Introduction to Astrophysics and Cosmology

Stellar Physics

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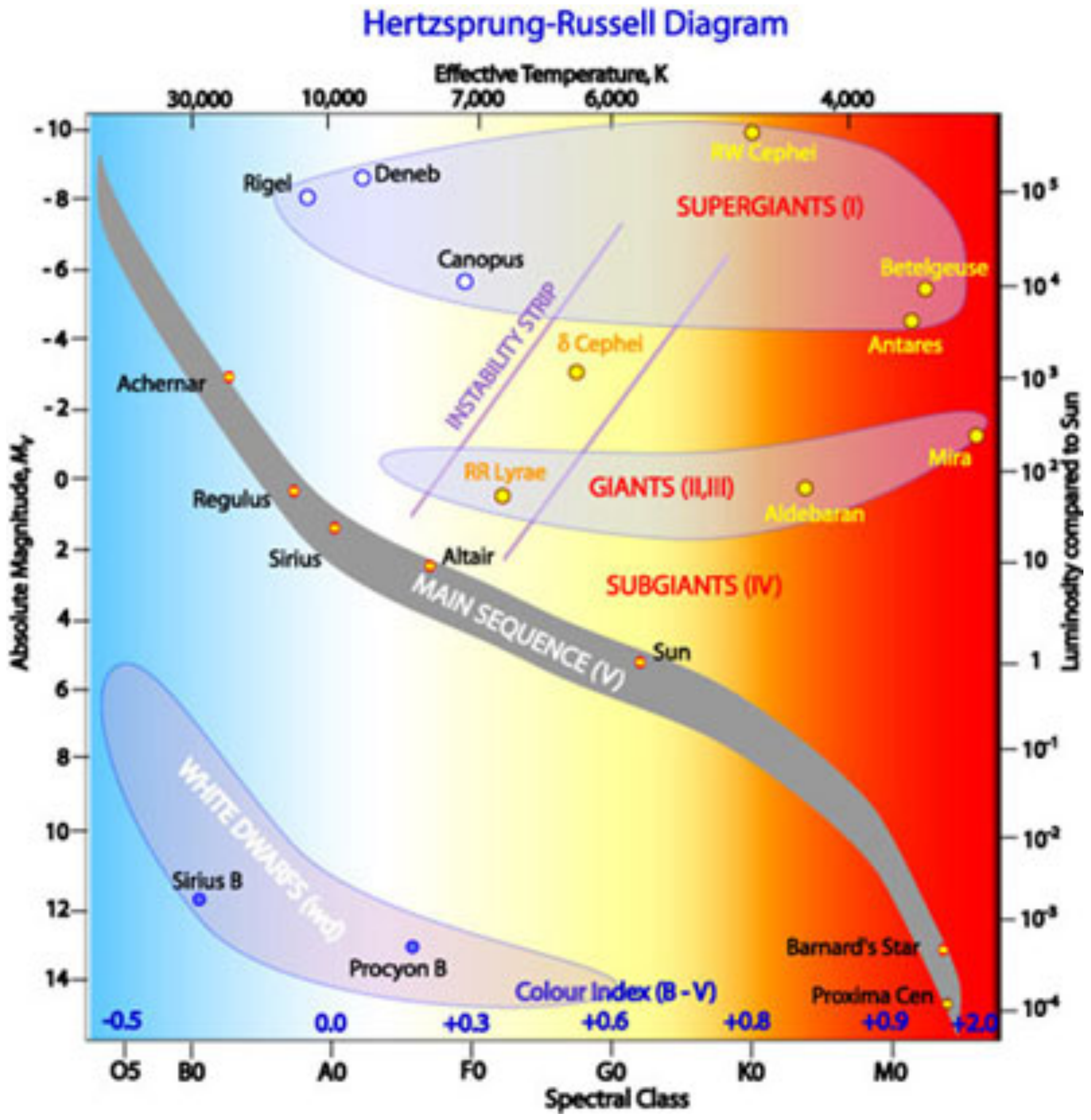
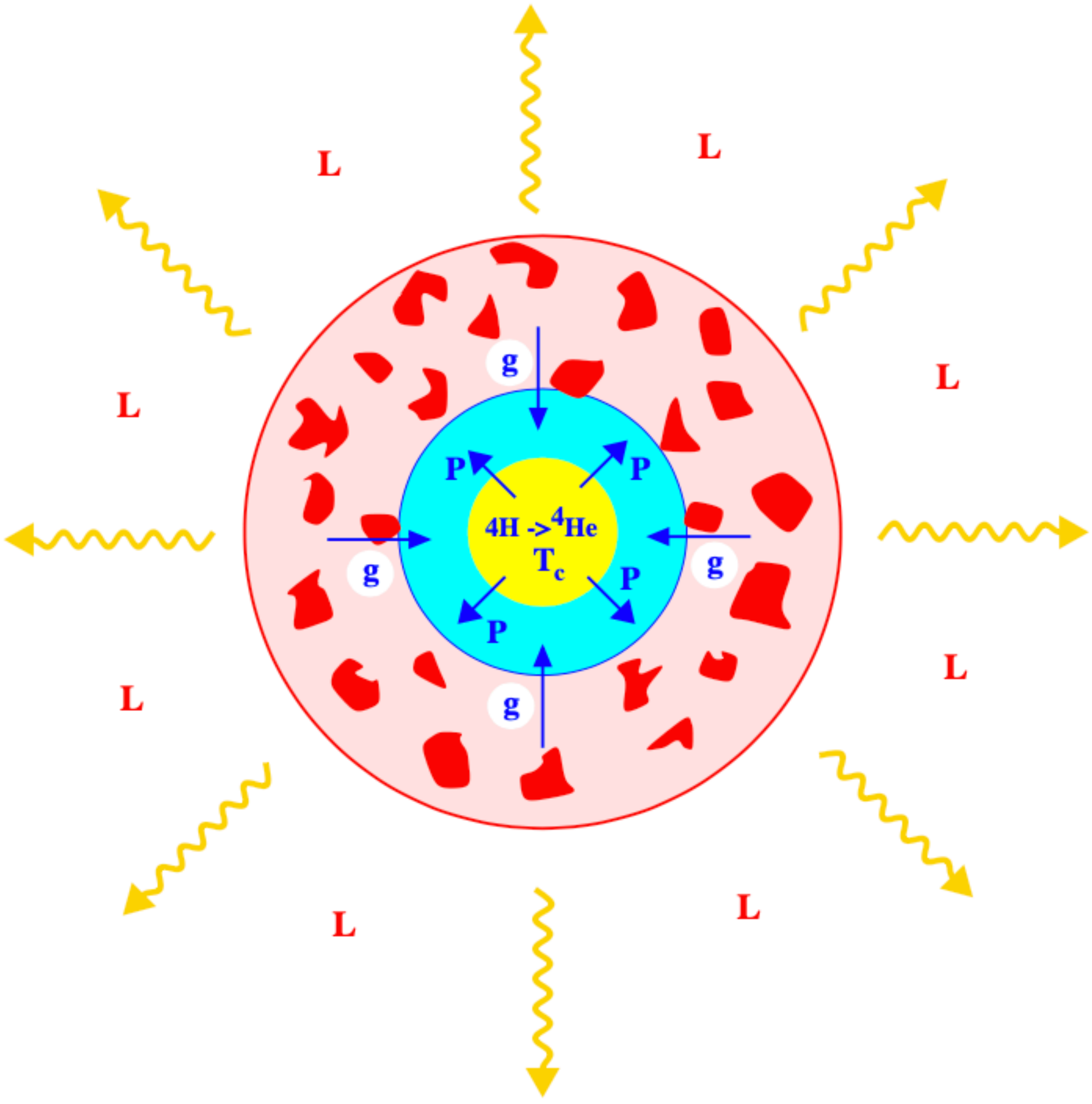
Herzsprung - Russel diagram

Describes the basic classification of stars



Basic equations of stellar structure

- **Stars are self-gravitating bodies in dynamical equilibrium** → balance of gravity and internal pressure forces (hydrostatic equilibrium);
- **stars lose energy by radiation from the surface** → stars supported by thermal pressure require an energy source to avoid collapse, e.g. nuclear energy, gravitational energy (energy equation);
- the **temperature structure** is largely determined by the mechanisms by which energy is transported from the core to the surface, radiation, convection, conduction (energy transport equation);



