Introduction to Astrophysics and Cosmology

Stellar Physics

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Stellar nucleosynthesis

What is stellar nucleosynthesis?

Stellar nucleosynthesis

Let us consider a nucleus of atomic mass A and atomic number Z . It is made of Z protons and $A - Z$ neutrons. The mass m_{nuc} of the nucleus is always found to be less than the combined mass of these protons and neutrons. It is the energy equivalent of this mass deficit which provides the ***binding energy*** of the nucleus and is given by

$$E_B = [Zm_\mathrm{p} + (A - Z)m_\mathrm{n} - m_\mathrm{nuc}]c^2$$

how tightly bound a nucleus is, we need to consider the binding energy per nucleon

$$f = \frac{E_B}{A}$$

Stellar nucleosynthesis

energy is released in two kinds of nuclear reactions:

- the fusion of very light nuclei into somewhat heavier nuclei or the fission of very heavy nuclei into intermediate mass nuclei. Energy production in the interiors of stars is believed to be due to nuclear fusion.

- nuclear fission, e.g. nuclear power plants

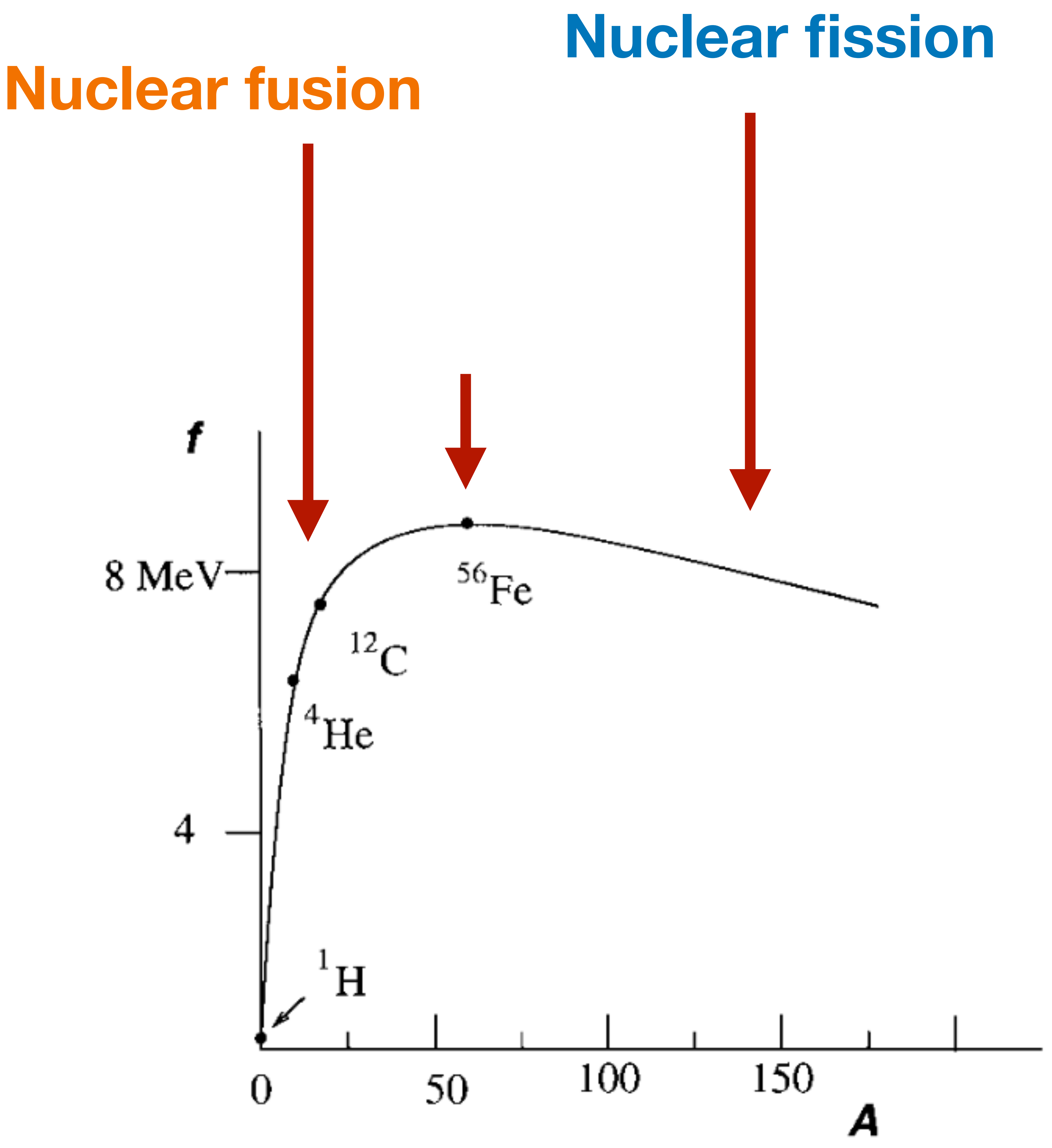

Fig. 4.1 A smooth curve showing the binding energy per nucleon, plotted against the atomic mass number.

Stellar nucleosynthesis

f for helium is 6.6 MeV, which is about 0.007 of the mass of a nucleon. Hence, if a mass $M_\odot$ of hydrogen is fully converted into helium, the total amount of energy released will be $0.007 M_\odot c^2$. Dividing this by the solar luminosity $L_\odot$, we get an **estimate of the lifetime of a star** which shines by converting hydrogen into helium, i.e.

$$\tau_{\rm nuc} \approx \frac{0.007 M_\odot c^2}{L_\odot} \qquad \tau_{\rm nuc} \approx 10^{11}\,\mathrm{yr},$$

Stellar nucleosynthesis

All nuclei are positively charged and normally repel each other. Only when two nuclei are brought within about 10^{-15} m (~1fm), can the short-range **nuclear forces overcome the electrical repulsion and the nuclei can fuse.**

