

Introduction to Astrophysics and Cosmology

Stellar Physics

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Relations between stellar quantities

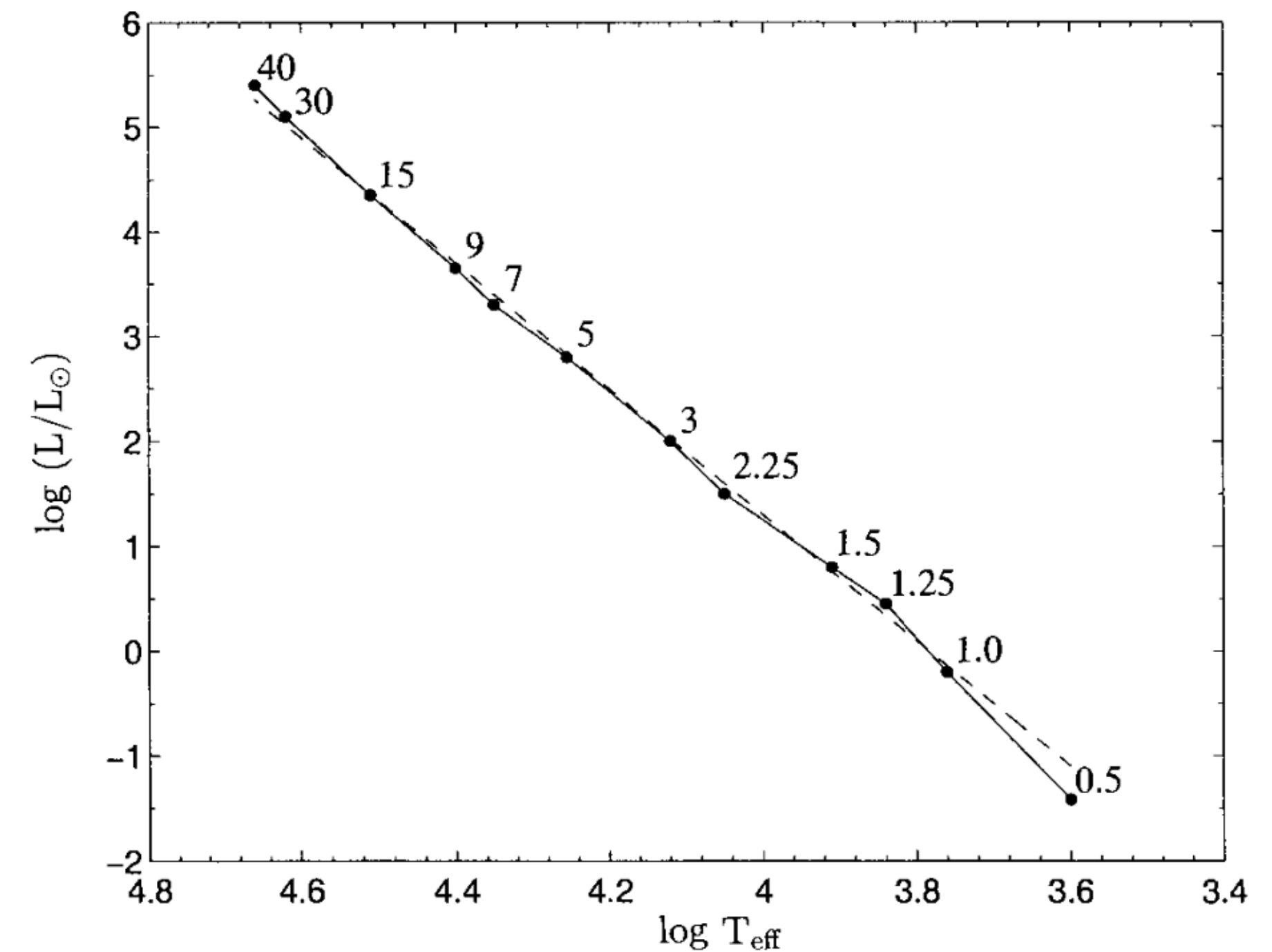
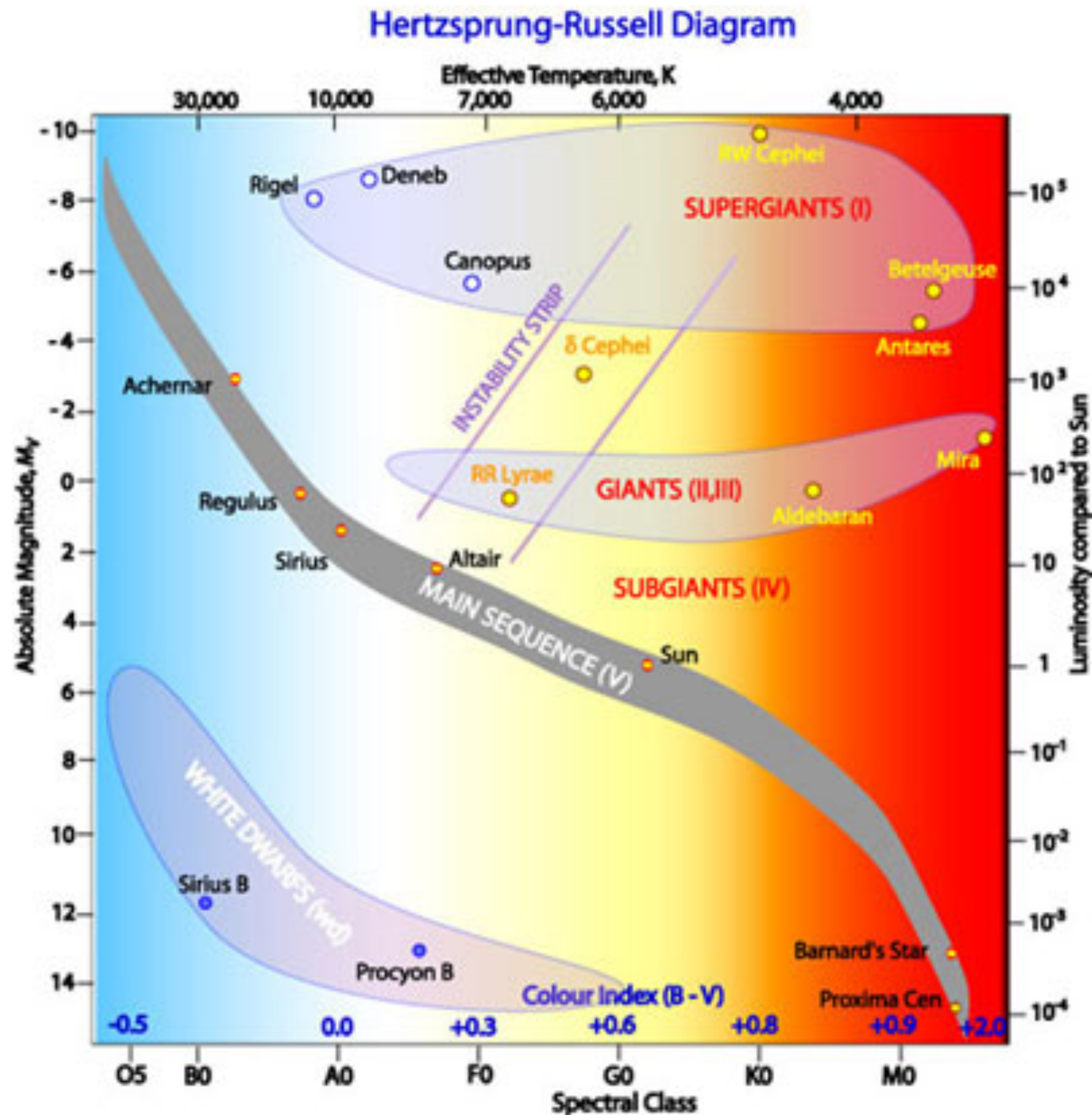


Fig. 3.3 The relation between luminosities and surface temperatures of stars as computed by detailed stellar models. The dashed line indicates the slope that would result if L varied as T_{eff}^6 . The masses of stars corresponding to different points on the curve are also shown. Adapted from [Hansen and Kawaler \(1994, p. 40\)](#) who use the results of [Iben \(1965\)](#) and [Brunish and Truran \(1982\)](#).

Determination of stellar parameters

Stellar spectra: surface temperature

- The surface of a star behaves approximately like a blackbody, with some spectral lines -> by fitting the spectrum of a star to a blackbody spectrum, it is possible to estimate the effective surface temperature T_{eff} of the star.
- One of the easy things to measure of a star is its apparent magnitude in the U , B and V bands. The quantity $B - V$ is a measure of the star's colour.
 - The effective surface temperature T_{eff} determines where the peak of the spectrum will be and thereby determines the colour of the star (a hotter star being bluish and a colder star reddish).
 - A one-to-one correspondence between $B - V$ and T_{eff} , for stars of similar properties.
 - The theoretical HR diagram had T_{eff} plotted on the horizontal axis, observational HR diagrams usually have the directly measurable quantity $B - V$ on their horizontal axes.

Determination of stellar parameters

Stellar spectra: composition

- The composition of the star can be found out from its spectral lines.
- However not entirely straightforward. For example: Since all stars are predominantly made of hydrogen, we may expect hydrogen lines to be present in the spectra of all stars. In reality, hydrogen lines are found only in stars of intermediate temperature.
 - Hydrogen lines in the visible part of the spectrum consist of Balmer lines, which are produced due to atomic transitions to the $n = 2$ atomic state from higher states ($n = 3, 4, \dots$).
 - If the stellar surface temperature is too high, then hydrogen is completely ionized and such atomic transitions do not take place.
 - On the other hand, a low surface temperature would imply that all hydrogen atoms are mostly in the ground state $n = 1$, with very few atoms occupying the states $n = 3, 4, \dots$.
 - Only for intermediate stellar surface temperatures, the levels $n = 3, 4, \dots$ are well populated and appropriate atomic transitions take place to produce the Balmer lines.

Determination of stellar parameters

Stellar spectra: composition

- Saha (1921) realized that the strengths of spectral lines by themselves do not give us the composition of a stellar atmosphere. **Matter of the same composition can produce very different spectra when kept at different temperatures.** Saha's of thermal ionization theory provided the explanation why spectra of stars with different surface temperatures look different.
- Around 1890, a group of astronomers at Harvard Observatory had developed a scheme of classifying stellar spectra in which a particular type of spectrum would be denoted by a Roman letter.
- Saha's work led to the realization that different spectral classes corresponded to different surface temperatures of stars.
- The spectral classes in the order of progressively **decreasing surface temperature are O, B, A, F, G, K, M.**
- HR diagrams are plotted with surface temperature on the horizontal axis increasing leftward. This is because HR diagrams were originally constructed by plotting spectral classes on the horizontal axis.

