- Astro summary.

Stellar models - observed properties of stars, models vs. observation
Stellar evolution

Stellar models:
What is a star: self-gravitating bodys of dynamical equilibrium
internal pressure $\leftrightarrow$ gravity
hydrostatic equilibrium
- loose energy by radiation $\rightarrow$ need energy
production: nuclear energy, gravitational energy
- temperature structure $\rightarrow$ influences energy
· transport: radiation, conduct.
convection

Basic equations of stellar structure:

(1) mass conservation $\frac{dM_r}{dr} = 4\pi r^2 \rho$

(2) hydrostatic equilibrium: $\frac{dP}{dr} = -\frac{GM_r}{r^2}\rho$

$\hookrightarrow$ assuming perfect gas: $\rightarrow$ calculate P and T

Virial theorem: thermal energy $\leftrightarrow$ balances gravity
$$2E_T + E_G = 0$$

(3.) ~~Energy transport~~ - Energy conservation
$$\frac{dL}{dr} = 4\pi r^2 \rho \varepsilon$$

(4.) Energy transport
$$\frac{dT}{dr} = -\frac{3}{4 a_B c}\frac{\chi \rho}{T^3}\frac{L_r}{4\pi r^2}$$ radiative E transport

$$\frac{dT}{dr} = \left(1-\frac{1}{\gamma}\right)\frac{T}{P}\frac{dP}{dr}$$ convective E transport
γ - adiabatic pressure

$\hookrightarrow$ Schwarzschild stability condition / criteria
$$\left|\frac{dT}{dr}\right| < \left(1-\frac{1}{\gamma}\right)\frac{T}{P}\left|\frac{dP}{dr}\right|$$
if the temperature gradient is steeper than this than the atmosphere is unstable
$\rightarrow$ ~~and~~ convection

-> composition of the star is important $\kappa, \mu, \varepsilon\ (\rho, T, X_i)$

-> adding boundary conditions for the inside and outside of the star

-> solving the equations to get a model

Voigth - Russel theorem of unique solutions -> not true, possible degeneracy

=> models - simplifications => $\boxed{\text{relations between different quantities}}$

$P \propto \frac{M^2}{R^2}$ $P \propto \frac{M}{R^3} T$ => $\boxed{T \propto \frac{M}{R}}$ $\boxed{L \propto M^3}$

mass - luminosity relation -> more massive stars -> more luminous

$\boxed{M \propto T^2}$ $\boxed{L \propto T_{eff}^6}$ T_{eff} - surface temp.

$\Downarrow$

L vs. T_{eff} => Herzsprung Russel diagram

$\tau \propto \frac{M}{L}$ -> $\boxed{\tau \propto M^{-2}}$ liftem of a star is related to the mass inversely

Surface temperature -> spectral lines -> composition -> spectral classes O, B, A, F, G, K, M

-> colour -> blackbody

O, B - young massive

G - Sun

Spectra -> doppler shift -> moton of the star (binary, planet, etc.)

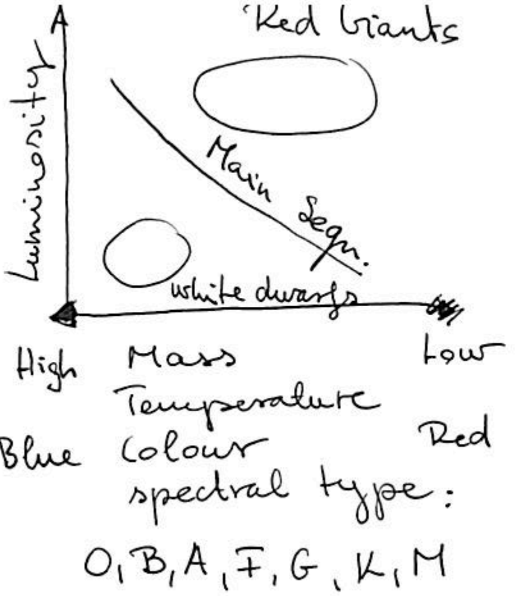
-> Zeeman effect -> magnetic field (e.g. sunpots)

Luminosity - needs distance measurment:
- parallax
- globular clusters
- variable stars

distance -> absolute magnitude, abs. bolometric magnitude, luminosity

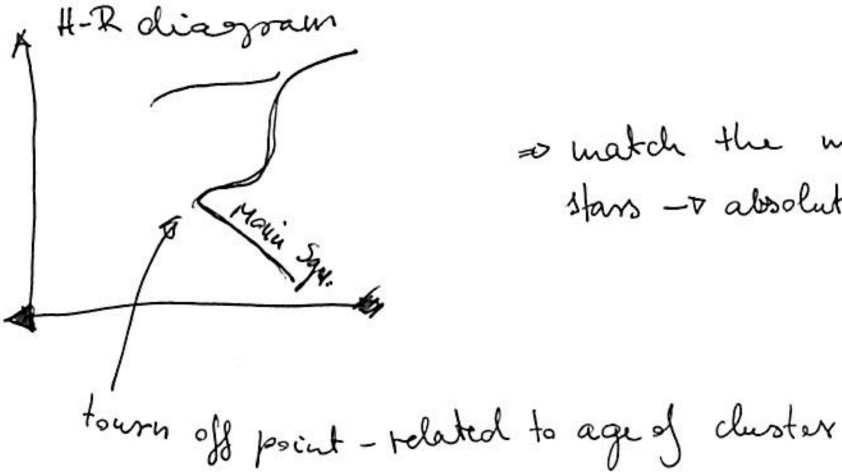
Stellar mass - binary systems (dynamic measurment)
- eclipsing binarie
- spectroscopic binaries
- Mass - Luminosity relation

