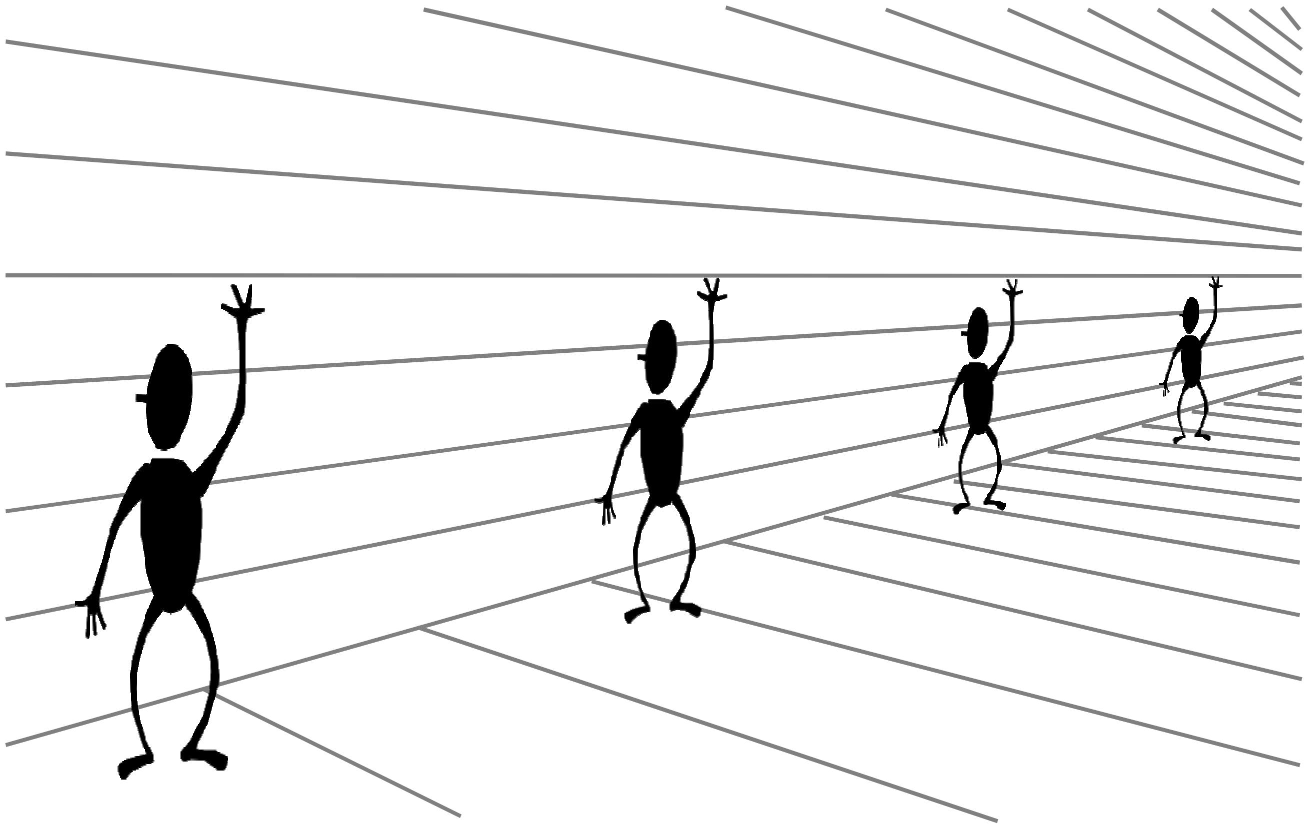
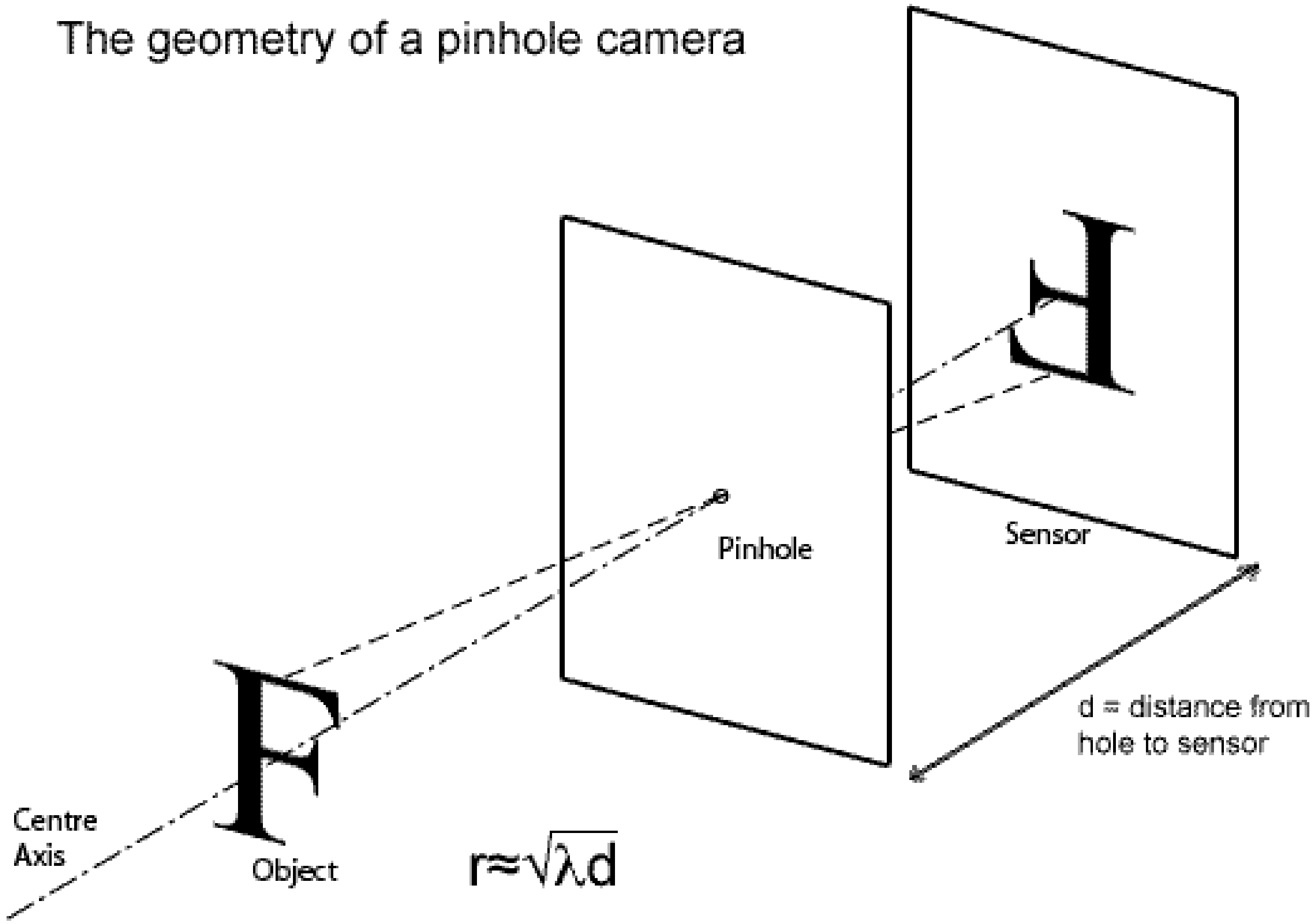
3D Perspectives
Single View Metrology

