Hoja de trabajo 3

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1 Ejercicio 1

$$n \oplus m := \begin{cases} m & \text{si } n = o \\ n & \text{si } m = o \\ s(i \oplus m) & \text{si } n = s(i) \end{cases}$$

$$s(s(s(0))) \oplus s(s(s(s(0))))$$

$$s(s(s(s(0))) \oplus s(s(s(0))))$$

$$s(s(s(s(s(0)))) \oplus s(s(0))))$$

$$s(s(s(s(s(s(0))))) \oplus s(0))))$$

$$s(s(s(s(s(s(s(0)))) \oplus 0))))$$

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$$s(s(s(s(s(s(s(s(0))))))))$$

2 Ejercicio 2

$$n \otimes m := \begin{cases} 0 & \text{si } n = o \\ 0 & \text{si } m = o \\ m & \text{si } n = 1 \\ n & \text{si } m = 1 \\ s(i) \otimes s(j) & \text{si } s(i) \oplus (s(i) \otimes j) \end{cases}$$

3 Ejercicio 3

1. $s(s(s(0))) \otimes 0$

$$s(s(s(0))) \otimes 0 = 0$$

2. $s(s(s(0))) \otimes s(0)$

$$s(s(s(0))) \otimes s(0) = s(s(s(0))) \oplus (s(s(s(0))) \otimes 0) = s(s(s(0)))$$

3. $s(s(s(0))) \otimes s(s(0))$

$$s(s(s(0))) \oplus (s(s(s(0))) \otimes s(0))$$

$$s(s(s(0))) \oplus s(s(s(0)))$$

$$s(s(s(s(0))) \oplus s(s(0)))$$

$$s(s(s(s(s(0))) \oplus s(0)))$$

$$s(s(s(s(s(s(0))) \oplus 0)))$$

$$s(s(s(s(s(s(0 \oplus 0))))))$$

$$s(s(s(s(s(s(0))))))$$

4 Ejercicio 4

1. $a \oplus s(s(0)) = s(s(a))$

Caso base: $a = 0$

$$0 \oplus s(s(0)) = s(s(0))$$

$$s(s(0)) = s(s(0))$$

Caso inductivo: $a = s(i)$

$$s(i) \oplus s(s(0)) = s(s(s(i)))$$

$$s(s(s(i \oplus 0))) = s(s(s(i)))$$

$$s(s(s(i))) = s(s(s(i)))$$

2. $a \otimes b = b \otimes a$

caso base: $a = 0$

$$0 \otimes b = b \otimes 0$$

$$0 = 0$$

Caso inductivo: $a = s(i)$

$$s(i) \otimes b = b \otimes s(i)$$

$$s(i) \oplus (s(i) \otimes b) = s(i) \oplus (s(i) \otimes b)$$

$$s(i) \otimes b = s(i) \otimes b$$

3. $a \otimes (b \otimes c) = (a \otimes b) \otimes c$

Caso base: $a = 0$

$$0 \otimes (b \otimes c) = (0 \otimes b) \otimes c$$

$$0 \otimes (bc) = (0) \otimes c$$

$$0 = 0$$

Caso inductivo: $a = s(i)$

$$s(i) \otimes (b \otimes c) = (s(i) \otimes b) \otimes c$$

$$s(i) \oplus (s(i) \otimes (b \otimes c)) = (s(i) \oplus (s(i) \otimes b)) \otimes c$$

$$s(i) \oplus (s(i) \otimes (b \otimes c)) = s(i) \oplus (s(i) \otimes (b \otimes c))$$

4. $(a \otimes b) \otimes c = (a \otimes c) \oplus (b \otimes c)$

Caso base: $c = 0$

$$(a \otimes b) \otimes 0 = (a \otimes 0) \oplus (b \otimes 0)$$

$$(ab) \otimes 0 = (0) \oplus (0)$$

$$0 = 0$$

Método inductivo: $c = n \oplus 1$

$$(a \otimes b) \otimes (n \oplus 1) = (a \otimes (n + 1)) \oplus (b \otimes (n \oplus 1))$$

$$(a \otimes (n \oplus 1) \oplus (b \otimes (n \oplus 1)) = (an \oplus a) \oplus (bn \oplus b)$$

$$(an \oplus a) \oplus (bn \oplus b) = (an \oplus a) \oplus (bn \oplus b)$$

$$an \ominus an \oplus bn \ominus bn \oplus a \ominus a \oplus b \ominus b = 0$$

$$0 = 0$$