Introduction to Astrophysics and Cosmology

Stellar Physics - end state of stars

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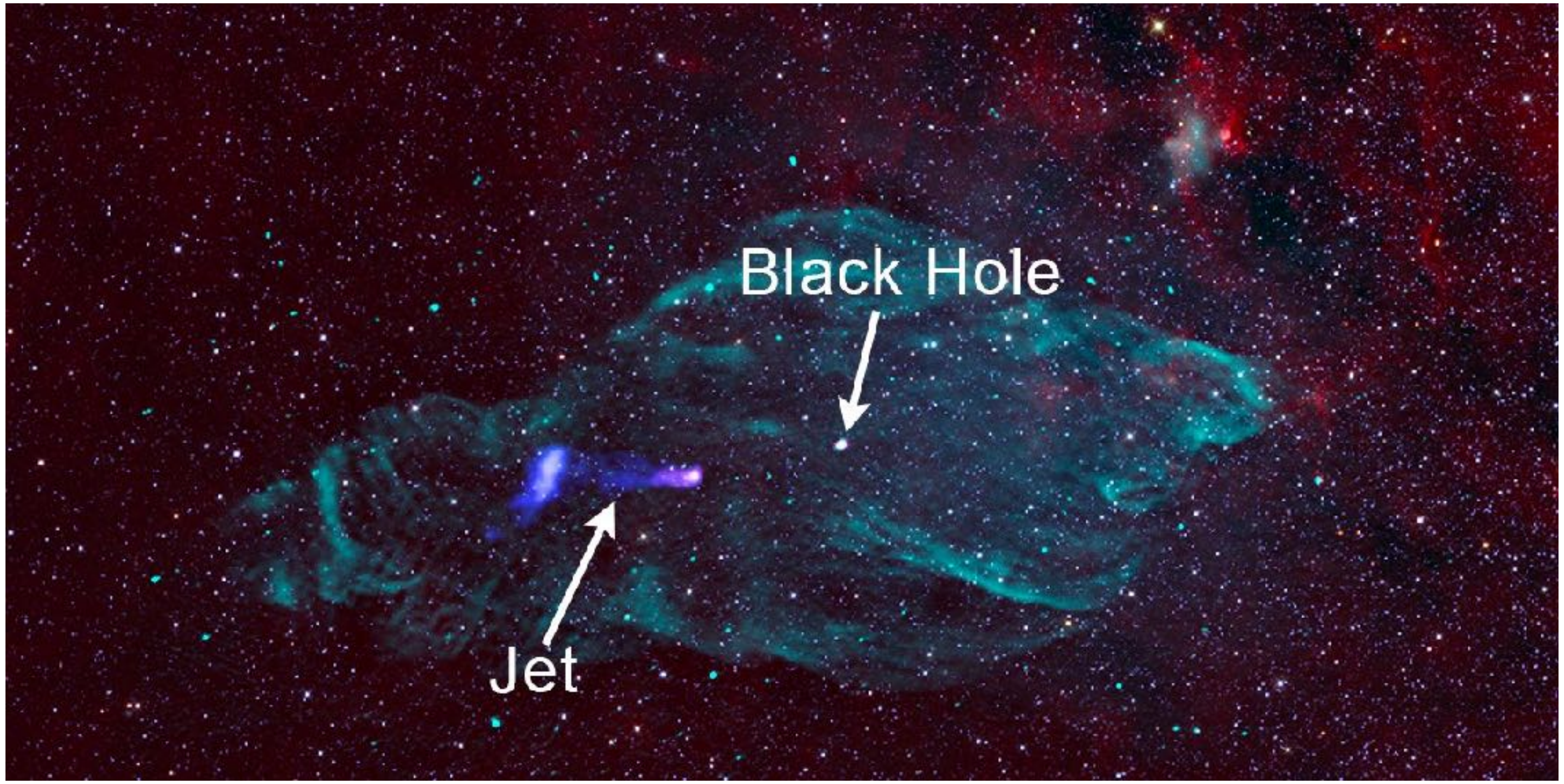
End state of stars

What are the options?

End state of stars



supernova remnant SS 433,
Also an X-ray binary

From statistical studies of various kinds of stars, it is inferred that:

- Stars **less massive than about 4$M_\odot$** eventually become **white dwarfs**
- Stars with initial masses **in the range of 4$M_\odot$ to 10$M_\odot$** end up as **neutron stars**, typically after undergoing a supernova explosion
- Stars with **initial masses more than 10$M_\odot$** probably cannot shed enough mass to become white dwarfs or neutron stars. They have to go on contracting until the gravitational attraction is so strong that even light cannot escape and they turn into a ***black hole***

Degeneracy pressure - Fermi gas

- The pressure in a gas arises from the random motions of the particles constituting the gas.
- If $4\pi f(p)p^2dp$ is the number of particles having momentum between p and $p + dp$ (assuming the distribution function to be isotropic), whereas v is the velocity of a particle having momentum p, then the **pressure** P of the gas is given by a standard expression in kinetic theory:

$$P = \frac{1}{3}\int vpf(p)\,4\pi p^2\,dp$$

- For an ordinary gas, on substituting the **Maxwellian distribution**, the pressure is found to be given by $n\kappa_B T$, where n is the number of particles per unit volume.
- The pressure of **stellar material containing different types of particles** is given by

$$P = \frac{\kappa_\mathrm{B}}{\mu\, m_\mathrm{H}}\rho\, T.$$

Degeneracy pressure - Fermi gas

$$P = \frac{\kappa_B}{\mu m_H} \rho T$$

- This **pressure,** which arises out of thermal motions of particles, should go to zero at $T = 0$ – provided we assume the validity of classical physics.
- However, when a gas of **Fermi particles is compressed** to very high density, many of the particles are forced to remain in **non-zero momentum states even at T = 0**, thereby giving rise to the **degeneracy pressure.**
- When stellar matter is compressed, **electrons become degenerate first,** before protons and other nuclei.
- The reason is that if the kinetic energy $p^2/2m$ is equally partitioned amongst different types of particles, the lighter electrons are expected to have smaller momenta. Hence **they occupy a much smaller volume of the momentum space** and consequently their number density in this region of momentum space is higher than the corresponding number density of heavier particles.
- At a density which makes electrons degenerate, the heavier particles still remain non-degenerate (i.e. their phase space occupancy remains well below the theoretical limit).

Degeneracy pressure - Fermi gas

- Electrons which occupy real space volume V and have momenta in the range d^3p in momentum space have $2Vd^3p/h^3$ states in phase space available to them (two being due to the two spin states). If d^3p corresponds to the shell between p and p + dp, then the number of states per unit volume within this shell is clearly $8\pi p^2 dp/h^3$.
- The **occupancies of these states** are given by the **Fermi–Dirac statistics.**
- To simplify things, we shall neglect the finite-temperature effects and assume that all states below the **Fermi momentum** p_F are occupied, whereas all states above p_F are unoccupied. Then the **number density n_e of electrons** is given by

$$n_\mathrm{e} = \int_0^{p_\mathrm{F}} \frac{8\pi}{h^3} p^2 dp = \frac{8\pi}{3h^3} p_\mathrm{F}^3.$$

If all states between p and $p + dp$ are occupied, then $8\pi p^2 dp/h^3$ must equal $4\pi f(p) p^2 dp$, implying that $f(p)$ in should be $2/h^3$ if $p < p_F$ and 0 if $p > p_F$. Hence

$$P = \frac{8\pi}{3h^3} \int_0^{p_\mathrm{F}} v\, p^3 dp$$

Degeneracy pressure - Fermi gas

We now use the **relativistic** expression that the **momentum** of a particle is given by $p = m\gamma$ v, where **γ is the Lorentz factor**

$$v = \frac{p}{m\gamma} = \frac{pc^2}{E} = \frac{pc^2}{\sqrt{p^2c^2 + m^2c^4}}.$$

$$P = \frac{8\pi}{3h^3} \int_0^{p_\mathrm{F}} v\, p^3 dp$$

the **pressure due to the degenerate electron gas** is finally given by

$$\boxed{P = \frac{8\pi}{3h^3} \int_0^{p_\mathrm{F}} \frac{p^4c^2}{\sqrt{p^2c^2 + m_\mathrm{e}^2c^4}} dp}$$

Degeneracy pressure - Fermi gas

- Our aim is to **derive an equation of state connecting the pressure and density.**
- **Protons and other heavier nuclei** present in the stellar material **contribute to density**, but not to pressure because they are non-degenerate.
- Let us first find out the relation between the density ρ and the electron number density n_e.
- If X is the hydrogen mass fraction, then the number density of hydrogen atoms (which are ionized and no longer exist in atomic form) is $X\rho/m_H$. These atoms contribute $X\rho/m_H$ electrons per unit volume.
- A helium atom has atomic mass 4 and contributes two electrons, i.e. the number of electrons contributed is 0.5 per atomic mass unit. For heavier atoms also, the number of electrons contributed is usually very close to 0.5 per atomic mass unit. **For helium and atoms heavier than helium, the number of electrons is half the number of nucleons.** In a unit volume of stellar matter, these atoms provide a mass $(1-X)\rho$, which corresponds to $(1-X)\rho/m_H$ nucleons. There are $(1-X)\rho/\ 2m_H$ corresponding electrons. Hence the electron number density is given by

Mean molecular weight

$$n_{\mathrm{e}} = \frac{X\rho}{m_{\mathrm{H}}} + \frac{(1-X)\rho}{2m_{\mathrm{H}}} = \frac{\rho}{2m_{\mathrm{H}}}(1+X).$$

$$n_{\mathrm{e}} = \frac{\rho}{\mu_{\mathrm{e}} m_{\mathrm{H}}}$$

$$\mu_{\mathrm{e}} = \frac{2}{1+X}$$

Degeneracy pressure - Fermi gas

Fermi momentum p_F is given by

$$p_{\mathrm{F}} = \left(\frac{3h^3\rho}{8\pi\mu_{\mathrm{e}}m_{\mathrm{H}}}\right)^{1/3}$$

We want to **solve this for non-relativistic and relativistic case:**

$$P = \frac{8\pi}{3h^3}\int_0^{p_{\mathrm{F}}} \frac{p^4c^2}{\sqrt{p^2c^2 + m_{\mathrm{e}}^2c^4}}dp.$$

When the electrons are **non-relativistic**, we can write:

$$\sqrt{p^2c^2 + m_{\mathrm{e}}^2c^4} \approx m_{\mathrm{e}}c^2$$

$$P = \frac{8\pi}{15h^3m_{\mathrm{e}}}p_{\mathrm{F}}^5.$$

Degeneracy pressure - Fermi gas

$$P = \frac{8\pi}{3h^3} \int_0^{p_F} \frac{p^4 c^2}{\sqrt{p^2c^2 + m_e^2 c^4}} dp.$$

When the electrons are **non-relativistic**, we can write:

$$\boxed{P = K_1 \rho^{5/3}}$$

$$K_1 = \frac{3^{2/3}}{20\pi^{2/3}} \frac{h^2}{m_e m_H^{5/3} \mu_e^{5/3}} = \frac{1.00 \times 10^7}{\mu_e^{5/3}}$$

