

# Astrophysics - Summary - Introduction

①

1. Units :- mass : solar mass

- length : AU, pc, kpc, Mpc, Gpc

- time : regular time

- coordinates : Ra. Dec Equatorial system  $\rightarrow$  precession  
l, b Galactic system

- brightness : magnitudes - log scale (absolute, apparent, bolometric)

$$m - M = 5 \log_{10} \frac{d}{10} \rightarrow \text{distance measurements}$$

+ colour of objects (B-V)

## Sources of astronomical information:

- EM radiation

- neutrinos  $\rightarrow$  Sun + supernovas

- gravitational waves  $\rightarrow$  black hole & neutron star mergers

- cosmic rays  $\rightarrow$  energetic objects

EM radiation : - various wavelengths

- atmospheric windows (throughput)

- Imaging + Spectroscopy

- Blackbody or synchrotron + spectral lines

Optical : resolution  $\theta = 1.22 \frac{\lambda}{D}$

Radio astronomy : single dish vs. interferometry

X-ray astronomy : only from space

Infrared astronomy :

Gamma-ray astronomy : directly only from space

} name a few  
sources  
+ where are  
the observatories  
in general

## 2. Radiation transfer - Summary

Describes how radiation interacts with matter

- Macroscopic: using emission and absorption coefficients
- Microscopic: calculating the emission & absorption coefficients

Planck's law - blackbody radiation

Radiation transfer - how does radiation propagate  
amount of radiation

$$dE_\nu d\nu = I_\nu \underbrace{\cos\theta}_{\substack{\text{solid} \\ \text{angle} \\ \text{of direction}}} dA \underbrace{dt}_{\text{time}} d\Omega d\nu$$

Radiation Flux:  $F_\nu = \int I_\nu \cos\theta d\Omega$   
↳ integrate the intensity

$$F = \int F_\nu d\nu$$

↳ integrate over frequency

Energy density:  $U_\nu = \int \frac{I_\nu}{c} d\Omega$  density of energy in a cylinder filled by radiation

Radiation pressure: pressure associated with radiation

$$P_\nu = \frac{1}{3} U_\nu$$

Radiative transport: how does radiation propagate through things

- in empty space: the radiation does not change
- through matter: emission and absorption

emission coef:  $j_\nu$   
absorption coef:  $\alpha_\nu$  } all matter has this

radiation transfer equation:  $\boxed{\frac{dI_\nu}{ds} = j_\nu - \alpha_\nu I_\nu}$

if the matter only absorbs:  $j_\nu = 0$   $\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu$

optical depth:  $d\tau_\nu = \alpha_\nu ds \rightarrow \tau_\nu = \int_{s_0}^s \alpha_\nu(s') ds'$

$$I_\nu(\tau_\nu) = I_\nu(0) e^{-\tau_\nu}$$

↳ ... absorption



- ③  $\tau \gg 1$  optically thick  
 $\tau \ll 1$  optically thin (can neglect optical depth)

source function:  $\boxed{S_\nu = \frac{j_\nu}{\kappa_\nu}}$  describes matter

Kirchoffs law: In thermodynamic equilibrium:

$$j_\nu = \kappa_\nu B_\nu(T)$$

\*  
 emission coef.      absorption coef  $\times$  blackbody

- Thermodynamic equilibrium  $\rightarrow$  Maxwellian velocity distribution (e.g. stars)

Boltzmann distribution law:  $\frac{n_e}{n_0} = \exp\left(-\frac{E}{k_B T}\right) \rightarrow$  fraction of excited atoms

Saha equation: fraction of ionised gas

- Local thermodynamical equilibrium

- Radiative transfer through stellar atmospheres  $\rightarrow$  general considerations

- plane parallel atmosphere  layers, no variation in  $\leftrightarrow$  direction

- LTE

- radiation field is anisotropic (radical temperature gradient)

- grey atmosphere model  $\rightarrow$  no dependence on  $\nu$  (simplification)

- limb darkening: intensity depends on direction

- Formation of spectral lines (absorption)

$\rightarrow$  layers with different  $\kappa_\nu$

$\rightarrow$  matter actually has  $\kappa$  dependent on  $\nu$

- Radiative energy transport inside stars:

opacity( $\kappa$ ):  $\kappa_R = \rho \kappa$  depends on density and absorption coef.