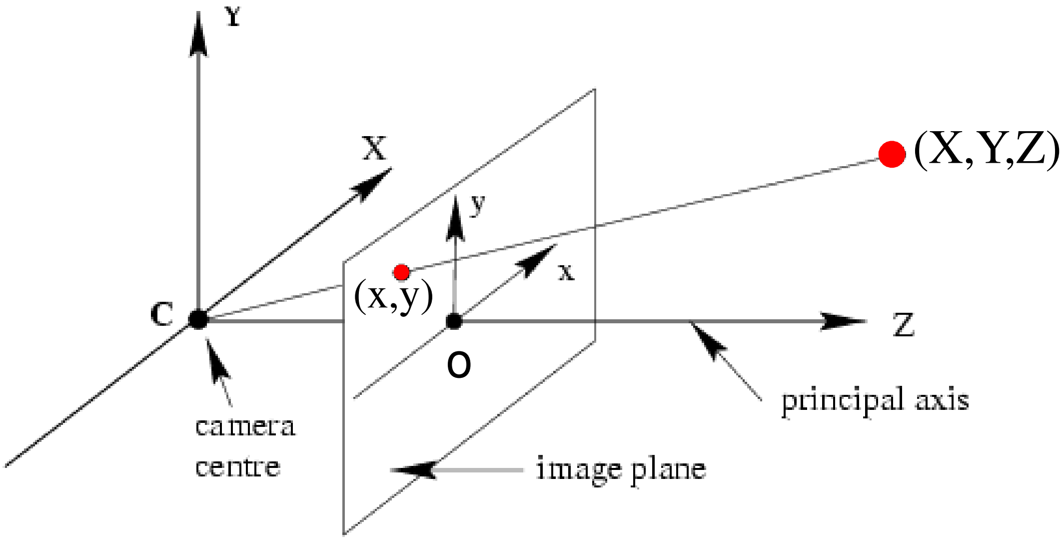


Many slides from Svetlana Lazebnik

Pinhole camera



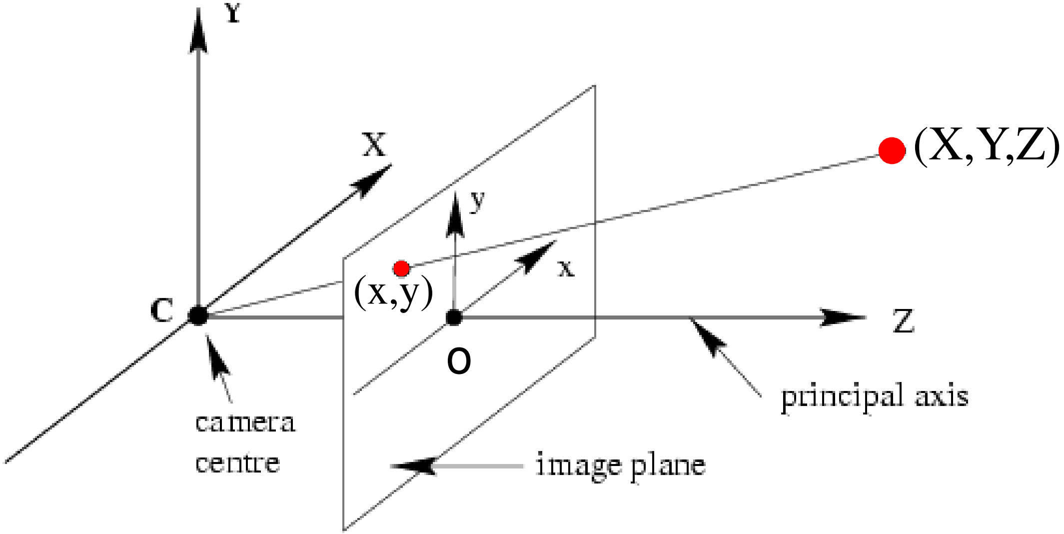
Modeling projection

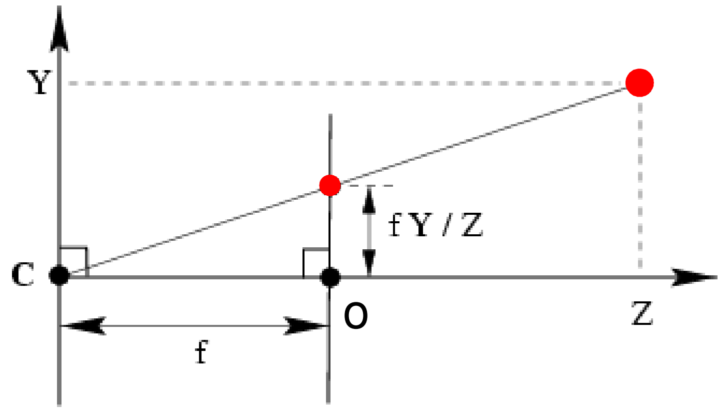


The coordinate system

- The optical center **C** (**C**enter **O**f **P**rojection) is at the origin
- Image plane (**P**rojection **P**lane) is *in front* of the COP
- The principle point **o** is at center of Image.
- **f** is the camera focal length.