When the electrons are **fully relativistic**, we can write

$$\sqrt{p^2c^2 + m_e^2 c^4} \approx pc$$

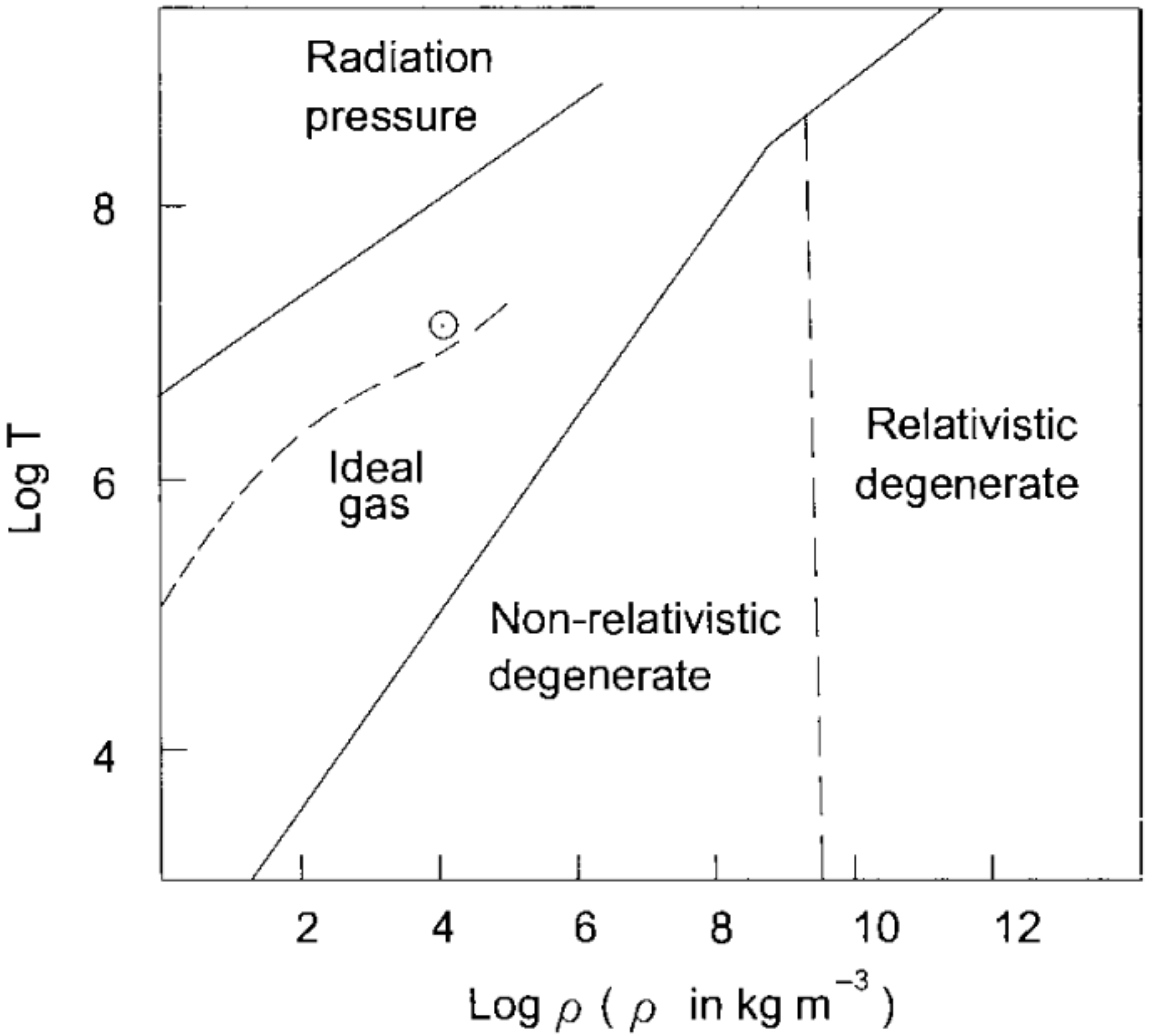
$$P = \frac{2\pi c}{3h^3} p_F^4.$$

$$\boxed{P = K_2 \rho^{4/3}}$$

$$K_2 = \frac{3^{1/3}}{8\pi^{1/3}} \frac{hc}{m_H^{4/3} \mu_e^{4/3}} = \frac{1.24 \times 10^{10}}{\mu_e^{4/3}}$$

Degeneracy pressure - Fermi gas

- **Two extreme limits of the equation of state** of degenerate stellar matter, whereas in normal circumstances we have the ideal gas equation of state.
- **which equation of state should be used when?**
- For a particular combination of ρ and T , one of the expressions would be the most appropriate.
- On a boundary between two such regions in the T versus ρ plot, the two different expressions for pressure valid on the two sides of the boundary should give the same value.
- In the figure radiation refers to the pressure from the blackbody radiation ($(1/3)a_B T^4$).



White dwarfs

- We now want to calculate the **structure of a star entirely made of degenerate matter** (such as a **white dwarf**). The main conservation and hydrostatic pressure equations alone suffice to formulate the problem completely if P is known as a function of ρ alone.
- Out of the three unknown variables ρ, P and M_r appearing in these two equations, one is no longer independent and the other two can be obtained by solving these two equations. The remaining two equations of stellar structure, become redundant.
- We can **combine into one single equation** by eliminating M_r:

$$\boxed{\frac{1}{r^2}\frac{d}{dr}\left(\frac{r^2}{\rho}\frac{dP}{dr}\right) = -4\pi G\rho}$$

$$\boxed{\begin{aligned} \frac{dM_r}{dr} &= 4\pi r^2\rho, \\ \frac{dP}{dr} &= -\frac{GM_r}{r^2}\rho, \end{aligned}}$$

$$\frac{dL_r}{dr} = 4\pi r^2\rho\varepsilon,$$

$$\left.\begin{aligned} \frac{dT}{dr} &= -\frac{3}{4a_{\mathrm{B}}c}\frac{\chi\rho}{T^3}\frac{L_r}{4\pi r^2} \\ \frac{dT}{dr} &= \left(1-\frac{1}{\gamma}\right)\frac{T}{P}\frac{dP}{dr} \end{aligned}\right\}.$$

White dwarfs

$$\boxed{\frac{1}{r^2}\frac{d}{dr}\left(\frac{r^2}{\rho}\frac{dP}{dr}\right) = -4\pi G\rho}$$

Given an equation of state of the form $P(\rho)$, we can easily integrate.

The two limiting equations of state both have the form:

$$P = K\rho^{(1+\frac{1}{n})}$$

with n equal to 3/2 and 3 respectively for the non-relativistic and fully relativistic cases. A **relation between density and pressure** is called a ***polytropic relation*****.** We now write the density inside the star in the form

$$\rho = \rho_c\theta^n,$$

where ρ_C is the density at the centre of the star and θ is a new dimensionless variable which clearly has to have the value 1 at the centre.

$$P = K\rho_c^{\frac{n+1}{n}}\theta^{n+1}.$$

White dwarfs

$$\boxed{\frac{1}{r^2}\frac{d}{dr}\left(\frac{r^2}{\rho}\frac{dP}{dr}\right) = -4\pi G\rho}$$

We also introduce another dimensionless variable ξ (xi) through

$$r = a\xi \qquad a = \left[\frac{(n+1)K\rho_c^{\frac{1-n}{n}}}{4\pi G}\right]^{1/2}$$

$$\boxed{\frac{1}{\xi^2}\frac{d}{d\xi}\left(\xi^2\frac{d\theta}{d\xi}\right) = -\theta^n}$$

known as the ***Lane–Emden equation.*** If the material inside a star satisfies the polytropic relation, the structure of the star can be found by solving the Lane–Emden equation. Since this is a second-order equation, we need two boundary conditions to integrate it:

$$\theta(\xi = 0) = 1. \qquad \left(\frac{d\theta}{d\xi}\right)_{\xi=0} = 0.$$

No cusp in the density in the centre

White dwarfs

$$a = \left[\frac{(n+1)K\rho_c^{\frac{1-n}{n}}}{4\pi G}\right]^{1/2}$$

What does this tell us about the **properties of the star?**

the **physical radius** of the star is given by:

$$R = a\xi_1$$

$$R \propto \rho_c^{\frac{1-n}{2n}}$$

The **mass of the star** is given by

$$M = \int_0^R 4\pi r^2 \rho\, dr = 4\pi a^3 \rho_c \int_0^{\xi_1} \xi^2 \theta^n d\xi$$

$$M \propto \left(\rho_c^{\frac{1-n}{2n}}\right)^3 \rho_c \qquad M \propto \rho_c^{\frac{3-n}{2n}}.$$

White dwarfs

On putting $n = 3/2$ in $\rightarrow$ $R \propto \rho_c^{-1/6}, \quad M \propto \rho_c^{1/2}$

$$\boxed{R \propto M^{-1/3}}$$

This is the mass–radius relation of white dwarfs within which matter satisfies the **non-relativistic equation of state**

It is clear that **white dwarfs of increasing mass are smaller in size.**

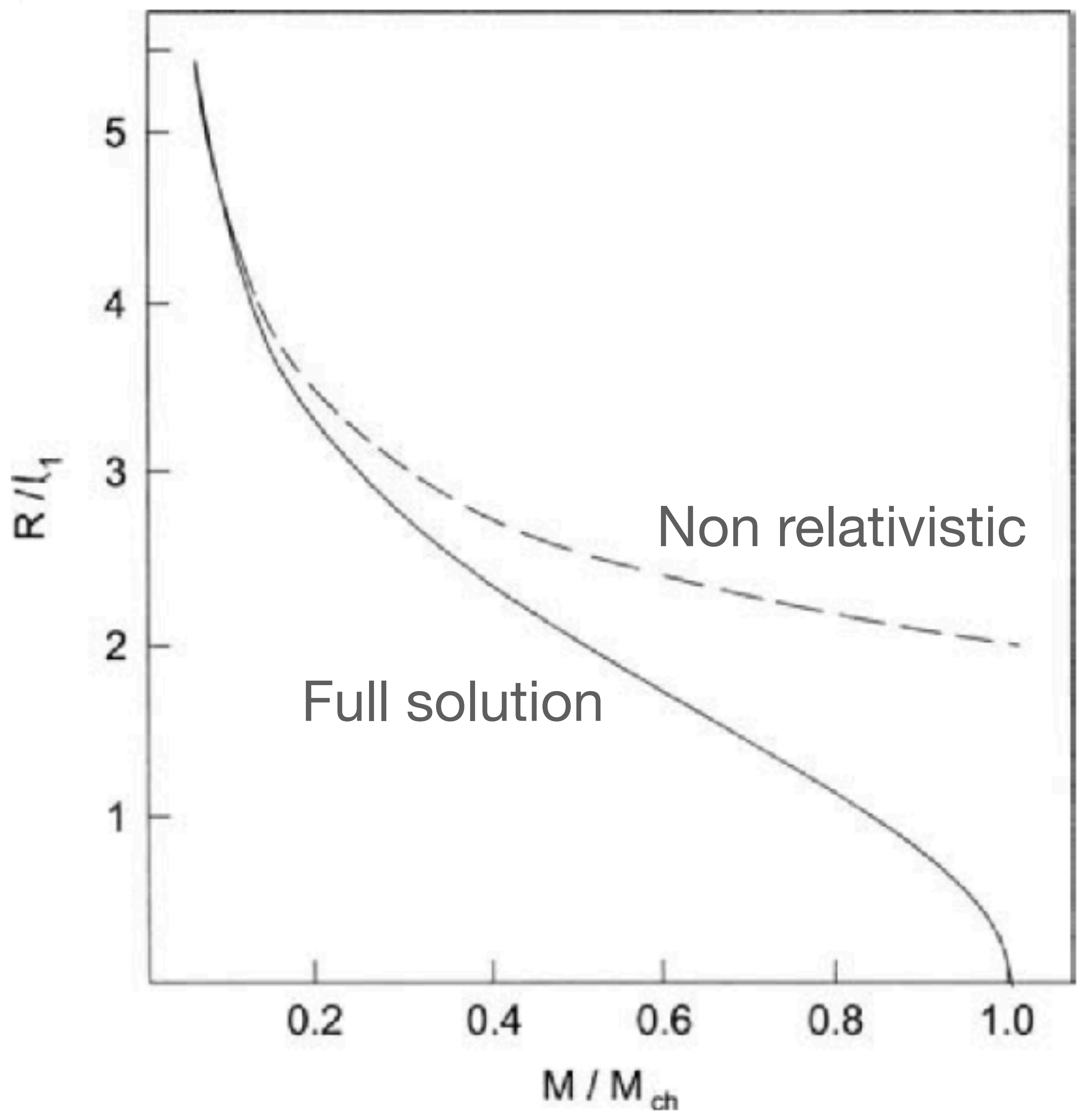


Fig. 5.2 The variation of radius with mass for white dwarfs. The solid curve corresponds to the full solution, where the dashed curve is obtained by using the non-relativistic equation of state (5.9). This figure is adapted from Chandrasekhar (1984), where the unit of radius l_1 used on the vertical axis is defined.

White dwarfs

Consider the case of the **relativistic equation of state** on taking $n = 3$. A very surprising result is that the **mass M becomes independent of** ρ_c on substituting $n = 3$