A typical internuclear potential is shown in Figure

For two nuclei with atomic numbers Z1 and Z2, the electrostatic potential is

$$\frac{1}{4\pi\epsilon_0}\frac{Z_1 Z_2 e^2}{r}.$$

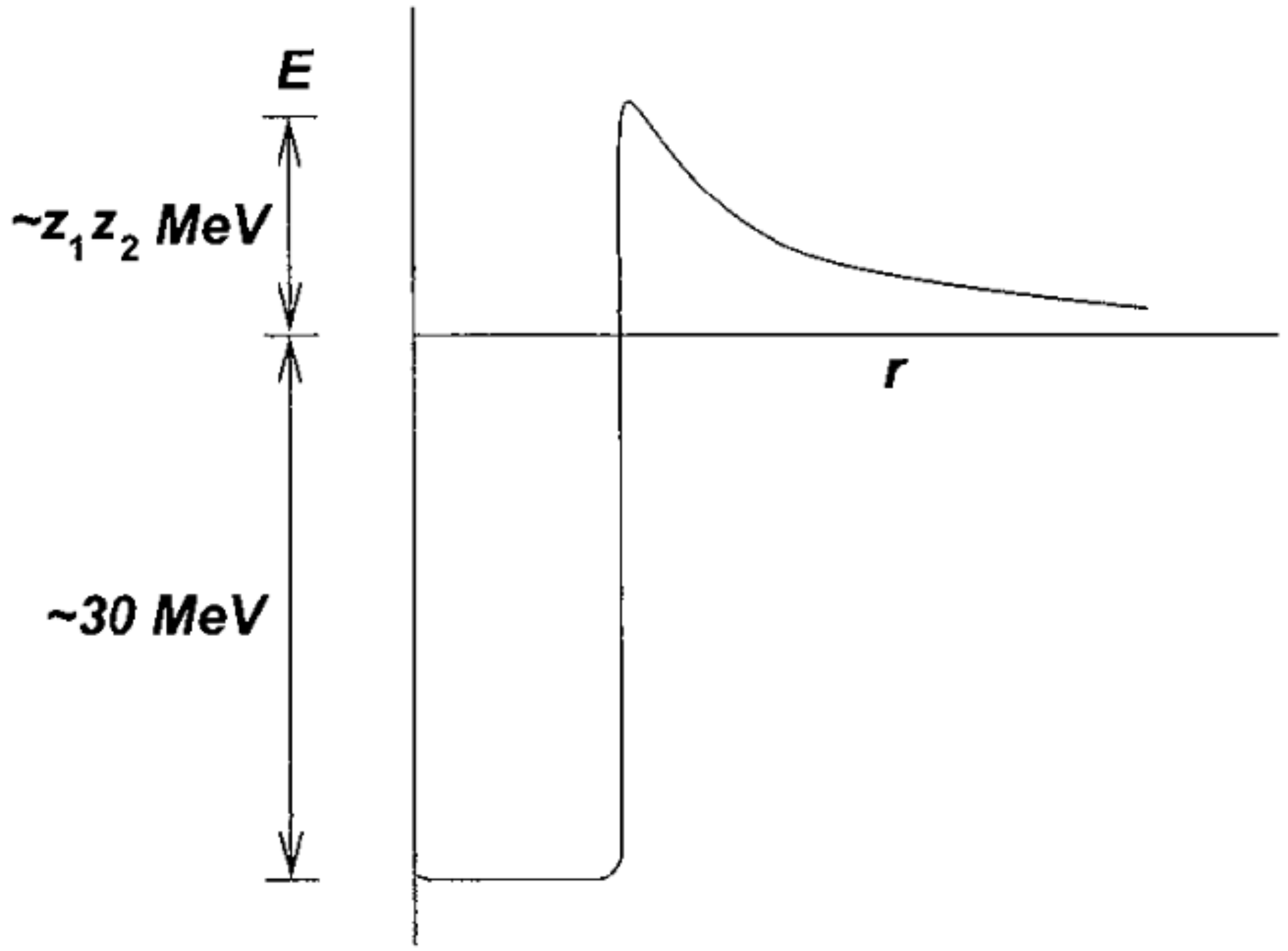

Fig. 4.2 A sketch of a typical nuclear potential.

The height of this potential at the nuclear radius ($r \approx 1$ fm) is about $Z1\ Z2$ MeV.

Stellar nucleosynthesis

- At the centre of the Sun where the temperature is of order $10^7\mathrm{K}$, the **typical kinetic energy** $\kappa_B T$ of a particle is about a keV, which is about 10^3 **lower than the electrostatic potential barrier between nuclei.**
- Even the centre of the Sun is not hot enough for the nuclei to overcome the mutual electrical repulsion and come close together for nuclear fusion – according to classical physics!

What is the solution?

Stellar nucleosynthesis

- At the centre of the Sun where the temperature is of order 10^7K, the typical kinetic energy $\kappa_B T$ of a particle is about a keV, which is about 10^3 lower than the electrostatic potential barrier between nuclei.
- Even the centre of the Sun is not hot enough for the nuclei to overcome the mutual electrical repulsion and come close together for nuclear fusion – according to classical physics!
- However, one of the standard results of **quantum mechanics is that a particle can tunnel through a potential barrier.**
- While studying α-decay, Gamow (1928) calculated the probability for the α-particle to tunnel from the inside of the nucleus to the outside by penetrating the potential barrier.
- The same probability should hold for a particle to tunnel from the outside through the potential barrier of the nucleus.
- On taking account of the **tunnelling probability, it was found that nuclear fusion can indeed take place in the interior of the Sun** (Atkinson and Houtermans, 1929).

Stellar nucleosynthesis

Suppose a nucleus having charge $Z_1 e$ can react with a nucleus having charge $Z_2 e$, their number densities per unit volume being n_1 and n_2. We want to **calculate the rate of the reaction,** i.e. the number of reactions taking place per unit volume per unit time.

If both types of nuclei have a **Maxwellian velocity distribution**, it is straightforward to show that the probability of the relative velocity between a pair being v also follows a Maxwellian distribution

$$f(v)\,dv = \left(\frac{m}{2\pi\kappa_B T}\right)^{3/2} \exp\left(-\frac{mv^2}{2\kappa_B T}\right) 4\pi v^2 dv,$$

where m is the reduced mass $m1m2/(m1 + m2)$. In terms of the kinetic energy $E = \frac{1}{2}mv^2$

$$f(E)\,dE = \frac{2}{\sqrt{\pi}} \frac{E^{1/2}}{(\kappa_B T)^{3/2}} \exp\left(-\frac{E}{\kappa_B T}\right) dE.$$

Stellar nucleosynthesis

If $\sigma(E)$ is the reaction cross-section between the two nuclei approaching each other with energy E, then it is easy to see that the **reaction rate is given by**

$$r = n_1 n_2 \langle \sigma v \rangle$$

$$\langle \sigma v \rangle = \int_0^\infty \sigma(E)\, v\, f(E)\, dE.$$

we need only the reaction cross-section $\sigma(E)$ to calculate the reaction rate

Stellar nucleosynthesis

- The typical particle energy in a stellar interior is much less than the height of the potential barrier.
- Hence the cross-section σ(*E*) has to **depend on the probability of tunnelling through this potential barrier.**
- For nuclei approaching each other with energy *E*, the probability of tunnelling through the potential barrier is given by

$$P \propto \exp\left[-\frac{1}{2\epsilon_0\hbar}\left(\frac{m}{2}\right)^{1/2}\frac{Z_1Z_2e^2}{\sqrt{E}}\right]$$

Now, without the tunnelling probability, the reaction cross-section is expected to go as approximately λ^2, where λ is the de Broglie wavelength. Since $\lambda^2 \propto 1/E$, we can write down the cross-section including the tunnelling probability in the form

$$\sigma(E) = \frac{S(E)}{E}\exp\left[-\frac{b}{\sqrt{E}}\right] \qquad b = \frac{1}{2\epsilon_0\hbar}\left(\frac{m}{2}\right)^{1/2} Z_1Z_2e^2$$

S(*E*) is a slowly varying function of *E*

Stellar nucleosynthesis

- The assumption of slow variation of $S(E)$ has its limitations. **Occasionally the cross-section of a nuclear reaction may become very large** for a certain energy. This is called a ***resonance***.
- Only in the absence of resonances, can we take $S(E)$ to be a slowly varying function. Usually $S(E)$ is determined from laboratory experiments.