Basic equations of stellar structure

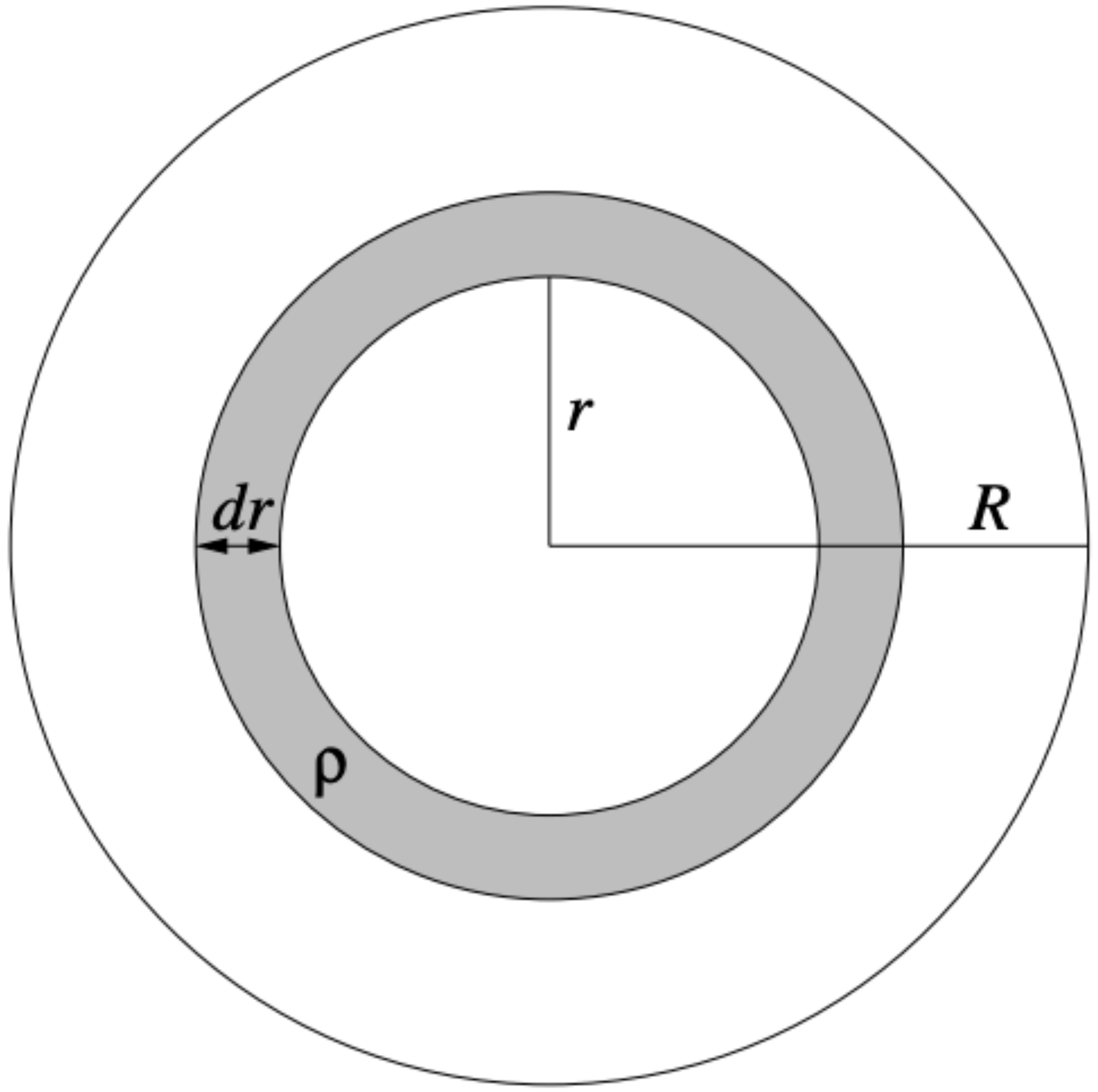
To establish the basic equations of stellar structure we **assume the star to be spherically symmetric**.

- If the star is rotating sufficiently rapidly, then there will be some flattening in the direction of the rotation axis.
- if the star has strong magnetic fields, that can be another cause of departure from spherical symmetry. e.g. the corona of the Sun

- We can assume that under the surface the spherical symmetry holds

Basic equations of stellar structure

Let M_r be the mass inside the radius r of a star. Then the mass inside radius $r + dr$ should be $M_r + dM_r$, which means that dM_r is the mass of the spherical shell between radii r and $r + dr$. If ϱ is the density at radius r, then the mass of this shell is:



$$dM_r = 4\pi r^2 \rho \, dr,$$

$$\boxed{\frac{dM_r}{dr} = 4\pi r^2 \rho}$$

first stellar structure equation, Equation of Mass Conservation

Basic equations of stellar structure

Hydrostatic equilibrium:

Let us now consider a small portion of the shell between r and $r + dr$.

If dA is the transverse area of this small element, the forces exerted by pressure acting on its inward and outward surfaces are $P\,dA$ and $-(P + dP)\,dA$, where P and $P + dP$ are respectively the pressures at radii r and $r + dr$.

So the net force arising out of pressure is $-dP\,dA$, which should be balanced by gravity under equilibrium conditions.

The gravitational field at r is caused by the mass M_r inside r and is equal to $-GM_r/r^2$. Since the mass of the small element under consideration is $\rho\,dr\,dA$, the force balance condition for it is

$$-dP\,dA - \frac{GM_r}{r^2}\rho\,dr\,dA = 0, \quad \longrightarrow \quad \boxed{\frac{dP}{dr} = -\frac{GM_r}{r^2}\rho}$$

Second stellar structure equation,

Equation of Hydrostatic Equilibrium

Basic equations of stellar structure

Hydrostatic equilibrium:

- Three variable functions of the radial coordinate r : M_r , ϱ and P . -> Two equations are not enough to solve for three variable functions.
  - There are special kinds of dense stars like white dwarfs and neutron stars inside which pressure becomes a function of density alone. In such cases, the number of independent variables becomes two rather than threes.
- In normal stars, however, the stellar material behaves very much like a perfect gas, and pressure is a function of both density and temperature, having the form $P \propto \varrho T$. In such a situation, we need additional equations for temperature and energy generation to solve the stellar structure.

Basic equations of stellar structure

Although the hydrostatic equilibrium does not tell us the whole story about the stellar interior, we now show that it can nevertheless provide us with valuable clues about the interior conditions of stars.

In the astrophysical Universe, we often have to deal with quantities for whose magnitudes we have no a priori feeling. For example, what are the temperature T_C and pressure P_C at the centre of the Sun?

For various order of magnitude estimates involving stars, we shall use the following **approximate values** of solar luminosity $L_\odot$ and solar radius $R_\odot$:

$\boldsymbol{L_\odot \approx 4 \times 10^{26}}$ **W,**

$\boldsymbol{R_\odot \approx 7 \times 10^{8}}$ **m.**

Table 1.1 Approximate conversion factors to be memorized.

$M_\odot$	$\approx$	2×10^{30} kg
pc	$\approx$	3×10^{16} m
yr	$\approx$	3×10^{7}s

A.2 Astronomical constants

1 astronomical unit	AU	=	1.50×10^{11} m
1 parsec	pc	=	3.09×10^{16} m
1 year	yr	=	3.16×10^{7} s
Mass of Sun	$M_\odot$	=	1.99×10^{30} kg
Radius of Sun	$R_\odot$	=	6.96×10^{8} m
Luminosity of Sun	$L_\odot$	=	3.84×10^{26} W
Mass of Earth	$M_\oplus$	=	5.98×10^{24} kg
Radius of Earth	$R_\oplus$	=	6.37×10^{6} m

Basic equations of stellar structure

For the purpose of an order of magnitude estimate, the derivative dP/dr can be replaced by $-P_c/R_\odot$. The various quantities on the right-hand side have to be replaced by their appropriate averages. Taking $M_\odot/2$ and $R_\odot/2$ to be the averages of M_r and r:

$$\frac{dP}{dr} = -\frac{GM_r}{r^2}\rho \quad \longrightarrow \quad \frac{P_c}{R_\odot} \approx \frac{G(M_\odot/2)}{(R_\odot/2)^2}\left(\frac{M_\odot}{\frac{4}{3}\pi R_\odot^3}\right)$$

Substituting the values of $M_\odot$ and $R_\odot$, we find

$$P_c \approx 6 \times 10^{14}\ \mathrm{N\ m^{-2}}$$

Basic equations of stellar structure

Since the gas inside the Sun behaves very much like a perfect gas, we can use $P = n\kappa_B T$, where n is the number density of gas particles.

Assuming the gas to contain hydrogen predominantly, the number of atoms per unit volume is ρ/m_H. Since hydrogen is completely ionized in the deep solar interior and each hydrogen atom contributes two particles (a proton and an electron), we have $n = 2\rho/m_H$ so that

$$P = \frac{2\kappa_{\mathrm{B}}}{m_{\mathrm{H}}}\rho\, T.$$

If we take the central density to be about twice the mean density, then at the centre of the Sun:

$$P_{\mathrm{c}} = \frac{4\kappa_{\mathrm{B}}}{m_{\mathrm{H}}}\left(\frac{M_{\odot}}{\frac{4}{3}\pi R_{\odot}^{3}}\right)T_{\mathrm{c}}.$$

Basic equations of stellar structure

On taking the value of P_C and substituting the values of other quantities, we obtain

$$T_c \approx 10^7 \text{ K}.$$

Thus we have estimated the values of central pressure and temperature of the Sun in a relatively simple way.