=> H-R diaram



⇒ mass limit of stars: 0.1 − 100 $M_\odot$
Lower limit: no fusion : brown dwarf
upper limit: Eddington luminosity limit
(based on radiation pressure

Star clusters − globular clusters : old, gravitationally bound
− open clusters : young, less bound
↓
born together
↓
same distance → distance measurment with the H-R diagram



⇒ match the main sequence to nearby stars → absolute magnitude → distance

Stellar nucleosynthesis:

mass-binding energy relation $E_B = \left[Z m_p + (A-Z) m_n - m_{nuc}\right] c^2$

tightest bound nucleus: Fe → lighter elements fusion } energy release
→ heavier elements fission

Stars: nuclear fusion − quantum mechanics tunneling

reaction rate − related to the cross-section $r = n_1 n_2 \langle \sigma v \rangle$

reaction rate → energy release ΔE, $\rho\varepsilon = r\Delta E = n_1 n_2 \langle \sigma v \rangle \Delta E$

↳ ε − nuclear energy generation function
↳ increases with temperature sharply

→ reaction of heavier nuclei needs higher temperature

- nuclear burning
- nuclear fuel

} main sequence burn H → He

proton-proton chain: $^1H + ^1H \rightarrow ^2H + e^+ + \nu$

$^2H + ^1H \rightarrow \underline{^3He} + \gamma$

^3He → pp1, pp2, pp3 chains

pp1: $^3He + ^3He \rightarrow ^4He + H_1 + H_1$ lowest temp

if ^4He & ^7Be & $T > 10^7 K$: $^3He + ^4He \rightarrow ^7Be + \nu$

pp2: $^7Be + e^- \rightarrow ^7Li + \nu$

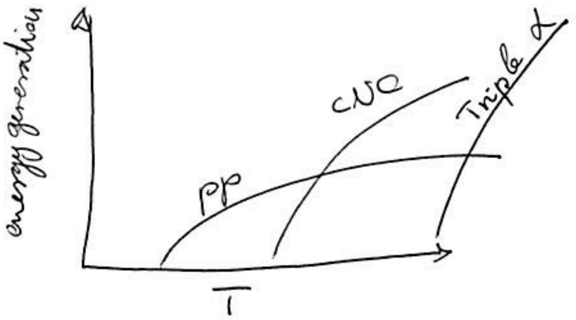
$^7Li + ^1H \rightarrow ^4He + ^4He$

pp3: $^7Be + ^1H \rightarrow ^8B + \nu$

$B^8 \rightarrow ^8Be + e^+ + \nu$

$^8Be \rightarrow ^4He + ^4He$

if C, N, O present CNO-cycle: higher temperature



low temp: pp
low mass

high temp: CNO → more energy
high mass

triple-α reaction $^4He + ^4He + ^4He \rightarrow ^{12}C + \gamma$

⇒ heavy elements are synthesised in stars

→ after H finished → He can burn in high mass stars

→ up to Si burning into Fe

heavier elements than Fe → in Supernovas

③ → composition of stars change with time
Composition of stars changes with time

→ more massive stars hotter → different temp gradient

→ massive stars : convective core

- low mass stars : convective envelop

<u>testing the inside of stars:</u>

stellar oscillations → sound speed inside → pressure and density

- helio seizmology
- astero seizmology

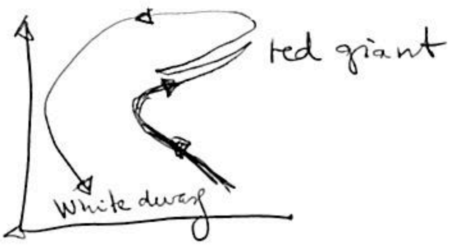
} test models

→ neutrino production in stars (nuclear fusion)

↳ test with observations → neutrino oscillation

^8B neutrinos depend on solar models

<u>Stellar evolution</u> on the H-R diagram



Binary system: equipotential surface → Roche lobe → mass transfer

Mass loss → stellar winds (solar wind)

- thermal driven wind
- radiative driven wind
- centrifugally driven winds

Extreme case: planetary nebula → White dwarf

Supernova explosion → neutron star or black hole

↳ Type I and Type II.

Type Ia supernova → mass transfer onto degenerate star

↳ identical mass → distance measurments

→ heavier elements than iron in supernova explosions (Type II)

↳ neutron capture + radioactive β decay

③ solar properties

-> differencial rotation

-> magnetic field - sunspots - magnetogram

-> solar cycle : 22 years -> min, max sunspot
+ magnetic field flips polarity

↳ related: solar flares, CMEs => space weather

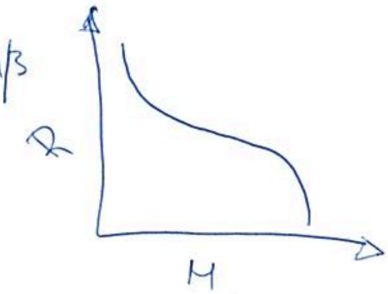
Exoplanets : 5 methods to detect:
- astrometry
- transit
- radial velocity
- direct imaging
- gravitational microlensing

End state of stars
-> white dwarf
-> neutron star
} degenerate stars
degenerate matter
-> black hole

WD : degenerate electron gas $R \propto M^{-1/3}$

higher mass -> smaller radius

planet sized star



chandrasekhar mass limit

Neutron star : degenerate neutron gas

to high pressure -> neutron drip

star of $M_\odot$ -> 1km city sized star

-> pulsars
-> rotation -> rotation period + mass => density
-> associated with supernova remnants
-> pulsar glitches
-> mili second pulsars (double system, mass accretion)
↳ X-ray sources