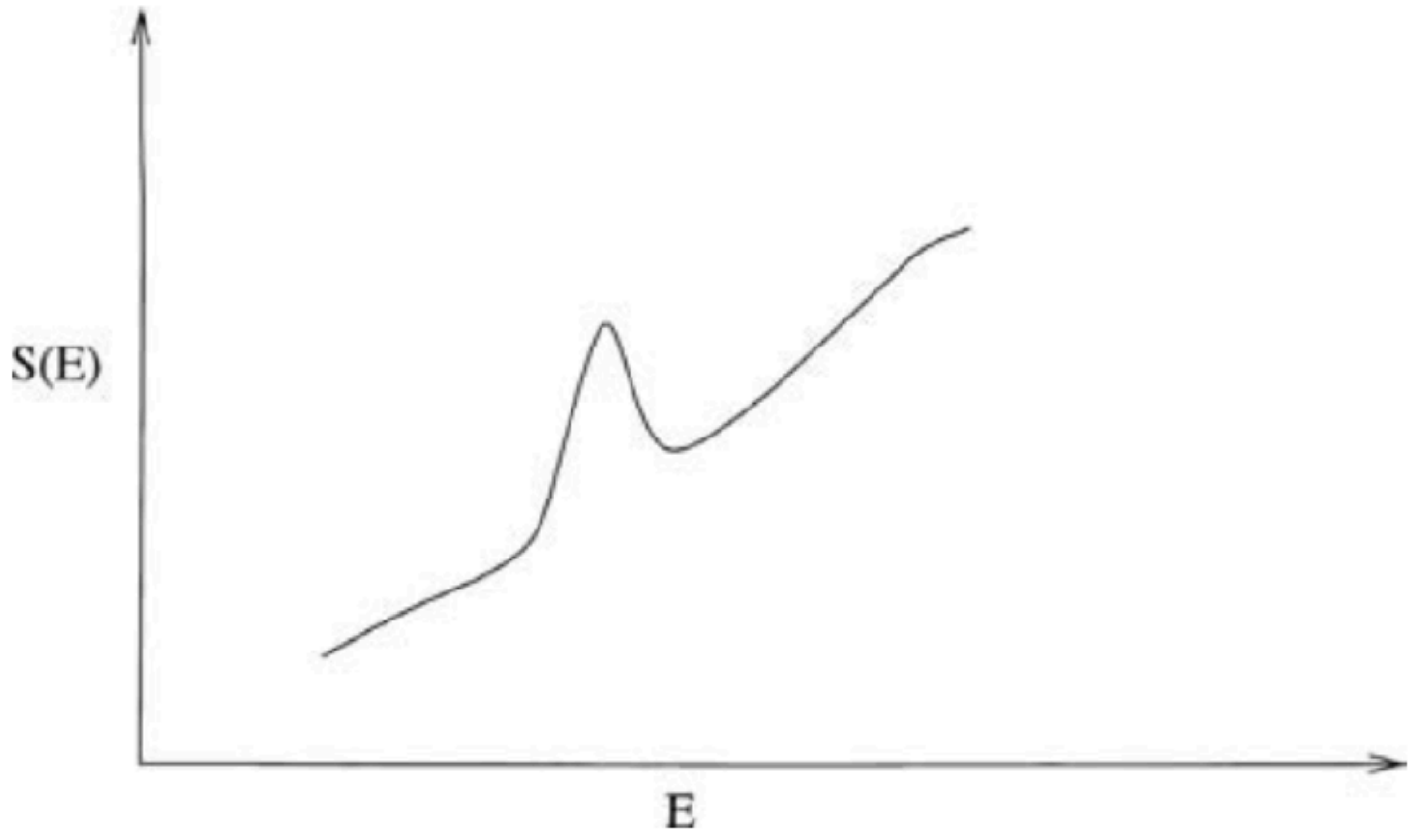

Fig. 4.3 A sketch showing the variation of a nuclear reaction cross-section with energy around a resonance.

Stellar nucleosynthesis

Velocity distribution (Maxwellian)

$$f(E)\,dE = \frac{2}{\sqrt{\pi}}\frac{E^{1/2}}{(\kappa_{\mathrm{B}}T)^{3/2}}\exp\left(-\frac{E}{\kappa_{\mathrm{B}}T}\right)dE.$$

$$\langle \sigma v\rangle = \int_0^\infty \sigma(E)\, v\, f(E)\, dE.$$

$$\sigma(E) = \frac{S(E)}{E}\exp\left[-\frac{b}{\sqrt{E}}\right]$$

Tunneling probability (Gammow)

$$\langle \sigma v\rangle = \frac{2^{3/2}}{\sqrt{\pi m}}\frac{1}{(\kappa_{\mathrm{B}}T)^{3/2}}\int_0^\infty S(E)\, e^{-E/\kappa_{\mathrm{B}}T} e^{-b/\sqrt{E}} dE.$$

Stellar nucleosynthesis

- The function $\exp(-E/\kappa_B T)$ decreases rapidly with E, whereas the other function $\exp(-b/\sqrt{E})$ increases rapidly with E
- Their product has an appreciable value only for a narrow range of energy around $E0$.

$$\langle \sigma v \rangle = \frac{2^{3/2}}{\sqrt{\pi m}} \frac{1}{(\kappa_B T)^{3/2}} \int_0^\infty S(E)\, e^{-E/\kappa_B T} e^{-b/\sqrt{E}} dE.$$