Kramers's law  $\kappa \propto \frac{1}{T^{3.5}}$  depends on temperature  $\rightarrow$  higher  $T$  lower  $\kappa$

However: opacity drops at low temperatures!

④ Thompson scattering : radiation scattering on free electrons  
elastic scattering - wavelength does not change

→ additional opacity due to Thompson scattering

Spectral lines : Equivalent width : the integral of the fractional dip in intensity for an absorption line

$$W_\lambda = \int \frac{I_c - I_\lambda}{I_c} d\lambda$$

$I_c$  - continuum intensity  
 $I_\lambda$  = absorption line

photon diffusion inside the Sun → it takes a very long time for photons to get out from the core  $\sim 10^4$  years



### Stellar Physics



### Solar phenomena :

rotation : differential rotation → from oscillations  
→ movement of sunspots



magnetic field: sunspots : colder spots associated with magnetic fields

solar cycle : 11 years of variation for number of sunspot  
+ polarity change ⇒ 22 years cycle

solar flares + CME's → space weather, Aurorae

Exo planets : 5 methods for detection:

- radial velocity
- transit
- direct imaging
- gravitational micro lensing
- astrometry

### End state of stars:

- White dwarf
  - neutron star
  - BH
- } discarded

White dwarfs : Fermi gas (not ~~Boltzmann~~ <sup>Maxwellian</sup> velocity) → degeneracy pressure  
↳ degenerate  $e^-$  gas

For the pressure we have two cases → relativistic case  $P = K_2 \rho^{4/3}$   
→ non relativistic case  $P = K_1 \rho^{5/3}$

5.

degenerate matter  $\rightarrow$  stellar structure simplifies: only need to solve

~~two~~<sup>3</sup> of the stellar structure equations  $\rightarrow$  the first 2 combine

$\Rightarrow R \propto M^{-1/3} \rightarrow$  increasing mass  $\rightarrow$  smaller star

depending on the ~~size~~<sup>mass</sup> of the star we can use ~~ideal~~<sup>non-rel</sup> for low mass and relativistic pressure for higher masses

$\rightarrow$  mass limit for white dwarf: Chandrasekhar mass limit

$\rightarrow$  mass of a star in the size of a planet ( $10^4 R_{\oplus}$ )

Neutron star: degenerate neutron gas  $\rightarrow$  "neutron drip" neutrons drip out of atoms at very high pressures,  $e^- + p^+ \rightarrow n + \nu$

$\rightarrow$  degenerate pressure

$\rightarrow$  also have a mass limit

$\rightarrow$  mass of a star in a size of a city ( $10 R_{\oplus}$ )

$\rightarrow$  rotating neutron star: pulsar

$\rightarrow$  binary pulsar  $\rightarrow$  gravitational waves

$\rightarrow$  millisecond pulsars  $\rightarrow$  binary system

$\rightarrow$  X-ray binaries

The Milky Way





# 'Astro summary:

Stellar models - observed properties of stars, models vs. observations  
Stellar evolution

Stellar models:

What is a star: self-gravitating bodies of dynamical equilibrium  
internal pressure  $\leftrightarrow$  gravity  
hydrostatic equilibrium

- lose energy by radiation  $\rightarrow$  need energy production: nuclear energy, gravitational energy
- temperature structure  $\rightarrow$  influences energy transport: radiation, conduction

Basic equations of stellar structure:

① mass conservation  $\frac{dm_r}{dr} = 4\pi r^2 \rho$

② hydrostatic equilibrium:  $\frac{dP}{dr} = -\frac{GM_r}{r^2} \rho$

$\rightarrow$  assuming perfect gas:  $\rightarrow$  calculate  $P$  and  $T$

Virial theorem: thermal energy  $\leftrightarrow$  balances gravity  
 $2E_T + E_G = 0$

③ ~~Energy transport~~ - Energy conservation

$$\frac{dL}{dr} = 4\pi r^2 \rho \epsilon$$

④ Energy transport

$$\frac{dT}{dr} = -\frac{3}{4a_B C} \frac{\chi_P}{T^3} \frac{L_r}{4\pi r^2}$$

radiative E transport

$$\frac{dT}{dr} = \left(1 - \frac{1}{\gamma}\right) \frac{T}{P} \frac{dP}{dr}$$

convective E transport  
 $\gamma$  - adiabatic pressure

$\rightarrow$  Schwarzschild stability condition

$$\left| \frac{dT}{dr} \right| < \left(1 - \frac{1}{\gamma}\right) \frac{T}{P} \left| \frac{dP}{dr} \right|$$

if the temperature gradient is steeper than this then the atmosphere is unstable  
 $\rightarrow$  convection

- ②
- composition of the star is important  $P, \chi, \varepsilon (p_i, T_i, \chi_i)$
  - adding boundary conditions for the inside and outside of the star
  - solving the equations to get a model

Voigt - Russell theorem of unique solutions → not true, possible degeneracy

⇒ model - simplifications ⇒ relations between different quantities

$$P \propto \frac{M^2}{R^2} \quad P \propto \frac{M}{R^3} T \Rightarrow \boxed{T \propto \frac{M}{R}} \quad \boxed{L \propto M^3}$$

mass-luminosity relation → more massive stars → more luminous  
 $\boxed{M \propto T^2} \quad \boxed{L \propto T_{\text{eff}}^6}$   
 $T_{\text{eff}}$  - surface temp.

↓  
 $L$  vs.  $T_{\text{eff}}$  ⇒ Hertzsprung Russell diagram

$\tau \propto \frac{M}{L} \Rightarrow \boxed{\tau \propto M^{-2}}$  lifetime of a star is related to the mass inversely

Surface temperature → spectral lines → composition → spectral classes  
 → colour → blackbody

O, B, A, F, G, K, M

↓  
 O, B - young massive  
 G - Sun

Spectra → doppler shift → motion of the star (binary, planet, etc.)

→ Zeeman effect → magnetic field (e.g. sunspots)

Luminosity - needs distance measurement:

- parallax
- globular clusters
- variable stars

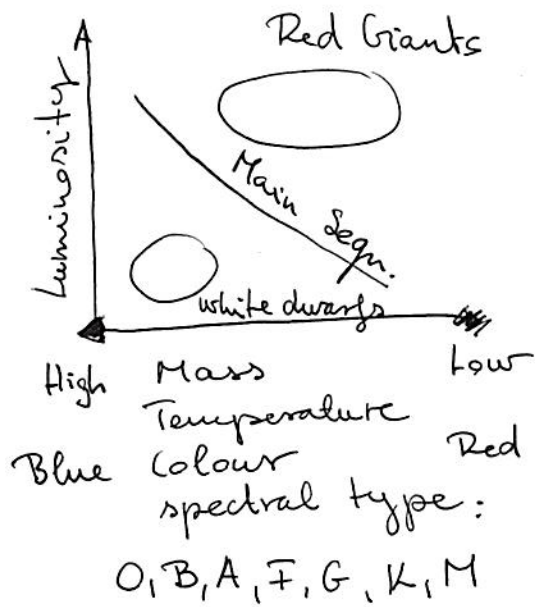
distance → absolute magnitude, abs. bolometric magnitude, luminosity

Stellar mass - binary systems (dynamic measurement)

