- Astro summary.

Stellar models - observed properties of stars, models vs. observation
Stellar evolution

Stellar models:
What is a star: self-graviteting bodys of dynamical equilibrium
internal pressure $\leftrightarrow$ gravity
hydrostatic equilibrium
- loose energy by radiation $\rightarrow$ need energy
production: nuclear energy, gravitational energy
- temperature structure $\rightarrow$ infulences energy
transport: radiation, conduct
convection

Basic equations of stellar structure:

(1) mass conservation $\frac{dM_r}{dr} = 4\pi r^2 \rho$

(2) hydrostatic equilibrium: $\frac{dP}{dr} = -\frac{GM_r}{r^2}\rho$

↳ assuming perfect gas: $\rightarrow$ calculte P and T

Virial theorem: thermal energy $\leftrightarrow$ balances gravity

$$2E_T + E_G = 0$$

(3.) ~~Energy transport~~ - Energy conservation

$$\frac{dL}{dr} = 4\pi r^2 \rho \varepsilon$$

(4.) Energy transport

$$\frac{dT}{dr} = -\frac{3}{4a_B c}\frac{\chi\rho}{T^3}\frac{L_r}{4\pi r^2}$$ radiative E transport

$$\frac{dT}{dr} = \left(1-\frac{1}{\gamma}\right)\frac{T}{P}\frac{dP}{dr}$$ convective E transport
γ - adiabatic pressure

↳ Schwarzschild stability condition criteria

$$\left|\frac{dT}{dr}\right| < \left(1-\frac{1}{\gamma}\right)\frac{T}{P}\left|\frac{dP}{dr}\right|$$

if the temperature gradient is steeper than this than the atmosphere is unstable
$\rightarrow$ ~~and~~ convection

→ composition of the star is important $\kappa, \mu, \varepsilon \; (\rho, T, X_i)$

→ adding boundary conditions for the inside and outside of the star

→ solving the equations to get a model

Voigth - Russel theorem of unique solutions → not true, possible degeneracy

⇒ models - simplifications ⇒ $\boxed{\text{relations between different quantities}}$

$P \propto \frac{M^2}{R^2}$ $P \propto \frac{M}{R^3} T$ ⇒ $\boxed{T \propto \frac{M}{R}}$ $\boxed{L \propto M^3}$

mass - luminosity relation → more massive stars → more luminous

$\boxed{M \propto T^2}$ $\boxed{L \propto T_{eff}^6}$ T_{eff} - surface temp.

⇓

L vs. T_{eff} ⇒ Herzsprung Russel diagram

$\tau \propto \frac{M}{L}$ → $\boxed{\tau \propto M^{-2}}$ liftem of a star is related to the mass inversely

Surface temperature → spectral lines → composition → spectral classes (O, B, A, F, G, K, M)

→ colour → blackbody

↓

O, B - young massive

G - Sun

Spectra → doppler shift → moton of the star (binary, planet, etc.)

→ Zeeman effect → magnetic field (e.g. sunpots)

Luminosity - needs distance measurment: - parallax
- globular clusters
- variable stars

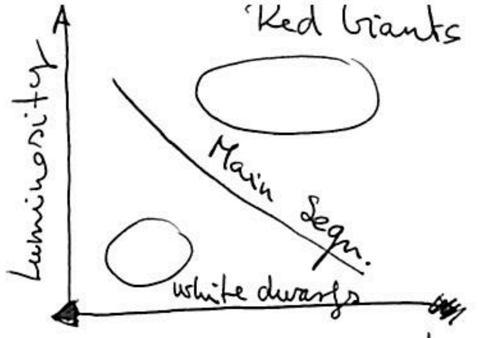
distance → absolute magnitude, abs. bolometric magnitude, luminosity

Stellar mass - binary systems (dynamic measurment)
- eclipsing binarie
- spectroscopic binaries

- Mass - Luminosity relation

⇒ H-R diaram

Herzsprung - Russell diagram



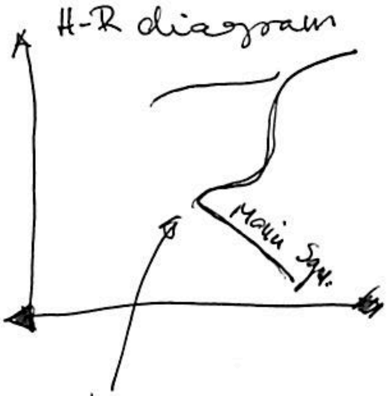
High Mass Low
Temperature
Blue Colour Red
spectral type:

O, B, A, F, G, K, M

⇒ mass limit of stars: 0.1 − 100 $M_\odot$
Lower limit: no fusion : brown dwarf
upper limit: Eddington luminosity limit
(based on radiation pressure)

Star clusters – globular clusters : old, gravitationally bound
– open clusters : young, less bound