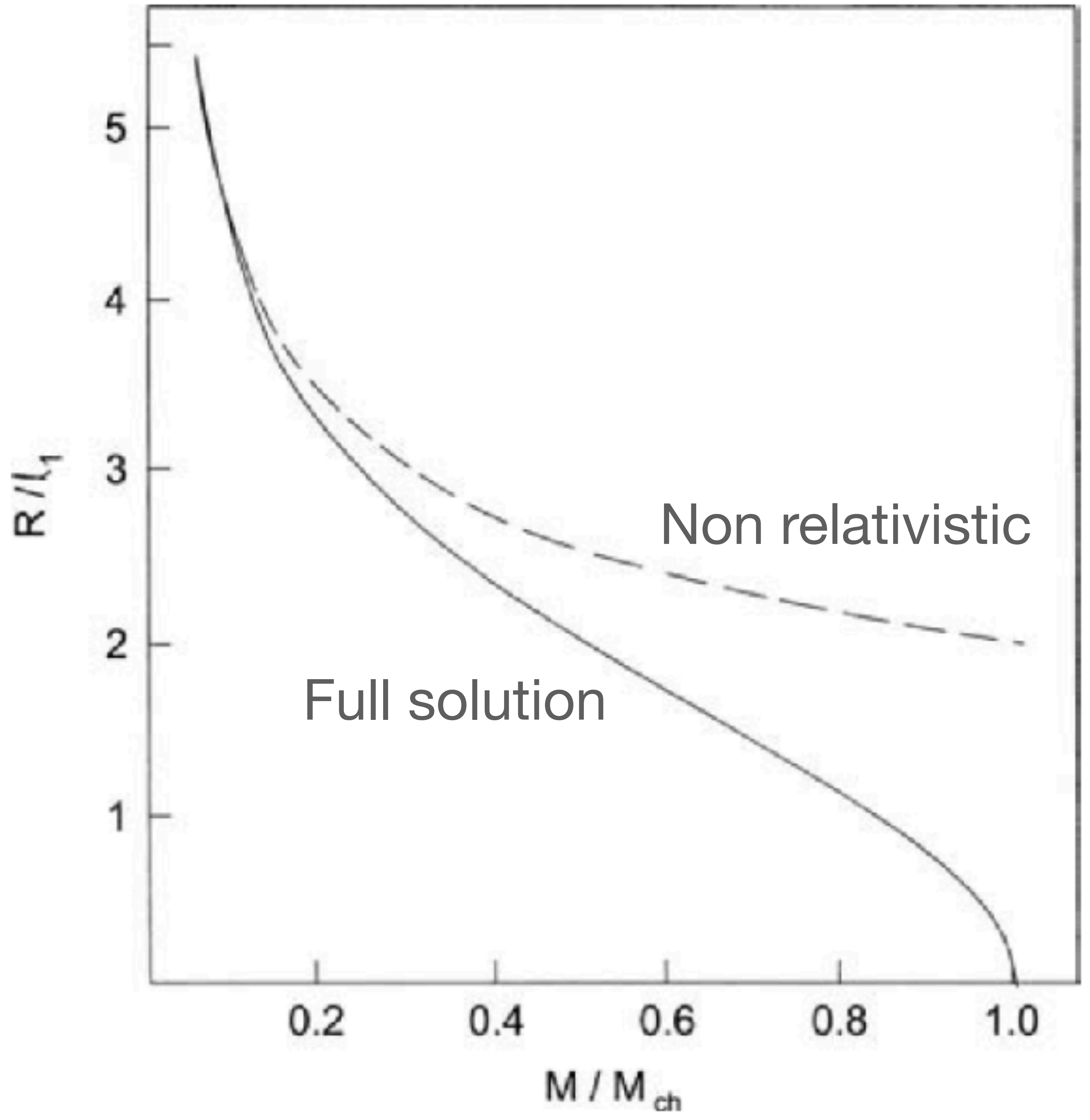
Chandrasekhar mass limit

$$M_{\mathrm{Ch}} = \frac{\sqrt{6}}{32\pi}\left(\frac{hc}{G}\right)^{3/2}\left(\frac{2}{\mu_{\mathrm{e}}}\right)^2 \frac{\xi_1^2|\theta'(\xi_1)|}{m_{\mathrm{H}}^2} \longrightarrow M_{\mathrm{Ch}} = 1.46\left(\frac{2}{\mu_{\mathrm{e}}}\right)^2 M_\odot.$$

We have come to the surprising conclusion that **only this fixed value of mass is possible if the stellar material satisfies the relativistic equation of state**.

White dwarfs

- **Using the full equation of state**, instead of considering the nonrelativistic and fully relativistic limits, one can find out the **variation of radius with mass.**
- The solid curve in Figure 5.2 indicates the results we get on using the full equation of state.
- **For white dwarfs of smaller masses (which also have larger sizes), the interior density is not so high and the non-relativistic limit of the equation of state holds**.
- For **increasing masses** and larger interior densities, the Fermi momentum p_F starts becoming larger. When $p_F c \approx m_e c^2$, the **relativistic effects become important** and the dashed curve deviates from the solid curve.



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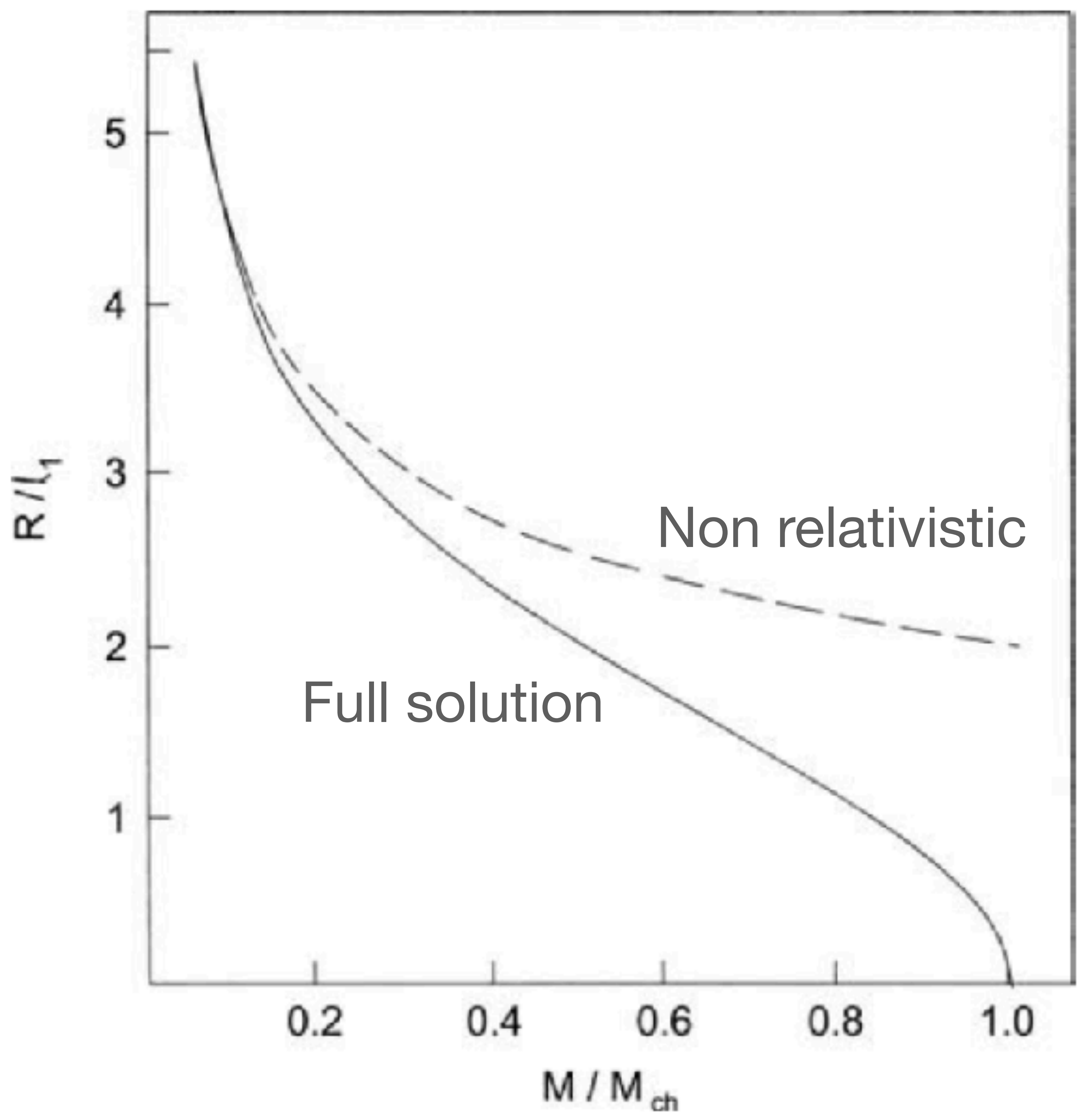
White dwarfs

- The **relativistic effects make the equation of state** 'less stiff' or **'softer'**, i.e. **the pressure does not rise with density as rapidly** as in the non-relativistic case.
- This is basically due to the fact that the speeds of particles saturate at c and the pressure, which results from the random motions of particles, cannot increase with density as rapidly as it was increasing before the saturation.
- Matter with a softer equation of state is **less efficient in counteracting gravity**. As a result, we find that the solid curve is below the dashed curve, which implies that the radius of a white dwarf of given mass is less when the complete equation of state (which is softer than the non-relativistic one) is used.



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White dwarfs

- Eventually, as we move towards the right side of the figure, the radius becomes too small and the interior density becomes too high so that the relativistic limit of the equation of state is approached. The mass M_{Ch} **corresponding to the relativistic limit of the equation of state is the limiting mass for which the radius goes to zero.** This is the ***Chandrasekhar mass limit***. It is not possible for white dwarfs to have larger masses.
- **White dwarfs usually form from the cores of stars** in which hydrogen has been **completely** burnt out to produce **helium** (and higher elements in some circumstances). If the hydrogen mass fraction $X \approx 0$, then it follows that $\mu_e \approx 2$. Hence the Chandrasekhar mass limit should be **around 1.4$M_{\odot}$**.
- It is seen in Figure that the equation of state starts becoming relativistic when the density is of order 10^9 kg m^{-3}. This can be taken as the typical density inside a white dwarf.
- If the mass is of order 10^{30}kg, then the **radius has to be about 10^7m ≈ 10^4km**. This is indeed the typical size of a white dwarf. (Earth's diameter is: 12,742 km, the Sun's diameter is: 1.4×10^6 km)

White dwarfs

NGC 2440 is a planetary nebula, one of many in our galaxy.
Its central star, HD 62166, is possibly **the hottest known white dwarf,** about 200,000°C.

Neutron stars

- Just as the degeneracy pressure of electrons supports a white dwarf against gravity, the **degeneracy pressure of neutrons supports a neutron star.**
- Unlike protons, neutrons are electrically neutral and hence many neutrons can be brought together without being disrupted by electrostatic repulsion. However, **neutrons are known to decay** according to the reaction

$$n \rightarrow p + e + \bar{\nu}$$

- with a **half-life of about 13 minutes**. A reverse reaction is also in principle possible:

$$p + e \rightarrow n + \nu.$$

- Since the neutron mass is more than the combined mass of a proton and an electron, the reaction can take place **only if some energy is supplied** to make up for this mass deficit. Therefore, under ordinary laboratory circumstances, is an unlikely reaction and free neutrons decay away.

Neutron stars

- When matter is compressed to very high densities, things change drastically. For simplicity, let us assume that the highly compressed matter consists of electrons, protons and neutrons (i.e. we do not include the possibility that nuclei form).
- The **electrons become degenerate** with the rise of density while the other heavier particles still remain non-degenerate. Suppose we want to put an additional electron in a region of high density. We know that all the **levels are filled up to the Fermi momentum** p_F, which is **related to the number density** n_e **of electrons**

$$E_{\mathrm{F}} = \sqrt{p_{\mathrm{F}}^2 c^2 + m_{\mathrm{e}}^2 c^4}$$

- E_F is the **Fermi energy associated with this Fermi momentum** p_F.
- Unless an energy $E_F - m_e c^2$ is added to an electron, it is not possible to put the electron in the region of high density, since all the lower energy states are filled.
- Consider the situation when this excess energy required becomes equal to or larger than $(m_n - m_p - m_e)c^2$, the amount by which the neutron mass exceeds the sum of the proton mass and the electron mass. In this situation, it will be **energetically favourable for the electron to combine with a proton to produce a neutron.**

Neutron stars

The condition is:

$$\sqrt{p_{\mathrm{F,c}}^2 c^2 + m_{\mathrm{e}}^2 c^4} - m_{\mathrm{e}} c^2 = (m_{\mathrm{n}} - m_{\mathrm{p}} - m_{\mathrm{e}}) c^2,$$

where $p_{F,c}$ **is the critical Fermi momentum.** From this

$$m_{\mathrm{e}} c^2 \left(1 + \frac{p_{\mathrm{F,c}}^2}{m_{\mathrm{e}}^2 c^2}\right)^{1/2} = Qc^2,$$

where Q = $m_n - m_p$. We can also **express the critical Fermi momentum** from this:

$$p_{\mathrm{F,c}} = m_{\mathrm{e}} c \left[\left(\frac{Q}{m_{\mathrm{e}}}\right)^2 - 1\right]^{1/2}$$

Neutron stars

- Since the **Fermi momentum increases with density**, we expect the Fermi momentum to be less than $p_{F,c}$ **when the density is below a critical density**. In this situation, **free electrons are energetically favoured** and we do not expect any neutrons to be present.
- The **critical density, at which the Fermi momentum becomes equal to** $p_{F,c}$, can be obtained by putting the values of fundamental constants into the equation to get $p_{F,c}$, then obtaining n_e and multiplying n_e by $m_p + m_e$. This gives:

$$\rho_c = 1.2 \times 10^{10} \text{ kg m}^{-3}$$

- **When the density is made higher than this, the electrons start combining with protons** to give neutrons. This phenomenon is called the ***neutron drip*.**
- At densities well above the critical density, matter would mainly consist of neutrons. **These neutrons do not decay, since there are no free states for the product electron** to occupy.

Neutron stars

- This simplified calculation of neutron drip without considering the possible formation of nuclei.
- When the existence of nuclei is taken into account, the calculation becomes much harder.
- On making various reasonable assumptions, the more realistic value of the critical density for neutron drip is found to be **3.2×10^{14} kg m^{-3}**. Strictly speaking, the term **'neutron drip' refers to neutrons getting out of nuclei when the density is raised above the critical density.**
- If a stellar core is compressed by some means to densities higher than what is needed for the neutron drip, the core will essentially consist of neutrons.

Neutron stars - mass

- Since neutrons are **Fermi particles** like electrons and obey the Pauli exclusion principle, **neutrons also can give rise to a degeneracy pressure**.
- While deriving the degeneracy pressure due to electrons we had used the Fermi–Dirac statistics, which tacitly assumes that the particles are non-interacting. This is not that bad an assumption for the electron gas inside a white dwarf. However, **when neutrons are packed to densities close to the density inside an atomic nucleus, the neighbouring neutrons interact with each other through nuclear forces and it is no longer justified to treat them as non-interacting particles**.
- Hence finding an **accurate equation of state for matter at such high densities is very difficult** and the subject of ongoing research.
- Like the Chandrasekhar limit of white dwarfs, **neutron stars also have a mass limit**. However, this mass limit is not known very accurately due to the uncertainty in our knowledge of the equation of state. One can get an **absolute theoretical limit of 3.2 $M_\odot$**, it is generally believed that the actual mass limit is somewhat less than this and **most likely around 2 $M_\odot$.**

Neutron stars - radius

- Calculations suggest that a neutron star typically has a radius of order 10 km and internal density close to 10^{18}kg m^{-3}. -> **star of mass $M_\odot$ and radius 10 km** (keep in mind that this mass refers to the **leftover of the core after the supernova explosion** and not the original mass of the star)
- Neutron stars remained a theorist's curiosity for many years. Baade and Zwicky (1934) made a remarkable suggestion that a **neutron star may form in a supernova explosion.**
- When a star of mass $M_\odot$ collapses to a radius of 10 km, the gravitational potential energy lost is of order 10^{46}J, which is tantalizingly close to the energy output of a supernova. If the gravitational energy lost in the collapse of the inner core to form a neutron star is somehow dumped into the outer layers of the star, then the outer layers can explode with this energy. Nobody took this idea seriously until a dramatic **confirmation** of this idea came in the late **1960s**.