Determination of stellar parameters

Stellar spectra: motion, magnetic field

- The star's velocity component along the line of sight would cause a Doppler shift of spectral lines, and by measuring this Doppler shift, the line of sight velocity of a star can be measured.
 - **What would be an example?**
 - **How can we learn about the magnetic field?**

Determination of stellar parameters

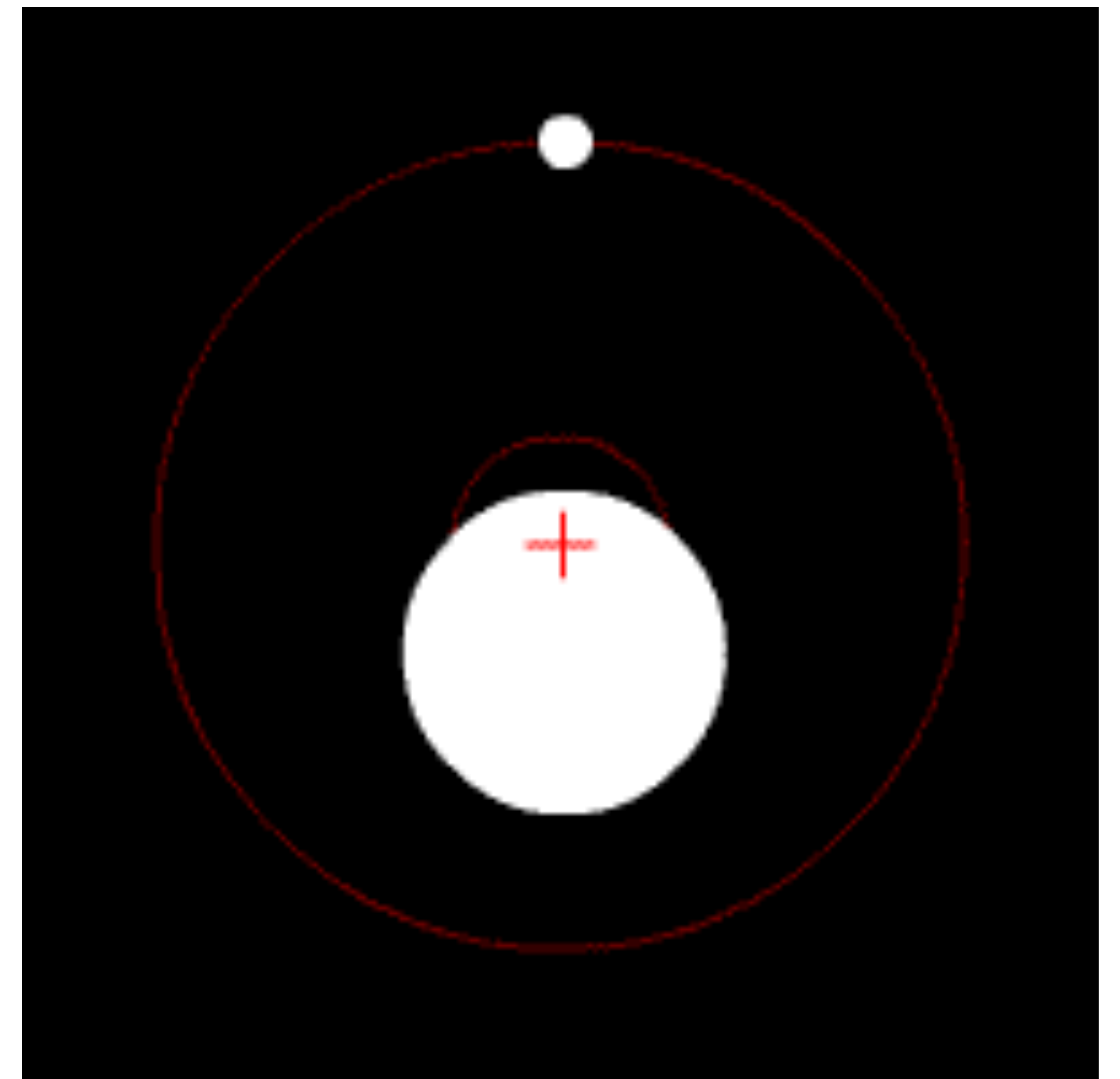
Stellar spectra: motion, magnetic field

- The star's velocity component along the line of sight would cause a Doppler shift of spectral lines, and by measuring this Doppler shift, the line of sight velocity of a star can be measured.
 - **What would be an example?**
- If the star is strongly magnetic, then one can hope to detect the Zeeman effect in the stellar spectra which would give information about the magnetic field.
 - **What is the Zeeman effect?**

Determination of stellar parameters

Stellar spectra: motion, magnetic field

- The star's velocity component along the line of sight would cause a Doppler shift of spectral lines, and by measuring this Doppler shift, the line of sight velocity of a star can be measured.
 - This method is widely used to detect exoplanets around stars
- If the star is strongly magnetic, then one can hope to detect the Zeeman effect in the stellar spectra which would give information about the magnetic field.
 - **What is the Zeeman effect?**



Determination of stellar parameters

Stellar spectra: motion, magnetic field

- The star's velocity component along the line of sight would cause a Doppler shift of spectral lines, and by measuring this Doppler shift, the line of sight velocity of a star can be measured.
 - This method is widely used to detect exo-planets around stars
- If the star is strongly magnetic, then one can hope to detect the Zeeman effect in the stellar spectra which would give information about the magnetic field.
 - The Zeeman effect describes the splitting of spectral lines in the presence of a magnetic field. In the absence of a magnetic field, emission is observed as a single spectral line and is dependent only on the principal quantum numbers of the initial and final states. In the presence of an external magnetic field, the principal quantum number of each state is split into different substates, resulting in permitted transitions that have frequencies above and below the transition that results in the absence of a magnetic field. The degree of the splitting depends on the field strength.

Determination of stellar parameters

Nearby stars: distance, luminosity

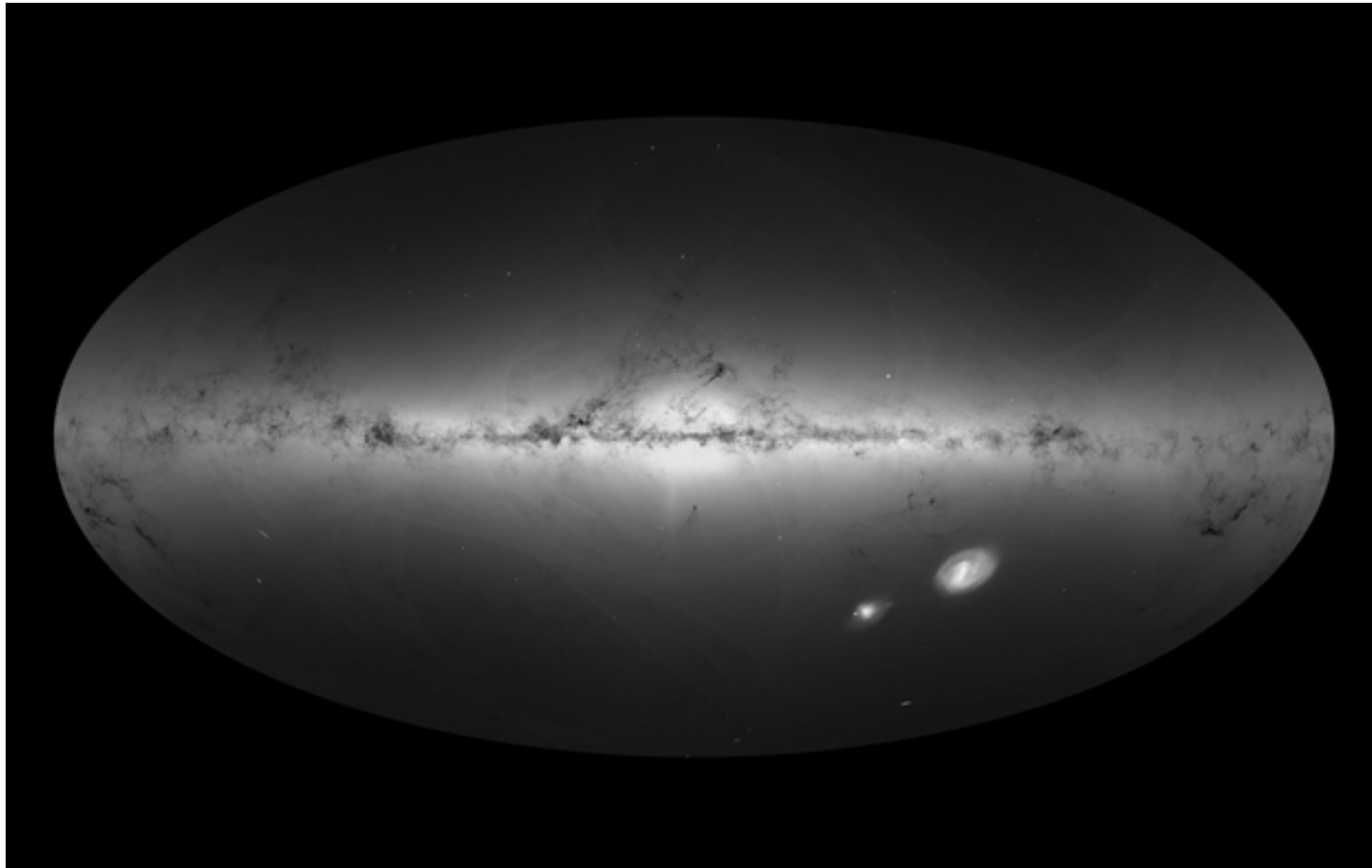
- **How can we measure the distances to nearby stars?**

Determination of stellar parameters

Nearby stars: distance, luminosity

- If a star is within a few pc, we can determine the distance of the star from a measurement of its parallax.
 - The Hipparcos satellite has measured distances for \sim hundred thousand stars.
 - The Gaia satellite has measured distances for \sim 1.8 billion stars

Determination of stellar parameters



map shows the density of stars observed by Gaia, Copyright: ESA/Gaia/DPAC, CC BY-SA 3.0 IGO

Determination of stellar parameters

Nearby stars: distance, luminosity

- Once the distance to the star is known, we can find the absolute magnitude in any band.
- The absolute magnitude in the V band, known as the *absolute visual magnitude*, is denoted by M_V and is a measure of the energy the star is giving out in visible light.
 - A star like the Sun may be giving out most of its energy in the visible light.
 - Stars with higher surface temperature may be giving out energy predominantly in the ultraviolet and stars with lower surface temperature in the infrared.
 - $\rightarrow M_V$ does not give a correct estimate of the total luminosity of the star.
- If we could measure the total energy received from the star in all wavelengths and calculate the absolute magnitude from that, that would be called the *absolute bolometric magnitude*, denoted by M_{bol} .
- If we know the surface temperature T_{eff} of the star, then we can estimate the fraction of the emitted energy which will go in the V band. \rightarrow from M_V , it is possible to infer M_{bol} which is related to the luminosity of the star. \rightarrow for nearby stars, if we know the distance, we also can infer the luminosity (or M_{bol}) from M_V .