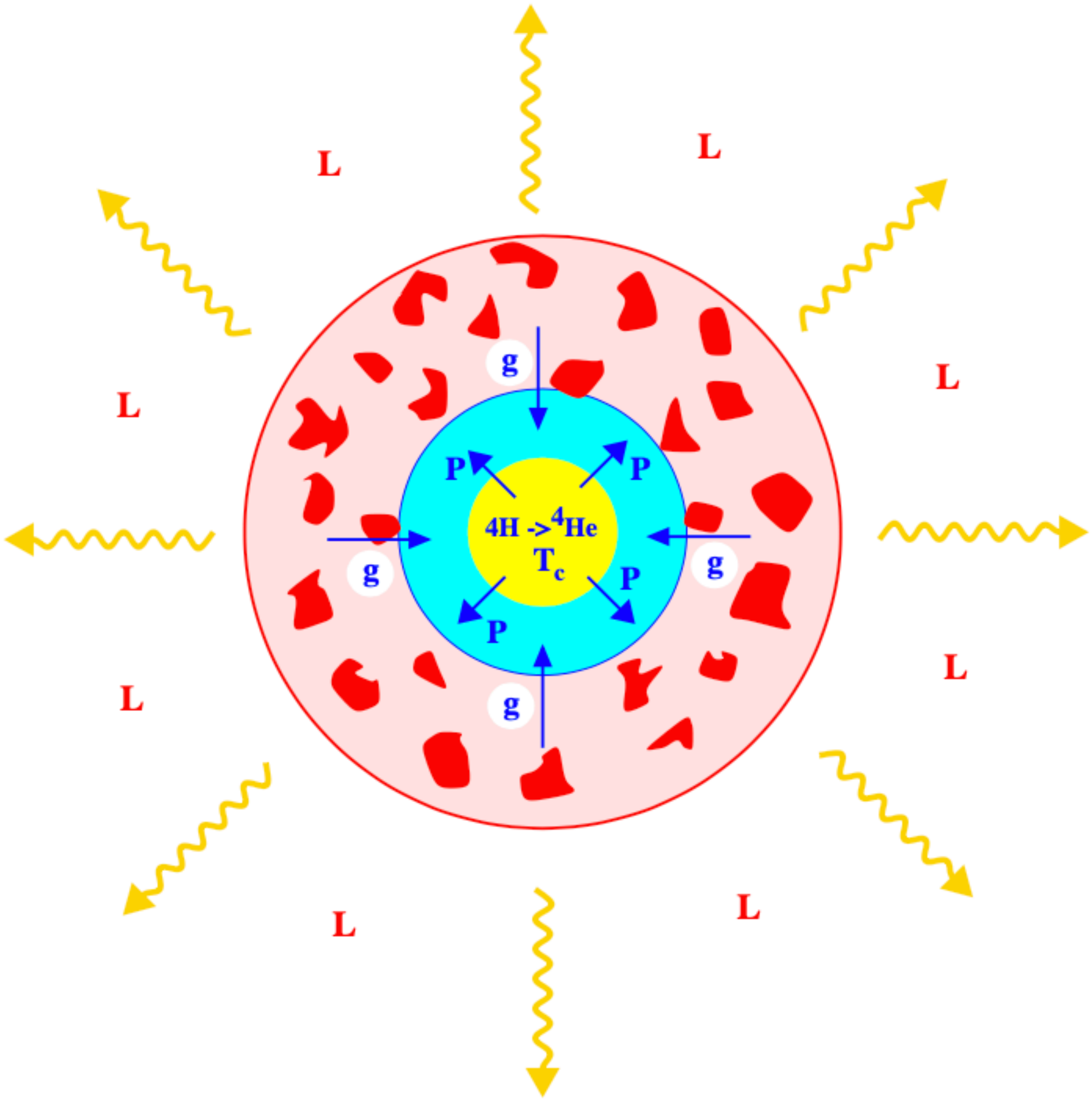
These values compare quite favourably with the values which one obtains from a detailed solution of all stellar structure equations.

Virial Theorem for stars

In ordinary stars like the Sun, the inward gravitational pull is balanced by the excess pressure of the hot interior. In other words, it is the thermal energy of the interior which balances gravity. We, therefore, expect that the total thermal energy should be of the same order as the total gravitational energy. This can be established from the hydrostatic equilibrium equation. We multiply both sides by $4\pi r^3$ and then integrate from the centre of the star to its outer radius R. This gives

$$\frac{dP}{dr} = -\frac{GM_r}{r^2}\rho$$

$$\int_0^R \frac{dP}{dr} 4\pi r^3 dr = \int_0^R \left(-\frac{GM_r}{r^2}\rho\right) 4\pi r^3 dr.$$



Virial Theorem for stars

$$\int_0^R \frac{dP}{dr} 4\pi r^3 dr = \int_0^R \left(-\frac{GM_r}{r^2}\rho\right) 4\pi r^3 dr.$$

$$-\int_0^R 3P \times 4\pi r^2 dr = \int_0^R \left(-\frac{GM_r}{r}\right) 4\pi r^2 \rho \, dr.$$

Total thermal energy

Total gravitational energy

$$E_{\mathrm{T}} = \int_0^R \frac{3}{2} n \kappa_{\mathrm{B}} T \times 4\pi r^2 dr = \int_0^R \frac{3}{2} P \times 4\pi r^2 dr.$$

$$E_{\mathrm{G}} = \int_0^R \left(-\frac{GM_r}{r}\right) 4\pi \rho \, r^2 dr.$$

Virial Theorem for stars

This is known as the **virial theorem**:

$$2E_{\mathrm{T}} + E_{\mathrm{G}} = 0.$$

$$E_{\mathrm{T}} = -\frac{1}{2}E_{\mathrm{G}} = \frac{1}{2}|E_{\mathrm{G}}|,$$

The sum of the thermal and gravitational energy:

$$E = E_{\mathrm{G}} + E_{\mathrm{T}} = \frac{1}{2}E_{\mathrm{G}} = -\frac{1}{2}|E_{\mathrm{G}}|$$

We now know that **a normal star radiates energy which is produced by nuclear reactions in the interior.** So, apart from thermal and gravitational energies, a star has another store of energy, i.e. nuclear energy.

If there was no such thing as nuclear energy, then all stars had to contract slowly. Half of the gravitational potential energy released in the process has to be converted to thermal energy, whereas the other half should leave the system, presumably in the form of radiation.

Energy transport inside a star

The energy generated by nuclear reactions in the central region of a star is transported outward. We now have to derive equations which describe this process.

Let L_r be the total amount of energy flux per unit time which flows outward across a spherical surface of radius r inside the star (the spherical surface being centred at the centre of the star).

We expect L_r to be equal to the luminosity L of the star at the outer radius $r = R$ of the star. If $L_r + dL_r$ is the outward energy flux at radius $r + dr$, then dL_r is obviously the additional input to the energy flux made by the spherical shell between r and $r + dr$. If ε is the rate of energy generation per unit mass per unit time (presumably by nuclear reactions), then we should have

$$dL_r = 4\pi r^2 dr \times \rho\,\varepsilon$$

$$\boxed{\frac{dL_r}{dr} = 4\pi r^2 \rho\,\varepsilon}$$

Third stellar structure equation,

Energy Conservation

Energy transport inside a star

The energy flux is driven by the temperature gradient inside the star. We need an equation for that as well.

We know that there are three important modes of heat transfer in nature: **conduction, convection and radiation.**

Although conduction is important in compact stars like white dwarfs, it turns out to be totally unimportant in the interiors of normal stars. In the next subsection, we shall discuss the possibility of convection. Right now, let us consider a region in the interior of a star where heat is transported outward only by radiative transfer. We have already derived an expression for the energy flux per unit area by radiative transfer in (2.78). Replacing z by r, the energy flux L_r across the spherical surface of radius r is given by

$$L_r = 4\pi r^2 F = -4\pi r^2 \frac{c}{\chi\rho} \frac{d}{dr} \left(\frac{a_B}{3} T^4 \right)$$