Neutron stars - pulsars

A definitive **observational confirmation** for the existence of neutron stars came when Hewish *et al. and Jocelyn Bell* (1968) discovered radio sources which were giving out radio pulses at intervals of typically a second. This signal came from an object called a **pulsar.**

Aliens??

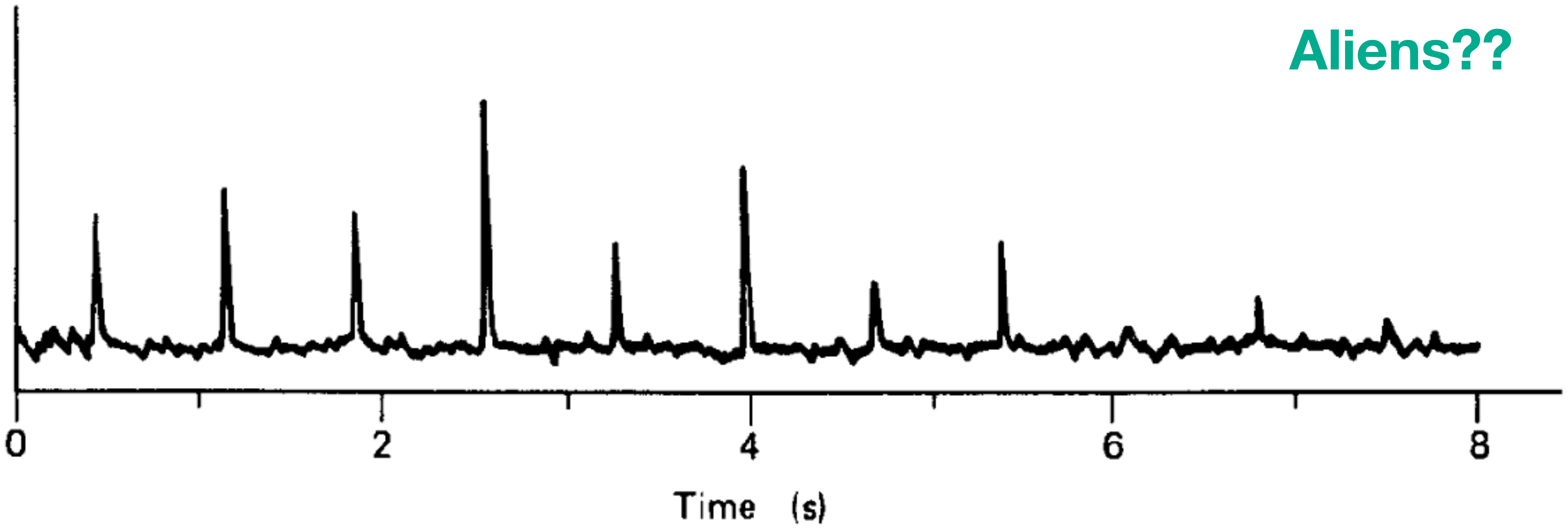


Fig. 5.3 Radio signals from the pulsar PSR 0329 + 54, which has a period of 0.714 s. Note that different pulses are not identical and some pulses are even missing.

Neutron stars - pulsars

- Soon after the discovery, the pulsars were identified as **rotating neutron stars**.
- The pulse period must be due to some physical mechanism like rotation or oscillation.
- Theoretical estimates of oscillation periods of white dwarfs or neutron stars show that they do not match the observed pulsar periods (oscillation periods of normal stars are much longer).
- If the pulsar period has to be identified with the rotation period of some object, one has to make sure that the **centrifugal force is not stronger than gravity**, i.e.

$$\Omega^2 r < \frac{GM}{r^2}, \longrightarrow \Omega < (G\rho)^{1/2}$$

- A **rotation period of 1 s** demands that the rotating object should have a density higher than 10^{11} kg m^{-3} if it is not to be disrupted by the centrifugal force.
- The pulsars with shortest periods could not be rotating white dwarfs (which have densities of order 10^9 kg m^{-3}). The **only possibility is that the pulsars are rotating neutron stars.**

Neutron stars - pulsars

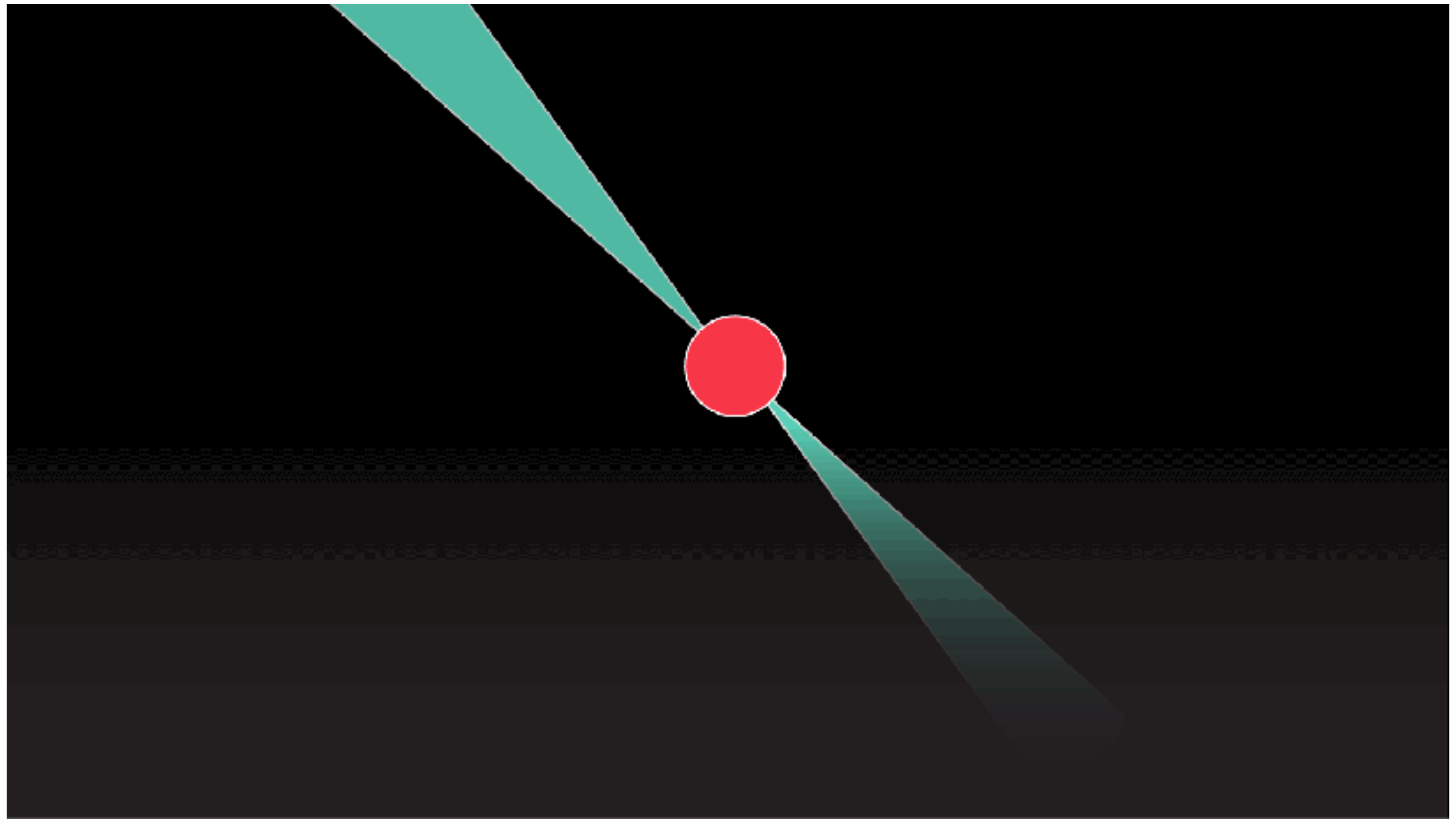
- When pulsars were found near the centres of Crab and Vela supernova remnants, the idea of neutron stars being born in supernova explosions got dramatic support.
- However, only a few clear pulsar and supernova remnant associations are known. Most of these cases are for supernova remnants which are not very old (less than 10^5 yr).
- One possibility is that many of the supernova explosions may be somewhat asymmetric and the neutron stars may be born with a net momentum. So **they move away from the centres of the supernova remnants and are found associated with the remnants only if not too much time elapsed since the explosion**.
- The other possibilities are: **many supernovae may not produce neutron stars (could produce black holes)**
- **or the neutron stars may not be visible to us as pulsars.**
    - To detect a pulsar the jet needs to point towards Earth.

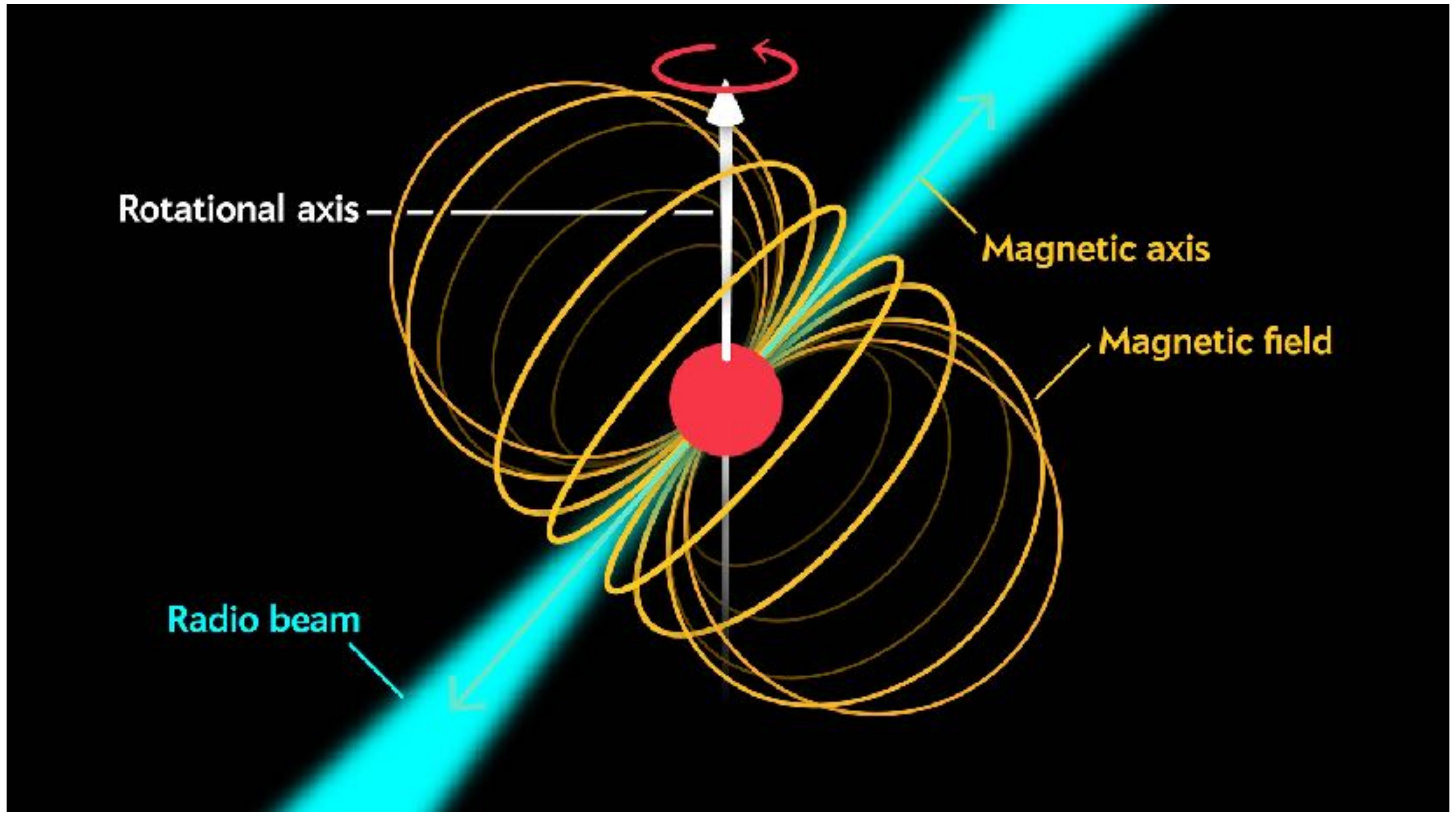
The Crab pulsar in X-rays

Composite image Crab Nebula (infrared) and the Crab pulsar (X-ray - blue colour)

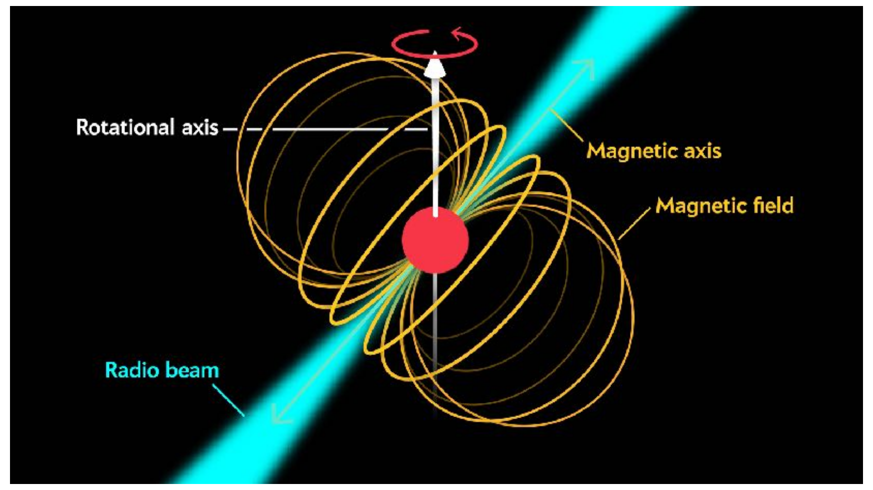
Neutron stars - pulsars

- **Why do rotating neutron stars become visible as pulsars?**
- The radio emission is produced at the magnetic poles of the neutron star by complicated plasma processes.
- Very often the **magnetic axis is inclined with respect to the rotation axis**.
- When the **magnetic pole gets turned towards the observer** during a rotation period, the observer receives the radio pulse. The duty cycle of a typical pulsar (i.e. the fraction of time during which the radio signal is received) is less than 10%.