Determination of stellar parameters

stellar mass determination

- **How can we measure stellar masses?**

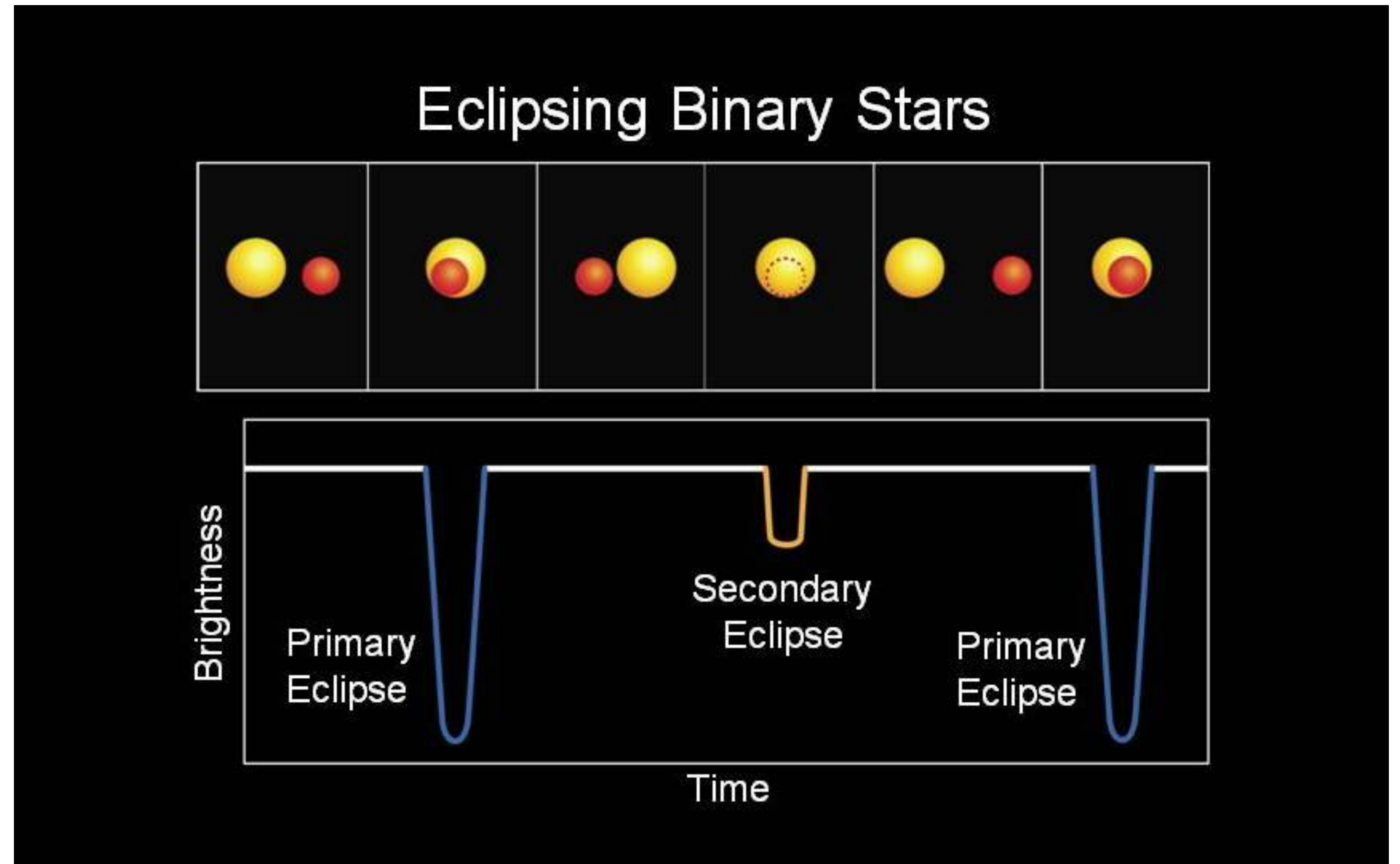
Determination of stellar parameters

Binary stars: stellar mass determination

- One of the fundamental parameters of a star is its mass.
- The mass of a star can be estimated only from the gravitational attraction it produces and we can observe the gravitational attraction only if there is a nearby object on which it acts.
- Many stars are found in binary systems and one can determine the masses of both the stars by observing the effect of each on the other.
- Some binary stars are resolved through powerful telescopes.
- In other cases, the binary nature is inferred from indirect evidence. If one star is much dimmer than the other and the dimmer star sometimes blocks the light coming from the brighter star, then we can observe a periodic variation of brightness. Such binaries are called *eclipsing binaries*.

Determination of stellar parameters

Binary stars: stellar mass determination



Determination of stellar parameters

Binary stars: stellar mass determination

- As the two stars in a binary system move around their common centre of gravity, one star may sometimes be moving towards us and sometimes away from us, leading to a periodic variation in the Doppler shift of spectral lines. Binaries displaying such periodically varying Doppler shifts in their spectra are known as *spectroscopic binaries*.
- Once the binary period and the velocities of the companions are known, it is straightforward to apply Newtonian gravitational mechanics to calculate the masses of the two companions

Observed relations

- **The mass - luminosity relation:**
- If a star is both nearby and in a binary, then both its luminosity and mass can be determined.
- Plotting luminosities and masses of such stars, we get the [Figure](#)
- Our simple theoretical considerations led to implying that L should go as M^3 .
- The fact that a straight line fits the data reasonably well implies that L indeed goes as M^n , the index n being given by the slope of the straight line having value 3.7.

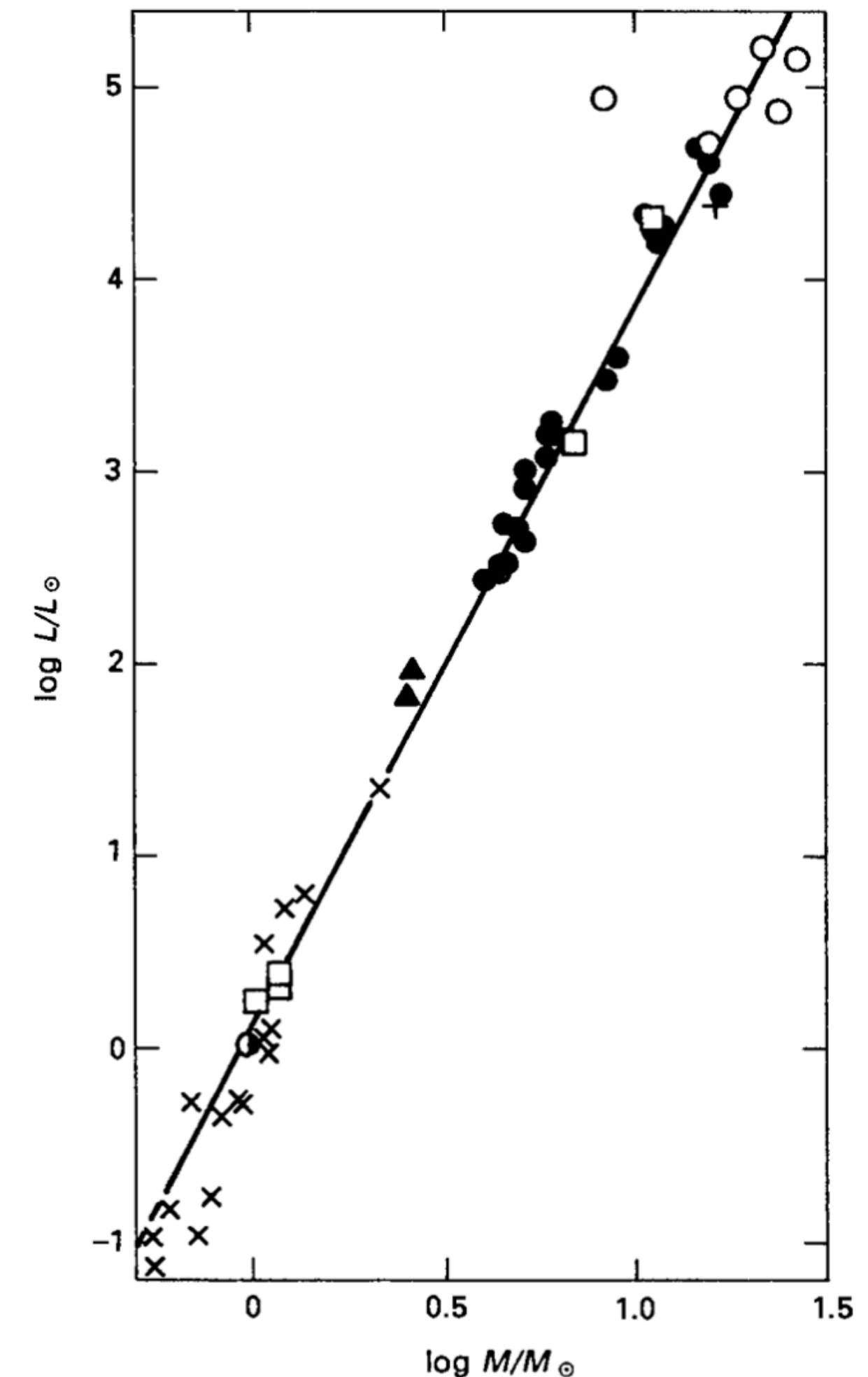


Fig. 3.4 The observational mass–luminosity relation. The different symbols correspond to different types of binaries (i.e. visual binaries are indicated by crosses, spectroscopic binaries by open squares, etc.). From Böhm-Vitense (1989, p. 87), based on the data presented by [Popper \(1980\)](#).

