Stellar Physics - Summary

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This summary is based on the book Chapters 3 - 5 from Arnab Rai Choudhuri: Astrophysics for physicists.

1 Stellar models

A star is a self gravitating body in dynamical equilibrium.

- The internal pressure (thermal pressure + radiation pressure from nuclear fusion in the core) is balancing out gravity.
- hydrostatic equilibrium
- stars loose energy trough radiation
- energy production trough nuclear fusion
- internal temperature structure - energy transport trough: convection and radiation

1.1 Stellar structure equations

1) **mass conservation**

$$\frac{dM}{dr} = 4\pi r^2 \rho$$

2) **hydrostatic equilibrium**

$$\frac{dP}{dr} = -\frac{GMr}{r^2}\rho$$

Virial theorem for stars: the thermal energy balances gravity: $2E_T + E_G = 0$
3) **energy conservation**

$$\frac{dL}{dr} = 4\pi r^2 \rho \epsilon$$

4) **energy transport**

radiative energy transport:

$$\frac{dT}{dr} = -\frac{3}{4a_B c}\frac{\chi\rho}{T^3}\frac{L_r}{4\pi r^2}$$

convective energy transport:

$$\frac{dT}{dr} = \left(1 - \frac{1}{\gamma}\right)\frac{T}{P}\frac{dP}{dr}$$

Schwarzschild stability condition: if the temperature gradient is steeper than this the atmosphere is unstable → convection

$$\left|\frac{dT}{dr}\right| = \left(1 - \frac{1}{\gamma}\right)\frac{T}{P}\left|\frac{dP}{dr}\right|$$

The composition of the star is important $P\chi, \epsilon(\rho, T, X_i)$. Solving the stellar structure equations → stellar models.

Voight-Russel theorem: stellar models have unique solutions e.g. stars of a certain mass and composition have the same internal structure. This is only true for non-degenerate stars. We can have a star with the same mass and composition as a regular star or as a degenerate star (white dwarf).

1.2 Relations between stellar properties

By simplifying and approximating the stellar models we can get some relations for the stellar properties:

- $T \propto \frac{M}{R}$
- $L \propto M^3$ - **mass-luminosity relation**: more massive stars are more luminous
- $M \propto T^2$
- $L \propto T_{eff}^6$ - **Herzsprung-Russel diagram**
- $\tau \propto M^{-2}$ - the **lifetime of a star is inversely related to the mass**, high mass stars live shorter

surface temperature → spectral lines → composition → spectral type (O, B, A, F, G, K, M). O and B stars are young massive stars, the Sun is a G star.

spectra → Doppler shift → motion of the star (binary systems, planets)
spectra → Zeeman effect → magnetic field (sunspots)

Luminosity (absolute magnitude, absolute bolometric magnitude) → requires distance measurement.

1.3 The Herzsprung-Russel diagram

The Herzsprumg-Russel (H-R) diagram shows the Temperature - Luminosity relation of stars. 1 shows the schematics of the diagram. The x-axis can typically show 3 different properties that are equivalent: temperature, colour and spectral type. The forth property that is equivalent to these 3 is mass. Note: the x-axis goes in reverse direction, where the hottest stars are on the left side and the cooler stars are on the right side.

There are 3 main groups of stars in the H-R diagram. The **main sequence,** the **red giants** and the **white dwarfs**.

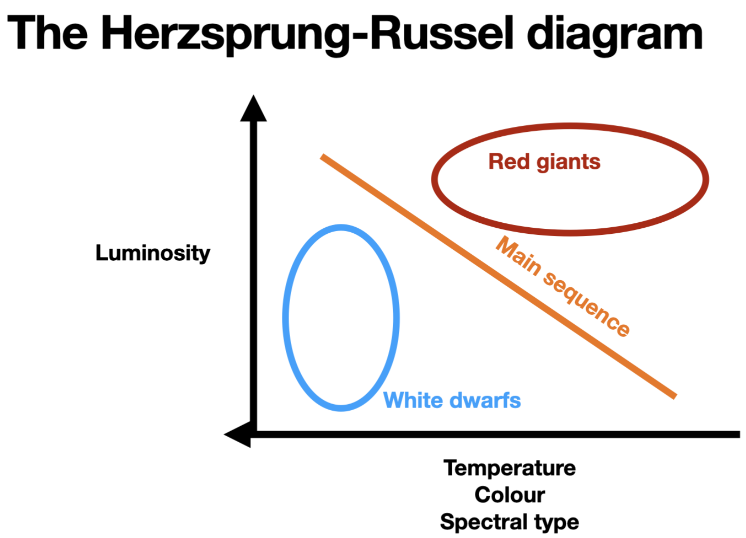

Figure 1: Thematics of the Herzsprumg-Russel diagram

There is a range of mass that stars can take (0.1 - 100 $M_{\odot}$) with a lower and an upper limit. The lower limit represents where nuclear fusion can start, the upper limit is related to the **Eddington Luminosity limit**, which represents where **radiation pressur**e form a high mass star would exceed the gravitational force keeping the star together.