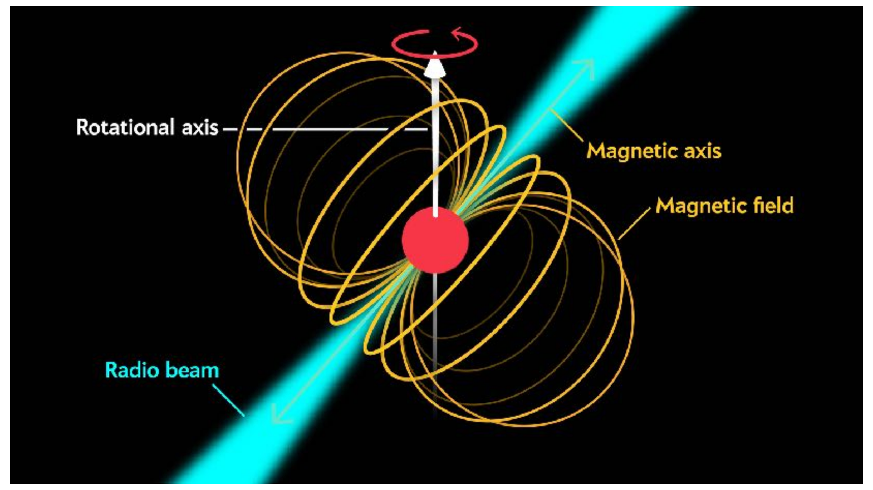


Neutron stars - pulsars



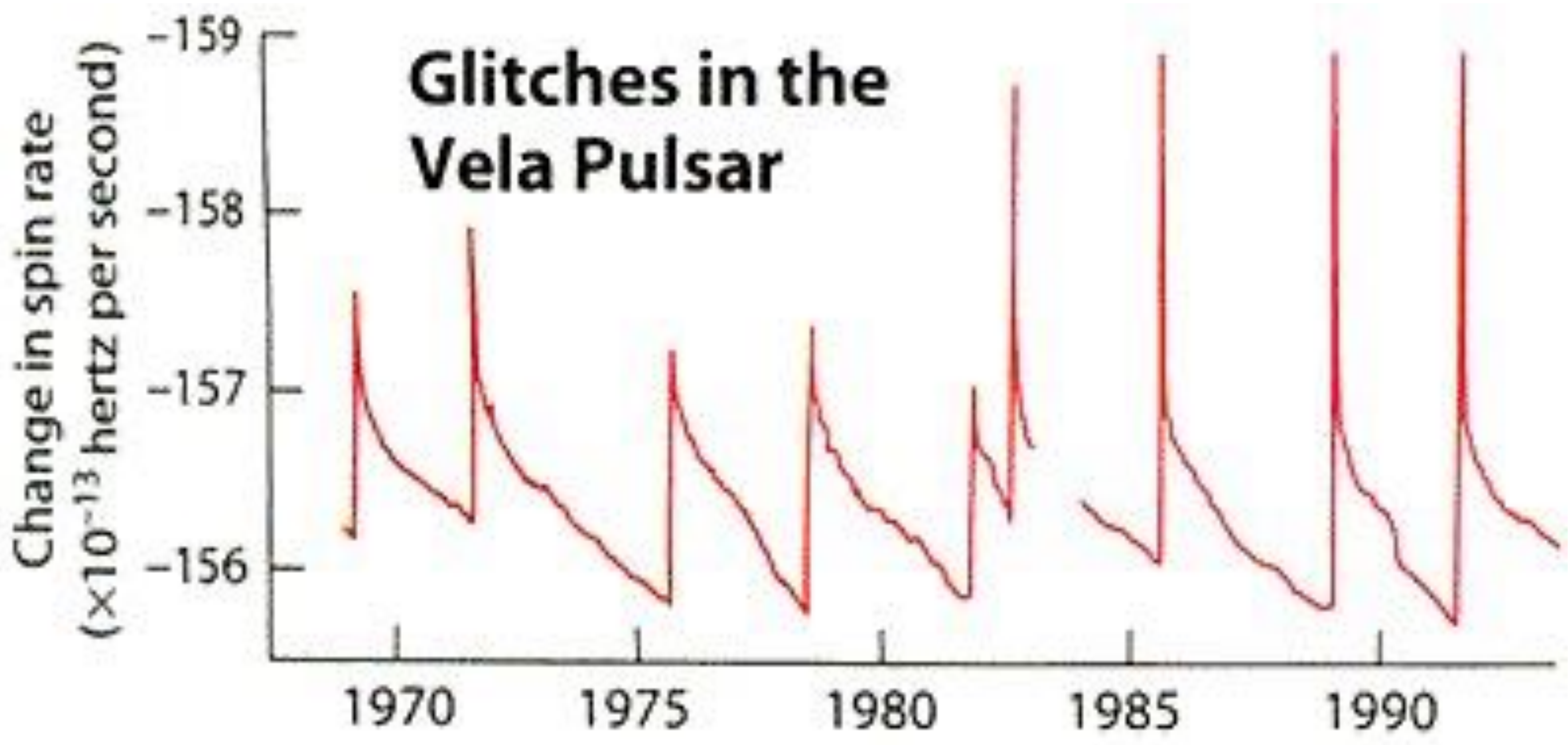
- **From where does the pulsar get the energy which is radiated away?**
- The **rotational kinetic energy** of the neutron star is believed to be the ultimate source of energy.
- As this energy source is tapped, the neutron star **rotation slows down**.
- The periods of all pulsars keep on increasing very slowly as a result of this.
- The typical **period increase rate is $\dot{P} \approx 10^{-15}$s s^{-1}. This gives the pulsar lifetime $P/\dot{P}$**, which is of order 10^7 yr.
- After a neutron star has existed as a pulsar for time of the order of 10^7 yr, presumably its rotation becomes so slow that it can no longer act as a pulsar.

Neutron stars - pulsars

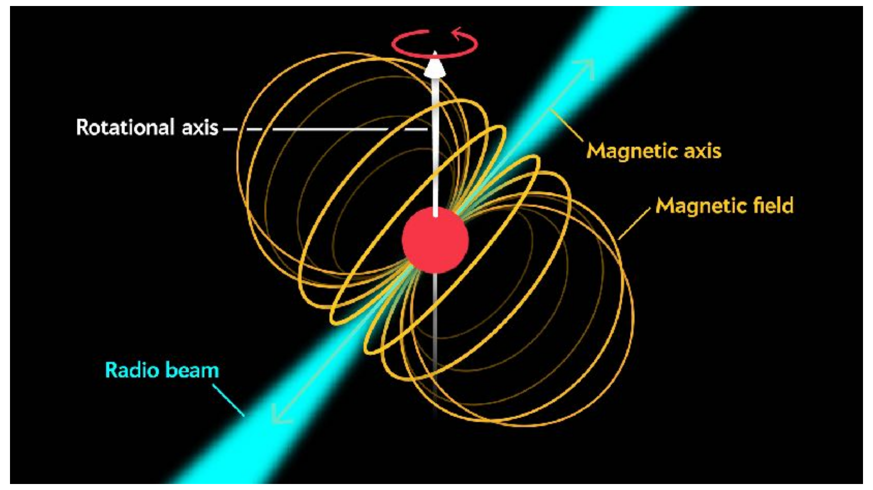


- A **rapidly rotating** object like a pulsar is expected to be somewhat **flattened near the poles**. As the **rotation slows down**, the pulsar tries to take up a **more spherical shape**.
- Since the **crust of a neutron star is believed to be solid**, the **shape of the neutron star cannot change continuously.**
- When sufficient stress builds up due to the slowing down of the neutron star, the **crust suddenly breaks** and the neutron star is able to take up a less flattened shape, causing a decrease in the moment of inertia because more material is brought near the rotation axis.

- When this happens, the moment of inertia changes abruptly and the angular velocity increases suddenly to conserve the angular momentum, leading to a decrease in pulsar period.
- Such **sudden decreases of pulsar periods have been observed and are known as *glitches*.**



Neutron stars - pulsars



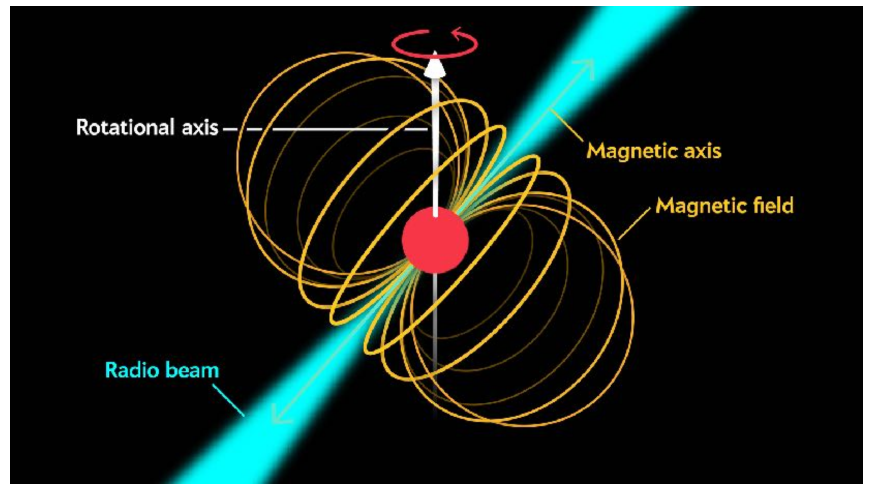
A simple way of modelling the emission from a pulsar is to treat it as a **rotating magnetic dipole**. If the magnetic field of the pulsar is of dipole nature, then the magnetic field at the pole is given by

$$B_{\mathrm{p}} = \frac{\mu_0 |\mathbf{m}|}{2\pi R^3},$$

where R is the radius of the neutron star. Writing $2\pi B_{\mathrm{p}} R^3/\mu_0$ for $|\mathbf{m}|$, we get

$$\dot{E} = -\frac{2\pi B_{\mathrm{p}}^2 R^6 \Omega^4 \sin^2\alpha}{3\mu_0 c^3}$$

Neutron stars - pulsars



If this energy comes from the rotational kinetic energy $\frac{1}{2}I\Omega^2$ (where I is the moment of inertia), then we have

$$I\dot{\Omega} = -\frac{2\pi B_{\mathrm{p}}^2 R^6 \Omega^3 \sin^2\alpha}{3\mu_0 c^3}.$$

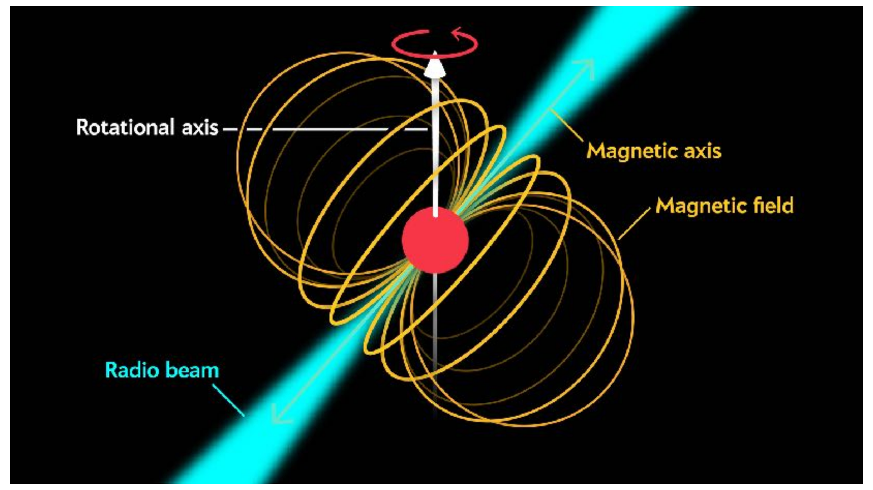
Where α is the inclination of the magnetic dipole to the rotation axis. Once Ω and $\dot{\Omega}$ of a pulsar have been determined, one can use (5.35) to obtain the pulsar magnetic field B_{p} by putting reasonable values of I and R. For the Crab pulsar, this yields

$$B_{\mathrm{p}} \approx 5 \times 10^8 \text{ T},$$

if we take $\sin\alpha \approx 1$. The **magnetic fields of pulsars are the strongest magnetic fields known.**

