

Radiative transfer

①

- how radiation interacts with matter

macroscopic ~~level~~ level: emission and absorption coefficients

microscopic ~~too~~ range: what is the origin and what are the emission and absorption coefficients

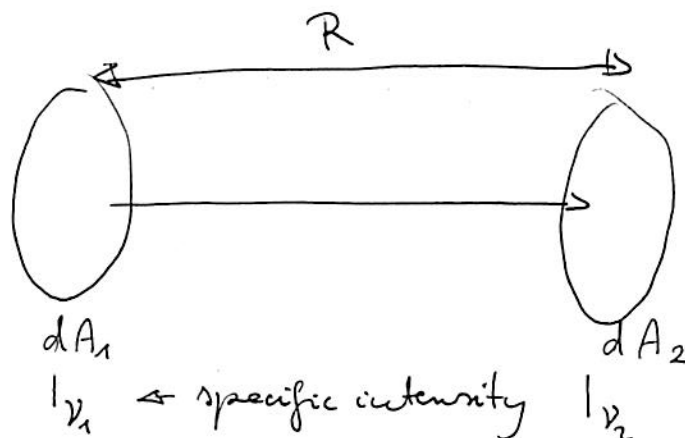
Radiation field

simplest case: blackbody radiation - homogenous and isotropic inside a container

Radiative transport

frequency range ν $\nu + d\nu$

one of the dA s can be the solid angle Ω



$$I_{\nu_2} dA_2 dt d\Omega_2 d\nu_2 = I_{\nu_1} dA_1 dt d\Omega_1 d\nu_1$$

~~with dA_1 and dA_2~~ with: $d\Omega_1 = \frac{dA_2}{R^2}$ $d\Omega_2 = \frac{dA_1}{R^2}$

$$\Rightarrow I_{\nu_1} = I_{\nu_2}$$

\rightarrow in empty space the specific intensity does not change
if s is the distance measured along a ray's path then

$$\frac{dI_{\nu}}{ds} = 0$$

specific intensity is the intensity divided by the solid angle

\rightarrow if things are far and appear as unresolved point sources the source is smaller than the resolution element and the intensity "washes out" decreases

(2.) matter in the path:

if the matter is emitting \rightarrow adding the emission coefficient j_ν
absorption is proportional to intensity, the absorption coef: α_ν

$$\frac{dI_\nu}{ds} = j_\nu - \alpha_\nu I_\nu \quad \Rightarrow \text{radiative transfer equation}$$

if the matter only absorbs $j_\nu = 0$

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu \quad \text{integrating this along } s_0 \rightarrow s$$

$$I_\nu(s) = I_\nu(s_0) \exp \left[- \int_{s_0}^s \alpha_\nu(s') ds' \right]$$

optical depth: $d\tau_\nu = \alpha_\nu ds$

$$\tau_\nu = \int_{s_0}^s \alpha_\nu(s') ds'$$

$$I_\nu(\tau_\nu) = I_\nu(0) e^{-\tau_\nu}$$

$\tau_\nu \gg 1$ optically thick

$\tau_\nu \ll 1$ optically thin

source function: $S_\nu = \frac{j_\nu}{\alpha_\nu}$

dividing the radiative transfer equ. by α_ν

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu$$

$$\frac{d}{d\tau_\nu} (I_\nu e^{\tau_\nu}) = S_\nu e^{\tau_\nu}$$

Integrating the optical path $\tau_0 \rightarrow \tau_\nu$

$$I_\nu(\tau_\nu) = I_\nu(0) e^{-\tau_\nu} + \int_0^{\tau_\nu} e^{-(\tau_\nu - \tau'_\nu)} S_\nu(\tau'_\nu) d\tau'_\nu$$

general solution to
the radiative
trans. eq.

③ if the matter has constant properties $\Rightarrow S_\nu$ is constant
 $\Rightarrow I_\nu(\tau_\nu) = I_\nu(0) e^{-\tau_\nu} + S_\nu (1 - e^{-\tau_\nu})$

if there is no background source: $I_\nu(0) = 0$

$$I_\nu(\tau_\nu) = S_\nu (1 - e^{-\tau_\nu})$$

if $\tau_\nu \ll 1$

$$I_\nu(\tau_\nu) = S_\nu \tau_\nu$$

$\tau_\nu = \alpha_\nu L$ where L is the total length of the ray

optically thin $I_\nu = j_\nu L$

optically thick: neglect $e^{-\tau_\nu}$ compared to 1

$$I_\nu = S_\nu$$