Modeling projection





$$(X, Y, Z)^T \mapsto (fX / Z, fY / Z)^T$$

Recap: Homogeneous Coordinates

- Homogeneous Coordinates is a mapping from R^n to R^{n+1}:

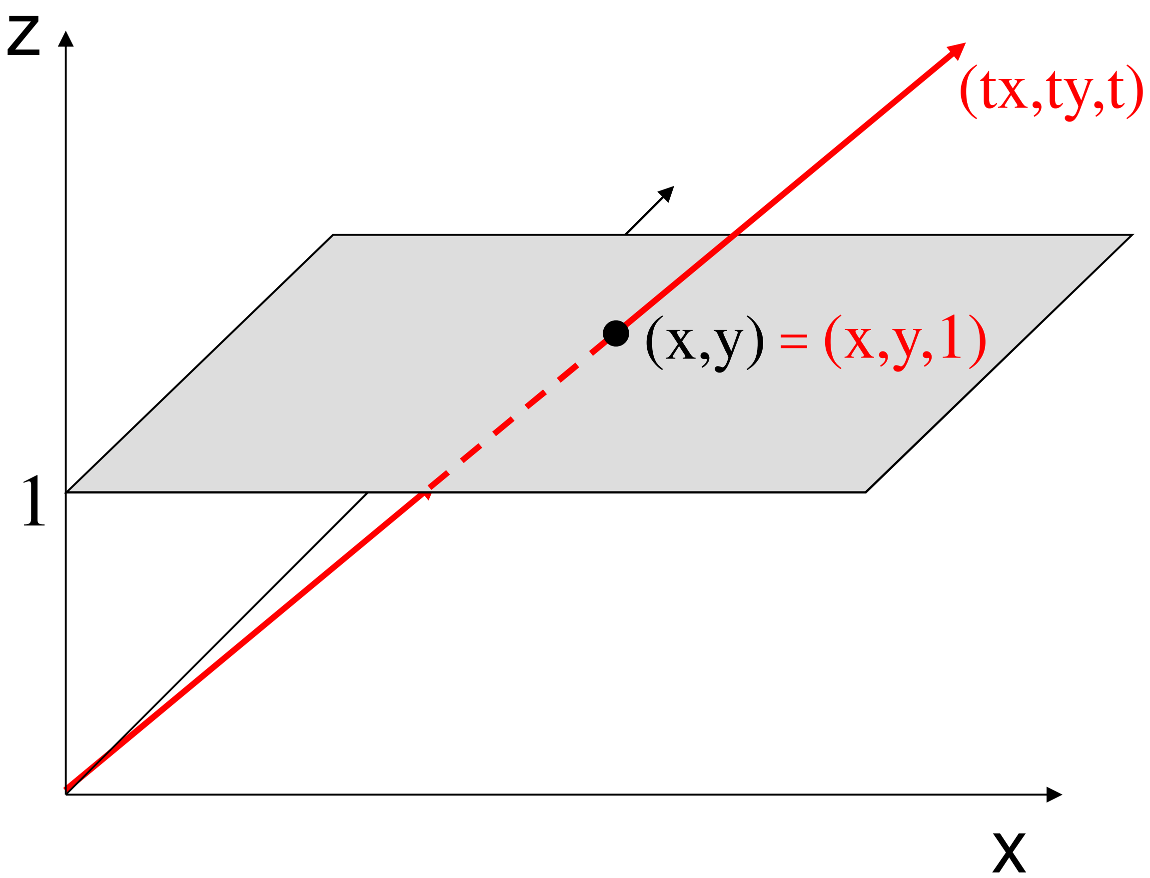
$$\boxed{(x, y) \rightarrow (X, Y, W) \equiv (tx, ty, t)}$$

- Note: (tx,ty,t) all correspond to the same non-homogeneous point (x,y). E.g. (2,3,1)≡(6,9,3) ≡(4,6,2).

- Inverse mapping:

$$(X, Y, W) \rightarrow \left(\frac{X}{W}, \frac{Y}{W}\right) = (\mathrm{x}, \mathrm{y})$$

Homogeneous Coordinates



$$(x, y) \rightarrow (x, y, 1) \equiv (tx, ty, t)$$

$$(X, Y, W) \rightarrow \left(\frac{X}{W}, \frac{Y}{W}\right) = (\mathrm{x}, \mathrm{y})$$

Recap: Homogeneous Coordinates

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

homogeneous image coordinates

$$(X, Y, Z) \Rightarrow \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

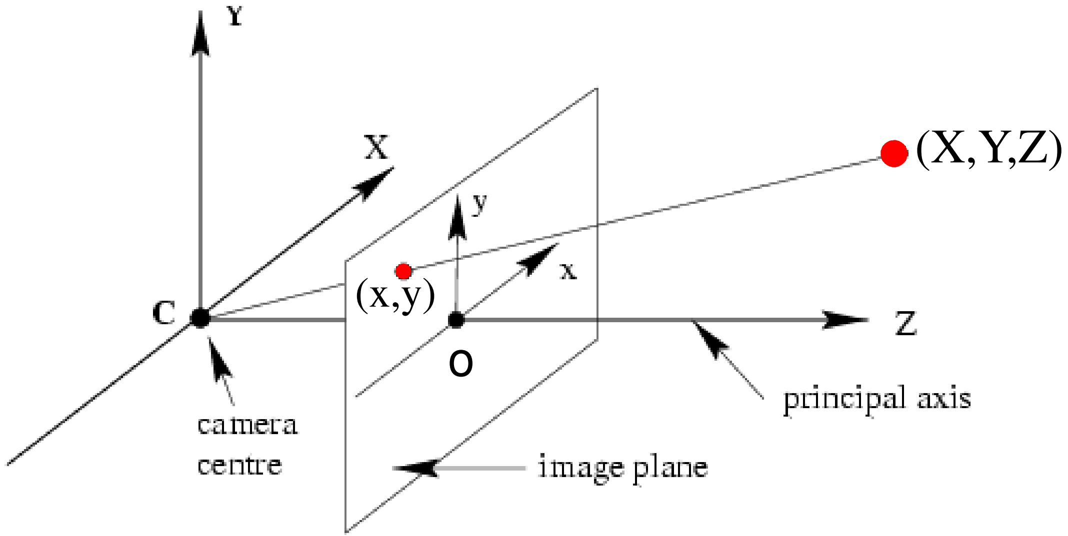
homogeneous scene coordinates

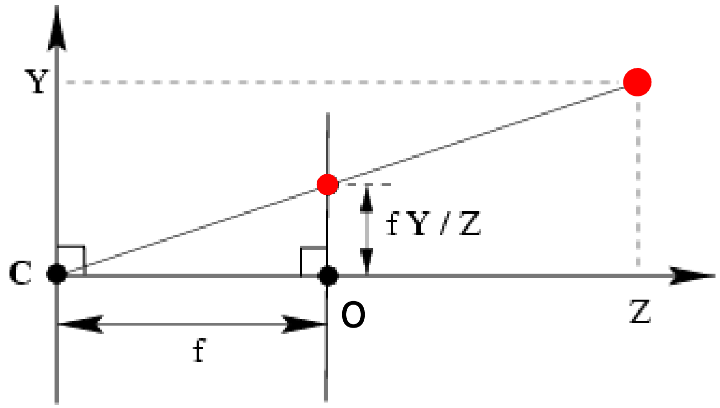
Converting *from* homogeneous coordinates