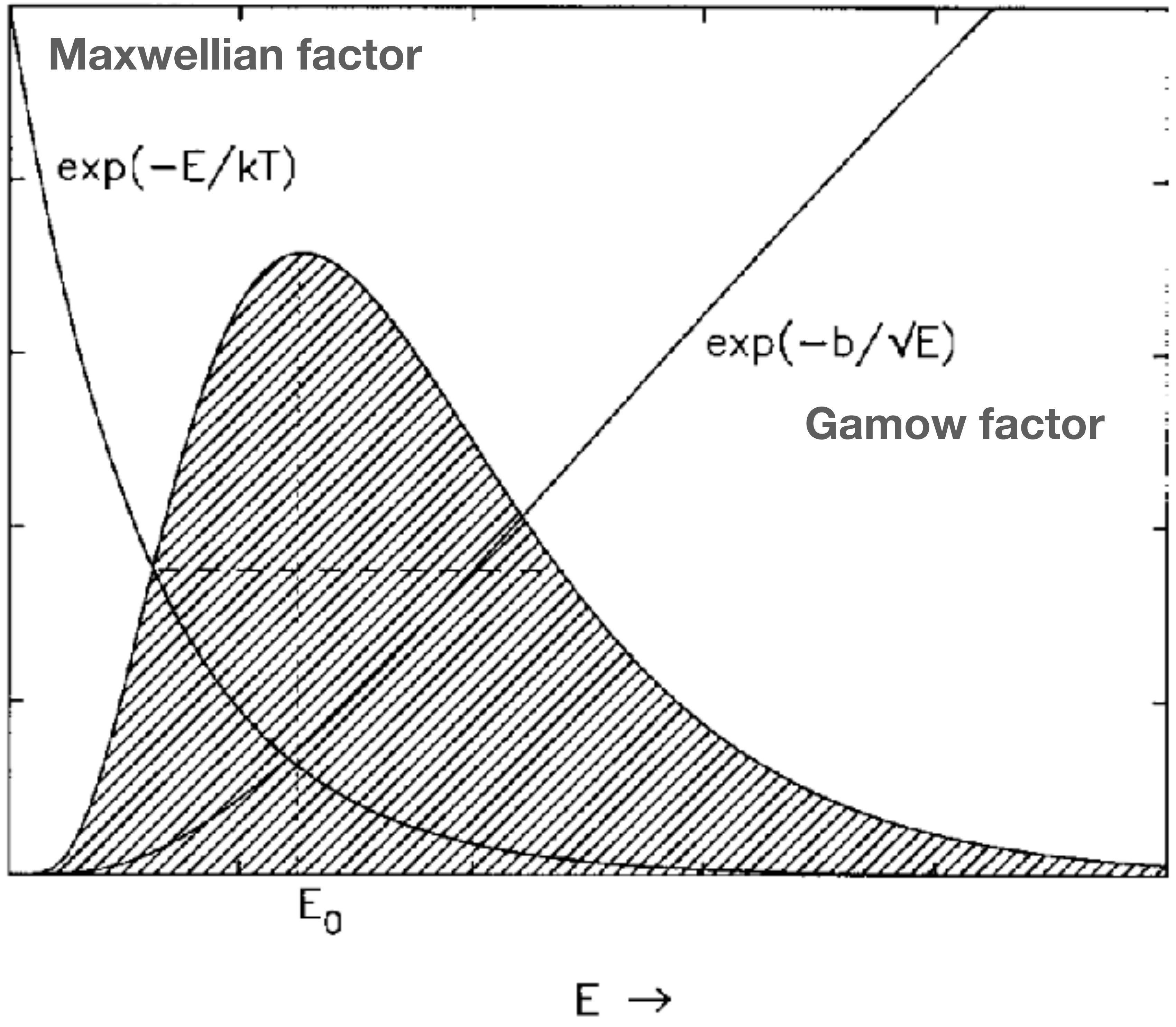

Fig. 4.4 Variation with energy of the Gamow factor, the Maxwellian factor and their product (the curve with the shading underneath).

Stellar nucleosynthesis

- We can replace the slowly varying function $S(E)$ by its value $S(E_0)$ at E_0 and take it outside the integral.

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$$J = \int_0^\infty e^{g(E)}\, dE, \qquad g(E) = -\frac{E}{\kappa_B T} - \frac{b}{\sqrt{E}}$$

The value of ***J* is given by the shaded area** in Figure. From $dg/dE = 0$, we can find the value of E_0 **where the function *g*(*E*) is maximum**, which gives

$$E_0 = \left(\frac{1}{2} b\kappa_B T\right)^{2/3} = \left[\left(\frac{m}{2}\right)^{1/2} \frac{Z_1 Z_2 e^2 \kappa_B T}{4\epsilon_0 \hbar}\right]^{2/3}$$

Let the value of $g(E)$ at E_0 be denoted by $-\tau$, i.e.

$$\tau = -g(E_0) = 3\frac{E_0}{\kappa_B T} = 3\left[\left(\frac{m}{2\kappa_B T}\right)^{1/2} \frac{Z_1 Z_2 e^2}{4\epsilon_0 \hbar}\right]^{2/3}$$

Stellar nucleosynthesis

- We can now expand $g(E)$ in a Taylor series around the point $E = E_0$, which gives

$$g(E) = g(E_0) + \left(\frac{dg}{dE}\right)_{E=E_0} (E - E_0) + \frac{1}{2}\left(\frac{d^2g}{dE^2}\right)_{E=E_0} (E - E_0)^2 + \cdots$$
$$= -\tau - \frac{\tau}{4}\left(\frac{E}{E_0} - 1\right)^2 + \cdots$$

on calculating d^2g/dE^2 and noting that $dg/dE = 0$ at $E = E_0$. Substituting this, we get

$$J \approx e^{-\tau} \int_0^\infty e^{-\frac{\tau}{4}\left(\frac{E}{E_0}-1\right)^2} dE$$

Stellar nucleosynthesis

keeping in mind that τ goes as $T^{-1/3}$

$$\langle \sigma v \rangle = \frac{2^{3/2}}{\sqrt{\pi m}} \frac{1}{(\kappa_B T)^{3/2}} \int_0^\infty S(E)\, e^{-E/\kappa_B T} e^{-b/\sqrt{E}} dE.$$

$$\langle \sigma v \rangle \propto \frac{S(E_0)}{T^{2/3}} \exp\left[-3 \left(\frac{e^4}{32\epsilon_0^2 \kappa_B \hbar^2} \frac{m Z_1^2 Z_2^2}{T} \right)^{1/3} \right]$$

Once ***S*(*E*) for the nuclear reaction** is found **from laboratory experiments**, the reaction rate can be obtained by substituting

$$\langle \sigma v \rangle \longrightarrow r = n_1 n_2 \langle \sigma v \rangle$$

Stellar nucleosynthesis

$$\frac{dL_r}{dr} = 4\pi r^2 \rho\, \varepsilon.$$

For calculating stellar models, we need to know the energy generation rate by the nuclear reaction. If $\Delta\epsilon$ **is the energy released** in this nuclear reaction, then the energy generation rate per unit volume is $r\Delta\epsilon$, with r given by

$$r = n_1 n_2 \langle \sigma v \rangle$$

This must be equal to $\varrho\varepsilon$, where ε

$$\rho\varepsilon = r\,\Delta\mathcal{E} = n_1 n_2 \langle \sigma v \rangle \Delta\mathcal{E}.$$

If X_1 and X_2 are the mass fractions of the two elements which take part in the nuclear reaction, then n_1 and n_2 should respectively be proportional to ϱX_1 and ϱX_2. The **nuclear energy generation function ε** should have the following functional dependence on various relevant quantities:

$$\boxed{\varepsilon = C\rho\, X_1 X_2 \frac{1}{T^{2/3}} \exp\left[-3 \left(\frac{e^4}{32\epsilon_0^2 \kappa_B \hbar^2} \frac{m Z_1^2 Z_2^2}{T} \right)^{1/3} \right]}$$

Stellar nucleosynthesis

$$\varepsilon = C\rho\, X_1 X_2 \frac{1}{T^{2/3}} \exp\left[-3\left(\frac{e^4}{32\epsilon_0^2 \kappa_{\mathrm{B}} \hbar^2}\frac{m Z_1^2 Z_2^2}{T}\right)^{1/3}\right]$$

Once the coefficient C is estimated from the experimentally determined cross section $S(E)$, we have the necessary input for stellar structure calculations.

The function **ε increases with temperature sharply** because of the exponential involving temperature.

Since $Z_1^2 Z_2^2 / T$ appears in the exponential, **it should be clear that reactions involving heavier nuclei are much less likely compared to reactions involving lighter nuclei at a given temperature.**

Stellar nucleosynthesis

Although nuclear reactions inside stars involve no chemical burning, it is quite customary to refer to energy generation by nuclear reactions as ***nuclear burning*** and the element which gets transformed in the nuclear reactions as ***nuclear fuel***.

- One needs an experimentally determined cross-section $S(E)$ for a nuclear reaction to calculate the energy generation by that reaction in stellar interiors.
- Typical particle energies in stellar interiors are of the order of keV.
- Laboratory experiments are usually done for energies of order MeV so that the Coulomb barrier does not pose a big problem and the nuclear reactions become more likely.
- From measurements of $S(E)$ at MeV energies, one has to extrapolate to keV energies for application to stellar interiors.
- Extensive spectroscopic analysis of the Sun, proved that the stars are mainly made up of hydrogen.
- Hydrogen can 'burn' at a temperature lower than the temperatures necessary to burn helium and other heavier elements with higher atomic number Z.
- **Main-sequence stars generate their energies by burning hydrogen into helium.**

Stellar nucleosynthesis

The energy generation inside the Sun primarily takes place due to the ***proton– proton* or *pp chain*.** In the first two reactions of this chain, deuterium ^2H and then ^3He are produced as follows:

$$^1\mathrm{H} + {}^1\mathrm{H} \longrightarrow {}^2\mathrm{H} + e^+ + \nu,$$

$$^2\mathrm{H} + {}^1\mathrm{H} \longrightarrow {}^3\mathrm{He} + \gamma.$$

After the production of ^3He, the reactions can proceed through three alternative branches: $pp1$, $pp2$, $pp3$.

