5) A partir de las definiciones de primera y segunda derivada central:

$$f'(x_i) = \frac{f(x_{i+1}) - f(x_{i-1})}{2h}$$

$$f''(x_i) = \frac{f(x_{i+2}) - 2f(x_i) + f(x_{i-2})}{4h^2}$$

Al derivar $f''(x_i)$ se obtiene la expresión de la tercera derivada:

$$f'''(x_i) = \frac{f'(x_{i+2}) - 2f'(x_i) + f'(x_{i-2})}{4h^2}$$

$$f'''(x_i) = \frac{\left(\frac{f(x_{i+3}) - f(x_{i+1})}{2h}\right) - 2\left(\frac{f(x_{i+1}) - f(x_{i-1})}{2h}\right) + \left(\frac{f(x_{i-1}) - f(x_{i-3})}{2h}\right)}{4h^2}$$

$$f'''(x_i) = \frac{\frac{f(x_{i+3}) - 3f(x_{i+1}) + 3f(x_{i-1}) - f(x_{i-3})}{2h}}{4h^2}$$

$$f'''(x_i) = \frac{f(x_{i+3}) - 3f(x_{i+1}) + 3f(x_{i-1}) - f(x_{i-3})}{8h^3}$$

Al derivar esta expresión nuevamente se obtiene la cuarta derivada:

$$f^{IV}(x_i) = \frac{f'(x_{i+3}) - 3f'(x_{i+1}) + 3f'(x_{i-1}) - f'(x_{i-3})}{8h^3}$$

$$f^{IV}(x_i) = \frac{\left(\frac{f(x_{i+4}) - f(x_{i+2})}{2h}\right) - 3\left(\frac{f(x_{i+2}) - f(x_i)}{2h}\right) + 3\left(\frac{f(x) - f(x_{i-2})}{2h}\right) - \left(\frac{f(x_{i-2}) - f(x_{i-4})}{2h}\right)}{8h^3}$$

$$f^{IV}(x_i) = \frac{f(x_{i+4}) - 4f(x_{i+2}) + 6f(x_i) - 4f(x_{i-2}) + f(x_{i-4})}{16h^4}$$

$$f^{IV}(x_i) = \frac{f(x_i+4h) - 4f(x_i+2h) + 6f(x_i) - 4f(x_i-2h) + f(x_i-4h)}{(2h)^4}$$

Dejando h sin coeficiente en el denominador:

$$2h \rightarrow h$$

$$f^{IV}(x_i) = \frac{f(x_i+2h) - 4f(x_i+h) + 6f(x_i) - 4f(x_i-h) + f(x_i-2h)}{h^4}$$

$$f^{IV}(x_i) = \frac{f(x_{i+2}) - 4f(x_{i+1}) + 6f(x_i) - 4f(x_{i-1}) + f(x_{i-2})}{h^4}$$