Introduction to Astrophysics and Cosmology

Radiation transfer

Helga Dénes 2022 Yachay Tech

hdenes@yachaytech.edu.ec

Radiation transfer

- describes how radiation interacts with matter
- Macroscopic: using emission and absorption coefficients
- Microscopic: calculating the emission and absorption coefficients

Radiation field
 - simples case: blackbody radiation (homogeneous and isotropic inside a container)

Planck's law - specifies energy density U_ν in given frequency range $\nu, \nu + d\nu$:

$$U_\nu d\nu = \frac{8\pi h}{c^3} \frac{\nu^3 d\nu}{\exp\left(\frac{h\nu}{\kappa_B T}\right) - 1}.$$

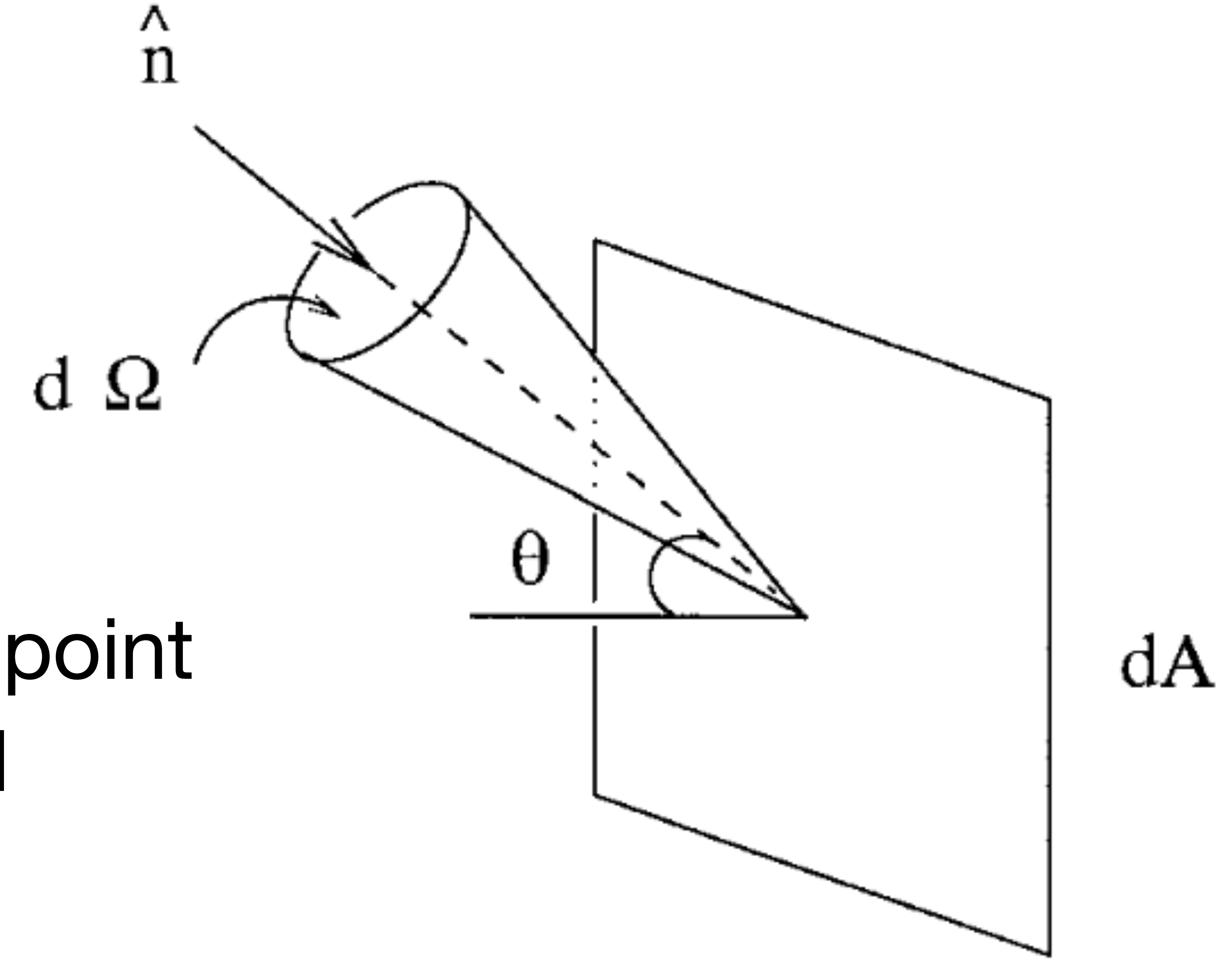
Radiation transfer

In general the radiation is not isotropic and we need to consider the direction.

