

# Formulas

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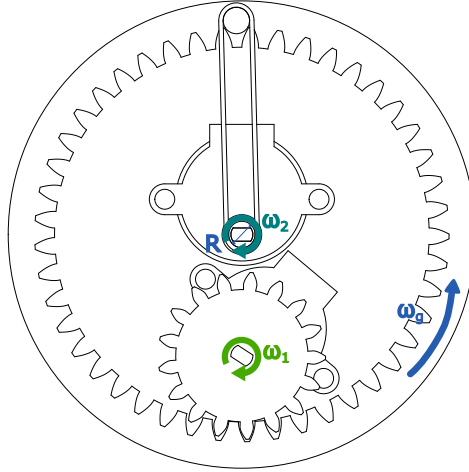
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This document will talk about the physical analysis used to control two motors in a polar coordinate system. Each motor generates motion: one for the radius, and the other one for the angle.

The main objective is to determine two independent functions that calculate the time required based on  $\Delta\theta$  and  $\Delta r$ . This analysis assumes: Constant angular velocity in both motors, identical radius on both axis of the belt, perfect transmission between gearbox, perfect transmission between the belt and the pulleys.

## 1 Inputs

- Angular Motor: The motor responsible for changing the angle.
- Radial Motor: The motor that adjusts the radius of the system.
- $Z_p$  : Number of teeth on the pinion.
- $Z_n$  : Number of teeth on the gear.
- $R$  : Radius of the axis
- $\omega_1$  : Angular velocity of the angular motor.
- $\omega_2$  : Angular velocity of the radial motor.
- $\omega_g$  : Angular velocity of the gear.



## 2 Angular Motor

To calculate the time the motor needs to be powered on, we will use the angular velocity of the gear as follows:

$$\omega_g = \Delta\theta/t \quad (1)$$

Now the  $\omega_g$  is equal to the ration of the gear teeth as a reduction gearbox, multiplied by  $\omega_1$ . By substituting this into equation (1) and solving for  $t$ , we get:

$$\begin{aligned} \frac{\Delta\theta}{t} &= (z_p/z_n) \cdot \omega_1 \\ t &= \frac{z_n}{z_p \cdot \omega_1} \Delta\theta \end{aligned} \quad (2)$$

### 3 Radial Motor

To calculate the time the motor needs to remain powered on to move a point  $r_i$  along a belt to a target point  $r_f$  we need the radius of the axis circle,  $R$ . Using the formula  $\Delta r = 2\pi R \cdot \text{revolutions}$  and the relationship between a revolution and the angular velocity of the radial motor ( $\omega_2$ ) we can state the following:

$$\begin{aligned}\Delta r &= 2R\pi \cdot \text{revolutions} \\ \text{revolutions} &= (\omega_2 \cdot t) / 2\pi\end{aligned}\tag{3}$$

Thus, solving for  $t$ , we obtain:

$$\begin{aligned}\Delta r &= 2R\pi \cdot \frac{\omega_2 \cdot t}{2\pi} \\ \Delta r &= R \cdot \omega_2 \cdot t \\ t &= \frac{\Delta r}{\omega_2 \cdot R}\end{aligned}\tag{4}$$

### 4 Code implementation with Julia

```
const w1 = deg2rad(50) # rad/s
const w2 = 15 # deg/s

Fposition = (20, 10) # Final position (mm, deg)
function time_radious_motor(r)
    time_r = (r) / (w1 * R)
    return time_r
end
function time_angle_motor(theta)
    t_theta = theta / (relation * w2)
    return t_theta
end
function times(final_position)
    return (time_radious_motor(final_position[1]), time_angle_motor(
        final_position[2]))
end
result = times(Fposition)
println("Seconds")
println("Time_to_go_to_the_target_position:
        Radial_Motor: $(result[1])
        Angular_Motor: $(result[2])"
)

(3.0, 13.23529411764706)
```