Maple Coursework Assignment

Task: you are asked to optimize a measurement procedure such that the total error of the entity of interest is minimized given the constraint of total measurement time.

Information: The write-up should be word processed. The analysis has to be done with the help of Maple. Any graphics have to be created using Maple. A **copy of the Maple file(s)** is required on a disk or CD. The programme should run without interruptions or errors by using the !!!-button! Note that **e-mailed versions are not acceptable**. Your write-up should contain

- **A main text,** including a one page summary of your analysis and results. This needs to be followed by a brief c**onclusion.**
- **References** (if you wish to draw on material of books, articles , web-sites (not recommended as these are not proper references) etc. you need to reference those)
- **An Appendix:** this should contain a paper copy of the programme code!

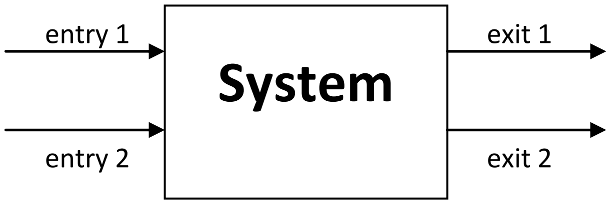
Deadline: The deadline for the submission of the coursework will be **Friday, 16th December 2011**, in the General Office W1.88 (Vici Webster). Please remember to use a cover sheet for your submission.

You will have to submit your coursework with a cover sheet. This is an essential part of the coursework. There you sign that the coursework **is your own and that any other sources are clearly indicated**. The University takes copying and plagiarism very seriously. As a consequence if you copy your coursework you may have to face serious consequences. Therefore: **Be honest!**

You should ensure that your **CD/disk, report and any other material is properly bound together**. Loose items cannot be accepted.

Optimization of measurement conditions

Consider a system which has two entry channels and two exit channels. This is graphically presented in the figure below.



An example of such a system could be a neutron scattering experiment which employs spin polarised neutrons. The entries are determined by the neutron spin direction (either up or down) and the exit channels also involve the counting of either spin-up or spin-down neutrons. If a sample is used which does not reverse the neutron spin, then one expects the neutrons to enter in entry channel one and to leave via exit channel 1. However, due to imperfections in the system there are a small number of neutrons which will exit with their spins reversed, i.e. via exit channel 2.

In order to characterise the imperfection of the instrument, or the system, it is meaningful to define a **ratio R**, which is defined by the number of neutrons exiting via the 'correct' channel over the number of neutrons exiting the instrument via the 'wrong' channel. Experimentally both numbers can be measured separately. A large value of R indicates a 'good' system while low values of R are indicative of loss of neutron spin polarisation (i.e. neutron depolarisation) in the system.

If the system is good then the number of neutrons in the correct exit channel will be high, and the number of wrong neutrons will be low. Each, of course, is proportional to the number of incident neutrons. The measurement process for determining R can be designed as follows:

Set up the system such that input is only via one channel. Measure the output, i.e. the number of neutrons in exit channel 1, $N_1 = N_1(\Delta T_1)$, counted for a time ΔT_1. Then measure neutrons exiting the system via exit channel 2, $N_2 = N_2(\Delta T_2)$, for a time ΔT_2. The ratio R is the ratio of the fluxes in these two channels, where flux is the number of neutrons divided by the time in which these were counted. This results in our entity of interest here, R, being given as:

$$R_1 = \frac{N_1(\Delta T_1)}{N_2(\Delta T_2)}$$

The error in R, δR, is determined by the counting statistics, i.e. the error in N_i, $\delta N_i = \sqrt{N_i}$, with i=1, 2. The error in the time intervals can be neglected in view of the dominance of the errors due to statistics of a neutron beam (counting statistics).

Task:

Given that the experimenter designates a particular total time interval, $\Delta T = \Delta T_1 + \Delta T_2$ e.g. an hour, for the measurement of this ratio, determine how he/she has to distribute the measurement time between exit channels 1 and 2 in order to determine R such that it has the smallest possible error. The error is subject to the constraint of having the total measurement time fixed. Find an expression for δR, the error in R, in terms of $N_1(\Delta T_1)$ and $N_2(\Delta T_2)$, that is ΔT_1 and ΔT_2 and solve the problem algebraically using Maple.

This assignment consists of writing appropriate Maple code to obtain the algebraic answer to the above question. It is important that you are able to justify your approach, the approximations used, and to clearly argue in a logical and convincing manner how the above problem is addressed. Once you have obtained an answer, and clearly stated (and justified) it in your write-up, ensure that you have clearly given all relevant information for the calculation of the error in R, δR. Justify your formulae and approach using suitable physics based arguments. In your write-up address the following questions:

- Do either R or δR depend on the flux of particles into the system? (Please state what happens to the values of R and δR when the number of incident particles in entry channel 1 is increased by a factor of n).
- Does the optimal condition depend on the value of R?
- What happens to δR when the total time interval is doubled?

Also demonstrate explicitly that the equations you use

- conserves the particle numbers (i.e. the number of particles going into the system is the same as those coming out)
- actually yield the ratio R for all possible time interval partitionings.

Your write-up should also include the following plots (even if these plots should turn out to be trivial):

- A plot of the error of R, for various values of R (say R= 5, 20 and 50) as a function of the partition variable p (defined as $p = \frac{\Delta T_1}{\Delta T_1 + \Delta T_2}$)
- A plot of the optimal value of p as a function of the value of R