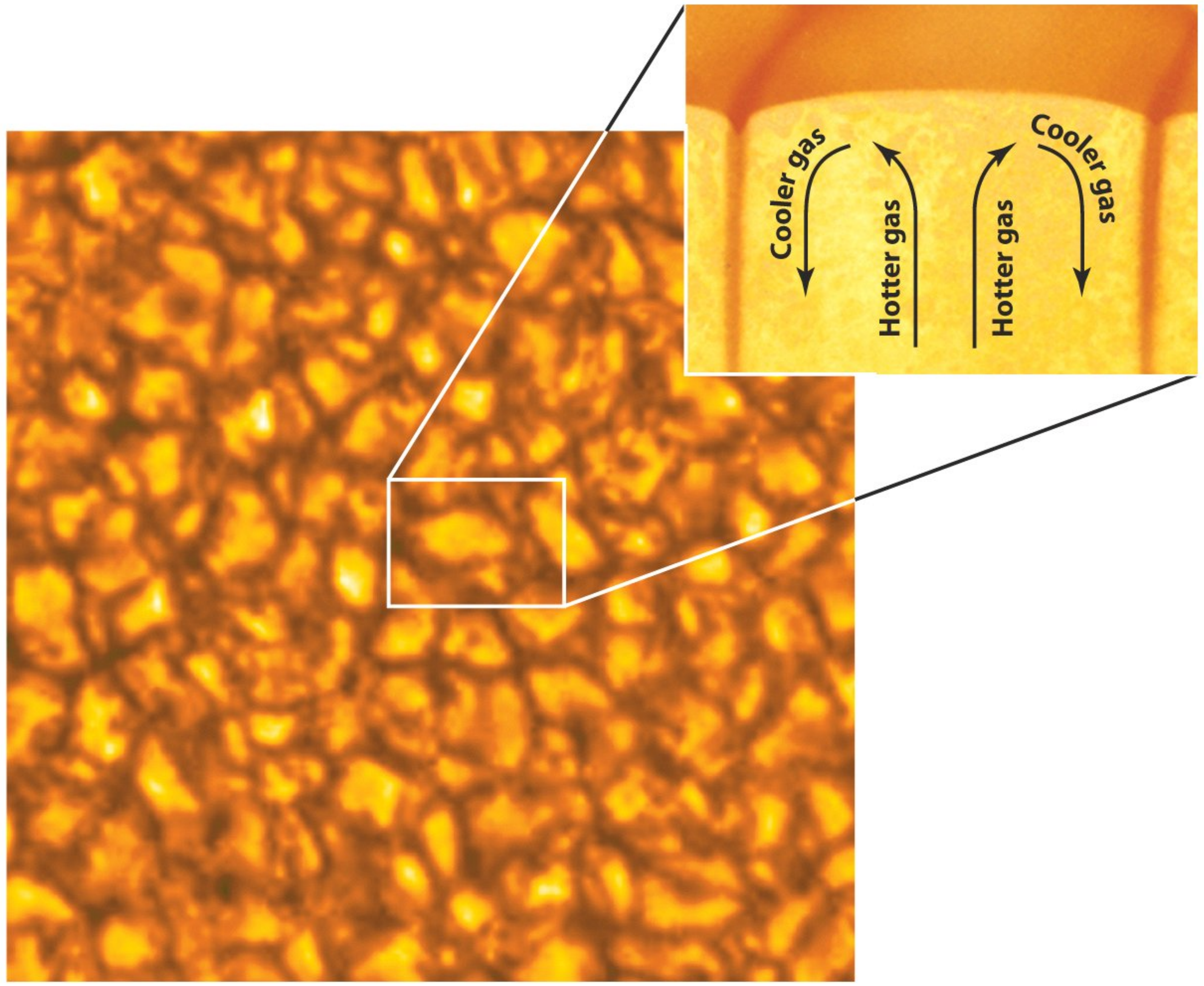
$$\boxed{\frac{dT}{dr} = -\frac{3}{4 a_B c} \frac{\chi\rho}{T^3} \frac{L_r}{4\pi r^2}.}$$

Forth stellar structure equation,

Energy transport equation, if the flux is carried outward via radiative transfer

Convection inside a star

In radiative transfer, energy is transported without any material motion. Convection, on the other hand, involves up and down motions of the gas. Hot blobs of gas move upward and cold blobs of gas move downward, thereby transporting heat.



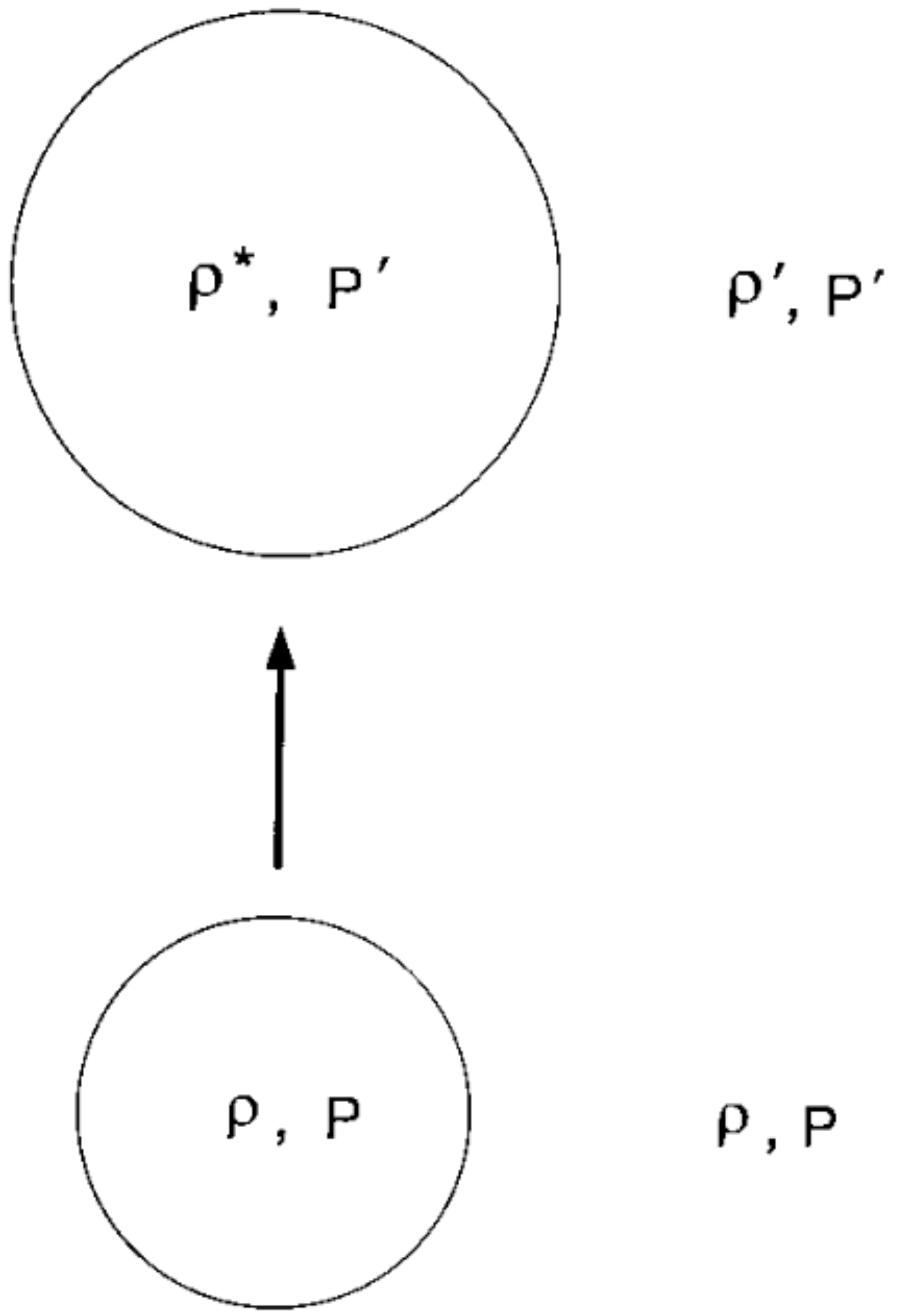


Fig. 3.1 Vertical displacement of a blob of gas in a stratified atmosphere.

Convection inside a star

- Suppose we have a perfect gas in hydrostatic equilibrium inside a star. We now consider a blob of gas which has been displaced vertically upward as shown in Figure 3.1.
- Initially the blob of gas had the same density ϱ and the same pressure P as the surroundings. The external gas density and pressure at the new position of the blob are ϱ' and P'.
- We know that pressure imbalances in a gas are rather quickly removed by acoustic waves, but heat exchange between different parts of the gas takes more time. Hence it is not unreasonable to consider the blob to have been displaced *adiabatically* and to have the same pressure P' as the surroundings in its new position.

