① Astro summary:

Stellar models - observed properties of stars, models vs. observations
Stellar evolution

Stellar models:
What is a star: self-gravitating bodys of dynamical equilibrium
internal pressure ↔ gravity
hydrostatic equilibrium
- loose energy by radiation → need energy production: nuclear energy, gravitational energy
- temperature structure → infulences energy transport: radiation, conduction

Basic equations of stellar structure:

① mass conservation $\frac{dM_r}{dr} = 4\pi r^2 \rho$

② hydrostatic equilibrium: $\frac{dP}{dr} = -\frac{GM_r}{r^2}\rho$

↳ assuming perfect gas: → calculate P and T

Virial theorem: thermal energy ↔ balances gravity

$$2E_T + E_G = 0$$

(3.) ~~Energy transport~~ - Energy conservation

$$\frac{dL}{dr} = 4\pi r^2 \rho \varepsilon$$

(4.) Energy transport

$$\frac{dT}{dr} = -\frac{3}{4a_B c}\frac{\chi\rho}{T^3}\frac{L_r}{4\pi r^2}$$ radiative E transport

$$\frac{dT}{dr} = \left(1-\frac{1}{\gamma}\right)\frac{T}{P}\frac{dP}{dr}$$ convective E transport

γ - adiabatic pressure

↳ Schwarzschild criteria stability condition

-> convection $$\left|\frac{dT}{dr}\right| < \left(1-\frac{1}{\gamma}\right)\frac{T}{P}\left|\frac{dP}{dr}\right|$$ if the temperature gradient is steeper than this than the atmosphere is unstable

②

→ composition of the star is important $P, \chi, \varepsilon\ (\rho, T, X_i)$

→ adding boundary conditions for the inside and outside of the star

→ solving the equations to get a model

Voigth - Russel theorem of unique solutions → not true, possible degeneracy

⇒ models - simplifications ⇒ $\boxed{\text{relations between different quantities}}$

$P \propto \frac{M^2}{R^2}$ $P \propto \frac{M}{R^3} T$ ⇒ $\boxed{T \propto \frac{M}{R}}$ $\boxed{L \propto M^3}$

mass - luminosity relation → more massive stars → more luminous

$\boxed{M \propto T^2}$ $\boxed{L \propto T_{eff}^6}$ T_{eff} - surface temp.

⇓

L vs. T_{eff} ⇒ Herzsprung Russel diagrams

$\tau \propto \frac{M}{L}$ → $\boxed{\tau \propto M^{-2}}$ liftime of a star is related to the mass inversely

Surface temperature → spectral lines → composition → spectral classes O, B, A, F, G, K, M
→ colour → blackbody

↓

O, B - young massive

G - Sun

Spectra → doppler shift → motion of the star (binary, planet, etc.)

→ Zeeman effect → magnetic field (e.g. sunspots)

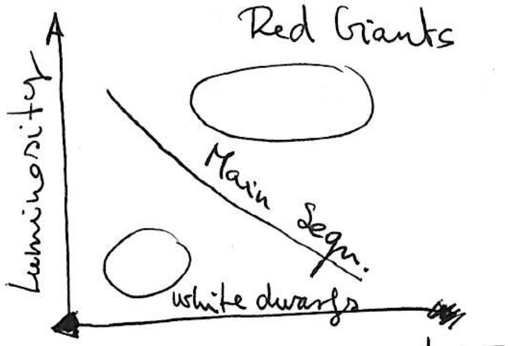
Luminosity - needs distance measurement:
- parallax
- globular clusters
- variable stars

distance → absolute magnitude, abs. bolometric magnitude, luminosity

Stellar mass - binary systems (dynamic measurement)
- eclipsing binaries
- spectroscopic binaries

- Mass - Luminosity relation **-> H-R digram**

(3)



High Mass Low
Temperature
Blue Colour Red
spectral type:

O, B, A, F, G, K, M

⇒ mass limit of stars: $0.1 - 100 M_\odot$
Lower limit: no fusion : brown dwarf
upper limit: Eddington luminosity limit
(based on radiation pressure)

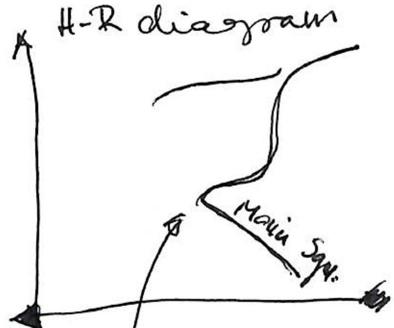
Star clusters – globular clusters : old, gravitationally bound
– open clusters : young, less bound

↓

born together

↓

same distance → distance measurement with the H-R diagram



⇒ match the main sequence to nearby stars → absolute magnitude → distance

tourn off point – related to age of cluster

<u>Stellar nucleosynthesis:</u>

mass-binding energy relation $E_B = \left[Z m_p + (A-Z) m_n - m_{nuc}\right] c^2$

tightest bound nucleus: Fe → lighter elements fusion
→ heavier elements fission } energy release