The branch $pp1$ is by far the dominant branch for conditions corresponding to the solar interior. It involves two nuclei of ^3He producing a nucleus of ^4He:

$$\boxed{pp1:\ {}^3\mathrm{He} + {}^3\mathrm{He} \longrightarrow {}^4\mathrm{He} + {}^1\mathrm{H} + {}^1\mathrm{H}}$$

Stellar nucleosynthesis

On considering all the reactions in the $pp1$ branch, it should be clear that effectively four ^1H nuclei combine to form one ^4He nucleus.

The other two branches ($pp2$ and $pp3$) start dominating only when the temperature is above 107 K. They require the prior existence of ^4He and first form ^7Be:

$$^3\mathrm{He} + {}^4\mathrm{He} \longrightarrow {}^7\mathrm{Be} + \gamma.$$

Afterwards ^7Be can lead to the following two kinds of reactions:

$$\boxed{\begin{aligned} pp2:\ {}^7\mathrm{Be} + e^- &\longrightarrow {}^7\mathrm{Li} + \nu, \\ {}^7\mathrm{Li} + {}^1\mathrm{H} &\longrightarrow {}^4\mathrm{He} + {}^4\mathrm{He}. \\ pp3:\ {}^7\mathrm{Be} + {}^1\mathrm{H} &\longrightarrow {}^8\mathrm{B} + \gamma, \\ {}^8\mathrm{B} &\longrightarrow {}^8\mathrm{Be} + e^+ + \nu, \\ {}^8\mathrm{Be} &\longrightarrow {}^4\mathrm{He} + {}^4\mathrm{He}. \end{aligned}}$$

Stellar nucleosynthesis

$$\varepsilon = C\rho\, X_1 X_2 \frac{1}{T^{2/3}} \exp\left[-3\left(\frac{e^4}{32\epsilon_0^2 \kappa_{\rm B}\hbar^2}\frac{mZ_1^2Z_2^2}{T}\right)^{1/3}\right]$$

To find the energy generation function ε for the whole chain of reactions. How can this be done?

We note that the first reaction is mediated by the weak interaction (the emission of a neutrino is usually the signature of a reaction being mediated by the weak interaction) and is a slow reaction with a small cross-section. Even though some of the other reactions may be faster, they cannot proceed without the ^2H nuclei which are produced in the first slow reaction.

It is thus the first reaction which determines the reaction rate in a steady state.

In general, when a series of reactions will have to take place, the slowest reaction determines the rate at the steady state. However, while calculating the energy generation, it is necessary to add up the energies released in all the reactions in the chain. When all these are done carefully, the energy generation rate ε is given by

$$\varepsilon_{pp} = 2.4 \times 10^{-1} \rho X^2 \left(\frac{10^6}{T}\right)^{2/3} \exp\left[-33.8\left(\frac{10^6}{T}\right)^{1/3}\right] \text{W kg}^{-1},$$

when the contributions of $pp2$ and $pp3$ branches are neglected. Here X is the mass fraction of hydrogen.

Stellar nucleosynthesis

If carbon, nitrogen and oxygen are already present and can act as catalysts, then hydrogen can be synthesized into helium by a completely different series of nuclear reactions. This series of reactions, known as the ***CNO cycle***. The reactions in this cycle are the following:

$$
\begin{aligned}
{}^{12}\mathrm{C} + {}^{1}\mathrm{H} &\longrightarrow {}^{13}\mathrm{N} + \gamma, \\
{}^{13}\mathrm{N} &\longrightarrow {}^{13}\mathrm{C} + e^{+} + \nu, \\
{}^{13}\mathrm{C} + {}^{1}\mathrm{H} &\longrightarrow {}^{14}\mathrm{N} + \gamma, \\
{}^{14}\mathrm{N} + {}^{1}\mathrm{H} &\longrightarrow {}^{15}\mathrm{O} + \gamma, \\
{}^{15}\mathrm{O} &\longrightarrow {}^{15}\mathrm{N} + e^{+} + \nu, \\
{}^{15}\mathrm{N} + {}^{1}\mathrm{H} &\longrightarrow {}^{12}\mathrm{C} + {}^{4}\mathrm{He}.
\end{aligned}
$$

Stellar nucleosynthesis

$$\varepsilon = C\rho\, X_1 X_2 \frac{1}{T^{2/3}} \exp\left[-3\left(\frac{e^4}{32\epsilon_0^2\kappa_B\hbar^2}\frac{mZ_1^2Z_2^2}{T}\right)^{1/3}\right]$$

On adding up these reactions, the net result again is that four ^1H nuclei have combined together to make one ^4He nucleus.

Again, the reaction rate in the steady state is governed by the slowest reaction in the cycle, which in this case happens to be the fourth reaction. The energy generation rate by the CNO cycle is found to be

$$\varepsilon_{\mathrm{CNO}} = 8.7 \times 10^{20}\, \rho X_{\mathrm{CNO}} X \left(\frac{10^6}{T}\right)^{2/3} \exp\left[-152.3\left(\frac{10^6}{T}\right)^{1/3}\right] \mathrm{W\ kg^{-1}}$$

where X_{CNO} is the sum of the mass fractions for carbon, nitrogen and oxygen.

Stellar nucleosynthesis

The variations of ϵ_{pp} and ϵ_{CNO} as functions of T are shown in the Figure for a typical stellar composition.

- It should be clear from this figure that for stars like the **Sun with the central temperatures of order 10^7, the *pp* chain should be the dominant** energy generation mechanism.
- On the other hand, **more massive stars with higher central temperatures generate energy predominantly by the CNO cycle**.