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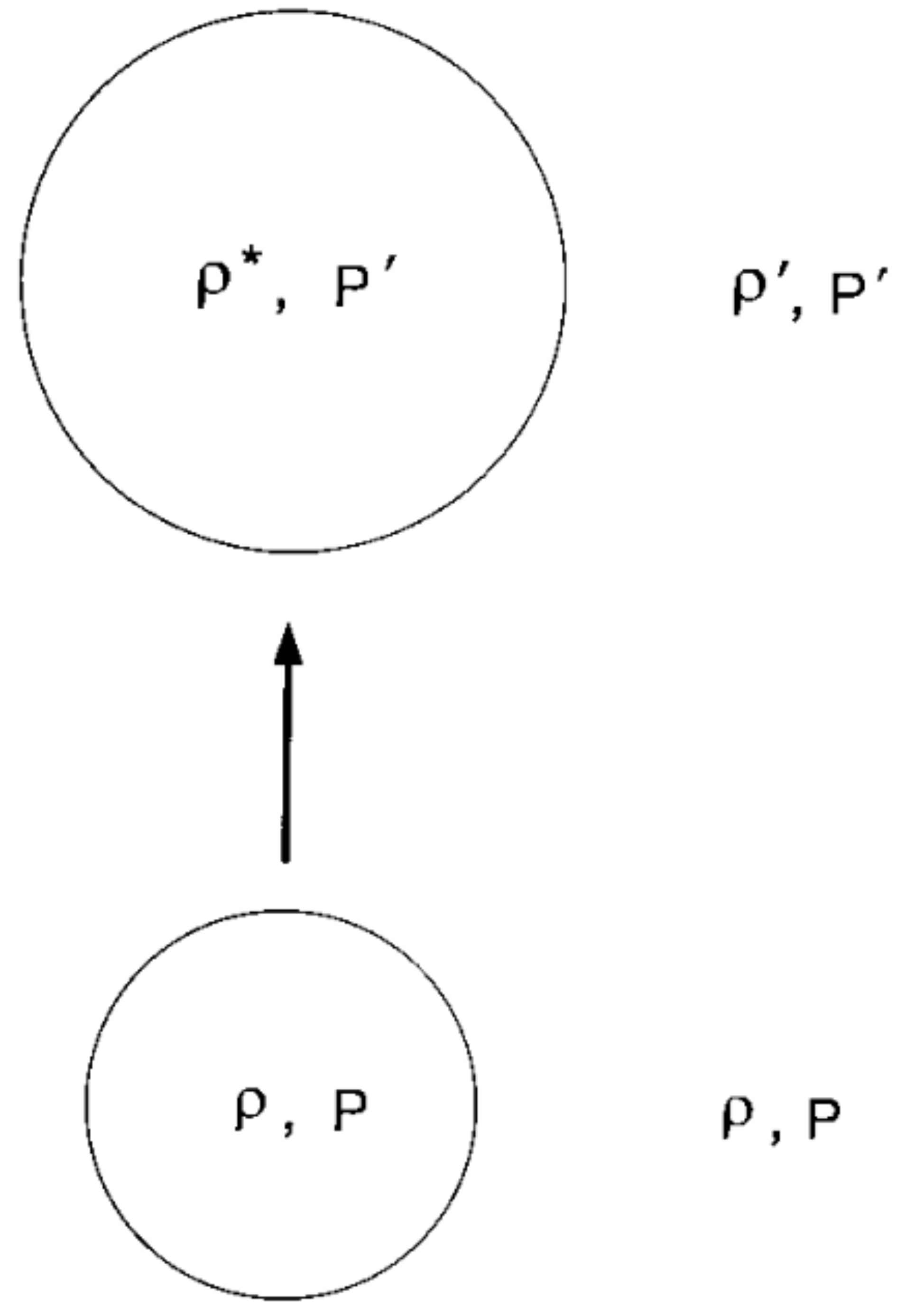


Fig. 3.1 Vertical displacement of a blob of gas in a stratified atmosphere.

Convection inside a star

- Let ϱ^* be its density in the new position.
- If $\varrho^* < \varrho'$, then the displaced blob will be buoyant and will continue to move upward further away from its initial position, making the system unstable and giving rise to convection.
- If $\varrho^* > \varrho'$, then the blob will try to return to its original position so that the system will be stable and there will be no convection. So convection is of the nature of an instability in the system. To find the condition for convective instability, we have to determine whether ϱ^* is greater than or less than the surrounding density ϱ'.

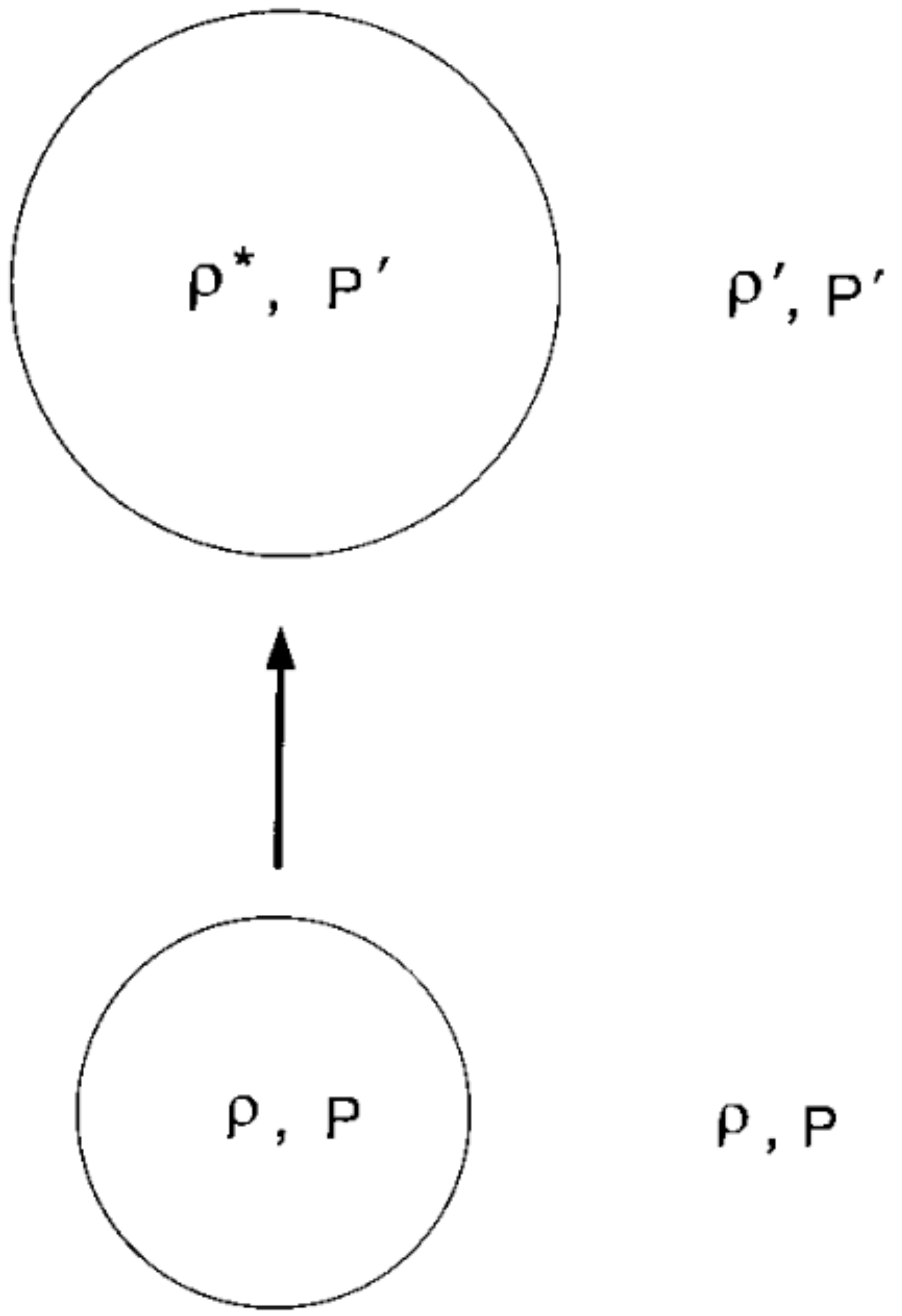


Fig. 3.1 Vertical displacement of a blob of gas in a stratified atmosphere.

Convection inside a star

From the assumption that the blob has been displaced adiabatically, it follows that: $\rho^* = \rho \left(\frac{P'}{P}\right)^{1/\gamma}$

If dP/dr is the pressure gradient in the atmosphere, we can substitute: $P' = P + \frac{dP}{dr}\Delta r$

If and make a binomial expansion keeping terms to the linear order in Δr. This gives:

$$\rho^* = \rho + \frac{\rho}{\gamma P}\frac{dP}{dr}\Delta r.$$

$$\rho' = \rho + \frac{d\rho}{dr}\Delta r.$$

Convection inside a star

Using $\varrho = P/RT$, we get:

$$\rho' = \rho + \frac{\rho}{P}\frac{dP}{dr}\Delta r - \frac{\rho}{T}\frac{dT}{dr}\Delta r.$$

Here $d\varrho/dr$ and dT/dr are the density and temperature gradients in the atmosphere.

$$\rho^* - \rho' = \left[-\left(1 - \frac{1}{\gamma}\right)\frac{\rho}{P}\frac{dP}{dr} + \frac{\rho}{T}\frac{dT}{dr}\right]\Delta r.$$

Keeping in mind that dT/dr and dP/dr are both negative, the atmosphere is stable if

$$\left|\frac{dT}{dr}\right| < \left(1 - \frac{1}{\gamma}\right)\frac{T}{P}\left|\frac{dP}{dr}\right|.$$

This is the famous ***Schwarzschild stability condition***. If the temperature gradient of the atmosphere is steeper than the critical value $(1 - 1/\gamma)(T/P)|dP/dr|$, then the atmosphere is unstable to convection.

Convection inside a star

Convection is an extremely efficient mechanism for transporting energy. The temperature gradient has to be only slightly steeper than the critical gradient to drive the typical stellar energy flux. A good approximation is:

$$\boxed{\frac{dT}{dr} = \left(1 - \frac{1}{\gamma}\right) \frac{T}{P} \frac{dP}{dr}}$$

Forth stellar structure equation,

Energy transport equation, if the flux is carried outward via convection

inside the convection zone. In order to make more accurate calculations, one has to take the help of ***mixing length theory***, which was developed. This theory assumes that upward-moving hot blobs or downward-moving cold blobs typically traverse a vertical distance called the mixing length, after which they lose their identity and mix their heat contents with their surroundings. By assuming a reasonable value of the mixing length, it is possible to calculate the small difference between the actual temperature gradient and the critical gradient, which is responsible for transporting the necessary heat flux.

