Parametric Statistics

Week 1 - Probability Review

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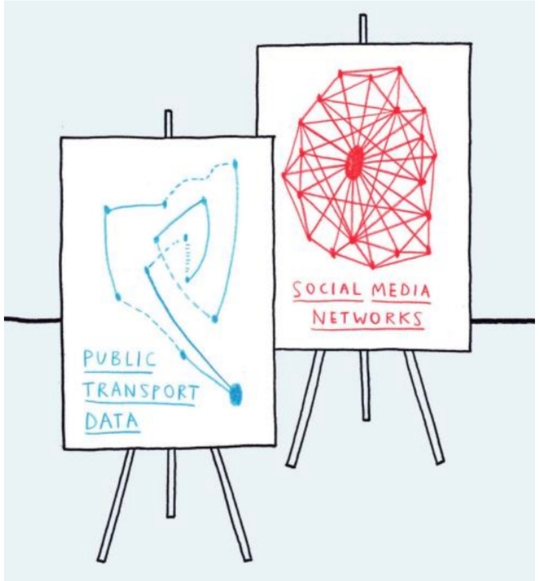


Definitions

Probability: How probable is an event of uncertain nature to take place?
Example: Probability that it rains next Tuesday

In a **random experiment**, S is the space of possible outcomes. We refer to an outcome as an event E.
Example Throwing a dice. S = {1,2,3,4,5,6} and getting a 4. E = {4}.

Random variable: They are not variables per se... They are *functions* that map outcomes to real values.
Example With the dice, there are 6 possible outcomes, BUT we can define a random variable in any way we want.
$X = \{1, 2, 3, 4, 5, 6\}$ is an identity function, another example is

$$X = \begin{cases} 0, if \quad outcome \quad is \quad odd \\ 1, if \quad outcome \quad is \quad even \end{cases}$$

Definitions

Sets: The curly brackets ($\{\}$) refer to a set.
Example $A = \{1,2,3\}, B = \{2,3,4,5\}$

- $A' = \{4,5,6\}$
- $A \cup B = \{1,2,3,4,5\}$
- $A \cap B = \{2,3\}$

Example New random variable $X = \{A, B\}$

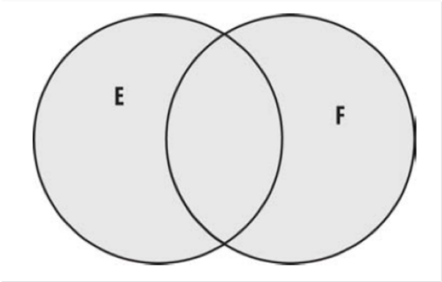
shortcut From now on, we write $P(X = A)$ as $P(a)$

- $P(A) = 3/6 = 1/2$
- $P(B) = 4/6 = 2/3$
- $P(A \cup B) = 5/6$
- $P(A \cap B) = 2/6 = 1/3$
- $P(A') = 1/2$

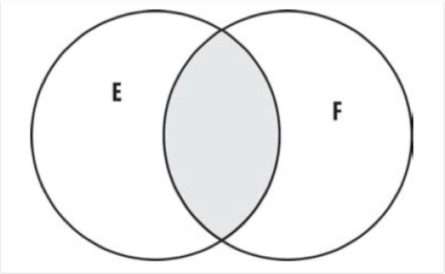
Axioms of Probability

$$P : \mathscr{F} \rightarrow [0,1]$$

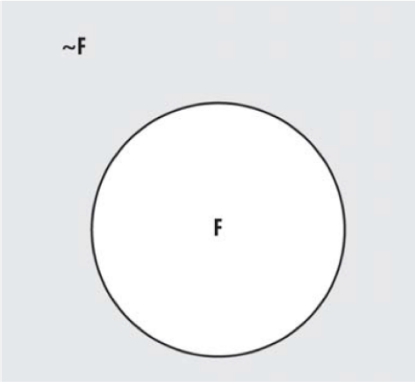
- $P(\alpha) >= 0$
- $P(S) = 1$
- $P(\alpha \cup \beta) = P(\alpha) + P(\beta) - P(\alpha \cap \beta)$
- $P(\alpha) + P(\alpha') = 1$



E ∪ F



E ∩ F



Probability Rules

shortcut From now on, we write $P(A \cap B)$ as $P(A, B)$

- Conditional Probability

$$P(X \mid Y) = \frac{P(X, Y)}{P(Y)}$$

- Multiplication

$$P(X, Y) = P(X \mid Y) * P(Y)$$

  - If X and Y are independent :

$$P(X, Y) = P(X) * P(Y)$$

- Total Probability

$$P(X) = \sum_i P(X, Y = i)$$

- Bayes Rule

$$\boxed{P(Y \mid X) = \frac{P(X|Y)P(Y)}{\sum_i P(X|Y_i) * P(Y_i)}}$$

Bayes Rule Example

Last semester 29 students presented the exam on Statistics, some of them went to the R tutorial (T), others did not (NT). Some students passed the exam (E), others did not (NE). This table summarizes the data:

	E	NE
T	21	4
NT	1	3

What is the probability of passing the exam, if you went to the tutorial? $P(E \mid T)$?

$$P(E \mid T) = \frac{P(T \mid E) * P(E)}{P(T \mid E) * P(E) + P(T \mid NE) * P(NE)}$$

$$P(E \mid T) = \frac{\frac{21}{22} * \frac{22}{29}}{\frac{21}{22} * \frac{22}{29} + \frac{4}{7} * \frac{7}{29}} = \frac{21}{25}$$

The same value can be obtained directly from the table without Bayes. This is only to show that Bayes works!

Probability Distributions

Maps the outcomes of a random variable to values between zero and one.

Discrete Distributions:

$$\sum P(X) = 1$$

Continuous Distributions:

$$\int_{-\infty}^{\infty} P(X) = 1$$

Probability distributions and frequency distributions are different! (Why?) Same difference as between histograms and density plots.

Throughout the course: Review of different probability distributions.

Probability Review

Literature

- Probability Theory Review for Machine Learning (on Moodle)
- Statistics in a Nutschell- Ch. 2