Stars: nuclear fusion – quantum mechanics tunneling

reaction rate – related to the cross-section $r = n_1 n_2 \langle \sigma v \rangle$

reaction rate → energy release ΔE, $\rho \varepsilon = r \Delta E = n_1 n_2 \langle \sigma v \rangle \Delta E$

epsilon -> nuclear energy generation function
-> increases with T sharply

(4) → reaction of heavier nuclei needs higher temperature

- nuclear burning
- nuclear fuel

} main sequence burn H → He

proton-proton chain : $^1H + ^1H \rightarrow ^2H + e^+ + \nu$
$^2H + ^1H \rightarrow \underline{^3He} + \gamma$

^3He → pp1, pp2, pp3 chains

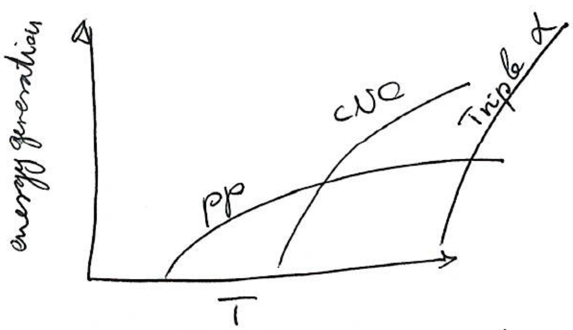
pp1 : $^3He + ^3He \rightarrow ^4He + H_1 + H_1$ lowest temp

if ^4He & ^7Be & $T > 10^7 K$: $^3He + ^4He \rightarrow ^7Be + \nu$

pp2 : $^7Be + e^- \rightarrow ^7Li + \nu$
$^7Li + H \rightarrow ^4He + ^4He$

pp3 : $^7Be + ^1H \rightarrow ^8B + \nu$
$B^8 \rightarrow ^8Be + e^+ + \nu$
$^8Be \rightarrow ^4He + ^4He$

if C, N, O present CNO-cycle : higher temperature



low temp : pp
low mass

high temp : CNO → more energy
high mass

triple-α reaction $^4He + ^4He + ^4He \rightarrow ^{12}C + \gamma$

⇒ heavy elements are synthesised in stars

→ after H finished → He can burn in high mass stars

→ up to Si burning into Fe **heavier elements than Fe in supernovae**

⑤ → composition of stars change with time

→ more massive stars hotter → different temp gradient

→ massive stars : convective core

- low mass stars : convective envelop

testing the inside of stars:

stellar oscillations → sound speed inside → pressure and density

- helio seizmology
- astero seizmology

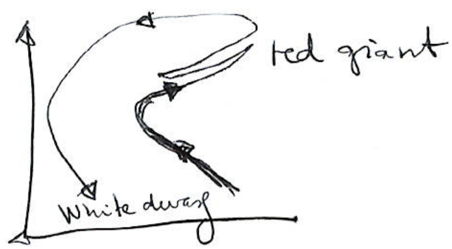
} test models

→ neutrino production in stars (nuclear fusion)

↳ test with observations → neutrino oscillation

^8B neutrinos depend on solar models

Stellar evolution on the H-R diagram



Binary system: equipotential surface → Roche lobe → mass transfer

Mass loss → stellar winds (solar wind)

- thermal driven wind
- radiative driven wind
- centrifugally driven winds

Extreme case: planetary nebula → white dwarf

Supernova explosion → neutron star or black hole

↳ Type I and Type II.

Type Ia supernova → mass transfer onto degenerate star

↳ identical mass → distance measurments

→ heavier elements than iron in supernova explosions (Type II)

↳ neutron capture + radioactive decay

⑥ Solar properties

-> differencial rotation

-> magnetic field - sunspots - magnetogram

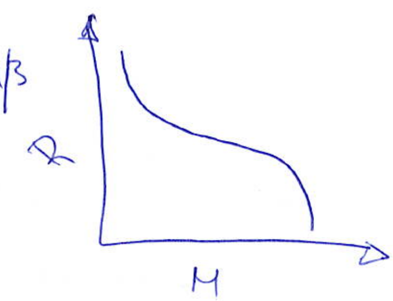
-> solar cycle : 22 years -> min, max sunspot
+ magnetic field flips polarity

↳ related: solar flares, CMEs => space weather

Exoplanets : 5 methods to detect:
- astrometry
- transit
- radial velocity
- direct imaging
- gravitational microlensing

End state of stars
-> white dwarf } degenerate stars
-> neutron star } degenerate matter
-> black hole

WD : degenerate electron gas $R \propto M^{-1/3}$



higher mass -> smaller radius
planet sized star

Chandrasekhar mass limit

Neutron star : degenerate neutron gas

to high pressure -> neutron drip

star of $M_\odot$ -> 1 km city sized star

-> pulsars
- pulsar glitches
-> millisecond pulsars (binary system with mass accretion) -> x-ray sources

-> rotation -> rotation period + mass => density

-> associated with supernova remnants