Convection inside a star

While constructing a model of a star, one has to proceed in the following way.

- First one assumes that there is no convection and heat transport is entirely due to radiative transport.
- After calculating the temperature distribution on the basis of this assumption, the next step is to check if the temperature gradient obtained in this way satisfies the Schwarzschild stability condition.
- If it is satisfied, then it can be taken as established that the heat flux is really carried by radiative transport.
- On the other hand, if the stability condition is not satisfied in some regions, then the heat flux is primarily carried by convection in those regions and one has to repeat the calculation by using the energy transport equation for the convective case.

Stellar models

- First of all, one has to specify the chemical composition of a star, since opacity and the nuclear energy generation rate depend on the chemical composition. The chemical composition can be given by specifying the mass fraction X_i of various elements present in the stellar material.
- The next step is to figure out the equation of state $P(\varrho, T, X_i)$, the opacity $\chi(\varrho, T, X_i)$ and the nuclear energy generation rate $\varepsilon(\varrho, T, X_i)$ as functions of density, temperature and chemical composition.

Equation of State

The density at the centre of the Sun is more than 100 times the density of water. However, still the material there behaves like a **perfect gas, because the temperature is so high** that the interatomic potential energies are negligible compared to the typical kinetic energies of the particles and atoms do not get a chance to bind together to form a solid or a liquid.

Stellar models

- If we can assume the gas to be completely ionized, then the equation of state becomes particularly simple.
  - X *is* be the mass fraction of hydrogen
  - Y *is* the mass fraction of helium
  - Z *is* the mass fraction of other heavier elements (referred to as ‘metals’ in astrophysics).
- The number of hydrogen atoms per unit volume is $X\varrho/m_H$. Since each hydrogen atom contributes two particles (one electron and one nucleus which is a proton), there will be $2X\varrho/m_H$ particles per unit volume from fully ionized hydrogen.
- The number density of helium atoms will be $Y\varrho/4m_H$ and they will contribute $3Y\varrho/4m_H$ particles.
- A heavy atom of atomic mass A approximately contributes $A/2$ particles, it is easy to see that the contribution to the number density from heavier elements is $Z\varrho/2\ m_H$.

From this, the number of particles per unit volume is:

$$n = \left(2X + \frac{3}{4}Y + \frac{1}{2}Z\right)\frac{\rho}{m_{\mathrm{H}}}$$

Stellar models

- The gas pressure is given by: $$P = \frac{\kappa_B}{\mu m_H} \rho T$$

mean molecular weight

$$\mu = \left(2X + \frac{3}{4}Y + \frac{1}{2}Z \right)^{-1}$$

The assumption that all the gas fully ionised is a good approximation for generally understanding stars.

For accurate stellar models, however, one needs to take account of the partial ionization, especially in the outer layers of the star, and should also include the radiation pressure, which becomes important for more massive stars.

Finally, when the density is very high, the electron gas becomes *degenerate*, i.e. it obeys the Fermi–Dirac distribution rather than the classical Maxwellian distribution. This gives rise to what is called the *degeneracy pressure*. This pressure can play a crucial role in balancing gravity when the nuclear fuel is exhausted in a star.

Stellar models

- Equations for stellar structure:

$$\frac{dM_r}{dr} = 4\pi r^2 \rho.$$

Mass Conservation

$$\frac{dP}{dr} = -\frac{GM_r}{r^2}\rho$$

Hydrostatic equilibrium

$$\frac{dL_r}{dr} = 4\pi r^2 \rho\,\varepsilon.$$

Energy Conservation

$$\left.\begin{aligned} \frac{dT}{dr} &= -\frac{3}{4a_{\mathrm{B}}c}\frac{\chi\rho}{T^3}\frac{L_r}{4\pi r^2} \\ \frac{dT}{dr} &= \left(1-\frac{1}{\gamma}\right)\frac{T}{P}\frac{dP}{dr} \end{aligned}\right\}$$

Energy transport

Stellar models

- In a typical star, the convection may take place in a certain range of radius, whereas heat is transported by radiative transfer in other regions. So, for the same stellar model, it may be necessary to use one form of energy transport in some regions and the other form elsewhere.
- Once the equation of state $P(\varrho, T, X_i)$, the opacity $\chi(\varrho, T, X_i)$ and the nuclear energy generation rate $\varepsilon(\varrho, T, X_i)$ are given, the stellar structure equations involve four independent functions of r: ϱ, T, M_r and L_r.

Boundary conditions:

We have the following two boundary conditions at the centre of the star:

$$M_r = 0 \text{ at } r = 0,$$

$$L_r = 0 \text{ at } r = 0.$$

The radius $r = R$ of the star is the point where both ϱ and T become very small compared to their values in the interior. Hence the simplest boundary conditions for them are:

$$\rho = 0 \text{ at } r = R,$$

$$T = 0 \text{ at } r = R.$$

Stellar models

The equations can be solved numerically

A brief outline of the method:

- If we want to construct a model of a star with a given central density ρ_C. Taking the central temperature to have a value T_C and using the boundary conditions, we can start integrating the stellar structure equations from $r = 0$.
- In general, ϱ and T will not become zero at the same value of r so that it will not be possible to satisfy the boundary conditions simultaneously. We then have to try out the procedure again and again by varying the value of the central temperature T_C, until we find a combination ρ_C and ρ_C for which the solution would be such that ϱ and T will become simultaneously zero for some particular r .
- We would regard that r to be the radius R of the star, and boundary conditions will be satisfied. The values of M_r and L_r at $r = R$ would give us the mass and the luminosity of the star.
- We thus see that in principle the structure of a star with a central density ρ_C can be found this way, and such a star would have a definite mass and definite luminosity.

Stellar models

- Although the procedure outlined above gives an idea of how a stellar structure can be found, this simple procedure unfortunately does not work properly. The equation of radiative energy transfer in has a factor T^3 in the denominator and this factor becomes very large near the surface where T is very small. This leads to a numerical instability. One can think of the alternative of starting the numerical integration from the stellar surface $r = R$. This leads to a numerical instability at the centre due to the factor r^2 in the denominator of the hydrostatic equilibrium.
- One possible way of getting around these difficulties is to start the numerical integrations both from $r = 0$ and $r = R$, and then match them smoothly at an intermediate point. Although this method works, it is not a particularly efficient method.
- A more efficient numerical algorithm the *Henyey method*, which is an iterative method for the integration. This is a standard method widely used in solving stellar structures.

Are the solutions unique?

- For simplicity consider stars of given uniform composition.
- Then the equation of state, the opacity and the nuclear energy generation rate all become functions of density and pressure alone.
- It would seem that it will be possible to construct a unique stellar structure solution starting from a given central density ρ_C. Such a solution would correspond to a star of given mass M. At first sight, it appears that there should be a unique stellar structure solution for a star of a given mass. This idea was known as the *Vogt–Russell theorem*.
- Further research showed that solving the stellar structure equations is a complicated problem and often solutions were not unique. The Vogt–Russell theorem could not be a mathematically correct.
- An example: a star of mass $M_\odot$. Such a star can have a structure like the Sun or it can have the white dwarf configuration. The white dwarf configuration arises when stellar matter is in a degenerate state. In principle it is possible to put the solar material into a degenerate state. -> a star of mass $M_\odot$ can have at least two distinct configurations and both of these should follow from the stellar structure equations.
- For practical purposes a normal star of a given mass M and standard composition may be taken to have a reasonably unique structure with a luminosity L and radius R.

Relations between stellar quantities

We shall now do a few drastic things with the stellar structure to extract some relations amongst various quantities pertaining to a star. Our aim will be to find how various quantities scale with each other. We shall, therefore, ignore the constant factors in our equations.

A slightly more sophisticated approach than ours is to construct what are called *homologous* stellar models, in which it is assumed that various quantities vary inside different stars in similar ways (they scale with the fractional mass).

Relations between stellar quantities

Let us replace the left-hand side of the hydrostatic equation by $-P/R$, where P can be taken as the typical pressure inside the star. Replacing M_r/r^2 on the right-hand side by M/R^2, we are led to:

$$\frac{dP}{dr} = -\frac{GM_r}{r^2}\rho \longrightarrow \frac{P}{R} \propto \frac{M}{R^2}\rho \longrightarrow P \propto \frac{M^2}{R^4}$$

on taking $\rho \propto M/R^3$. The equation of state $P \propto \rho T$ would imply:

$$P \propto \frac{M}{R^3}T \longrightarrow T \propto \frac{M}{R}$$

inside temperatures of different stars should be proportional to M/R

Relations between stellar quantities

If we assume that the radiative transfer equation holds throughout the star and further the variation of χ inside the star is not very appreciable, then we can write:

$$\frac{dT}{dr} = -\frac{3}{4a_{\mathrm{B}}c}\frac{\chi\rho}{T^3}\frac{L_r}{4\pi r^2} \longrightarrow \frac{T}{R} \propto \frac{M}{R^3T^3}\frac{L}{R^2} \longrightarrow L \propto \frac{(TR)^4}{M}$$

TR should be proportional to M: $$L \propto M^3.$$

This is called the ***mass–luminosity relation***, which implies that a more massive star should be more luminous.