We consider a small area dA, the amount of radiation is $dE_\nu d\nu$, passing trough this area in time dt, from the solid angle $d\Omega$ in the frequency range $\nu, \nu + d\nu$.

$dE_\nu d\nu$ is proportional to the projected area $dAcos\theta$ and to dt, $d\Omega$, $d\nu$

$$dE_\nu d\nu = I_\nu(\mathbf{r}, t, \hat{\mathbf{n}}) \cos\theta \, dA \, dt \, d\Omega \, d\nu,$$



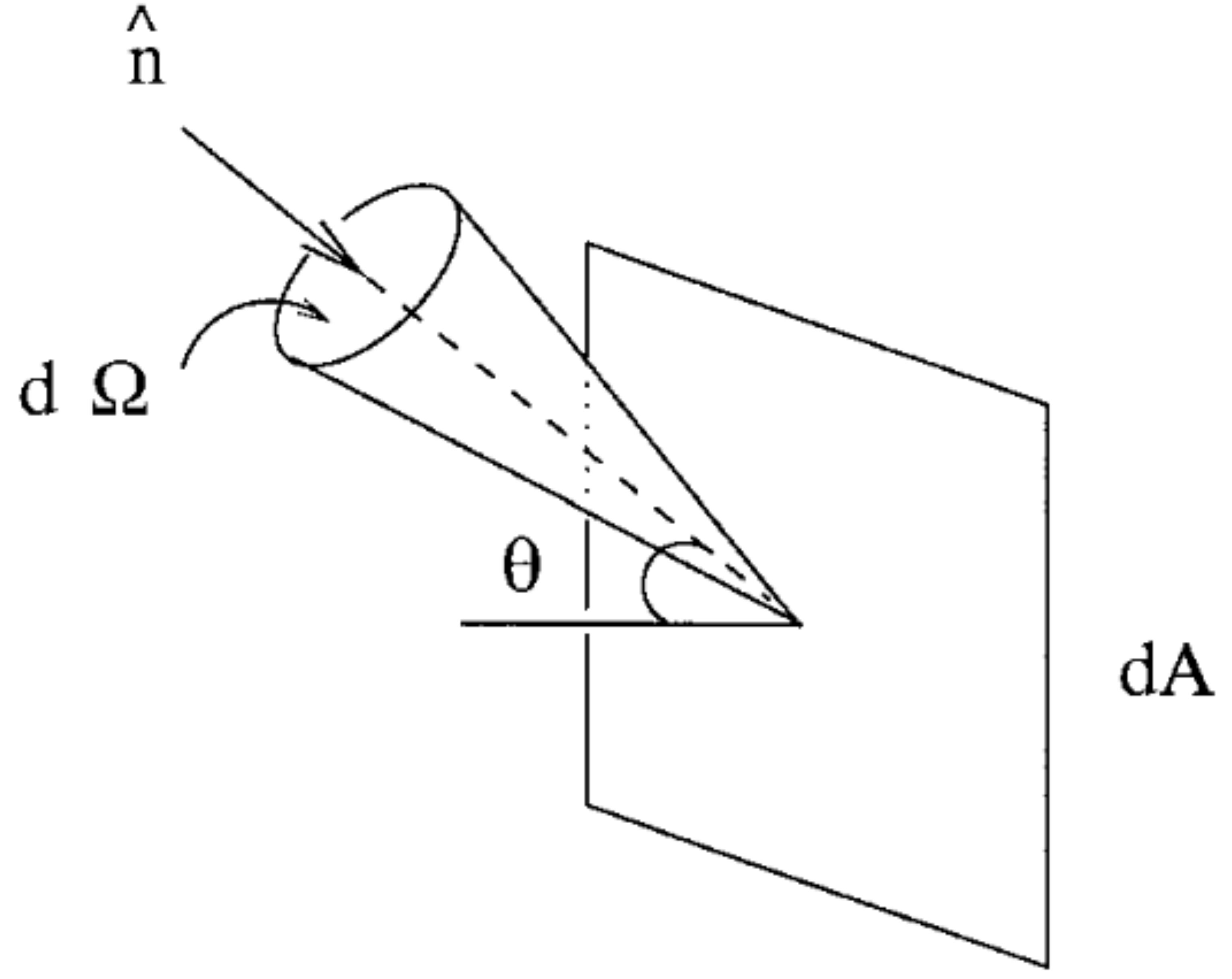
n is the unity vector that represents the direction of the radiation
I_ν is the specific intensity. If I_ν is specified in all directions in every point of a region at a specific time than we have a radiation field. We will consider now radiation fields in depend of time.