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w)$$

$$\begin{bmatrix} X \\ Y \\ Z \\ w \end{bmatrix} \Rightarrow (X/w, Y/w, Z/w)$$

Modeling projection



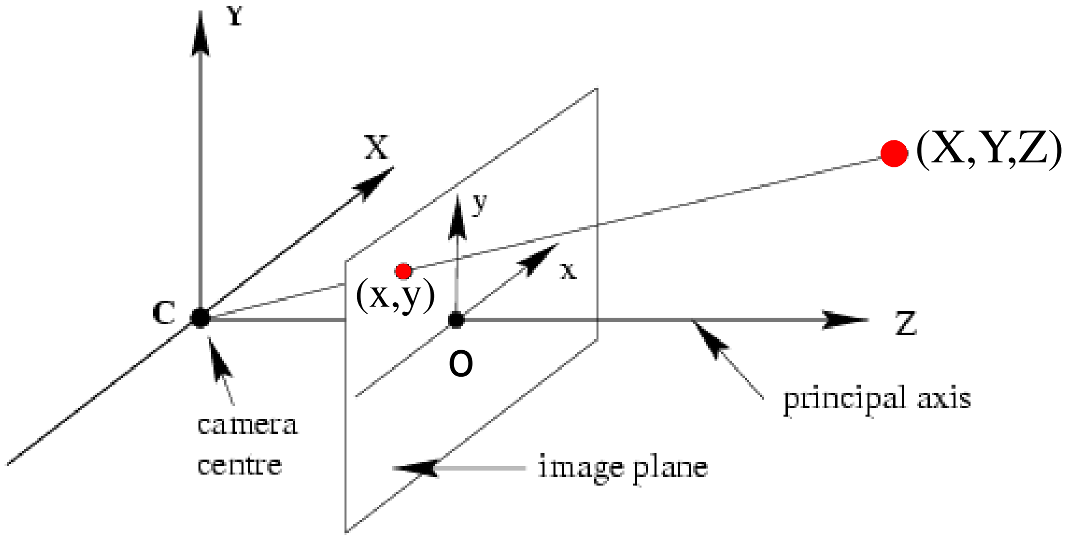


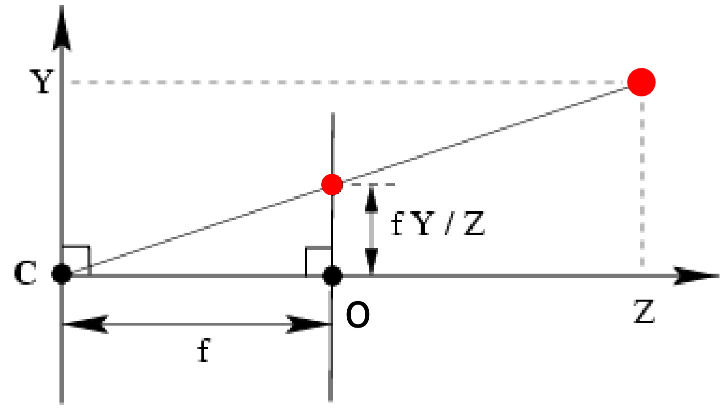
$$(X, Y, Z)^T \mapsto (fX / Z, fY / Z)^T$$

$$\begin{pmatrix} \lambda x \\ \lambda y \\ \lambda \end{pmatrix} = \begin{pmatrix} fX \\ fY \\ Z \end{pmatrix} = \begin{bmatrix} f & & & 0 \\ & f & & 0 \\ & & 1 & 0 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix} \longrightarrow \begin{pmatrix} fX/Z \\ fY/Z \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

Linear projection in homogeneous coordinates!