Observed relations

Table 3.1 The relationship amongst colour index $B - V$, absolute visual magnitude M_V , effective surface temperature T_{eff} and absolute bolometric magnitude M_{bol} for main sequence stars. Adapted from [Tayler \(1994, p. 17\)](#).

$B - V$	M_V	$\log T_{\text{eff}}$	M_{bol}
0.0	0.8	4.03	0.4
0.2	2.0	3.91	1.9
0.4	2.8	3.84	2.8
0.6	4.4	3.77	4.3
0.8	5.8	3.72	5.6
1.0	6.6	3.65	6.2
1.2	7.3	3.59	6.6

This straight line corresponds to a scaling relation $L \propto T_{\text{eff}}^n$ with $n = 5.6$. Our approximation had given a close power law index of 6

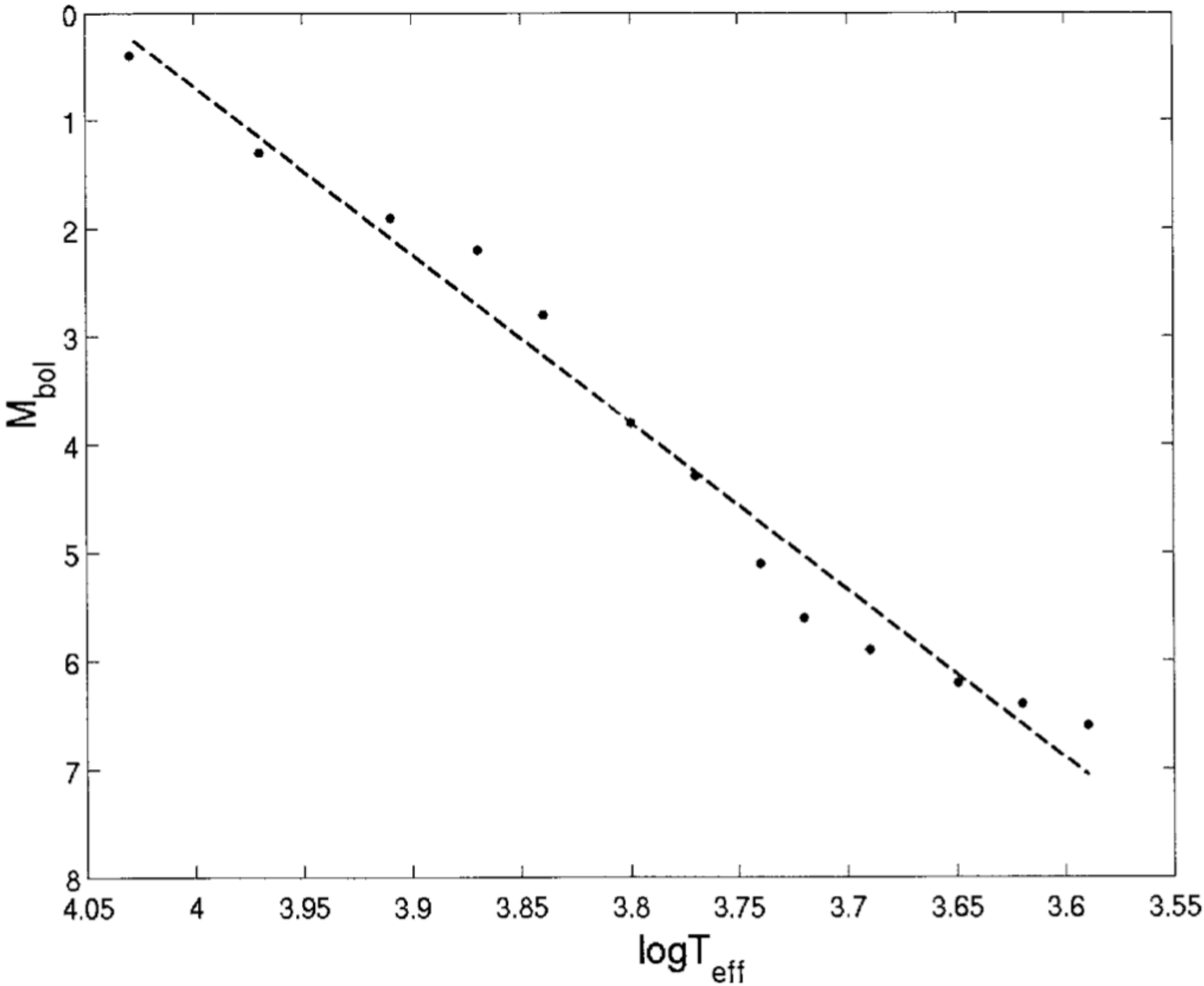
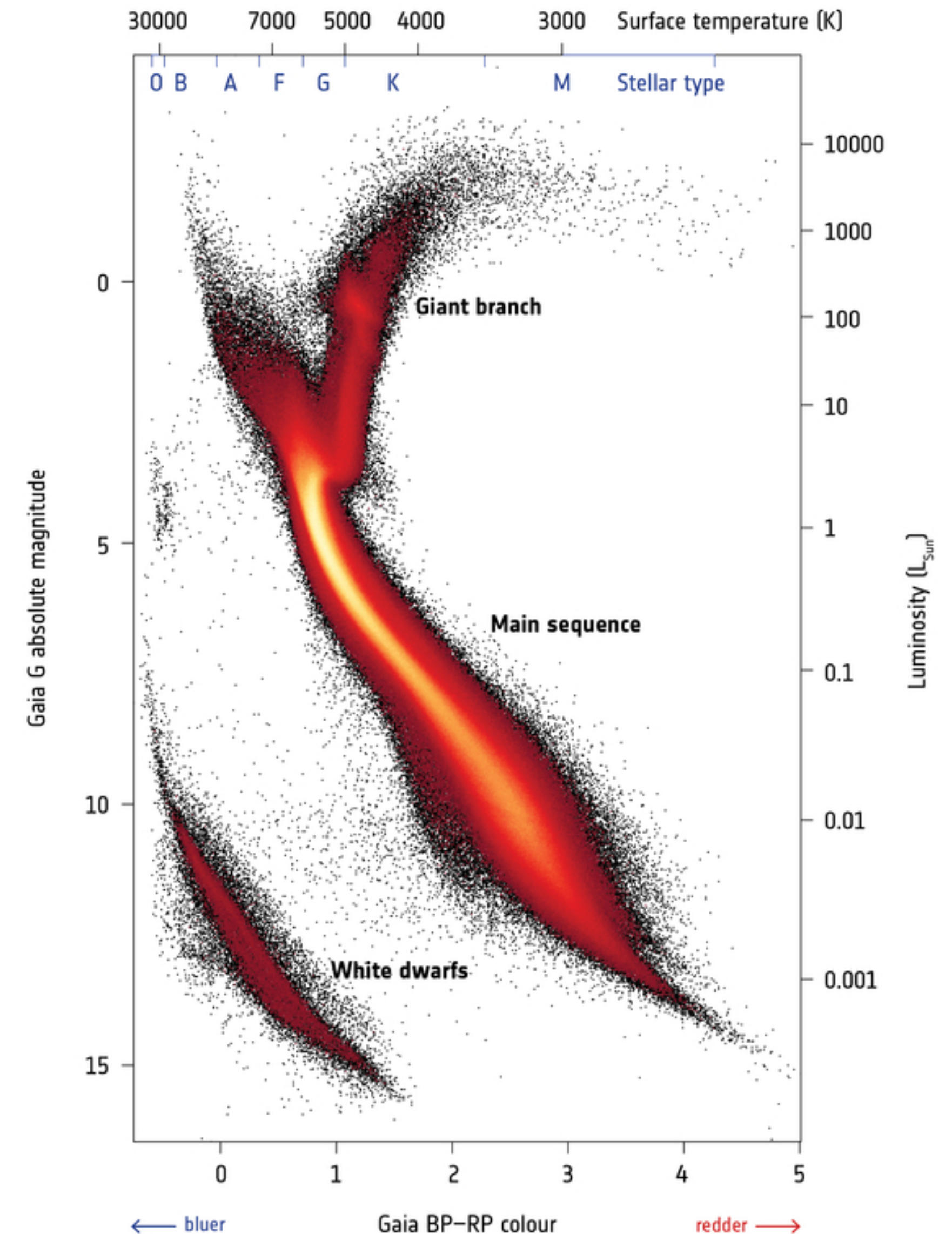


Fig. 3.6 The relation between M_{bol} and T_{eff} for stars lying on the median of the main sequence, with a best fit straight line. Based on data given by [Tayler \(1994, p. 17\)](#).