Relations between stellar quantities

Since we derived this relation by making some drastic assumptions, one may express doubts about the correctness of this relation.

The figure shows a plot of log L versus log M as obtained from detailed numerical solutions of stellar structure equations. On this figure, we superpose a dashed line with a slope corresponding to the relation. This line is not too far off from what we get from detailed stellar models

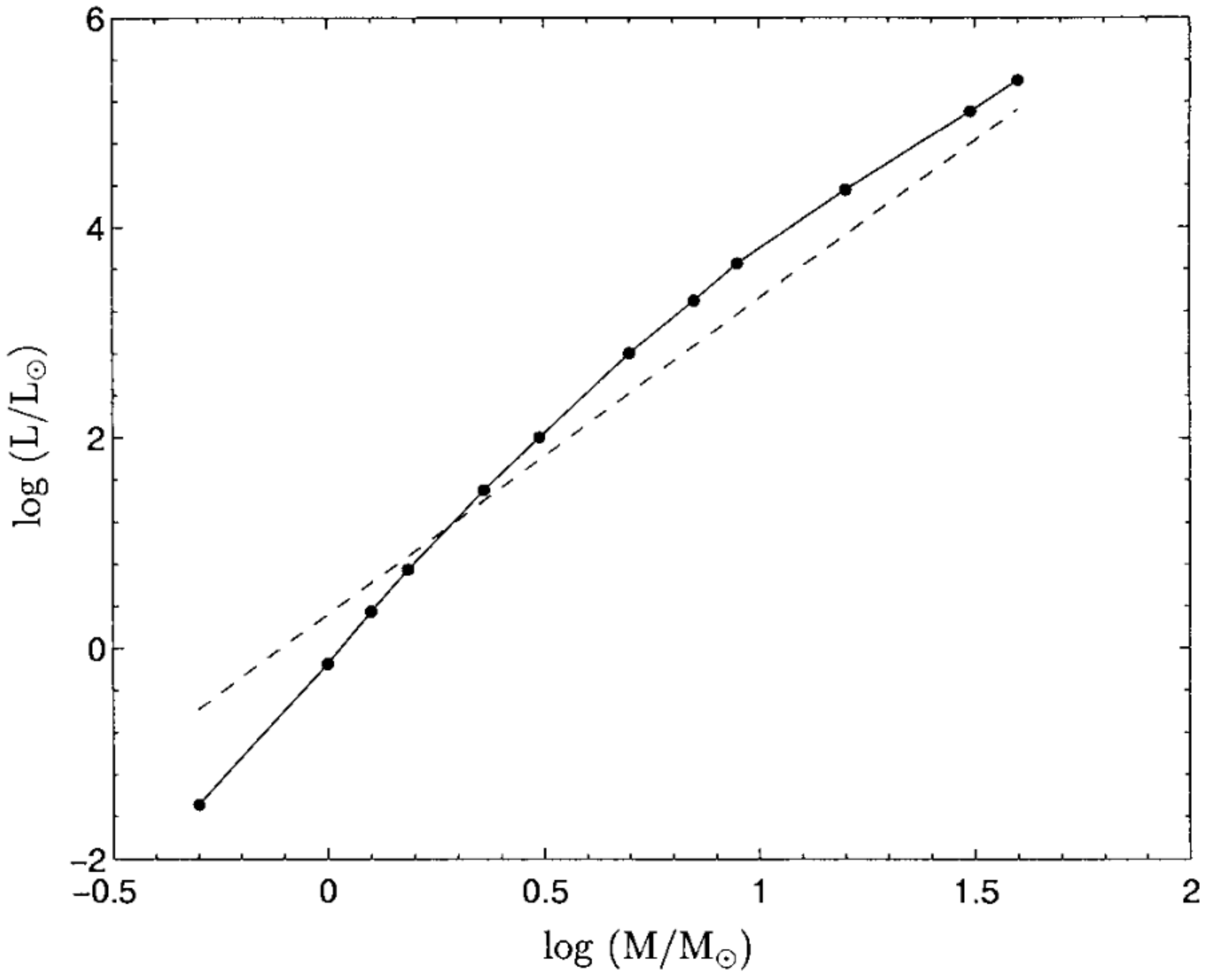


Fig. 3.2 Luminosity as a function of mass computed by detailed stellar models. The dashed line indicates the slope that would result if L varied as M^3. Adapted from Hansen and Kawaler (1994, p. 43) who use the results of Iben (1965) and Brunish and Truran (1982).

Relations between stellar quantities

We saw in the previous chapter that the surface of a star behaves approximately like a blackbody. Hence, if T_{eff} is the effective surface temperature, then we must have

$$L = 4\pi R^2 \sigma T_{\mathrm{eff}}^4,$$

where σ is the Stefan–Boltzmann constant. If we assume that T_{eff} is a measure of the typical interior temperature of the star (i.e. if hotter stars have hotter surface temperatures), then we can write from the above

$$L \propto R^2 T^4$$

Since L goes as M^3 by and RT goes as M: $M^3 \propto M^2 T^2 \longrightarrow M \propto T^2.$

the luminosities and the effective surface temperatures are related: $L \propto T_{\mathrm{eff}}^6$

Relations between stellar quantities

A plot of luminosity versus surface temperature for a number of stars is known as the ***Hertzsprung–Russell diagram***

- For historical reasons, the convention is to plot the effective surface temperature T_{eff} increasing towards the left!
- The figure compares results from theoretical models to the simplified relation between L and T_{eff}