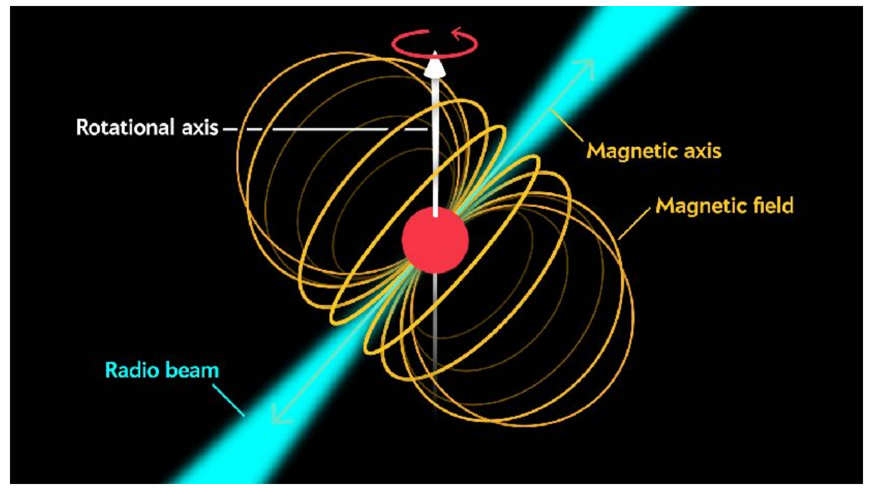
More details on pulsar properties: https://www.cv.nrao.edu/~sransom/web/Ch6.html

Binary pulsars



- We now discuss a very intriguing object which was first discovered in 1975. A pulsar with a mean period of 0.059 s.
- However, the actual value of the period was found to vary above and below this mean value periodically, with a period of about 8 hours.
- The most obvious explanation is that the **pulsar is orbiting around an unseen companion** and the variation in the pulsar period is due to the Doppler effect.
- One can **determine the masses of both** the pulsar and the unseen binary companion **by analysing the various orbit parameters.**
- Both the masses are found to be close to $1.4 M_\odot$.
- The unseen companion seems to have exactly the mass -> the **unseen companion is very likely to be another neutron star.**

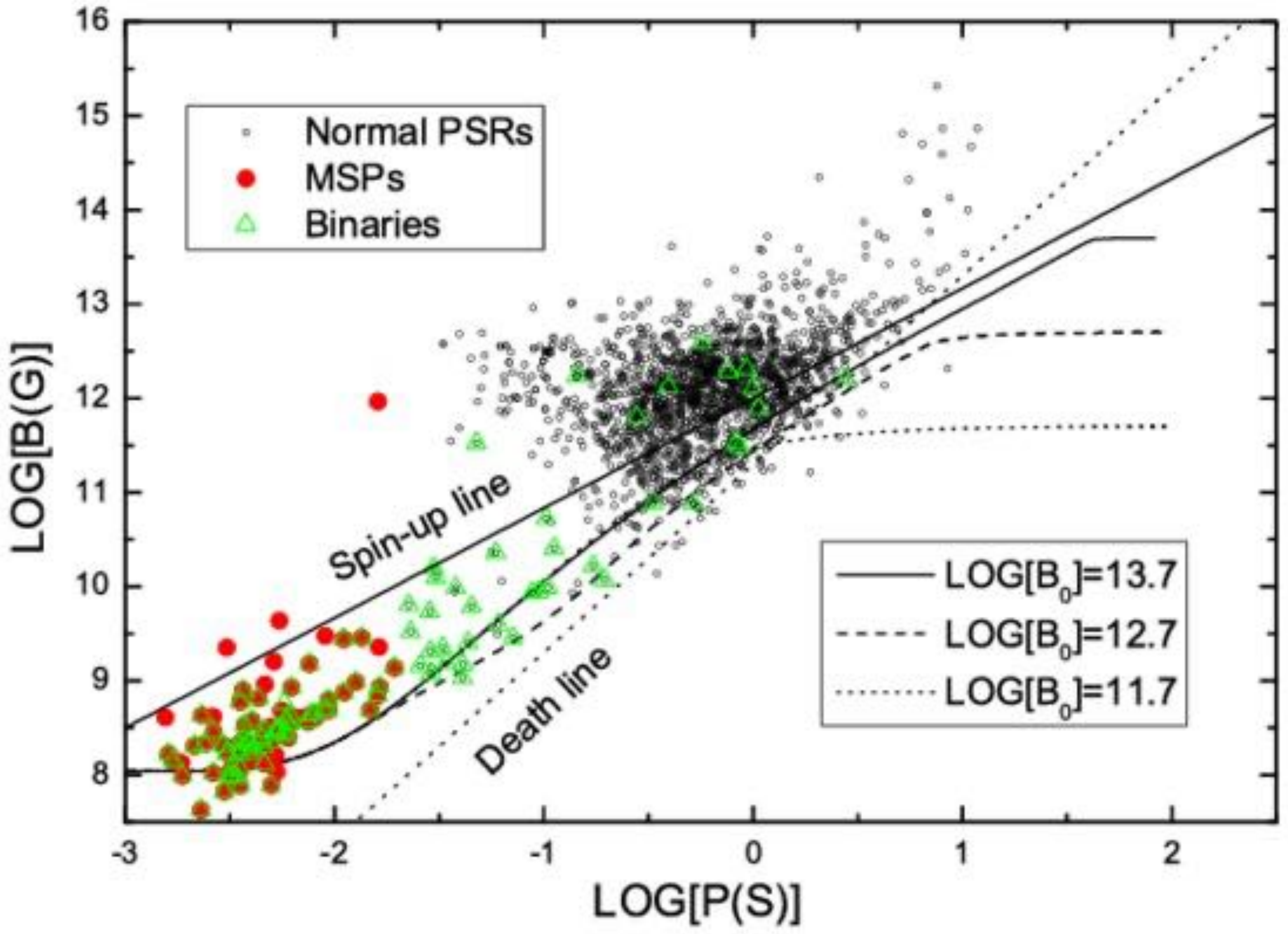
Binary pulsars - gravitational waves



- A system in which two neutron stars are orbiting around each other, one of them acting as a pulsar.
- **According to general relativity, such an object would emit gravitational radiation,** just as an orbiting charge would emit electromagnetic radiation according to classical electrodynamics.
- As the system loses energy in the gravitational radiation, the two neutron stars should come closer and **the orbital period should decrease**.
- General relativistic calculations suggest a value $\dot{P}_{orb} = -2.40 \times 10^{-12}$ for the orbital period change. The measured value $(-2.30 \pm 0.22) \times 10^{-12}$ is in very good agreement.
- This provides a test of general relativity to a high degree of precision and provides an **indirect confirmation of the existence of gravitational waves.**
  - Now we also have **direct detections of gravitational waves from LIGO.**

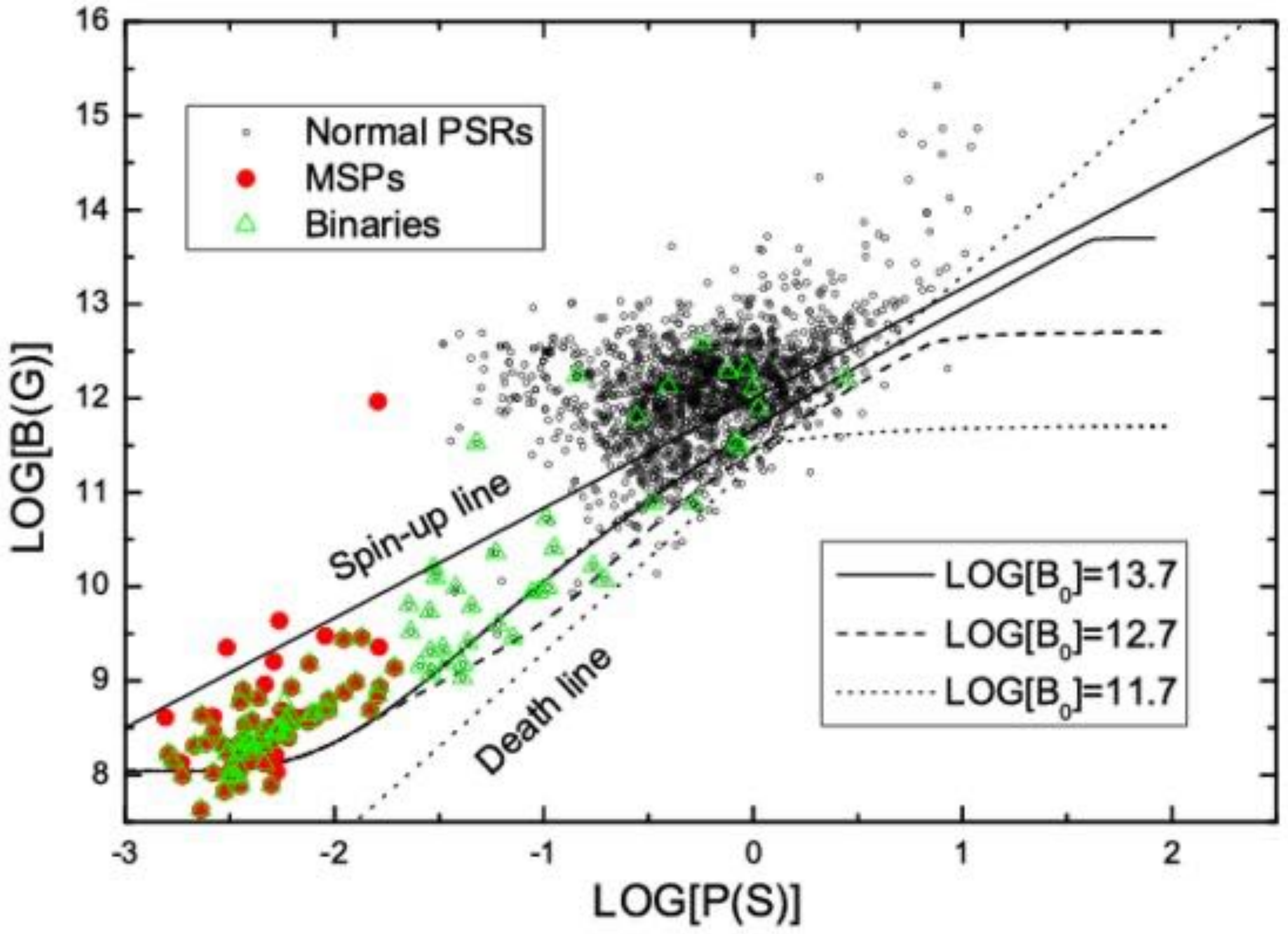
Millisecond pulsars

- There are several pulsars with ***periods less than 10ms.*** (Very short period)
- The Crab pulsar, had a period of 33.1ms.
- One striking feature is that a **majority of them were found in binary systems.**
- After measuring the period variation $\dot{P}$ of these millisecond pulsars, their magnetic fields could be estimated. Most of the millisecond pulsars were found to have magnetic fields around 10^4 T, considerably less than the typical magnetic fields of ordinary pulsars (around 10^8 T).
- Figure 5.5 is a plot of **magnetic field B against pulsar period of P.**