Observed relations

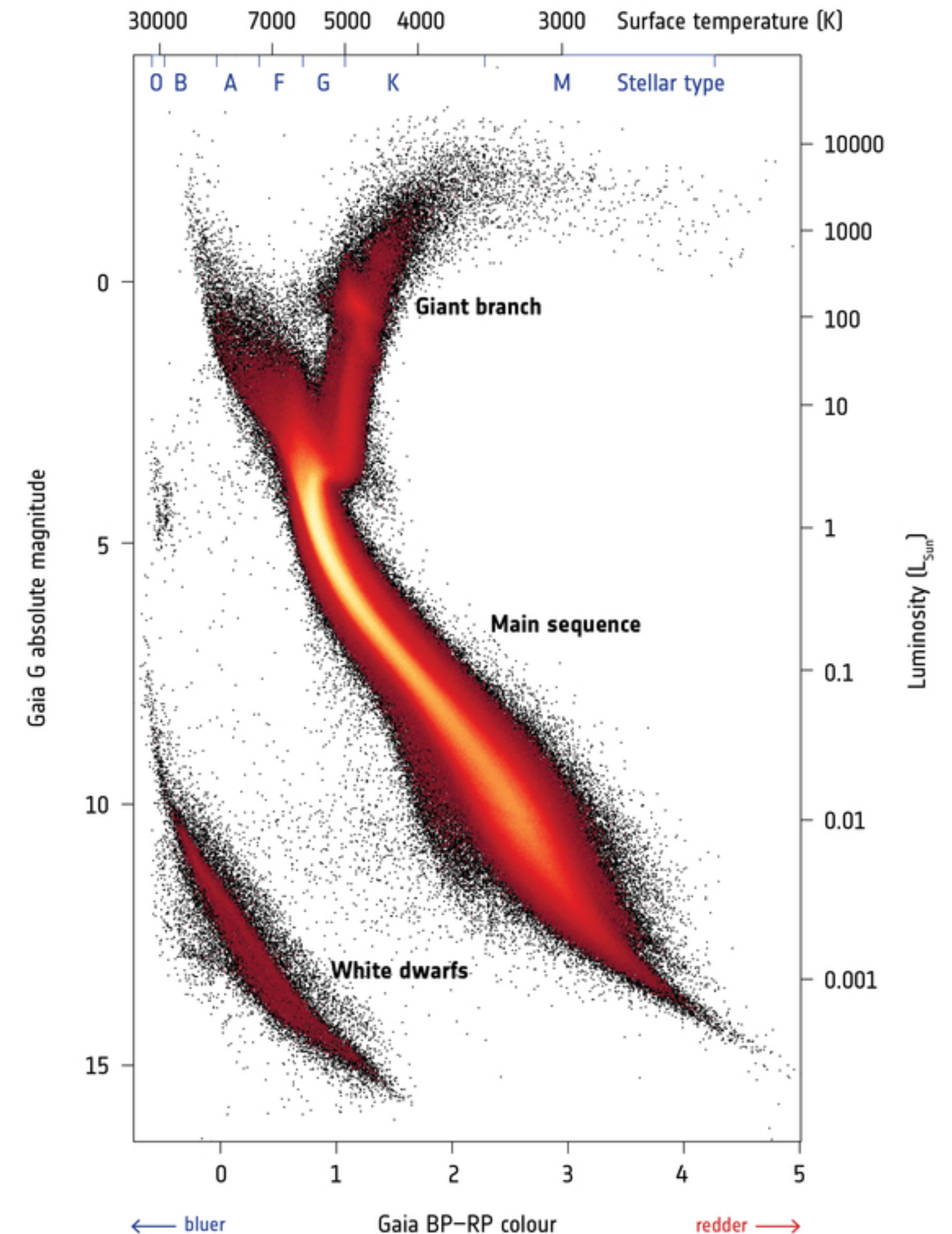
- The HR diagram:
- For nearby stars, we can determine the luminosities and then plot the luminosities against surface temperatures (obtained from the spectra) -> known as the HR diagram.
- HR diagram of nearby stars based on the distance measurements by the Gaia satellite
- note Gaia uses different filters compared to $B - V$

→ GAIA'S HERTZSPRUNG-RUSSELL DIAGRAM



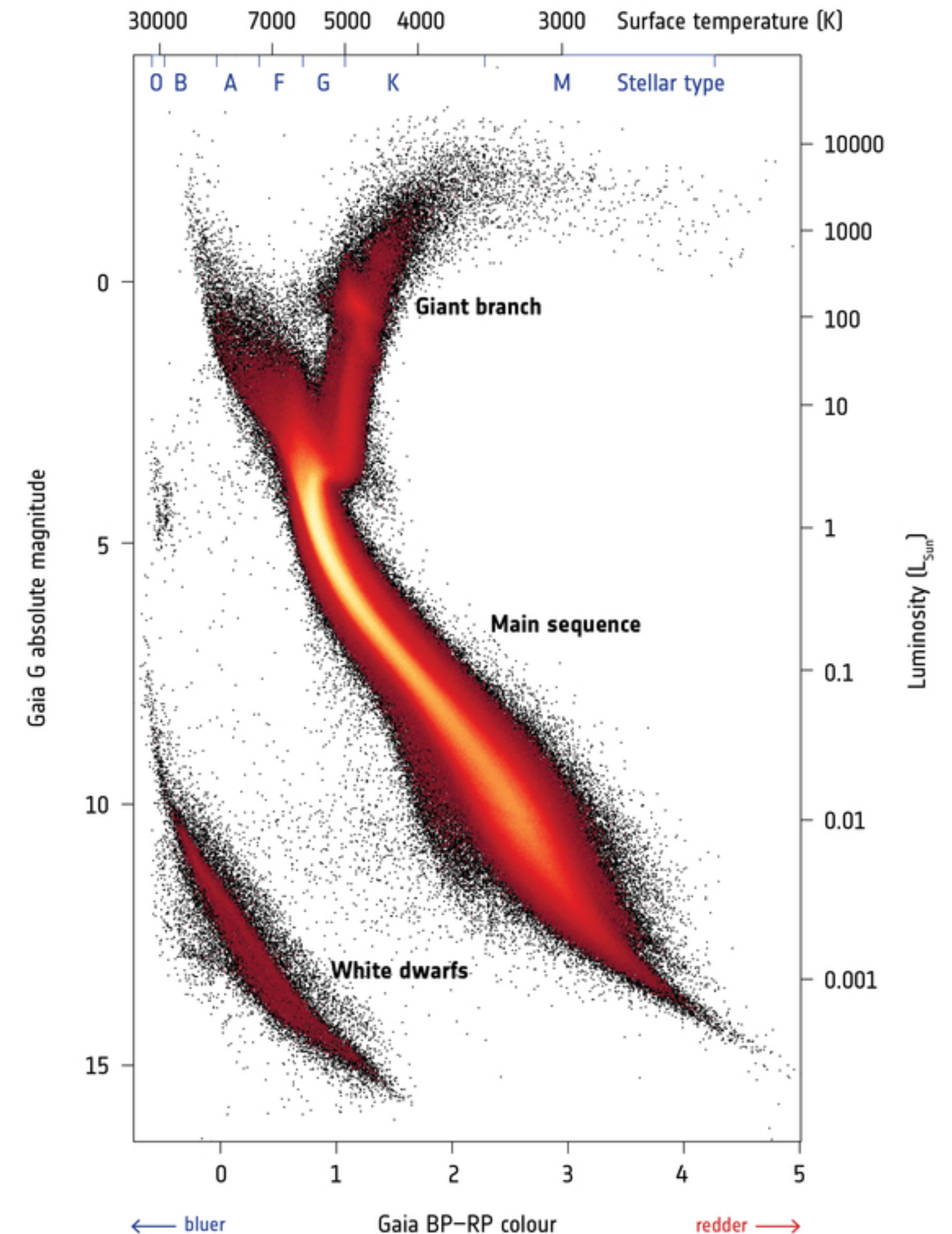
Observed relations

- Data points on the diagonal strip called the **main sequence**,
- We know that more luminous stars are more massive. So the upper left corner of the main sequence corresponds to more massive stars and the lower right corner to less massive stars. The mass of a star determines at which point of the main sequence the star would lie. **The main sequence is essentially a sequence of stellar masses**, with the mass increasing from the lower right towards the upper left.
- Stars in the main sequence generate energy by converting hydrogen into helium. While a steady energy generation goes on in this way, the internal thermal energy balances the gravity and the structure of the star does not change much with time. This means that the position of the star in the HR diagram does not change much while hydrogen is being converted into helium in its interior.



Observed relations

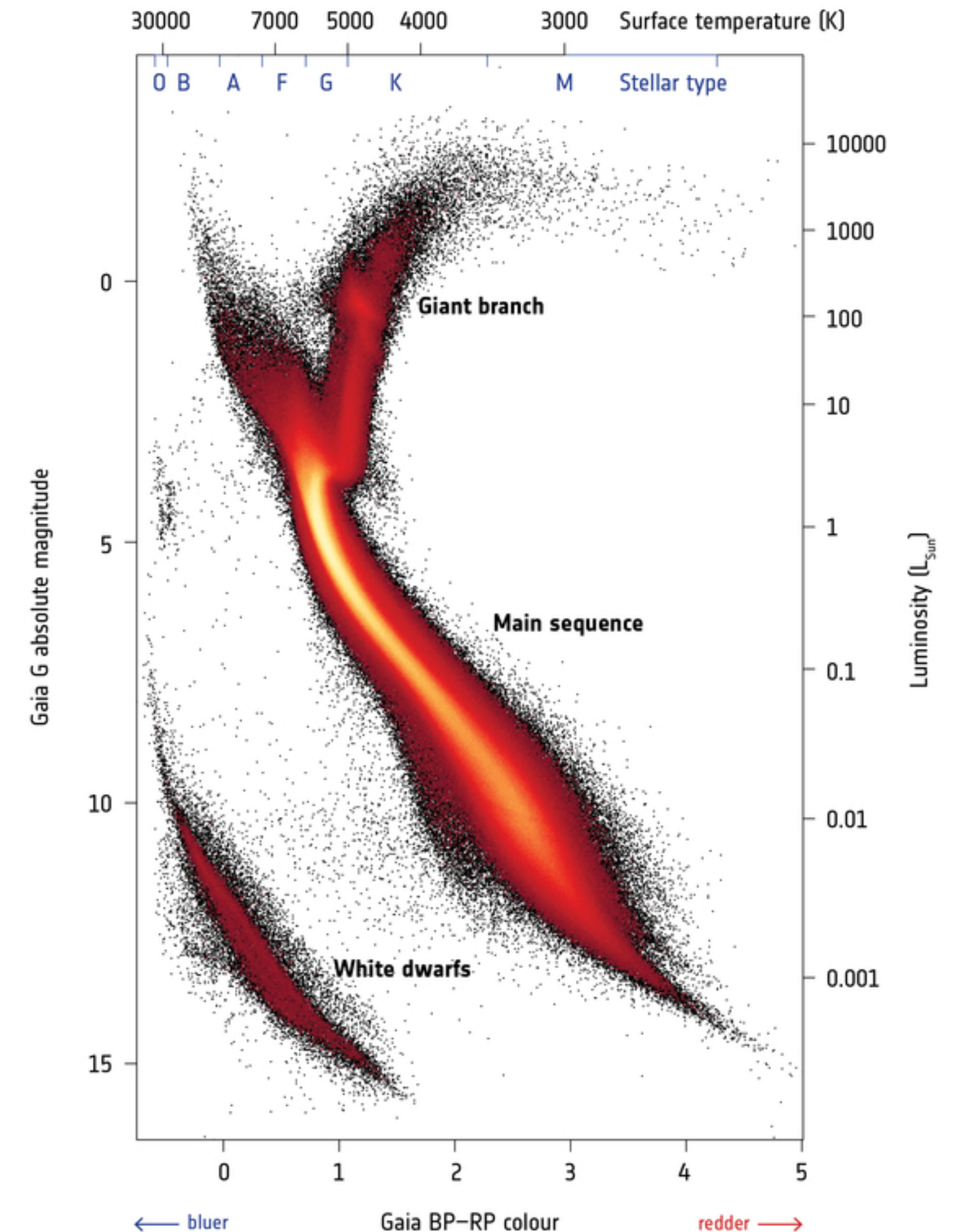
- data points in the upper right corner are red in colour and have luminosities much larger than the luminosities of red stars on the main sequence.
- Since unit areas of all stars with the same surface temperature give out energy at approximately the same rate (due to the Stefan–Boltzmann law of surface emission from blackbodies), the stars in the upper right corner have to be much larger in size than the red stars on the main sequence, in order to be much more luminous. The stars lying in the upper right corner of the HR diagram are, therefore, called *red giants*.
- When hydrogen is significantly depleted in the stellar core, the nuclear energy generation drops and is not able to balance the inward pull of gravity completely. This leads to a contraction of the core and this will cause the core to heat up. This process also dumps some heat in the surrounding layers of the star and inflates those layers. **Red giant** stars are generated in this way.