Radiation flux

We can calculate varios quantities from the radiation field: Flux, energy density, radiation pressure.

Radiaton flux is the total energy of radiation coming from all directions per unit area, per unit time.

$$F_\nu = \int I_\nu \cos\theta \, d\Omega,$$



Total radiaton flux is:

$$F = \int F_\nu \, d\nu.$$

Energy density

Energy passes to area dA in dt time in a certain direction **n**. The radiation travels a distance cdt in dt time, we expect the. Radiation to fill a cylinder with a bas of dA and length of cdt. The volume of such a cylinder is $cos\theta dAcdt$, from this the energy density is:

$$\frac{dE_\nu}{\cos\theta\, dA\, c\, dt} = \frac{I_\nu}{c} d\Omega$$

To get the total energy density at a point we need to integrate over all directions where radiation is coming from:

$$U_\nu = \int \frac{I_\nu}{c} d\Omega.$$

Energy density

Now we apply this to blackbody radiation. Where $B_\nu(T)$ is the specific intensity of blackbody radiation.

$$U_\nu = \frac{4\pi}{c} B_\nu(T),$$

From this, the specific intensity of blackbody ration is:

$$B_\nu(T) = \frac{2h\nu^3}{c^2} \frac{1}{\exp\left(\frac{h\nu}{\kappa_\mathrm{B} T}\right) - 1}.$$

Radiation pressure

The pressure of the radiation field over a surface is given by the flux of momentum perpendicular to that surface. The momentum associated with energy dE_ν is dE_ν/c and its component normal to the surface dA is $dE_\nu cos\theta/c$ by dividing this by $dA\ dt$ we get the momentum flux associated with dE_ν

$$\frac{dE_\nu \cos\theta}{c} \frac{1}{dA\, dt} = \frac{I_\nu}{c} \cos^2\theta\, d\Omega$$

The pressure is obtained by integrating over all directions:

$$P_\nu = \frac{1}{c} \int I_\nu \cos^2\theta\, d\Omega .$$

If the field is isotropic:

$$P_\nu = \frac{I_\nu}{c} \int \cos^2\theta\, d\Omega = \frac{4\pi}{3} \frac{I_\nu}{c} .$$

Radiation pressure

This can also be written as:

$$U_\nu = 4\pi \frac{I_\nu}{c}$$

$$P_\nu = \frac{1}{3} U_\nu$$