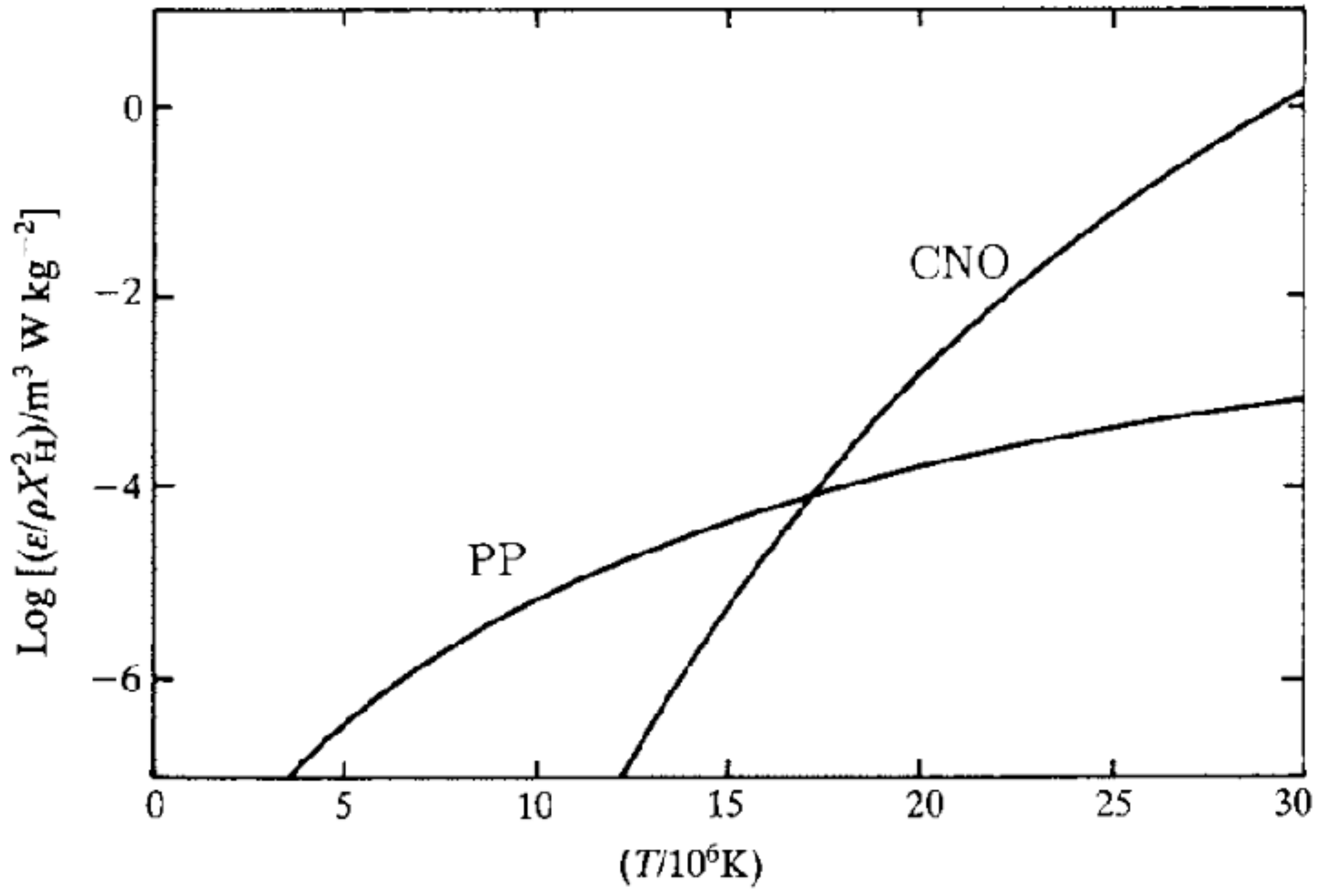


Fig. 4.5 The variation with temperature of the energy generation rate by hydrogen burning, for the two major reaction chains. From Tayler (1994, p. 92).

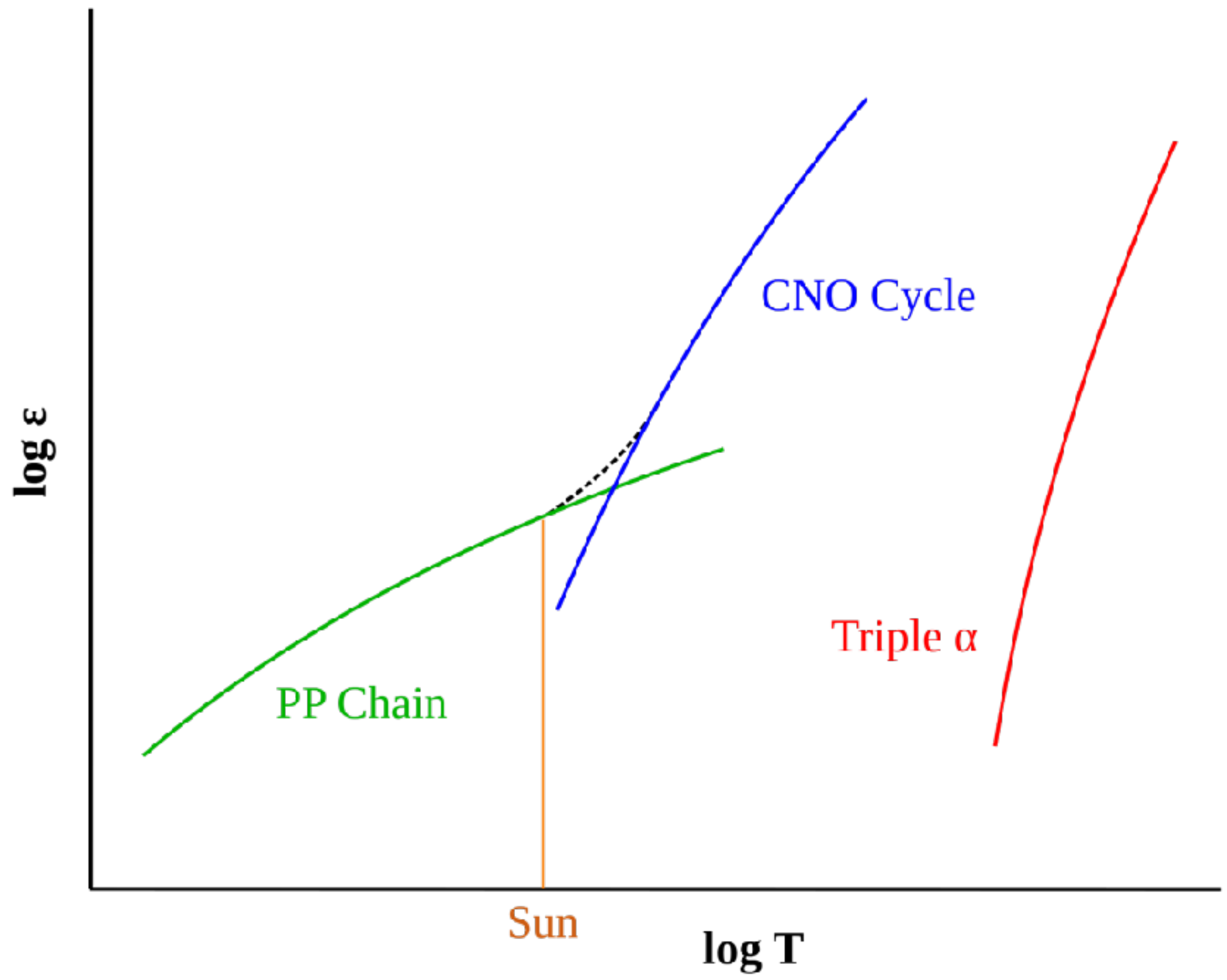
Stellar nucleosynthesis

- Apart from explaining the energy generation mechanism in stars, the other important goal of nuclear astrophysics is to explain the abundances of various elements in the Universe.
- Nuclear reactions likely took place in the early Universe and some significant fraction of baryonic matter was converted into helium. The helium synthesized in stars makes additions to this primordial helium.
- The next important question is how the heavier elements are produced.
- **Heavier elements are synthesized in stars.**
- After some helium has been synthesized from hydrogen by say *pp* chain reactions, we shall have a mixture of hydrogen and helium nuclei. If heavier nuclei have to be built up from this, then the obvious first step may be either of these two reactions:
    - one hydrogen and one helium nuclei combine to produce a nucleus of mass 5;
    - two helium nuclei combine to produce a nucleus of mass 8.
    - However, laboratory experiments failed to discover any stable nucleus of mass 5 or 8. It became clear that these two obvious nuclear reactions could not provide the next step of synthesizing heavier nuclei. Then how are heavier nuclei produced?