Observed relations

- The stars lying in the lower left corner of the HR diagram are bluish-white in colour and have much smaller luminosities compared to blue stars on the main sequence.
- These stars have to be much smaller in size compared to bluish-white stars on the main sequence. So these stars are called *white dwarfs*.
- Once all possible nuclear fuel is exhausted and the star can no longer produce thermal energy by nuclear reactions to balance gravity. -> Since electrons are fermions, they obey Pauli's exclusion principle, i.e. two electrons cannot occupy the same quantum state. So electrons resist being pushed into very small volumes, once the density is sufficiently high and all the low-lying quantum states are filled. The pressure arising out of this factor, called the *electron degeneracy pressure*

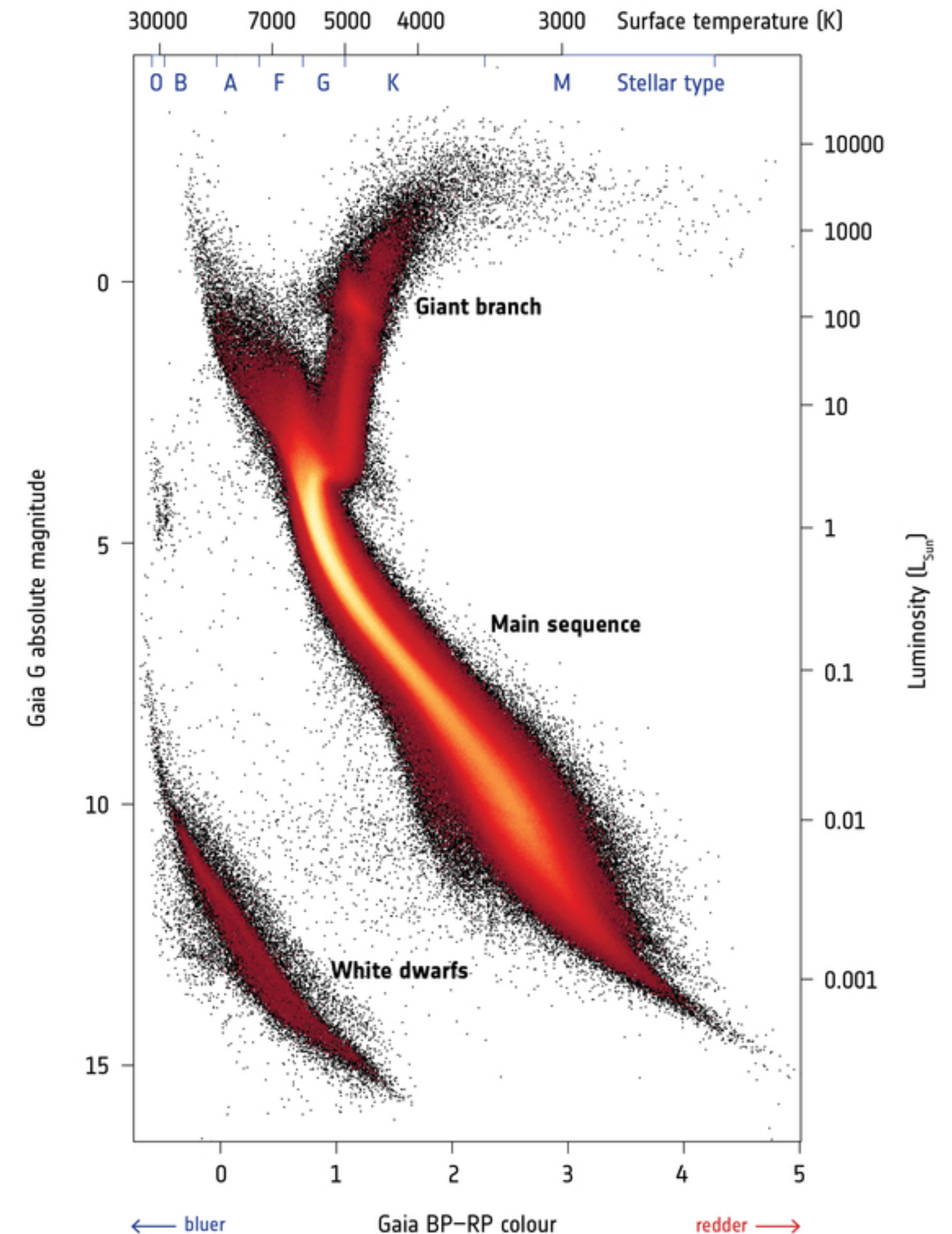
→ GAIA'S HERTZSPRUNG-RUSSELL DIAGRAM



Observed relations

- Even in the absence of other energy sources, the electron degeneracy pressure alone can balance gravity if the mass of the star is less than the Chandrasekhar mass limit (limit for black hole formation).
- **White dwarfs** are very dense, dead stars in which no more nuclear reactions are taking place and gravity is balanced by the electron degeneracy pressure of the dense stellar material. The surface temperature of white dwarfs is a remnant of the heat produced in the gravitational contraction. Eventually, after the white dwarf radiates out the heat, it will become a cold dark object.

→ GAIA'S HERTZSPRUNG-RUSSELL DIAGRAM



Eddington luminosity limit

The lightest stars on the main sequence at the lower right corner of the HR diagram have masses of order $0.1M_{\odot}$, whereas the most massive stars at the upper left corner have masses of order $100M_{\odot}$. **Why do all stars have masses in this narrow range of about three orders of magnitude?**

Eddington luminosity limit

lower limit:

- Stars form out of the gravitational collapse of interstellar gas clouds.
- When a newly forming proto-star shrinks gravitationally, the Kelvin–Helmholtz theory outlined should hold and the proto-star should become hotter while it shrinks. Eventually its interior may become hot enough for nuclear reactions to start, causing the gravitational contraction to halt. Thus the proto-star becomes a real star burning nuclear fuel inside.
- However, if the mass of the proto-star is less than a lower limit, then the interior does not become hot enough for nuclear reactions to start, because the electron degeneracy pressure halts the gravitational contraction before the temperature can become sufficiently high. Such an object is called a ***brown dwarf***.
- Theoretical calculations suggest that $0.08M_{\odot}$ is the lower limit for the mass of a star generating energy by nuclear reactions. A gravitationally contracting object with less mass becomes a brown dwarf. A brown dwarf will never have a surface temperature as high as that of even the least massive stars. However, after its formation, for some time a brown dwarf will be radiating away the heat produced during its gravitational contraction and can be detected.

Eddington luminosity limit

Upper limit:

- The radiation pressure becomes more important inside more massive stars.
- Eventually the very high radiation pressure inside a massive star can make the star unstable.
- **A high radiation pressure can lift the outer layers of a star.**
- The energy flux of radiation at the surface of a star with luminosity L and radius R is $L/4\pi R^2$. If χ is the opacity, then $\rho\chi$ is the absorption coefficient and the energy absorbed per unit volume per unit time is $\rho\chi(L/4\pi R^2)$. The momentum associated with this energy can be obtained by dividing this by c , which will give us the momentum absorbed per unit time in a unit volume, \rightarrow the force exerted on this unit volume.
- The star will be able to hold on to this outer layer of gas only if the inward force of gravity is stronger than this force exerted by radiation, i.e. if

$$\frac{GM}{R^2} \rho > \frac{L}{4\pi R^2} \frac{\rho\chi}{c}, \quad \longrightarrow \quad L < \frac{4\pi c GM}{\chi}, \quad \text{Eddington luminosity limit}$$

Eddington luminosity limit

Upper limit:

L goes as M^3 , we can write $L = \lambda M^3$. It then follows that the Eddington limit will be violated if the mass of the star were to be larger than M_{max} given by

$$L < \frac{4\pi c G M}{\chi} \quad \longrightarrow \quad \lambda M_{\text{max}}^2 = \frac{4\pi c G}{\chi}.$$

While M_{max} may be an absolute upper limit beyond which a star's outer layers would be blown off **by radiation**, in reality stars with mass considerably less than this M_{max} become unstable due to radiation pressure and are not able to exist

Star clusters

Why are star cluster important?

Star clusters

- Many stars are found in clusters. There are some relatively loosely bound clusters, each having a few dozens of stars. Such loosely bound clusters are called *open clusters*.
- *globular clusters* are very tightly bound almost spherical clusters, containing of the order of 10^5 stars. Globular clusters are found around the centre and in the halo of our Galaxy.
- Star cluster are important because they are groups of stars which have been born at about the same time and which are at roughly the same distance from us.



Pleiades open cluster

- Globular clusters can be more useful because the stars are gravitationally bound together for a long time and they typically have many more stars.



M87 globular cluster

Star clusters

- If all the stars in a cluster are **at the same distance d , then the difference between absolute magnitude and apparent magnitude will be the same for all stars.** Hence we can construct the HR diagram of a star cluster by plotting the apparent magnitude (instead of absolute magnitude) against $B - V$.
- The [Figure](#) shows such an HR diagram of a globular cluster. One can clearly see the main sequence. **We can calculate the distance to the globular cluster by matching the main sequence of the cluster to the main sequence of nearby stars.** This can be used also to determine distances to our neighbouring galaxies.