Objects that are larger than planets, but are to small to fuse hydrogen into helium are called **Brown Dwarfs.**

2 Stellar Nucleosynthesis

All heavy elements (than H and He) are shyntesised in stars.
mass - binding energy relation

the tightest bound nucleus is Fe → the heaviest element that can be produced trough nuclear fusion. The heavier elements are produced trough **neutron capture and** β **decay** in supernova explosions.

Nuclear fusion in stars relies on quantum mechanical **tunneling**. The reaction rate depends on the cross section of the nuclear fusion, which depends on the likelihood of tunneling, which depends on the Coulomb potential and the kinetic energy of the particles involved in the fusion. The **energy release** depends on the reaction rate → which increases sharply with temperature.

The fusion of heavier nuclei needs higher temperature → produces more energy → fusion process is more efficient → burns up the fuel faster → higher mass stars have shorter lives. Depending on the temperature we have different reaction chains for the nuclear fusion:

- **proton-proton (pp) chain** - lowest temperature, smallest stars (including the Sun) - the first step is to make deuterium (this takes the longest). There are 3 sub chains: pp1, pp2 and pp3. During 3 of the fusion steps there is neutrino production.
- **CNO cycle** - requires higher temperatures (the Sun can not do this)
- t**riple alpha (3α) process** - 3He → ^{12}C - requires high temperatures

Main sequence stars burn (fuse) H into He. Red giants burn heavier elements. The composition of stars changes with time

More massive stars are hotter → different temperature gradients for low mass and high mass stars:

- the smallest stars can be fully convective
- low mass stars have convective envelops
- high mass stars have convective cores

2.1 Information from the inside of stars

Helio Seismology / Astero Seismology: Observing oscillations of the stellar surface (brightness fluctuation) due to propagation of sound waves inside of the stars. The propagation of sound waves (the sound speed) depends on the density inside of the star. By measuring oscillations we can calculate the sound speed and the density in different layers of the stars. This is how we know how deep the convective layers is.

Neutrinos: There are 3 different steps during the pp-chain that produce neutrinos. We can test models with observing the amount and the energy distribution of neutrinos that get produced.

Initial neutrino observations observed a much smaller neutrino flux than expected → neutrino oscillation - neutrinos can spontaneously change into different types (ν_e, ν_μ, ν_τ) of neutrinos.

3 Stellar evolution

Hydrogen burning on the main sequence → after the Hydrogen finishes the star starts to contract, the core heats up due to Kelvin-Helmholtz → if the star has enough mass the fusion of He can start → the star moves from the main sequence to the red giant branch → if the star is sufficiently high mass, then after the He fusion heavier elements can fuse as well, up until Fe → once all fusion is done the star dies, low mass stars push off their outer envelopes as planetary nebulae and turn into white dwarfs, high mass stars have a supernova explosion and the leftover core turns into a neutron star or a black hole.

Binary stars: in binary systems one star can transfer mass to the other star trough the **inner Lagrange point** → because the mass exchange the evolution of the stars changes. e.g. Ia type supernova explosions, millisecond pulsars

Mass loss: stars are constantly loosing mass trough stellar winds (solar wind - charged particles that are constantly leaving the surface of the Sun). Types of stellar winds:

- thermally driven wind (e.g. solar wind)
- radiativly driven wind
- centrifugally driven wind

Extreme cases of mass loss are at the end of the stars when stars produce **planetary nebulae** and **supernova** remnants.