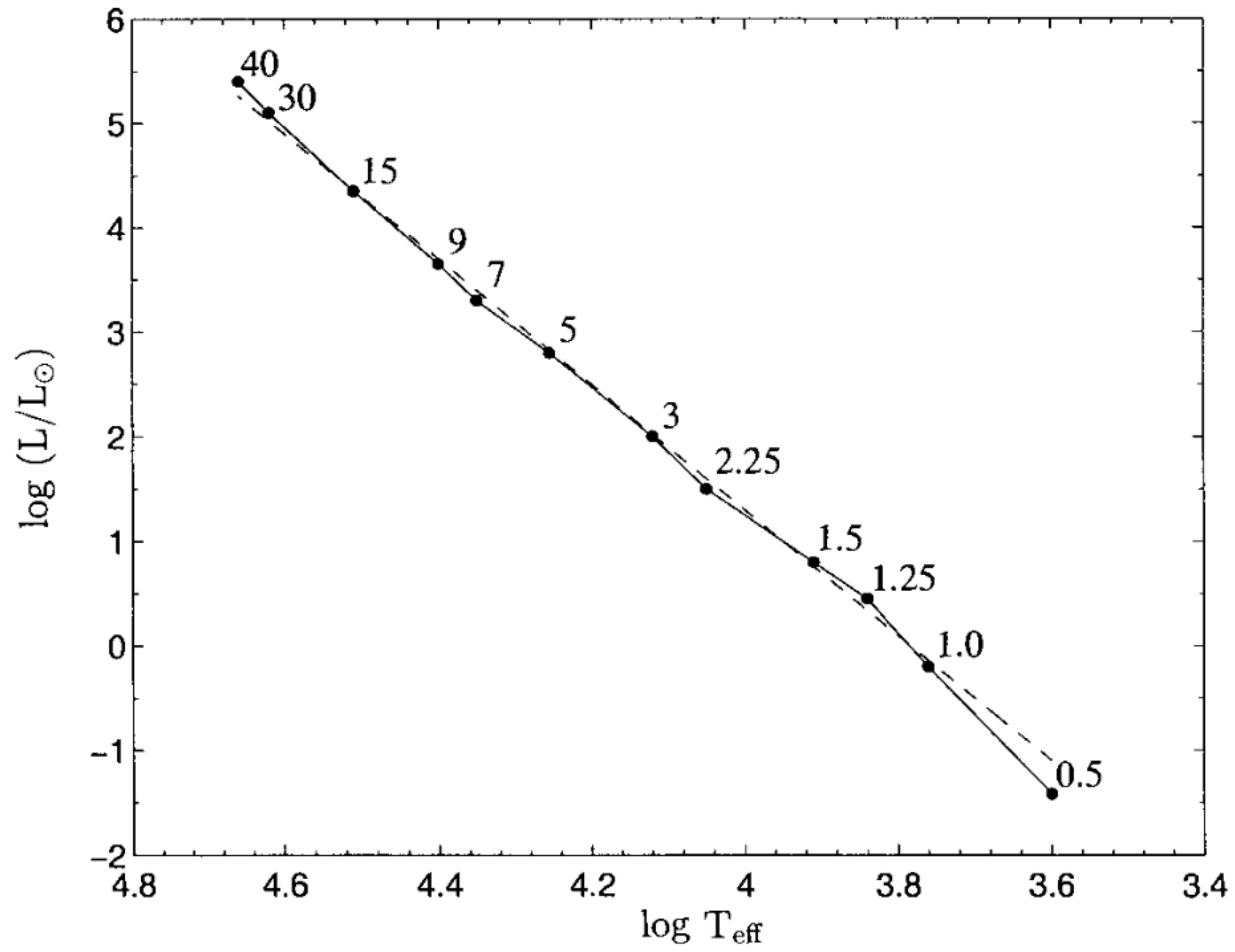
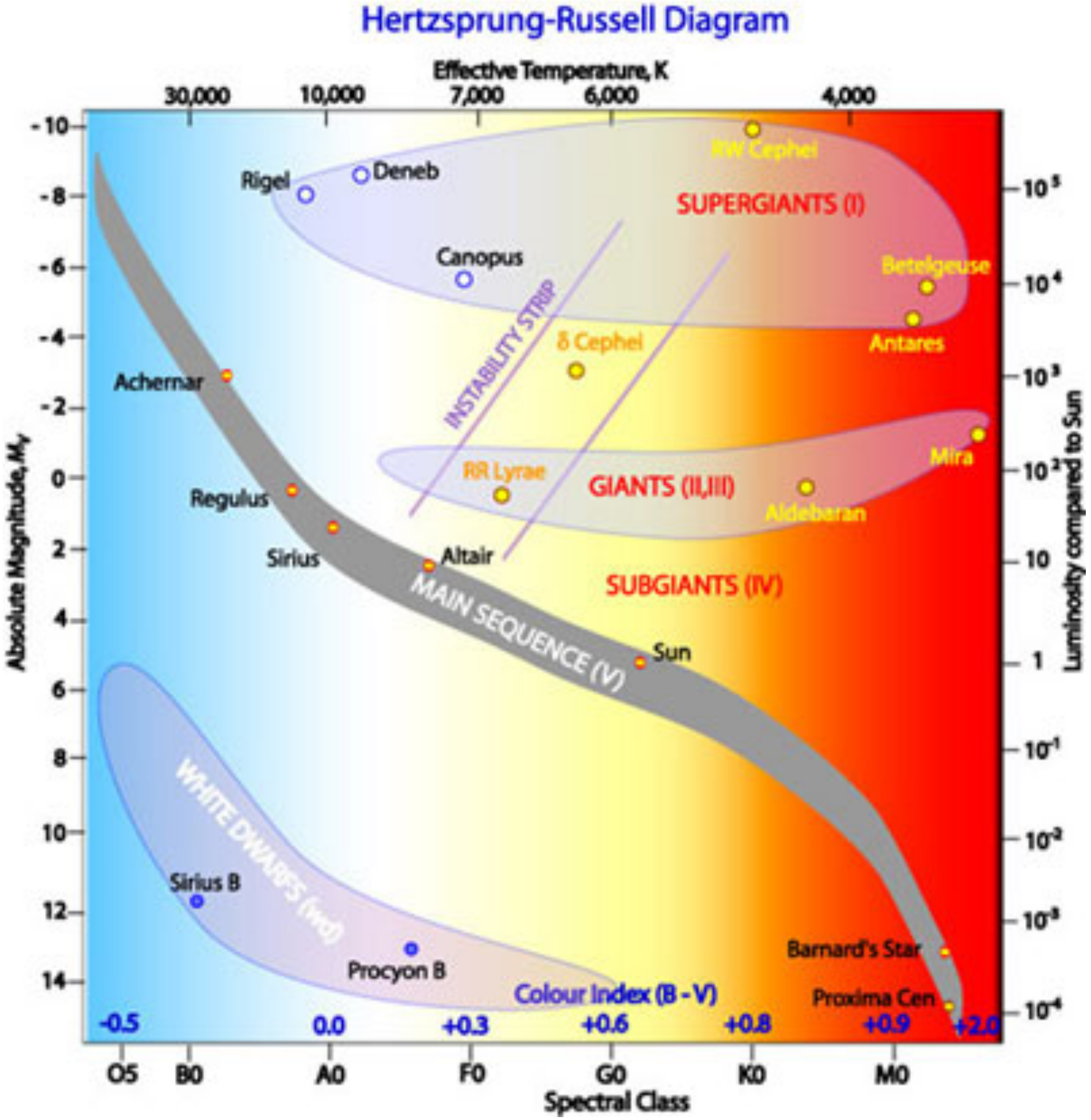
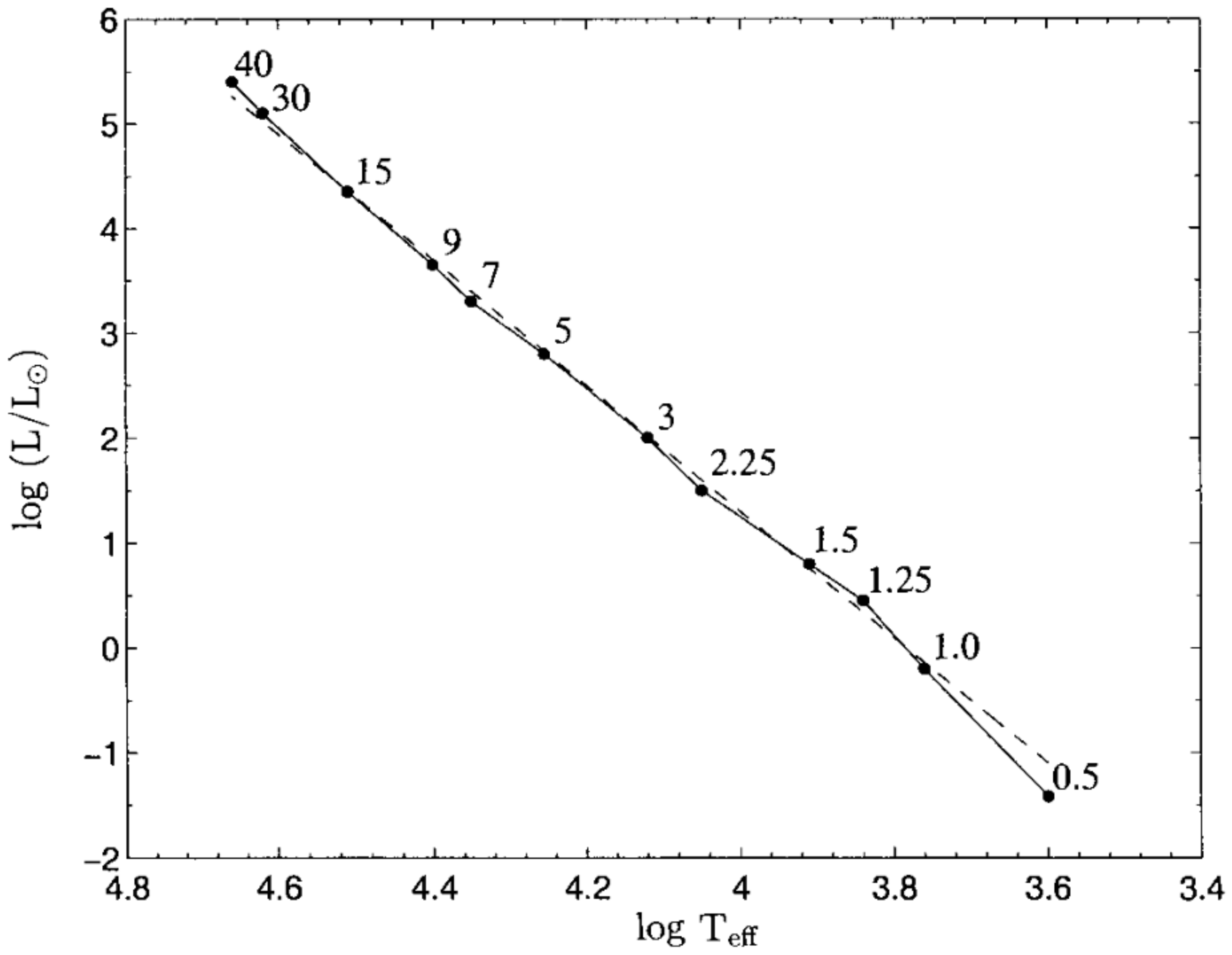


Fig. 3.3 The relation between luminosities and surface temperatures of stars as computed by detailed stellar models. The dashed line indicates the slope that would result if L varied as T_{eff}^6. The masses of stars corresponding to different points on the curve are also shown. Adapted from Hansen and Kawaler (1994, p. 40) who use the results of Iben (1965) and Brunish and Truran (1982).

Relations between stellar quantities





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Relations between stellar quantities

A star lives as a normal star as long as it has got nuclear fuel to burn. Since the amount of nuclear fuel is proportional to mass and the rate at which the fuel is burnt is proportional to luminosity, the lifetime τ of a star should be given by

$$\tau \propto \frac{M}{L} \longrightarrow \tau \propto M^{-2}$$

M**ore massive stars live for shorter times**. A more massive star has more nuclear fuel to burn; but it burns this fuel at such a fast rate that it runs out of the fuel in a shorter time. This very important result that massive stars are short-lived helps us understand many aspects of observational data