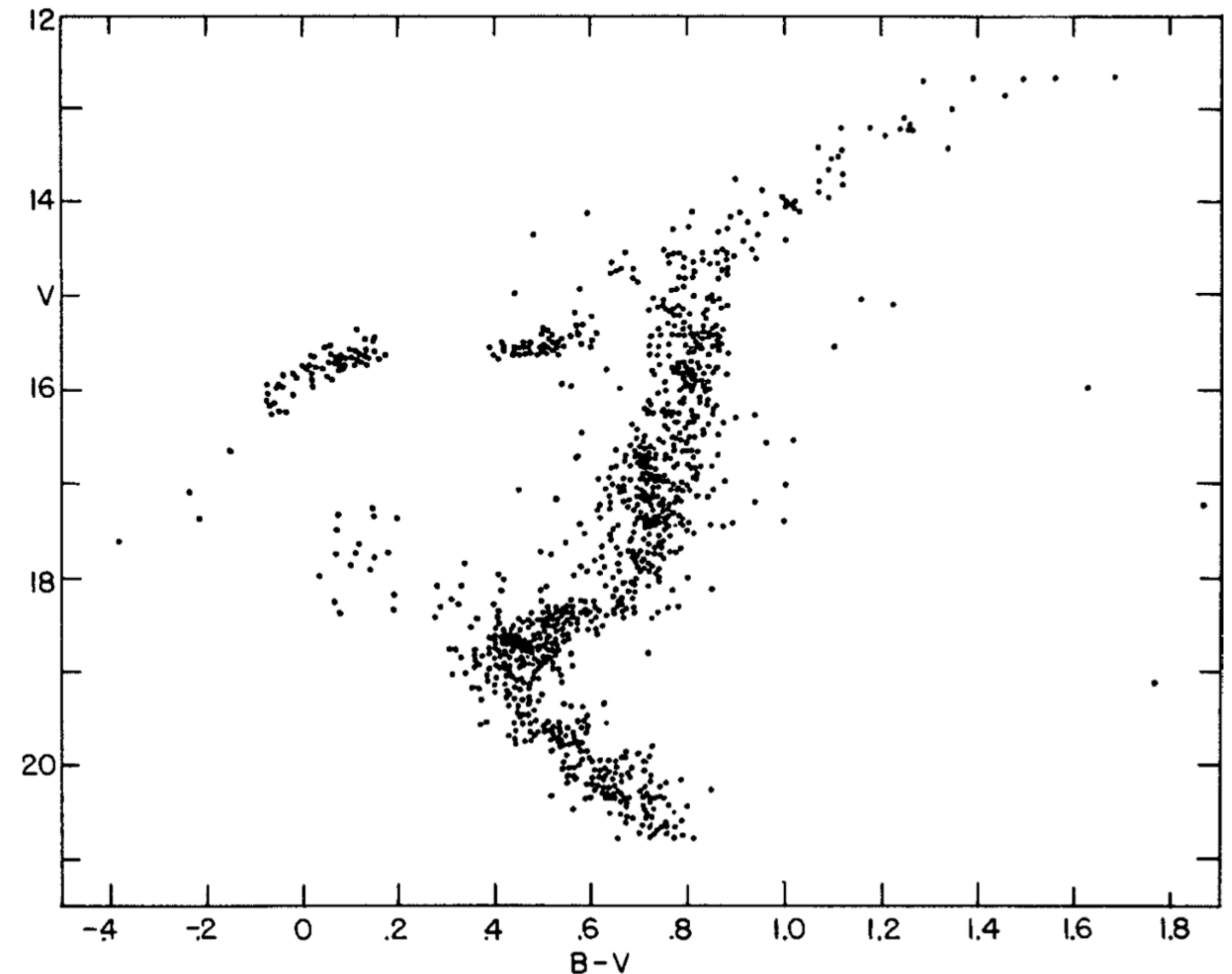


Fig. 3.8 The HR diagram of stars in the globular cluster M3. From [Johnson and Sandage \(1956\)](#). (©American Astronomical Society. Reproduced with permission from *Astrophysical Journal*.)

Star clusters

- The overall appearance of the [Figure](#) is quite different from [the other HR diagram](#). The **main sequence is much shorter**. Main sequence stars with lower values of $B - V$ correspond to more massive stars. So the globular cluster is basically missing stars on the main sequence heavier than a certain mass.
- **More massive stars have shorter lifetimes.** In a globular cluster of a certain age, stars heavier than a particular mass would have finished their lives on the main sequence.
- We see that there is a branch of stars proceeding towards the region of red giant stars (upper right corner) from the place where the main sequence seems to end abruptly.

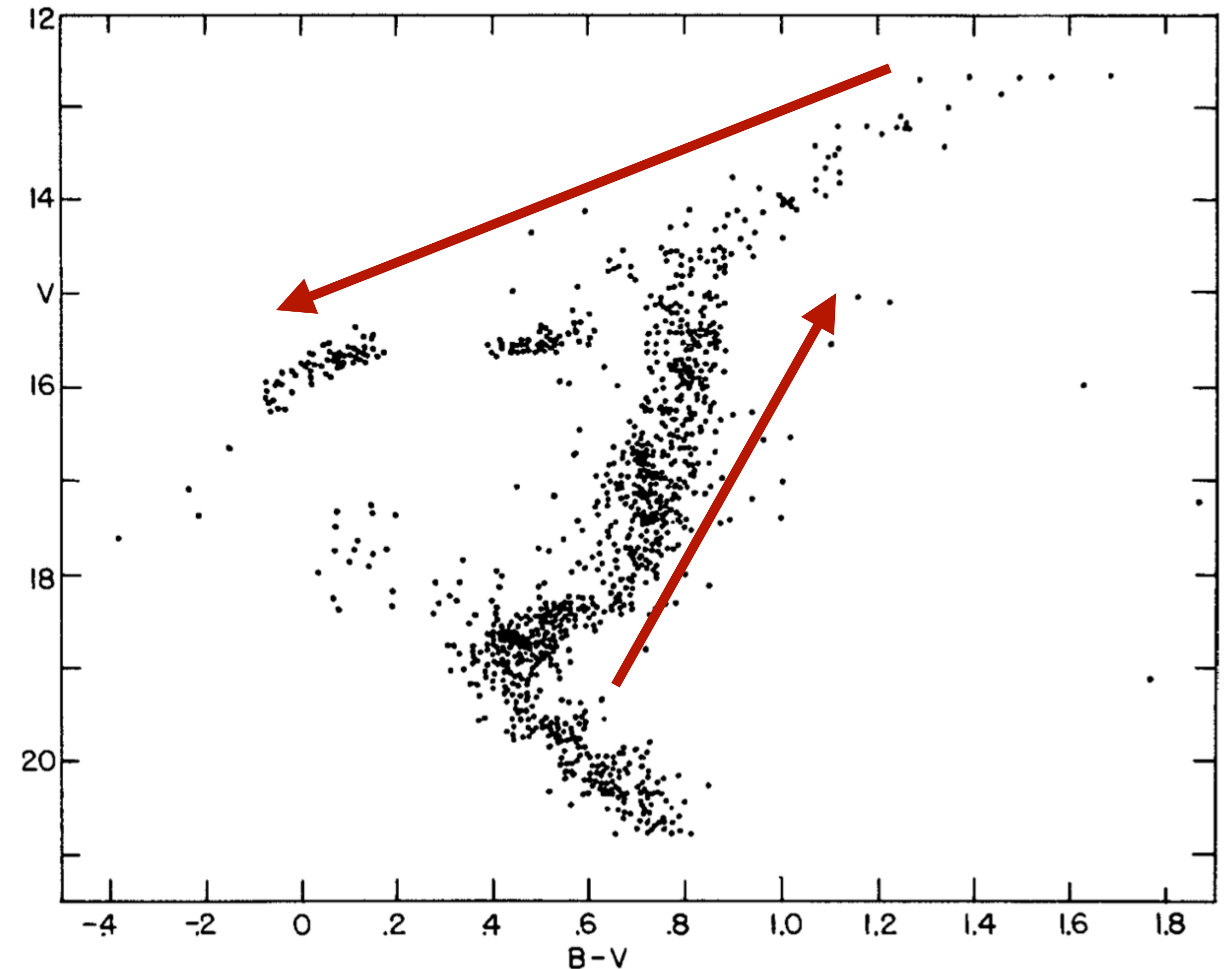


Fig. 3.8 The HR diagram of stars in the globular cluster M3. From [Johnson and Sandage \(1956\)](#). (©American Astronomical Society. Reproduced with permission from *Astrophysical Journal*.)

Star clusters

- The stars at the abrupt turning point of the main sequence are the stars which are just running out of hydrogen in the core. So **the age of the globular cluster is essentially equal to the main sequence lifetime of these stars at the turning point.**
- From a theoretical estimate of the lifetimes of stars, one can determine the age of a globular cluster simply by noting the turning point of the main sequence.
- The [Figure](#) is a composite HR diagram by superposing the HR diagrams of several star clusters. The vertical axis displays the absolute magnitude.