Modeling projection



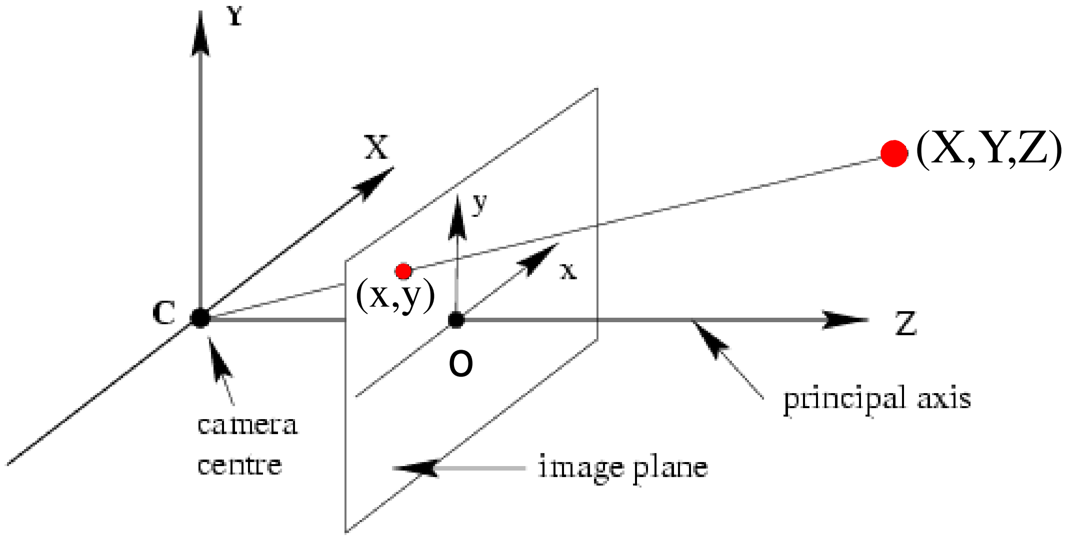


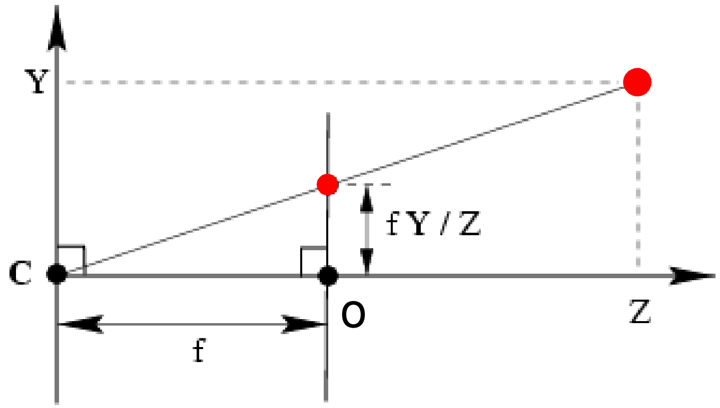
$$(X, Y, Z)^T \mapsto (fX/Z, fY/Z)^T$$

$$\begin{pmatrix} \lambda x \\ \lambda y \\ \lambda \end{pmatrix} = \begin{pmatrix} fX \\ fY \\ Z \end{pmatrix} = \begin{bmatrix} f & & \\ & f & \\ & & 1 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \longrightarrow \begin{pmatrix} fX/Z \\ fY/Z \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

Linear projection in homogeneous coordinates!

Modeling projection





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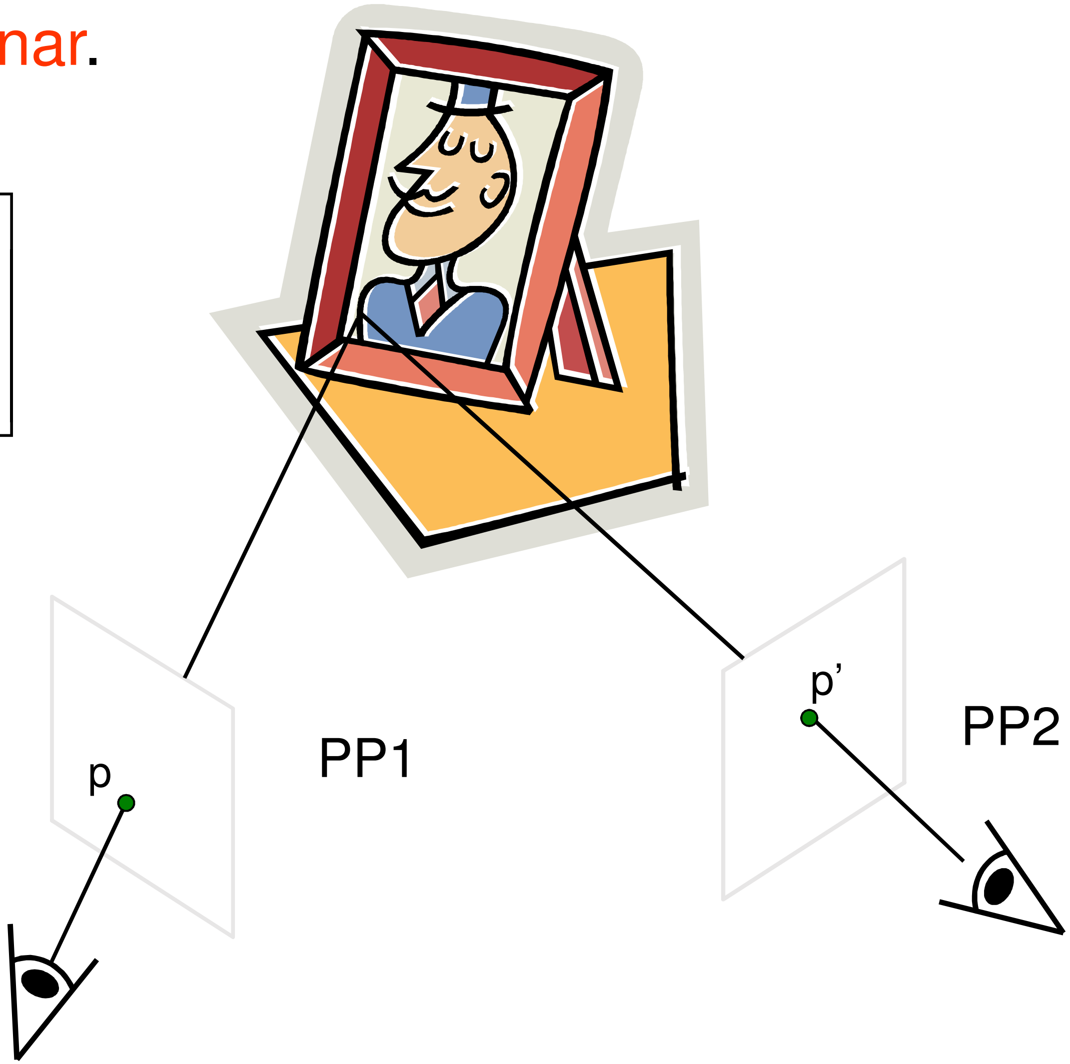
Linear projection in homogeneous coordinates!