- eclipsing binaries
- spectroscopic binaries

- Mass - Luminosity relation

⇒ H-R diagram



$\Rightarrow$  mass limit of stars:  $0.1 - 100 M_{\odot}$   
 Lower limit: no fusion: brown dwarf  
 upper limit: Eddington luminosity limit  
 (based on radiation pressure)

Star clusters - globular clusters: old, gravitationally bound  
 $\downarrow$   
 - open clusters: young, less bound  
 Born together

$\downarrow$   
 same distance  $\rightarrow$  distance measurement with the H-R diagram



$\Rightarrow$  match the main sequence to nearby stars  $\rightarrow$  absolute magnitude  $\rightarrow$  distance

turn off point - related to age of cluster

### Stellar nucleosynthesis:

mass-binding energy relation  $E_B = [Zm_p + (A-Z)m_n - m_{nuc}]c^2$   
 tightest bound nucleus: Fe  $\rightarrow$  lighter elements fusion } energy release  
 $\rightarrow$  heavier elements fission }

Stars: nuclear fusion - quantum mechanics tunneling

reaction rate - related to the cross section  $r = n_1 n_2 \langle \sigma v \rangle$

reaction rate  $\rightarrow$  energy release  $\Delta E$ ,  $pE = r \Delta E = n_1 n_2 \langle \sigma v \rangle \Delta E$

$\hookrightarrow E$  - nuclear energy generation function

$\hookrightarrow$  increases with temperature sharply

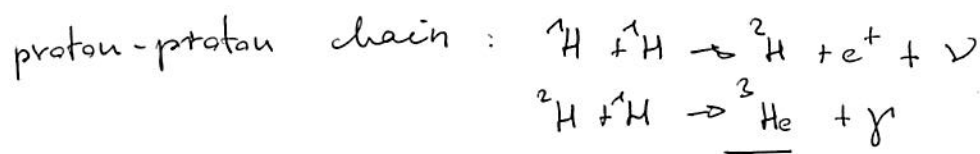


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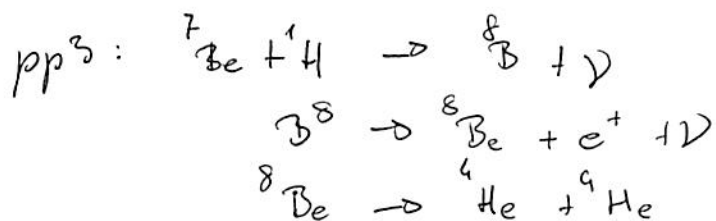
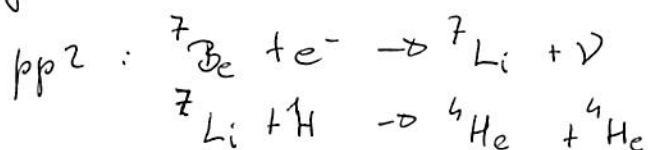
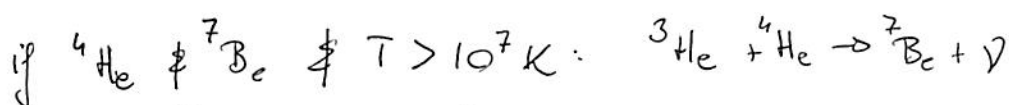
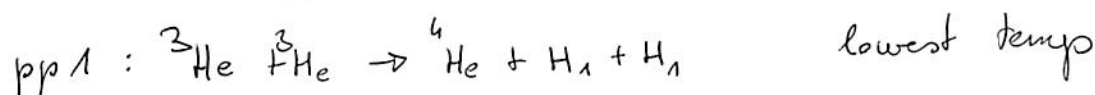
→ reaction of heavier nuclei needs higher temperature