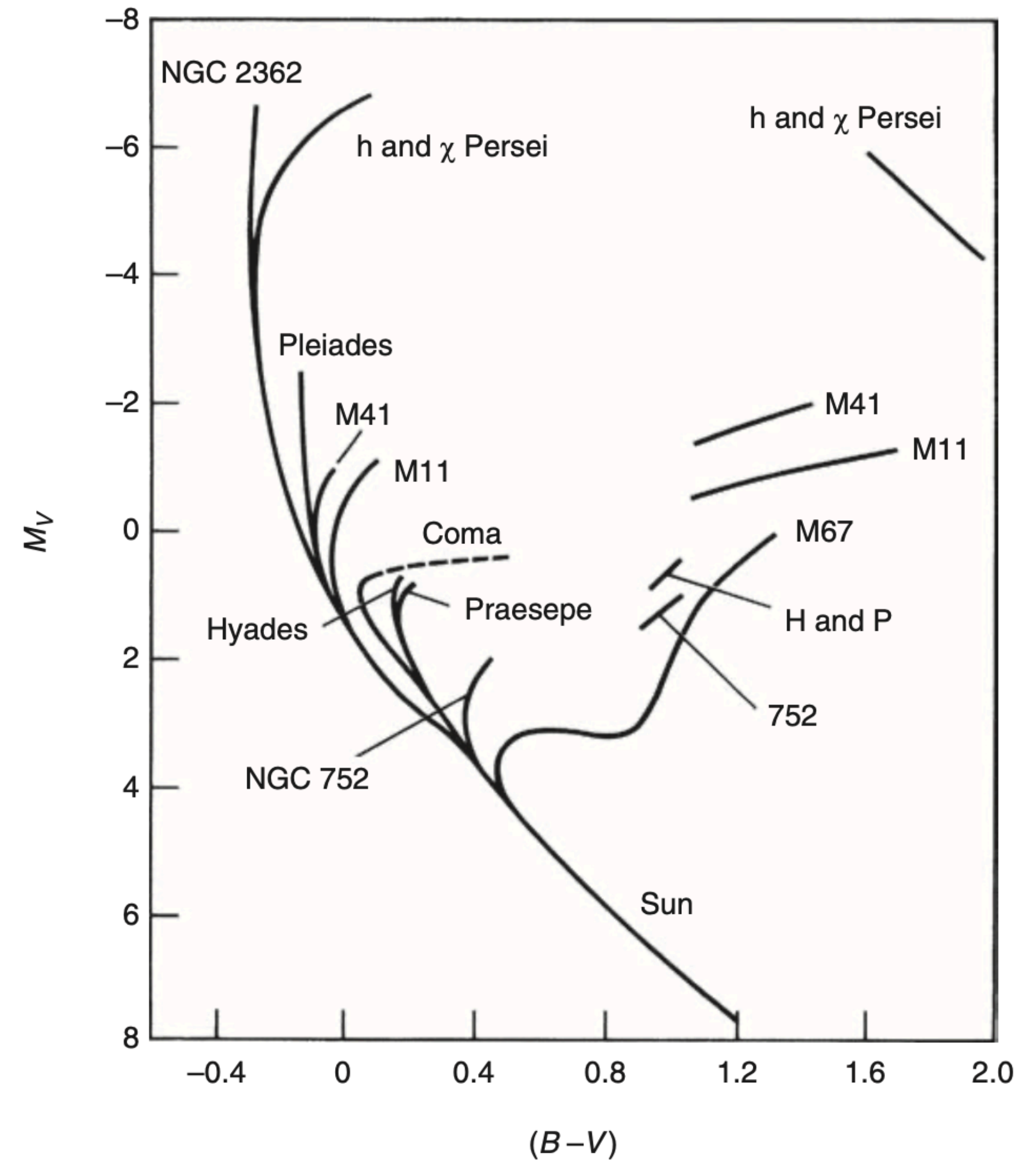


Fig. 3.9 A composite HR diagram sketching the extent of the main sequence for several star clusters. From [Sandage \(1957\)](#). (©American Astronomical Society. Reproduced with permission from *Astrophysical Journal*.)

Star clusters

- Which clusters are older?

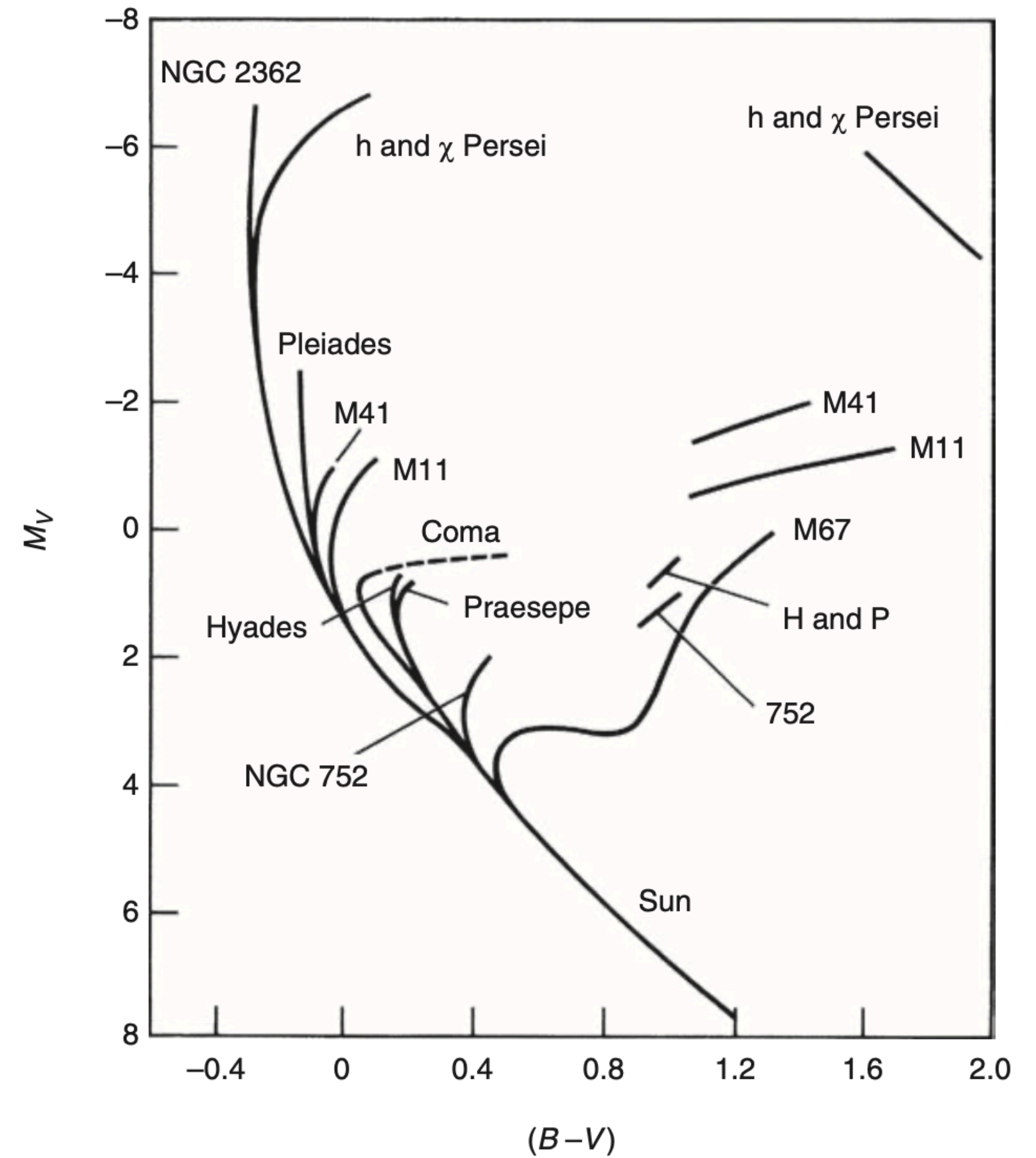


Fig. 3.9 A composite HR diagram sketching the extent of the main sequence for several star clusters. From [Sandage \(1957\)](#). (©American Astronomical Society. Reproduced with permission from *Astrophysical Journal*.)

Star clusters

- The clusters with the lower turning points are older.
- Theoretical calculations suggest that the oldest globular clusters are about 1.5×10^{10} yr old.
- This poses an important constraint on cosmology, since the Universe could not be younger than this

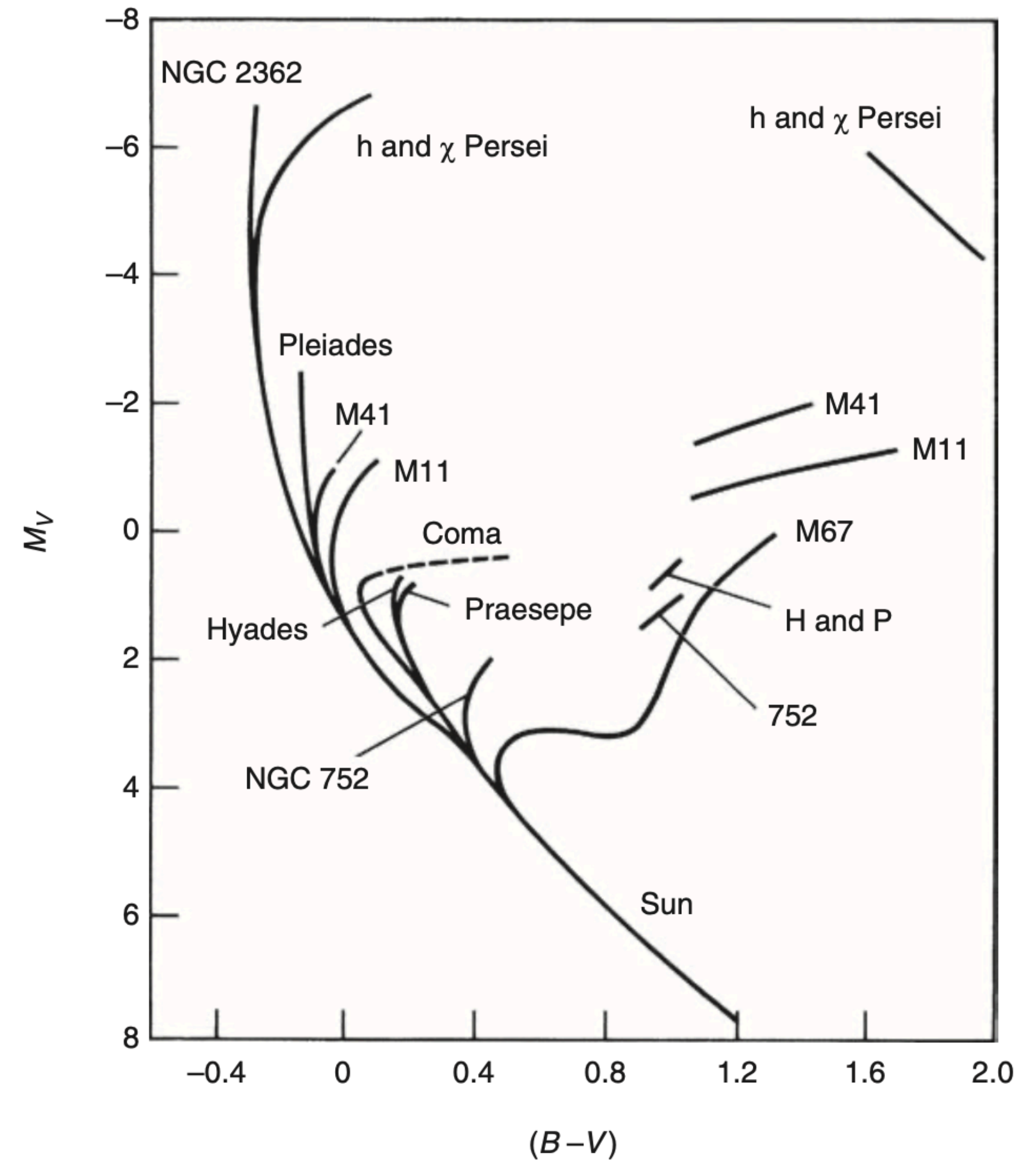


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