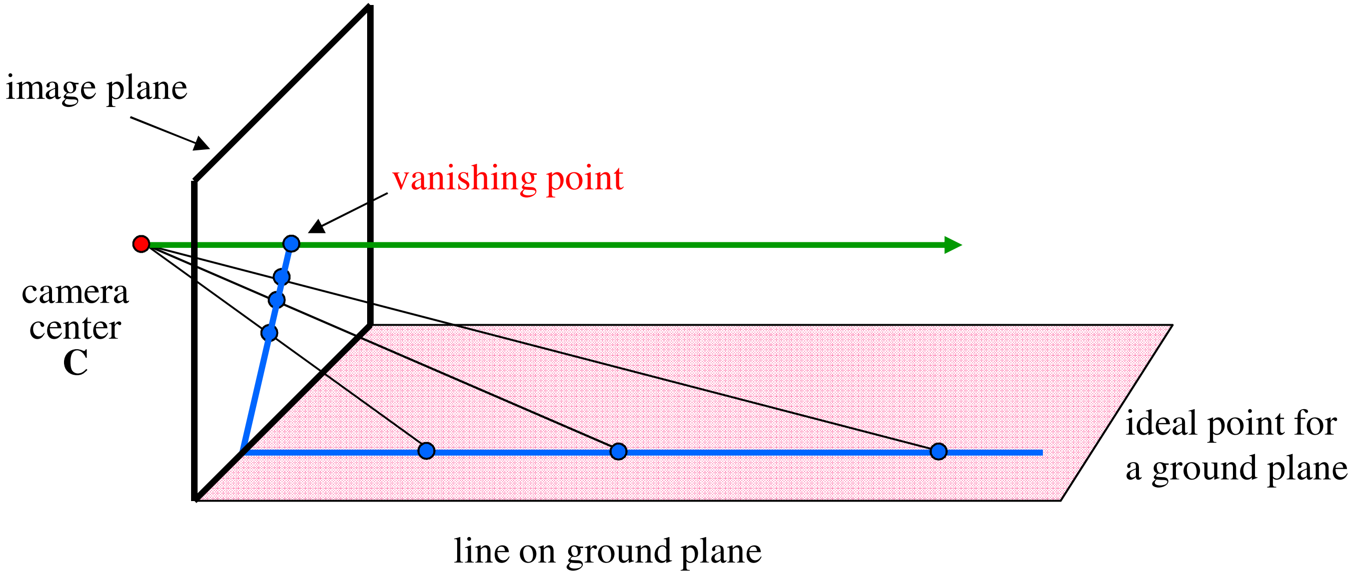
Homography Between Planar Objects

However: Mapping between any two arbitrary PPs IS possible
if the object is planar.

$$\underbrace{\begin{bmatrix} wx' \\ wy' \\ w \end{bmatrix}}_{p'} = \underbrace{\begin{bmatrix} * & * & * \\ * & * & * \\ * & * & * \end{bmatrix}}_{H} \underbrace{\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}}_{p}$$

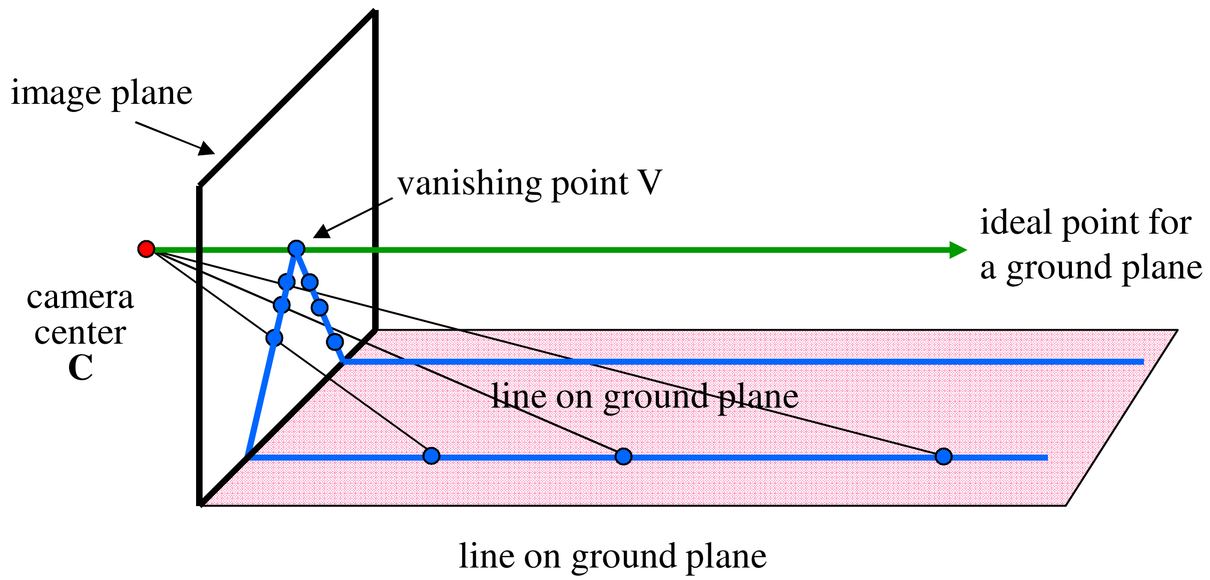


Vanishing points



Homography maps a point at infinity onto a finite point in the image plane called a vanishing point.