Types of supernovae:

- type II: the end of a massive star
- type I: the end of a star in a binary system. The type Ia is a special case when a white dwarf accrets material from a companion star → distance measurement

Star clusters - stars in star cluster are born together roughly at the same time → the stars in the cluster are the same age

- **Globular clusters** - old, spherical shape, gravitationally bound structures, with thousands of stars - stars are at the same distance from us → **distance measurement**. The H-R diagram of globular clusters has a shorter main sequence → the turn of point of the main sequence determins which globular cluster is older or younger
- **Open clusters** - young, not bound, typically a few stars

4 Distance determination

parallax range: within the Galaxy and the Magellanic Clouds

Globular cluster method stars are born together, they are all the same age - stars are at the same distance from us → construct an H-R diagram for the cluster using the apparent magnitude of the stars in the cluster → match the main sequence to the H-R diagram of nearby stars, for which we know the absolute magnitude → this calibrates the magnitudes for the globular cluster stars → once we know the absolute and the apparent magnitudes we can calculate the sistance to the cluster. range: nearby galaxies

Variable stars method - certain variable stars: RR Lyrae and Cepheids have a period - luminosity relation → from the period of the luminosity variation we can calculate the absolute magnitude → distance measurement. range: nearby galaxies

Ia supernova method - Type Ia suparnovas are the product of a white dwarf accreting material from a companion star → if the white dwarf grows to large (Chandrasekar mass limit) the white dwarf explodes as a supernova. This always happens at the same mass → the supernova has the same luminosity → we know the absolute magnitude, we measure the apparent magnitude → calculate distance. range: distant galaxies

5 Solar properties

- differential rotation - from oscillations and sunspot motion
- magnetic field → **sunspots** (colder spots of the sun, at the footpoints of magnetic loops) → **solar cycle** - an 11 year variation of sunspot activity + change of polarity in the magnetic field → full cycle is **22 years**
- solar phenomena due to magnetic field: solar flares, coronal mass ejections (CMEs)
- solar wind → space weather and aurora

6 Exo-planets

Currently there are about 5000 confirmed exo-plantes. The sizes range of Earth-sized to super-Jupiter sized planets. Methods to detect exo-planets:

- astrometry
- transit
- radial velocity
- direct imaging
- gravitational microlensing

7 End state of star

White dwarfs and neutron stars are dead stars with no nuclear fusion. They are kept in equilibrium trough degeneracy pressure. Because of the degeneracy the stellar structure equations simplify. Important: **the higher mass these objects have the smaller the radius** $R \propto M^{-1/3}$**.**

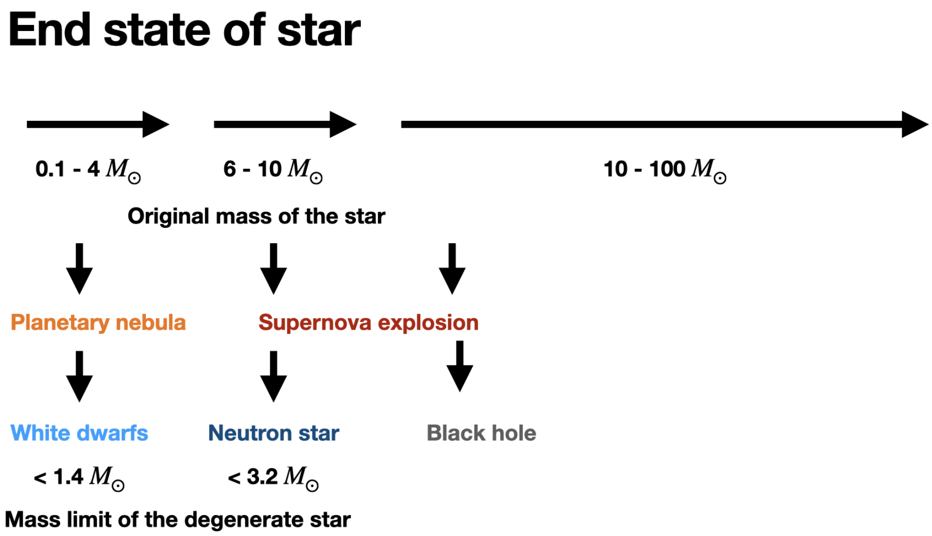

Figure 2: Options for the end state of stars.

7.1 White dwarfs

White dwarfs have degenerate electron gas, which produces degeneracy pressure. The Chandrasekhar mass limit determines the highest theoretical mass that a white dwarf can have. A white dwarf with a Chandrasekhar mass would have zero radius.

Typical white dwarfs have about the mass of the Sun in a 10000 km radius (like a planet).

7.2 Neutron stars

Neutron stars have degenerate neutron gas, which produces degeneracy pressure. **Neutron drip**: neutrons drip out of atoms at very high pressure. $e^- + p^+ \to n + \nu$

Typical neutron stars have about the mass of the Sun in a 1000 km radius (like a city).

Pulsars - rotating neutron stars with jets that periodicly point towards Earth

binary pulsars $\to$ indirect evidence of gravitational waves

millisecond pulsars - mass transfer from a companion star spins up the pulsar, also detected as X-ray binary sources.