1)

$$I = \int_a^b f(x)\,dx$$

$f(x)$ se aproxima a un Polinomio interpolador de grado uno:

$$f(x) \approx P_1(x) = \frac{x-b}{a-b} f(a) + \frac{x-a}{b-a} f(b)$$

$$I = \int_a^b P_1(x)\,dx = \int_a^b \frac{x-b}{a-b} f(a) + \frac{x-a}{b-a} f(b)\,dx$$

$$I = \int_a^b \frac{x-b}{a-b} f(a)\,dx + \int_a^b \frac{x-a}{b-a} f(b)\,dx$$

$f(a)$, $f(b)$ y los denominadores de las fracciones son constantes:

$$I = \frac{f(a)}{a-b}\int_a^b x-b\,dx + \frac{f(b)}{b-a}\int_a^b x-a\,dx$$

$$I = \frac{f(a)}{a-b}\left(\frac{x^2}{2} - bx\Big|_a^b\right) + \frac{f(b)}{b-a}\left(\frac{x^2}{2} - ax\Big|_a^b\right)$$

$$I = \frac{f(a)}{a-b}\left(\frac{b^2}{2} - \frac{a^2}{2} - b^2 + ba\right) + \frac{f(b)}{b-a}\left(\frac{b^2}{2} - \frac{a^2}{2} - ab + a^2\right)$$

$$I = \frac{f(a)}{a-b}\left(\frac{b^2 - a^2 - 2b^2 + 2ba}{2}\right) + \frac{f(b)}{b-a}\left(\frac{b^2 - a^2 - 2ab + 2a^2}{2}\right)$$

$$I = \frac{f(a)}{-(b-a)}\left(\frac{-(b-a)^2}{2}\right) + \frac{f(b)}{(b-a)}\left(\frac{(b-a)^2}{2}\right)$$

$$I = \left(\frac{b-a}{2}\right)\left(f(a) + f(b)\right)$$