Formulas

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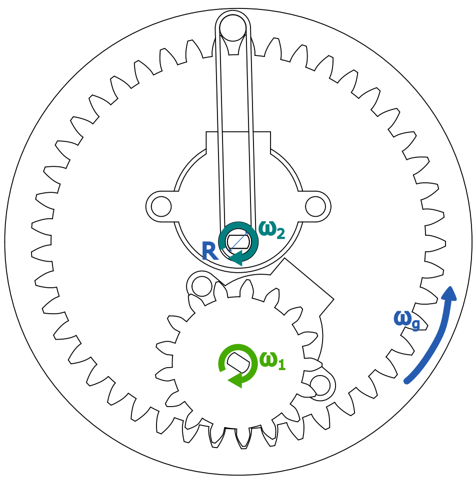
This document will talk about the physical analysis used to control two motors in a polar coordinate system. Each motor generates motion: one for the radius, and the other one for the angle.
The main objective is to determine two independent functions that calculate the time required based on $\Delta\theta$ and Δr. This analysis assumes: Constant angular velocity in both motors, identical radius on both axis of the belt, perfect transmission between gearbox, perfect transmission between the belt and the pulleys.

1 Inputs

- Angular Motor: The motor responsible for changing the angle.
- Radial Motor: The motor that adjusts the radius of the system.
- Zp : Number of teeth on the pinion.
- Zn : Number of teeth on the gear.
- R : Radius of the axis
- ω_1 : Angular velocity of the angular motor.
- ω_2 : Angular velocity of the radial motor.
- ω_g : Angular velocity of the gear.



2 Angular Motor

To calculate the time the motor needs to be powered on, we will use the angular velocity of the gear as follows:

$$\omega_g = \Delta\theta/t \tag{1}$$

Now the ω_g is equal to the ration of the gear teeth as a reduction gearbox, multiplied by ω_1. By substituting this into equation (1) and solving for t, we get:

$$\begin{aligned} \frac{\Delta\theta}{t} &= (zp/zn)\cdot\omega_1 \\ t &= \frac{zn}{zp\cdot\omega_1}\Delta\theta \end{aligned} \tag{2}$$

3 Radial Motor

To calculate the time the motor needs to remain powered on to move a point r_i along a belt to a target point r_f we need the radius of the axis circle, R.
Using the formula $\Delta r = 2\pi R \cdot \mathit{revolutions}$ and the relationship between a revolution and the angular velocity of the radial motor (ω_2) we can state the following:

$$\begin{aligned} \Delta r &= 2R\pi \cdot \mathit{revolutions} \\ \mathit{revolutions} &= (\omega_2 \cdot t)/2\pi \end{aligned} \tag{3}$$

Thus, solving for t, we obtain:

$$\begin{aligned} \Delta r &= 2R\pi \cdot \frac{\omega_2 \cdot t}{2\pi} \\ \Delta r &= R \cdot \omega_2 \cdot t \\ t &= \frac{\Delta r}{\omega_2 \cdot R} \end{aligned} \tag{4}$$

4 Code implementation with Julia

```julia
const zp, zn = 17, 45
const relation = zp / zn
const R = 5 # mm
const w1 = 2 # deg/s
const w2 = 1 # deg/s
Iposition = (0, 0) # Initial position (mm, deg)
Fposition = (30, 10) # Final position (mm, deg)
function times(Iposition, Fposition)
  r = Fposition[1] - Iposition[1] # Delta r
  time_r = (r) / (w2 * R)
  theta = Fposition[2] - Iposition[2] # Delta theta
  t_theta = theta / (relation * w1)
  return (time_r, t_theta)
end
println(times(Iposition, Fposition))
```

```
(3.0, 13.23529411764706)
```