- nuclear burning
- nuclear fuel

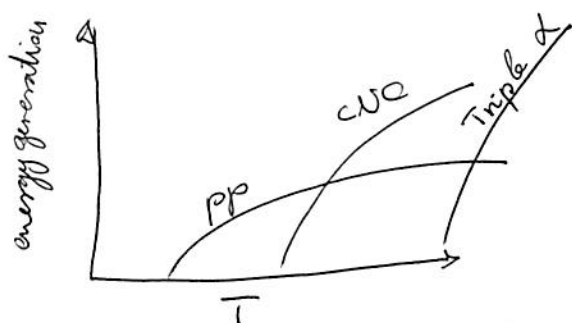
} main sequence burn  $H \rightarrow He$



${}^3_2He \rightarrow pp1, pp2, pp3$  chains

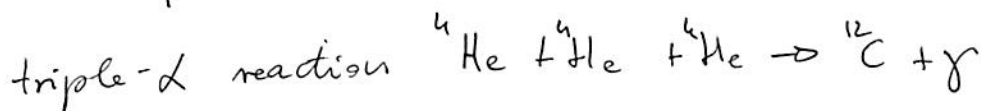


if C, N, O present CNO-cycle: higher temperature



low temp: pp  
low mass

high temp: CNO → more energy  
high mass



⇒ heavy elements are synthesised in stars

→ after H finished → He can burn in high mass stars

→ up to Si burning into Fe

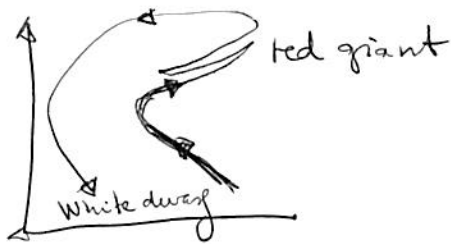
heavier elements than Fe → in Supernovas

- ⑤ → composition of stars change with time
- more massive stars hotter → different temp gradient
    - massive stars: convective core
    - low mass stars: convective envelop

### testing the inside of stars:

- stellar oscillations → sound speed inside → pressure and density
- helio seismology
  - astero seismology
- } test models
- neutrino production in stars (nuclear fusion)
- ↳ test with observations → neutrino oscillation
  - ↳ 3 neutrinos depend on solar models

### Stellar evolution on the H-R diagram



Binary systems: equipotential surface → Roche lobe → mass transfer

Mass loss → stellar winds (solar wind)

- thermal driven wind
- radiative driven wind
- centrifugally driven winds

Extreme case: planetary nebula → white dwarf

Supernova explosion → neutron star or black hole

- ↳ Type I and Type II.

Type Ia supernova → mass transfer onto degenerate star

- ↳ identical mass → distance measurements

→ heavier elements than iron in supernova explosions (Type II)

- ↳ neutron capture + radioactive decay

## ⑥ Solar properties

- differential rotation
- magnetic field - sunspots - magnetogram
- solar cycle: 22 years → min, max sunspot  
+ magnetic field flips polarity

↳ related: solar flares, CMEs ⇒ space weather

Exoplanets: 5 methods to detect:

- astrometry
- transit
- radial velocity
- direct imaging
- gravitational microlensing

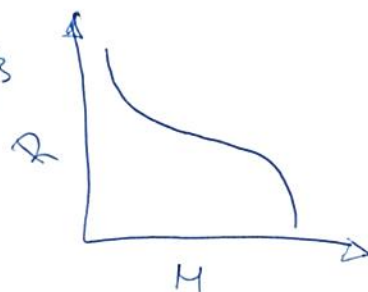
## End state of stars

- white dwarf
  - neutron star
  - black hole
- } degenerate stars  
degenerate matter

WD: degenerate electron gas  $R \propto M^{-1/3}$

higher mass → smaller radius  
planet sized star

Chandrasekhar mass limit



Neutron star: degenerate neutron gas

to high pressure → neutron drip

star of  $M_{\odot}$  → 12km city sized star

- pulsars
- rotation → rotation period + mass ⇒ density
- associated with supernova remnants
- pulsar glitches
- millisecond pulsars (double system, mass accretion)  
↳ X-ray sources