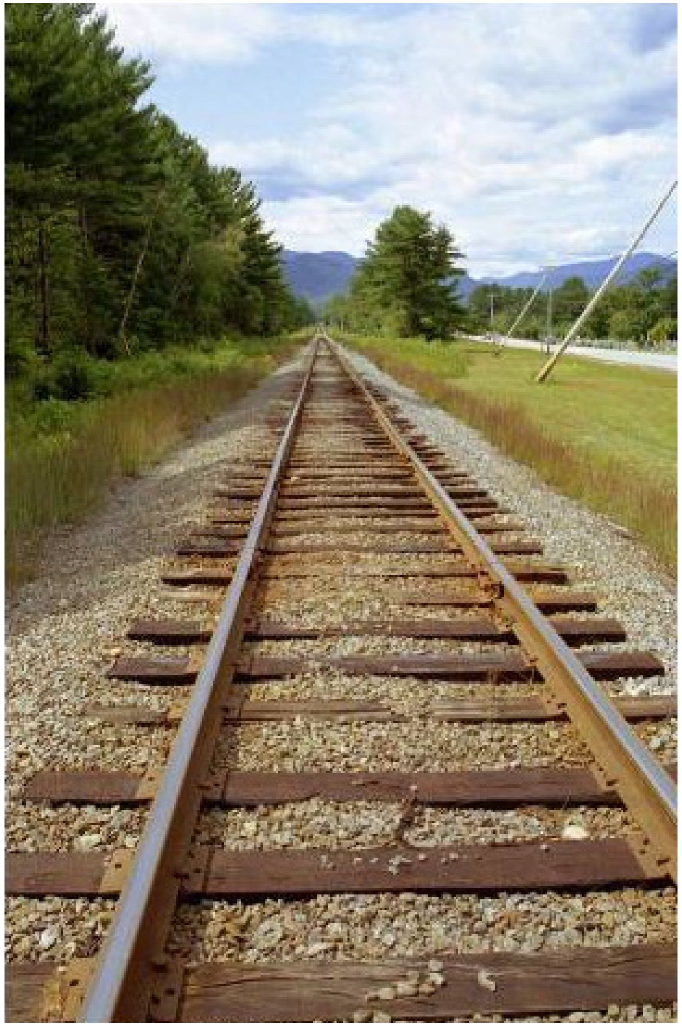
Vanishing points



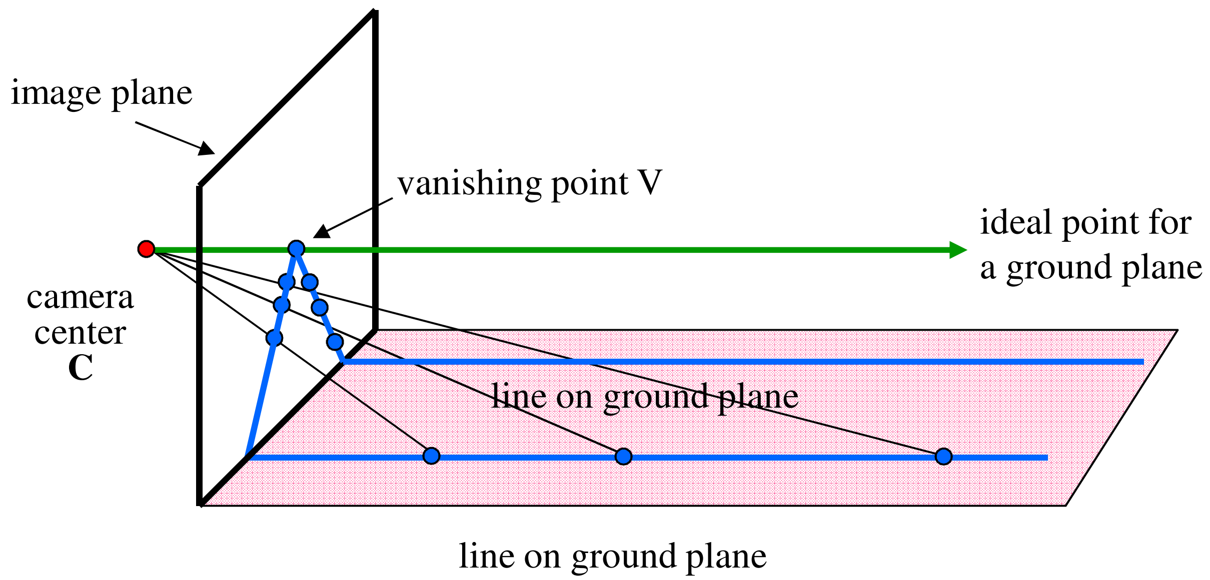
Properties

– 2 parallel lines on a plane are projected onto 2 lines that meet at the Vanishing Point (נקודת מגוז)

Vanishing points



Vanishing points

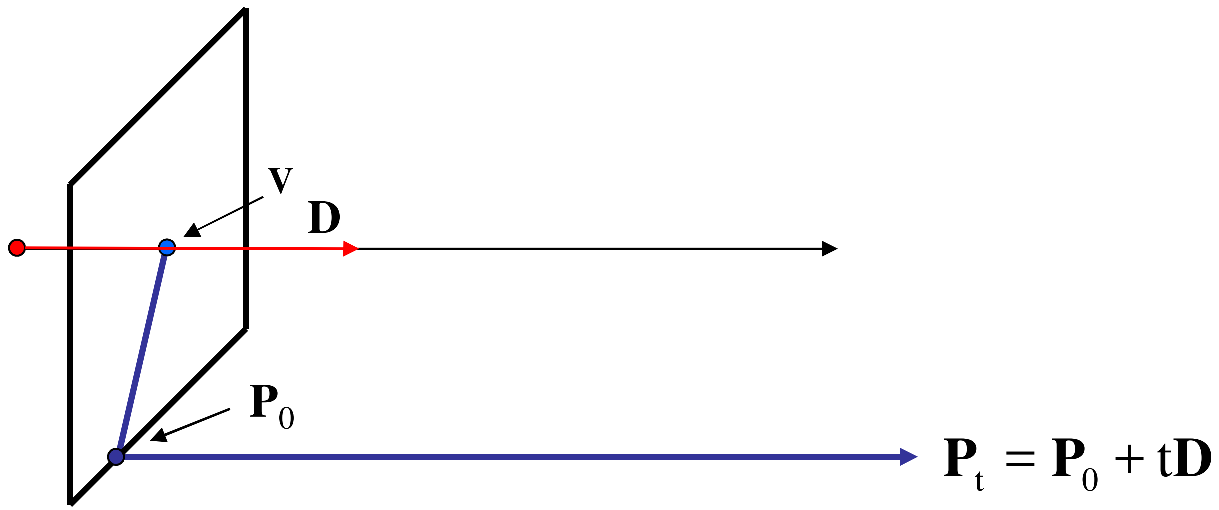


Properties

- any parallel lines have the same vanishing point v
- v is projection of their intersection (ideal point)
- v corresponds to a ray from C parallel to these lines


pxleyes.com

Computing vanishing points

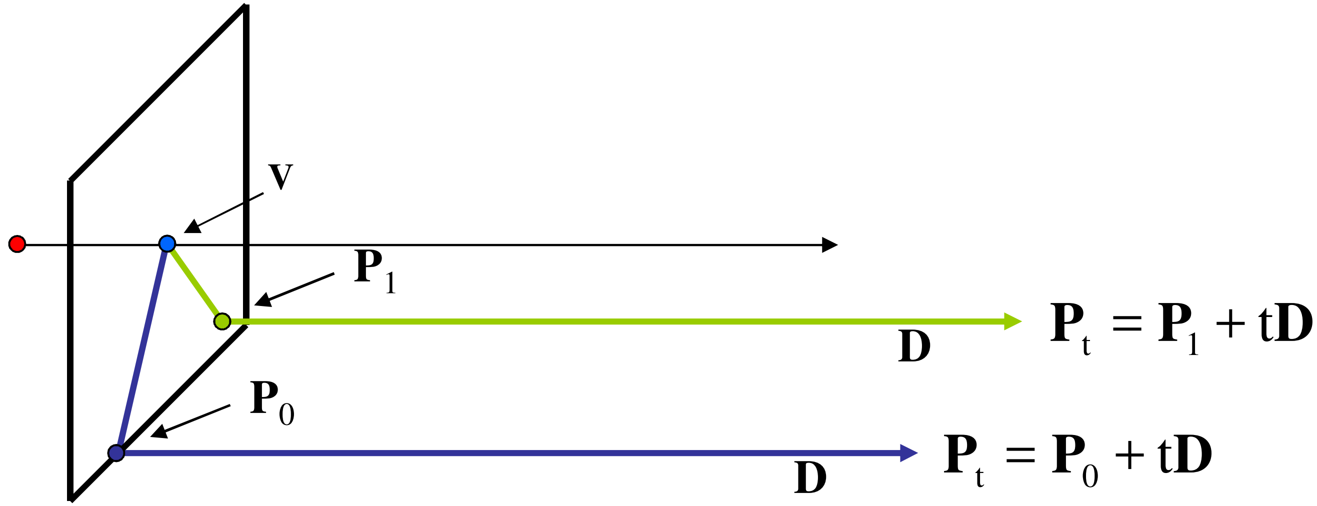


$$\mathbf{P}_t = \begin{bmatrix} P_X + tD_X \\ P_Y + tD_Y \\ P_Z + tD_Z \\ 1 \end{bmatrix} \cong \begin{bmatrix} P_X / t + D_X \\ P_Y / t + D_Y \\ P_Z / t + D_Z \\ 1/t \end{bmatrix} \qquad t \to \infty \qquad \mathbf{P}_\infty \cong \begin{bmatrix} D_X \\ D_Y \\ D_Z \\ 0 \end{bmatrix}$$

Properties $\mathbf{v} = K\mathbf{P}_\infty$ where K is a projection matrix.