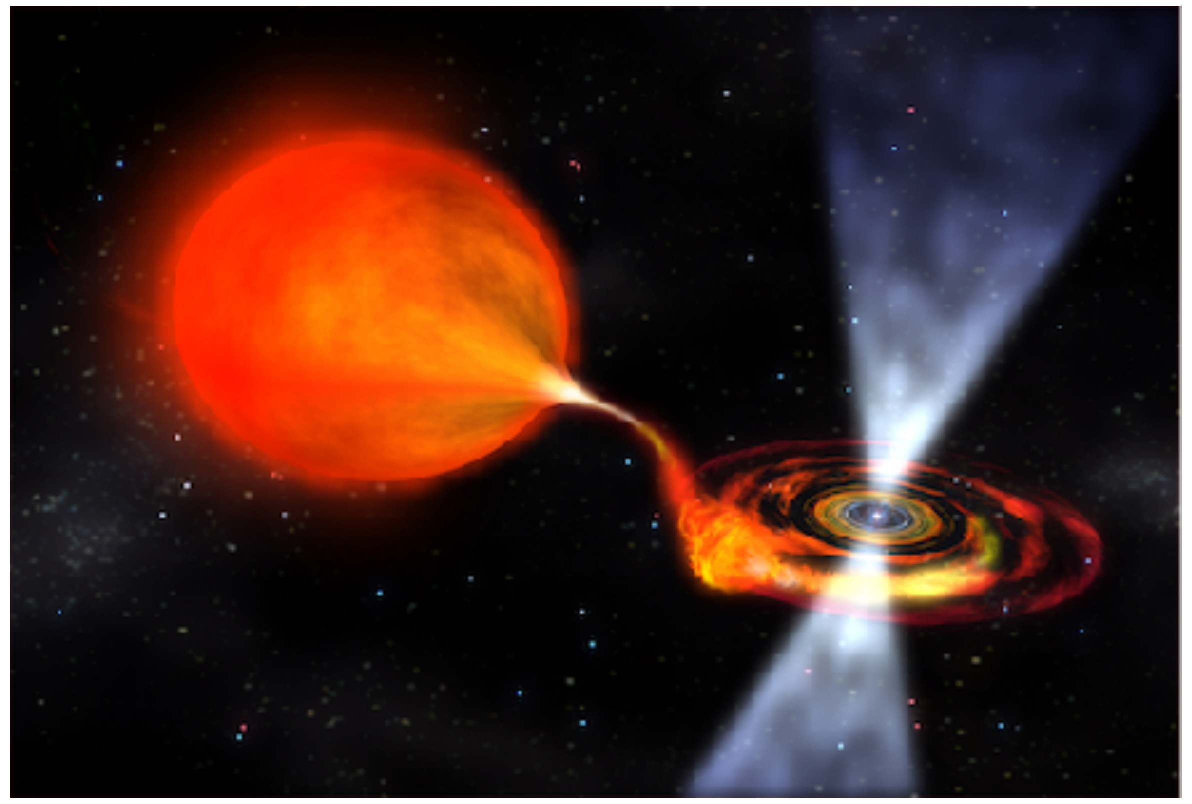


Millisecond pulsars

- The ordinary pulsars are towards the upper right part of the figure,
- the millisecond pulsars are towards the lower left.
    - two **very distinct population** groups.
- If a neutron star is rotating too slowly or has a too weak magnetic field, then presumably it would not act as a pulsar, marked by the ***death line***.
- As a **pulsar becomes olde**r, its period becomes longer and it follows a trajectory moving towards the right.
- Eventually it **crosses the death line** and is **no longer visible as a pulsar.**



Millisecond pulsars

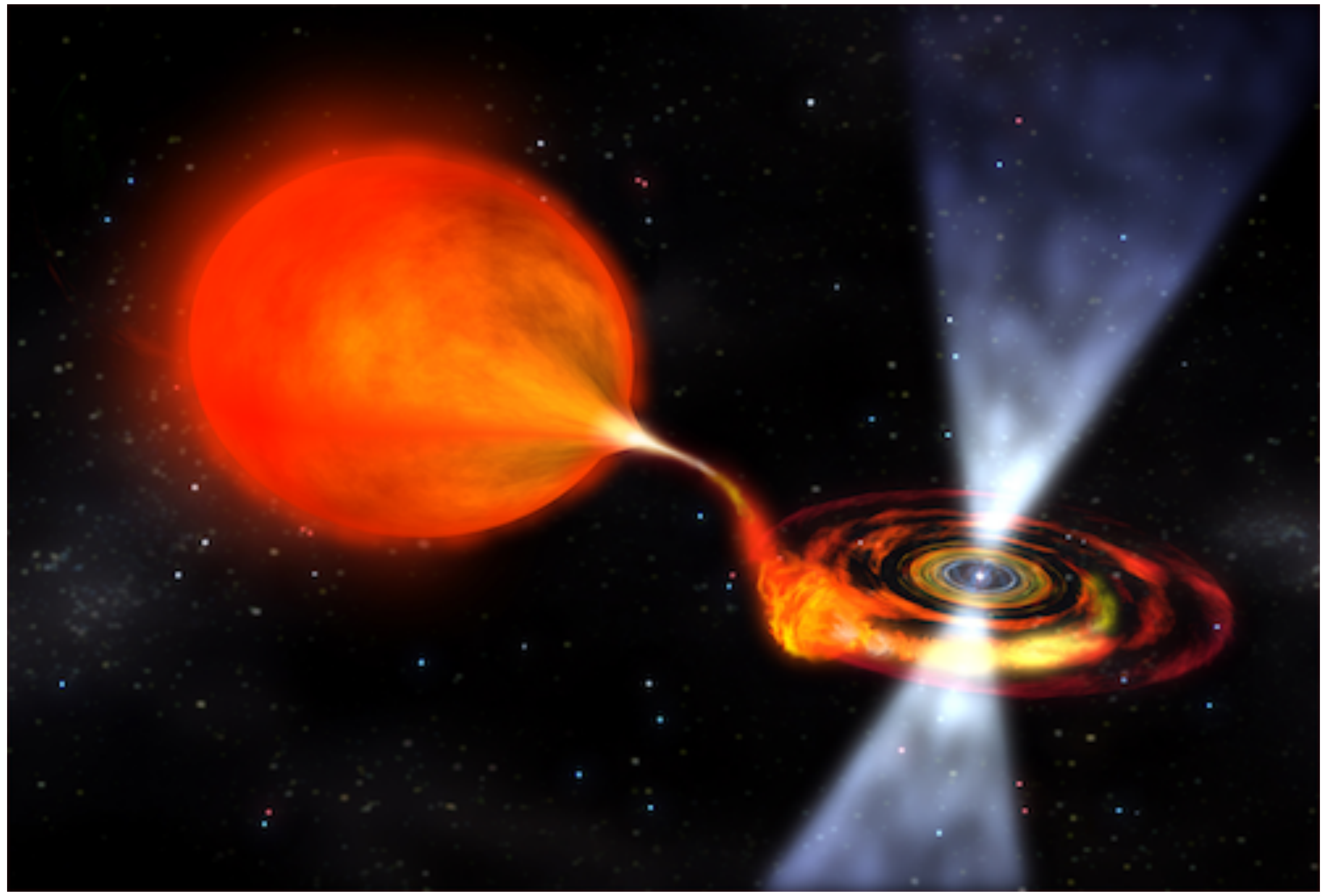


- What is the relation of **millisecond pulsars** with ordinary pulsars?
- Millisecond pulsars are usually found in binary systems.
- When a neutron star is born, it is expected to have values of rotation period P and magnetic field B typical of an ordinary pulsar.
- Suppose the neutron star is in a **binary system**. At some stage, the binary companion may become a red giant and fill up the Roche lobe. This would lead to a **transfer of mass** from the inflated companion star to the neutron star.
- There are **binary X-ray sources** are believed to be **neutron stars accreting matter** from inflated binary companions.
- Because of the orbital motion of the companion, the matter accreting onto the neutron star from its companion will carry a considerable amount of angular momentum. This is **expected to increase the angular velocity of the accreting neutron star.**
- Eventually, when the red giant phase of the companion star is over (it may become a white dwarf or another neutron star), **the neutron star which has been spun up** by accreting matter with angular momentum **becomes visible as a millisecond pulsar** with a short period P.

Binary X-ray sources

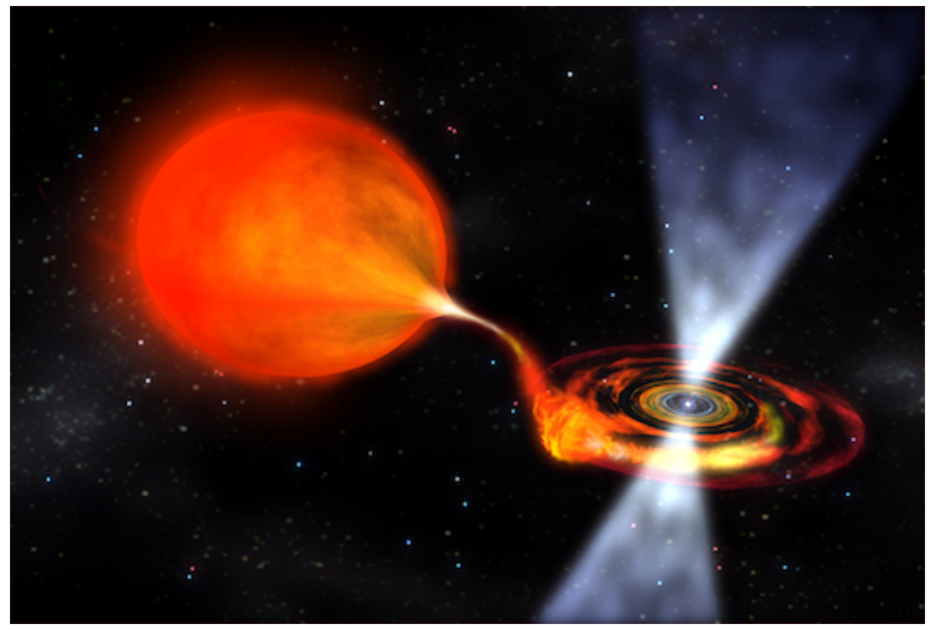
- Many X-ray sources were found in the galactic plane, indicating that they are galactic objects.
- The optical counterparts were invariably binary stellar systems. Something must be happening in these binaries to produce the X-rays.
- There can be mass transfer between the two stars in a binary system.
- If, the mass m is dropped from infinity to a star of mass M and radius R, then the gravitational energy lost is:

$$\frac{GM}{R}m = \frac{GM}{c^2 R}mc^2.$$



- For a typical neutron star of mass $1M_\odot$ and radius 10 km, the factor $GM/c^2 R$ turns out to be about 0.15. Hence the loss of gravitational energy may be a very significant fraction of the rest mass energy, making such an **infall of matter into the deep gravitational well** of a compact object like a neutron star a **very efficient process for energy release.**

Binary X-ray sources



- Suppose one member of a binary is a compact object like a neutron star or a black hole, whereas the other member is a star which has filled up the Roche lobe.
- Then the **compact star will accrete matter from its companion**. The accreted matter loses a large amount of gravitational potential energy while falling towards the compact star and this energy presumably is radiated away.
- This seems to be the likely mechanism by which most of the X-ray sources are powered.
- Millisecond pulsars are believed to be neutron stars spun up by the deposition of angular momentum in a binary mass transfer process. T
- the X-ray binary sources are basically such systems caught in the act of such mass transfer. A millisecond pulsar is a possible end product after the mass transfer is over.
- Since the **accreting material carries angular momentum, it is unlikely to fall radially inward, but is expected to move inward slowly in the form of a disk**. Such a disk is called an ***accretion disk***.

Binary X-ray sources

- **Do all X-ray binaries have neutron stars?**
- Figure shows the masses of several neutron stars which could be determined with reasonable accuracy. All the masses are presumably below the upper mass limit of neutron stars.
- However, **there are a few binary X- ray sources with accreting objects which possibly have masses higher than 3$M_\odot$**. E.g. Vela X-1.
- The **central accreting object is believed to be a black hole** rather than a neutron star, since its estimated mass is well above what would be the neutron star mass limit based on any reasonable equation of state.

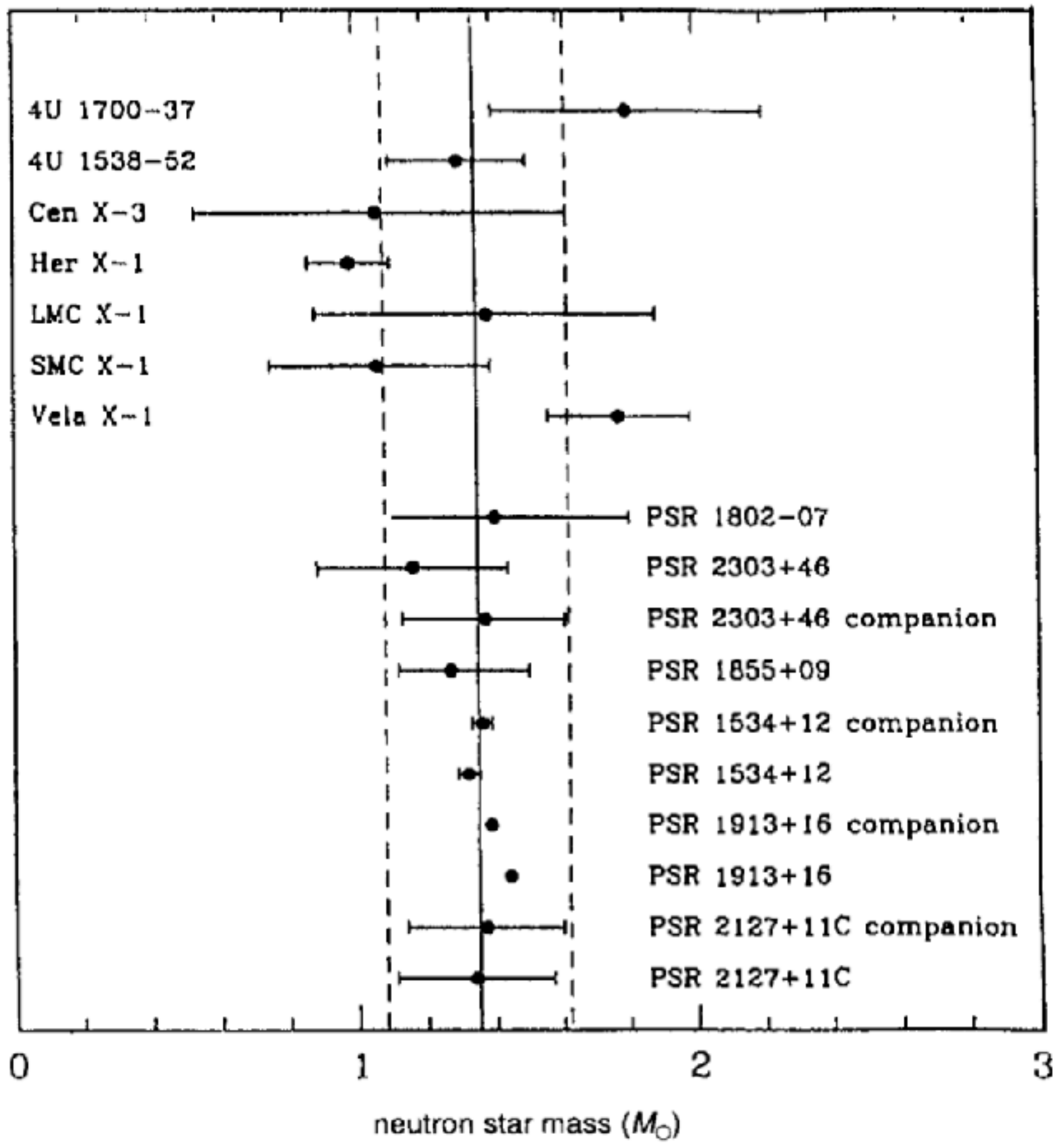


Fig. 5.7 Mass estimates of neutron stars in binary X-ray systems and in binary pulsars. From Longair (1994, p. 114) who credits J. Taylor for the figure. (©Cambridge University Press.)

