distributed throughout the Universe without clumping in the gravitational structures** like galaxies or galaxy structures. If we want **dark matter to be bound** in these gravitational structures, then we need to have ***cold dark matter***, in which **particles are more massive, move more slowly at a given temperature** (since the thermal velocity is given by $\sqrt{2\kappa_B T/m}$) and can get gravitationally bound in galaxies or galaxy clusters.

Cold dark matter helps to fulfil some requirements of structure formation as well.

The nature of dark matter

$$m_\nu < 9.4h^2 \text{ eV}. \tag{11.32}$$

$$n = \frac{g}{\hbar^3}\left(\frac{m\kappa_B T}{2\pi}\right)^{3/2} \exp\left(-\frac{mc^2-\mu}{\kappa_B T}\right). \tag{11.16}$$

In view of (11.32), **is it possible to have cold dark matter with particles more massive than this limit?**

Note that we obtained (11.32) by using (11.9), which would be applicable only if the particles were relativistic at the time of decoupling. **If the particles are so heavy that they were already non-relativistic at the time of decoupling, then the number density would be given by (11.16)**.

Some **supersymmetric theories** of particle physics suggest the possibility of some **particle with mass of about a few GeV**, which acts with other particles only through the weak interaction. Such a particle is expected to get decoupled from the other constituents of the Universe when the temperature was of order MeV. This particle would remain in thermodynamic equilibrium before the decoupling and its number density at the time of decoupling would be given by (11.16). The exponential factor in (11.16) would ensure a low number density of this particle.

The nature of dark matter

If the particle is **more massive than about 3 GeV**, detailed calculations show that its number density would be sufficiently suppressed by this exponential factor and it would not make a contribution to the density parameter more than what is estimated from observations (i.e. $\Omega_{M,0} \approx 0.3$).

The limit that the particles of cold dark matter have to be more massive than 3 GeV is known as the *Lee–Weinberg limit*.

We basically conclude that dark matter particles could not have mass in the range 10 eV to 3 GeV. **They either have to be lighter than 10 eV (hot dark matter) or heavier than 3 GeV (cold dark matter).**

Current evidence suggests the second possibility that the dark matter in the Universe should be cold dark matter made up of particles heavier than 3 GeV.

WIMPs

Weakly interacting massive particles (WIMPs) are **hypothetical particles** that are one of the proposed **candidates for dark matter.**

There exists no formal definition of a WIMP, but broadly, it is **an elementary particle which interacts via gravity** and any other force (or forces), which are **as weak as or weaker than the weak force**, but also non-vanishing in strength.
Many WIMP candidates are **expected to have been produced thermally in the early Universe**, similarly to the particles of the Standard Model according to Big Bang cosmology, and usually will constitute **cold dark matter**. The expected mass for the WHIMPs is in the 100 GeV mass range.

Experimental efforts to detect WIMPs include the **search for products of WIMP annihilation,** including gamma rays, neutrinos and cosmic rays in nearby galaxies and galaxy clusters; **direct detection experiments** designed to measure the collision of WIMPs with nuclei in the laboratory, as well as attempts to directly produce WIMPs in colliders, such as the Large Hadron Collider at CERN.

Supersymmetry

Supersymmetry is a theoretical framework in physics that **suggests the existence of a symmetry between particles with integer spin (bosons) and particles with half-integer spin (fermions)**. It proposes that for every known particle, there exists a partner particle with different spin properties. There have been multiple experiments on supersymmetry that have failed to provide evidence that it exists in nature.

In supersymmetry, each particle from the class of fermions would have an associated particle in the class of bosons, and vice versa, known as a superpartner. The spin of a particle's superpartner is different by a half-integer. For example, if the electron exists in a supersymmetric theory, then there would be a particle called a selectron (superpartner electron), a bosonic partner of the electron. In the simplest supersymmetry theories, with perfectly "unbroken" supersymmetry, each pair of superpartners would share the same mass and internal quantum numbers besides spin. More complex supersymmetry theories have a spontaneously broken symmetry, allowing superpartners to differ in mass.

https://en.wikipedia.org/wiki/Supersymmetry