↓
born together
↓
same distance → distance measurement with the H-R diagram



⇒ match the main sequence to nearby stars → absolute magnitude → distance

tourn off point - related to age of cluster

Stellar nucleosynthesis:

mass-binding energy relation $E_B = \left[Z m_p + (A-Z) m_n - m_{nuc}\right] c^2$

tightest bound nucleus: Fe → lighter elements fusion } energy release
→ heavier elements fission

Stars: nuclear fusion - quantum mechanics tunneling

reaction rate - related to the cross-section $r = n_1 n_2 \langle \sigma v \rangle$

reaction rate → energy release ΔE, $\rho \varepsilon = r \Delta E = n_1 n_2 \langle \sigma v \rangle \Delta E$

↳ ε - nuclear energy generation function
↳ increases with temperature sharply

→ reaction of heavier nuclei needs higher temperature

- nuclear burning
- nuclear fuel

} main sequence burn H → He

proton-proton chain : $^1H + ^1H \to ^2H + e^+ + \nu$

$^2H + ^1H \to \underline{^3He} + \gamma$

$^3He \to$ pp1, pp2, pp3 chains

pp1 : $^3He + ^3He \to ^4He + H_1 + H_1$ lowest temp

if ^4He & ^7Be & $T > 10^7 K$: $^3He + ^4He \to ^7Be + \nu$

pp2 : $^7Be + e^- \to ^7Li + \nu$

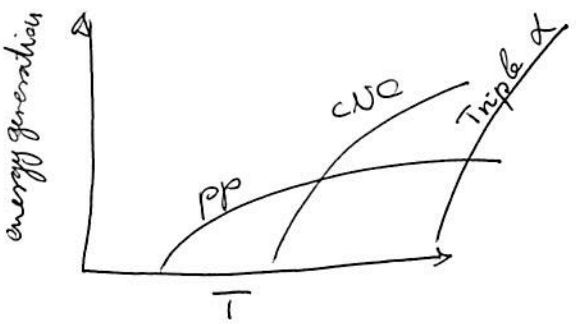
$^7Li + ^1H \to ^4He + ^4He$

pp3 : $^7Be + ^1H \to ^8B + \nu$

$B^8 \to ^8Be + e^+ + \nu$

$^8Be \to ^4He + ^4He$

if C, N, O present CNO-cycle : higher temperature



low temp : pp
low mass

high temp : CNO → more energy
high mass

triple-α reaction $^4He + ^4He + ^4He \to ^{12}C + \gamma$

⇒ heavy elements are synthesised in stars

→ after H finished → He can burn in high mass stars

→ up to Si burning into Fe

heavier elements than Fe → in Supernovas

③ → composition of stars change with time Composition of stars changes with time

→ more massive stars hotter → different temp gradient

→ massive stars : convective core

- low mass stars : convective envelop

testing the inside of stars :

stellar oscillations → sound speed inside → pressure and density

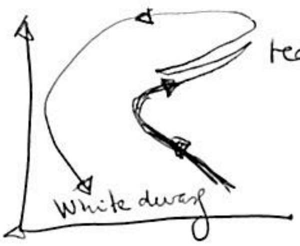
- helio seizmology } test models
- astero seizmology }

→ neutrino production in stars (nuclear fusion)

↳ test with observations → neutrino oscillation

^8B neutrinos depend on solar models

Stellar evolution on the H-R diagram



Binary system: equipotential surface → Roche lobe → mass transfer

Mass loss → stellar winds (solar wind)

- thermal driven wind
- radiative driven wind
- centrifugally driven winds

Extreme case: planetary nebula → White dwarf

Supernova explosion → neutron star or black hole

↳ Type I and Type II.

Type Ia supernova → mass transfer onto degenerate star

↳ identical mass → distance measurments

→ heavier elements than iron in supernova explosions (Type II)

↳ neutron capture + radioactive/β decay

④ solar properties

→ differencial rotation

→ magnetic field – sunspots – magnetogram

→ solar cycle : 22 years → min, max sunspot
+ magnetic field flips polarity

↳ related: solar flares, CMEs ⇒ space weather

Exoplanets: 5 methods to detect:
- astrometry
- transit
- radial velocity
- direct imaging
- gravitational microlensing

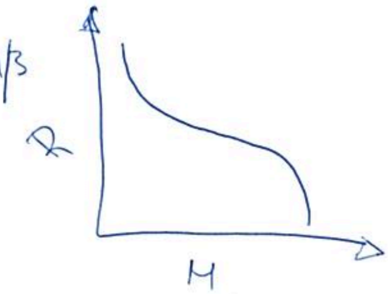
End state of stars

→ white dwarf
→ neutron star } degenerate stars / degenerate matter
→ black hole

WD: degenerate electron gas $R \propto M^{-1/3}$

higher mass → smaller radius

planet sized star



Chandrasekhar mass limit

Neutron star: degenerate neutron gas

to high pressure → neutron drip

star of $M_\odot$ → 1km city sized star

→ pulsars
→ rotation → rotation period + mass ⇒ density
→ associated with supernova remnants
→ pulsar glitches
→ mili second pulsars (double system, mass accretion)
↳ X-ray sources