Binary X-ray sources

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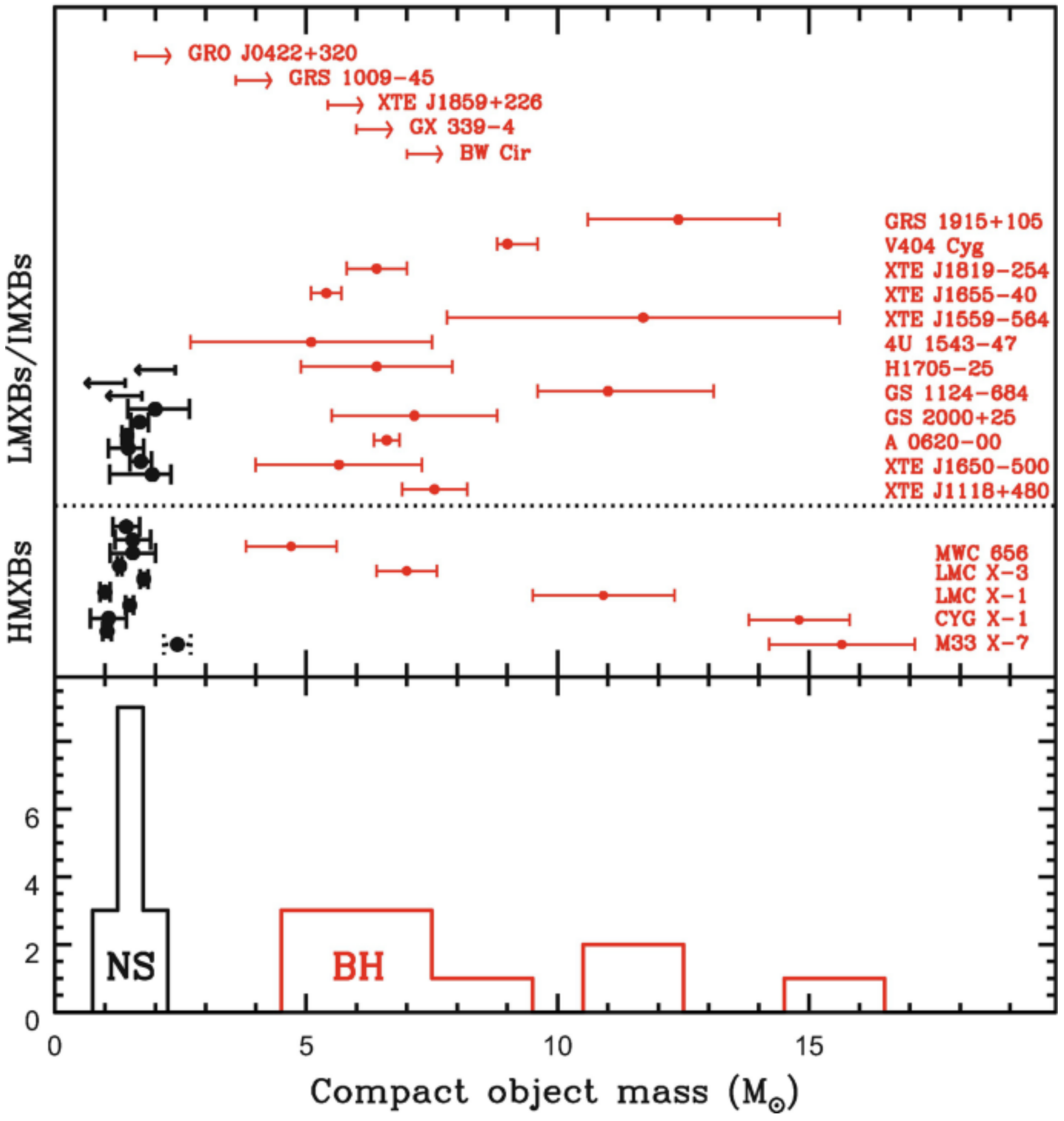
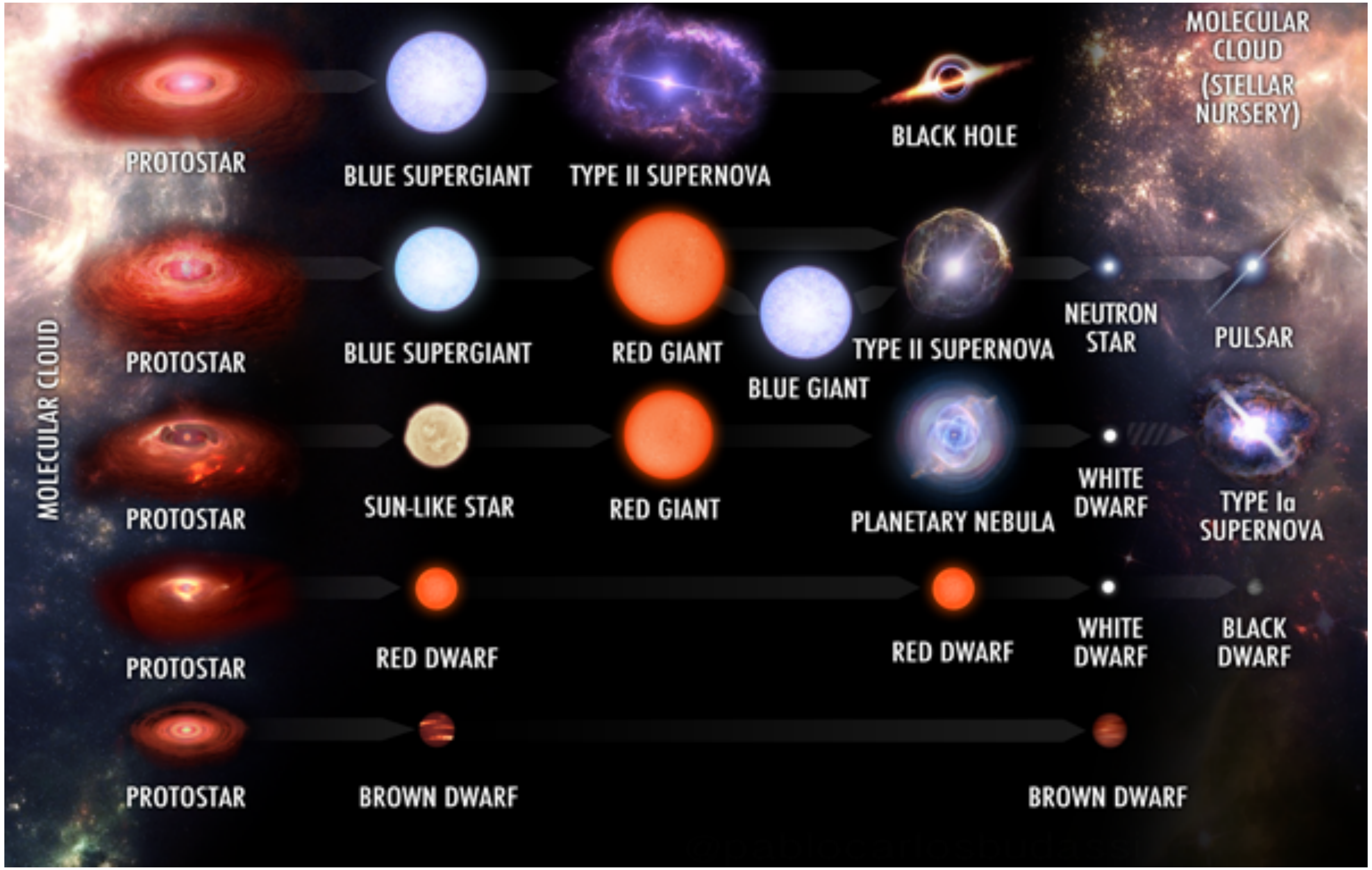


https://link.springer.com/referenceworkentry/10.1007/978-3-319-21846-5_111

Stellar evolution summary

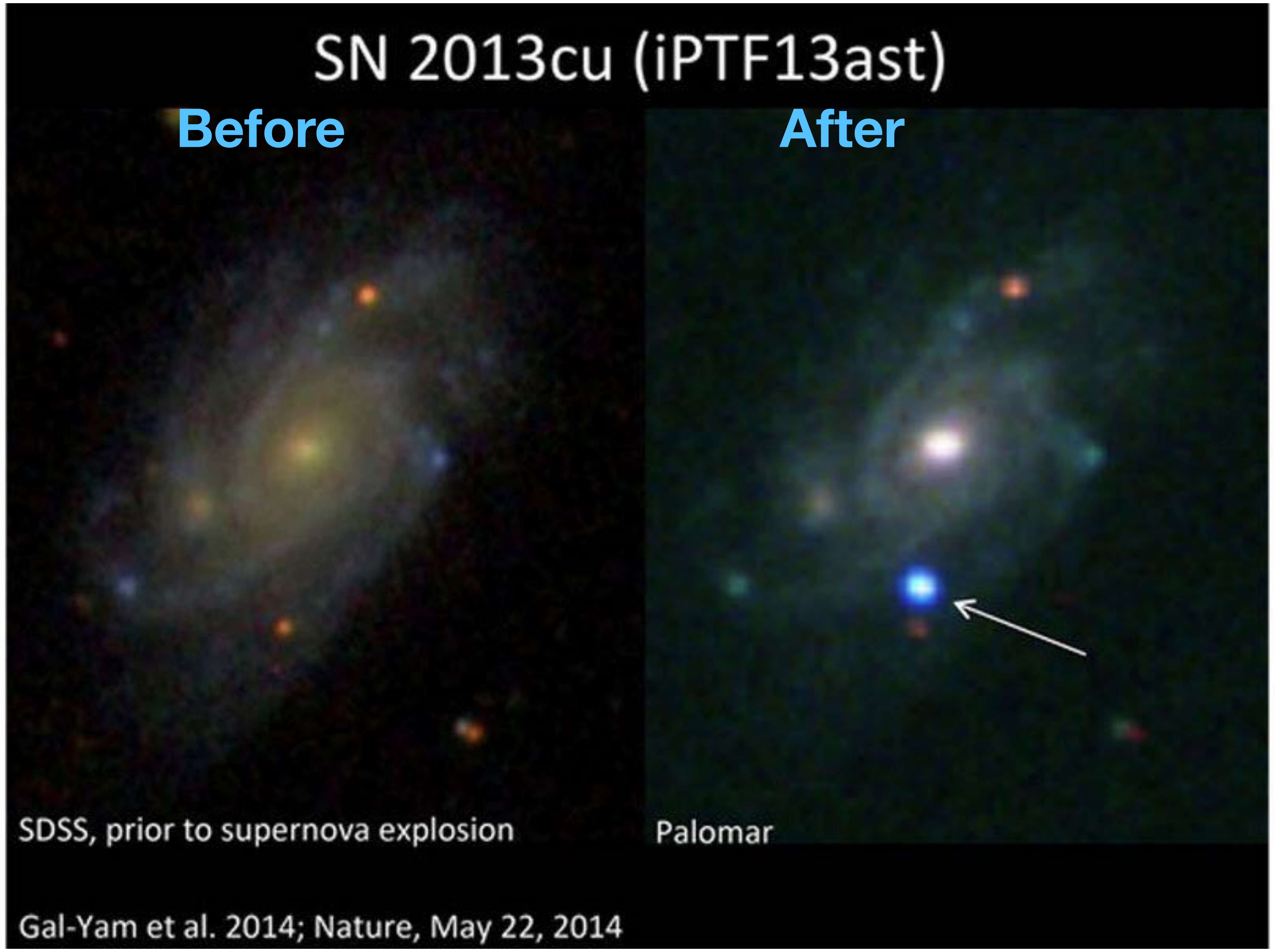
Evolution (time)

Increasing mass



Discovering supernovae

hypernova SN 1998bw in a spiral arm of galaxy ESO 184-G82



Discovering supernovae

A hypernova is a more energetic version of supernova. They are in most cases Type II supernova explosions of a massive star with a black hole as a final product.