Stellar nucleosynthesis

- This problem was solved by the ***triple alpha reaction***. In this reaction, three ^4He nuclei combine together as follows:

$$^4\mathrm{He} + {}^4\mathrm{He} + {}^4\mathrm{He} \longrightarrow {}^{12}\mathrm{C} + \gamma.$$



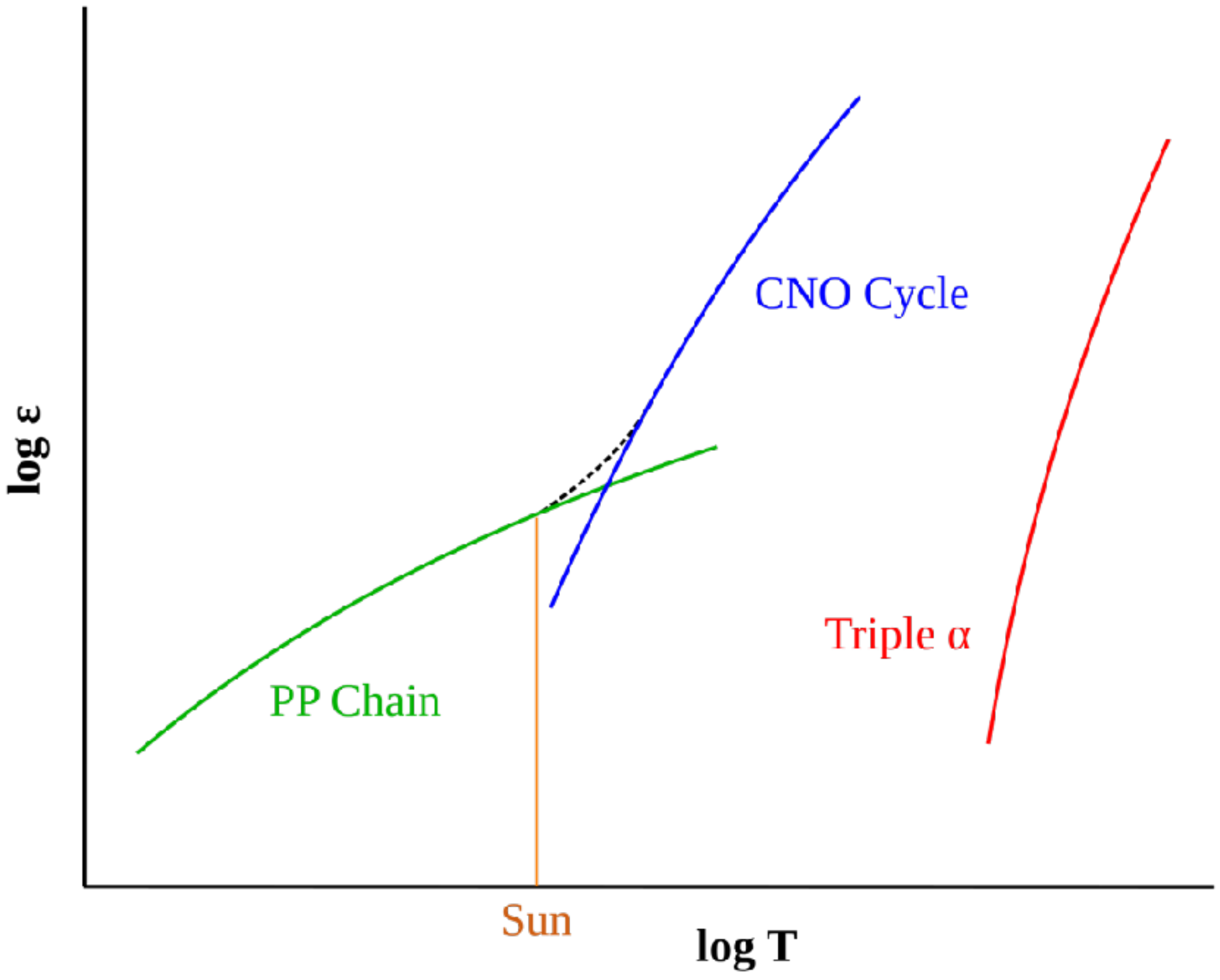
Stellar nucleosynthesis

$$^{4}He + {^{4}He} + {^{4}He} \longrightarrow {^{12}C} + \gamma.$$

Since this reaction involves three particles, it is much less likely to occur compared to reactions involving two particles.

Also, the Coulomb repulsion is stronger between helium nuclei than between the nuclei involved in *pp* chain reactions, requiring a higher temperature. In the conditions prevailing in the early Universe, this reaction is found to be highly improbable and nucleosynthesis could not possibly proceed beyond helium.

Inside **stellar cores, however, this reaction can take place when the temperature is higher than 10^8K.** But, even then, the rate would have been too slow if the cross-section of this reaction was non-resonant. This resonance was found in laboratory experiments.



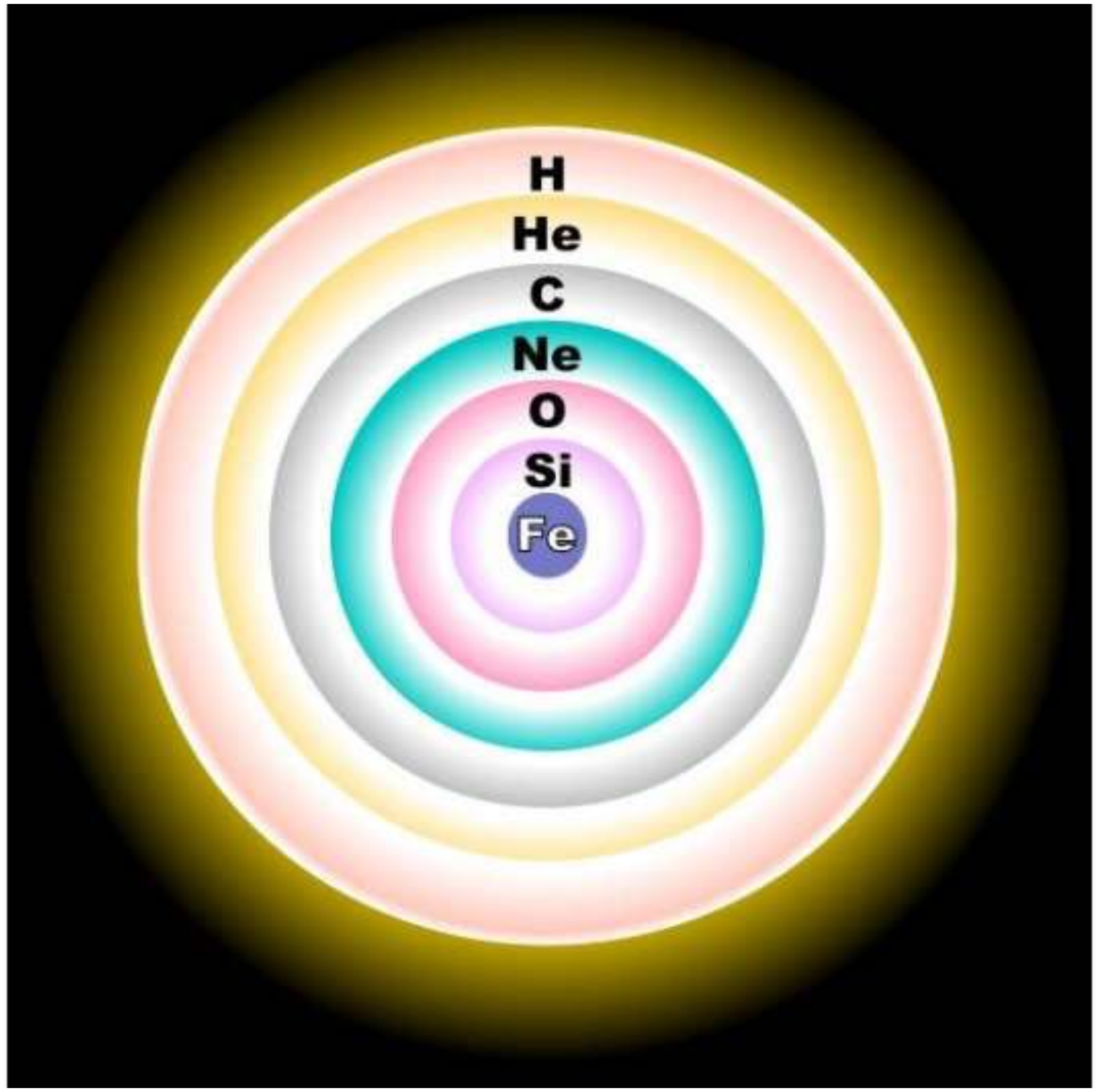
Stellar nucleosynthesis

- Detailed calculations show that central temperatures of main-sequence stars are not high enough for the triple alpha reaction. **So stars in the main sequence generate energy by the *pp* chain (less massive stars) or CNO cycle (more massive stars).**
- When, however, hydrogen is exhausted in the core, hydrogen- burning reactions can no longer halt the inward pull of gravity. The core then starts shrinking. It is possible for gravity to be eventually balanced by degeneracy pressure when the core density is sufficiently high (provided the core mass does not exceed the famous Chandrasekhar limit to collapse into a black hole).
- While the core shrinks, its temperature rises by the Kelvin–Helmholtz arguments. If the star is not too massive, then its central temperature may never become high enough to start the triple alpha reaction and the star may end up as a white dwarf with a helium core.
- In the case of very massive stars, on the other hand, the temperature of the shrinking core may become very high for other nuclear reactions involving heavier nuclei to start. Once a new nuclear reaction is ignited, it can halt the inward pull of gravity.

Stellar nucleosynthesis

- After carbon has been synthesized by the triple alpha process, the next heavier nuclei can be built up from carbon.
- In sufficiently massive stars, it is believed that **nuclear reactions can go all the way up to the most stable nucleus, iron.** So such stars may eventually have an iron core, beyond which it is not possible to generate energy by nuclear reactions.
    - After the He the star can go to stages of Li, C, Ne, O, Si burning
- One important question is why we see elements heavier than iron in the Universe, or why we even see elements higher than helium in the solar system, since the Sun has not yet gone beyond the stage of synthesizing helium from hydrogen.

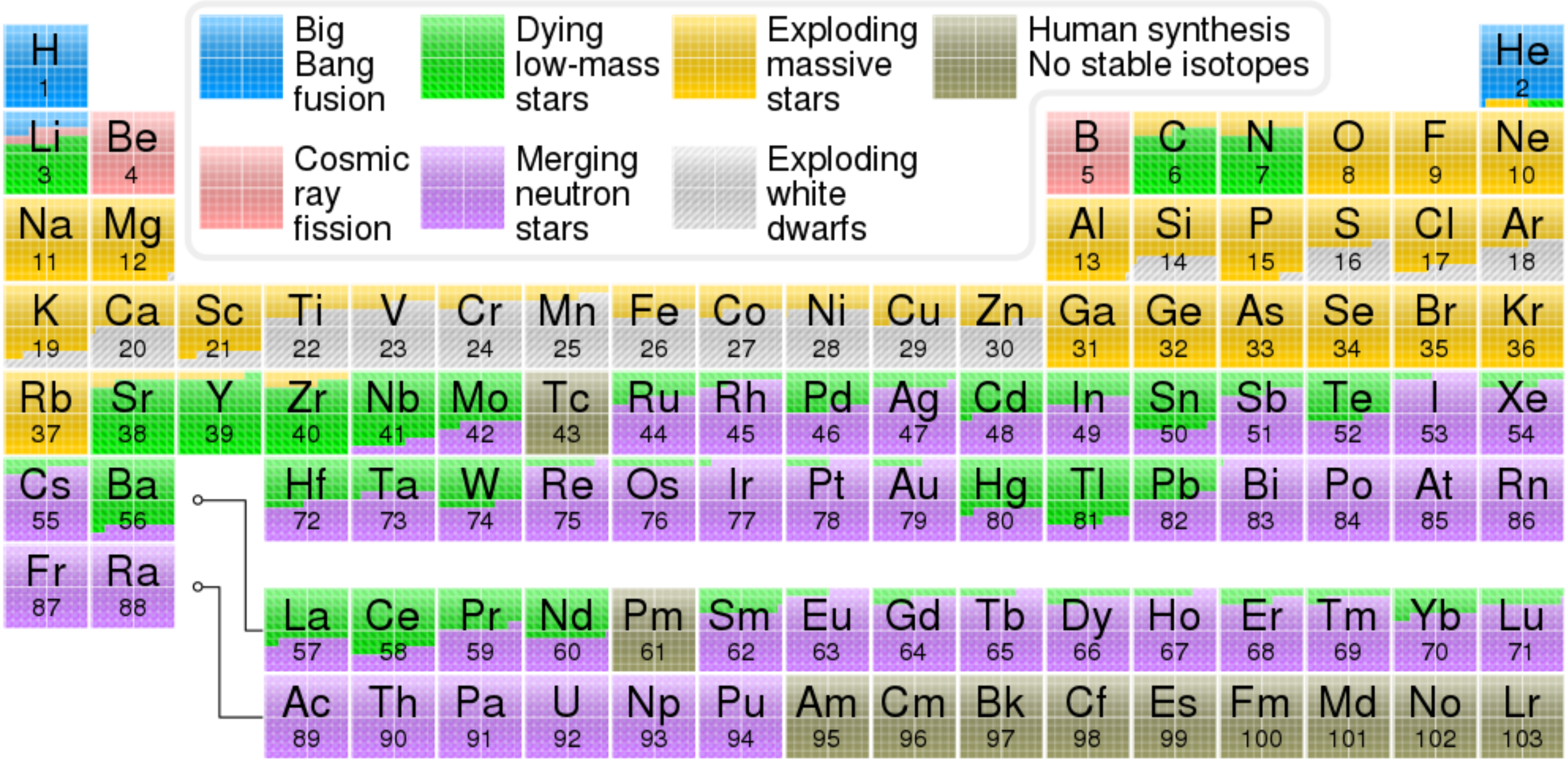
Inside layers of giant stars



https://phys.org/news/2020-05-peek-giant-star-dies.html

Stellar nucleosynthesis

A version of the periodic table indicating the origins – including stellar nucleosynthesis – of the elements.



https://en.wikipedia.org/wiki/Nucleosynthesis