- $\mathbf{P}_\infty$ is a point at *infinity*, **v** is its projection
- $\mathbf{P}_\infty$ depends only on line *direction*

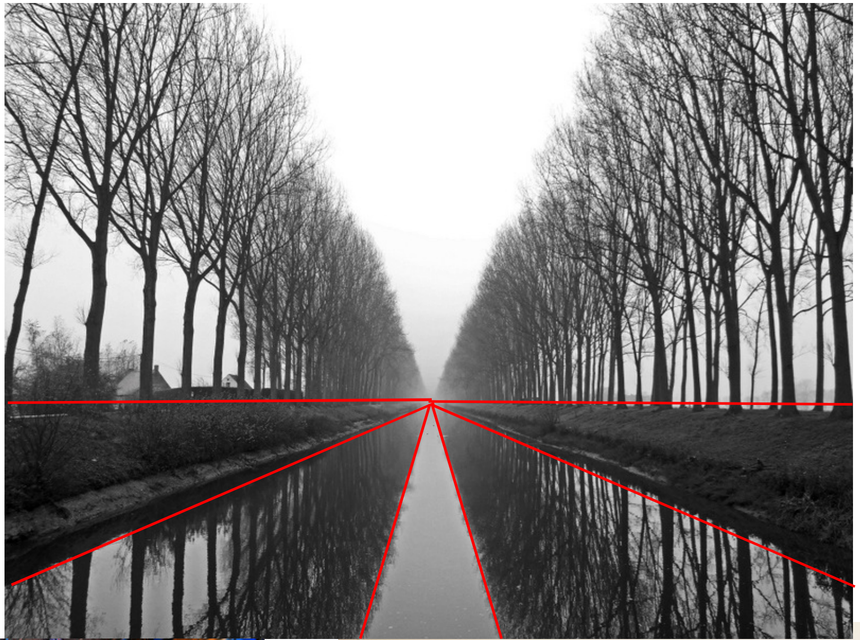
Computing vanishing points



$$\begin{bmatrix} v_X \\ v_Y \\ w \end{bmatrix} = \begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \end{bmatrix} \begin{bmatrix} D_x \\ D_y \\ D_z \\ 0 \end{bmatrix}$$

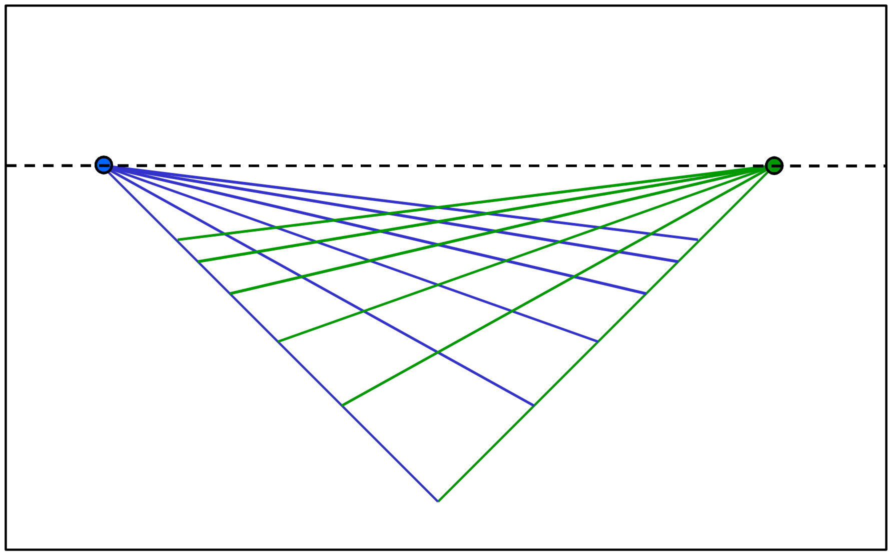
Properties

- Parallel lines $\mathbf{P}_0 + t\mathbf{D}$, $\mathbf{P}_1 + t\mathbf{D}$ intersect at $\mathbf{P}_\infty$
- Vanishing point v is same for all parallel lines in direction D: $\mathbf{P}_0 + t\mathbf{D}$, $\mathbf{P}_1 + t\mathbf{D}$



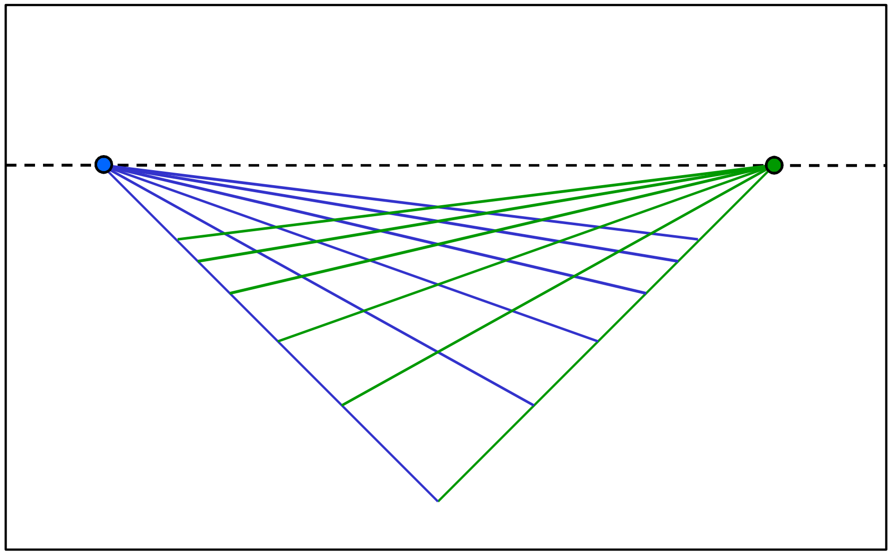
Vanishing points

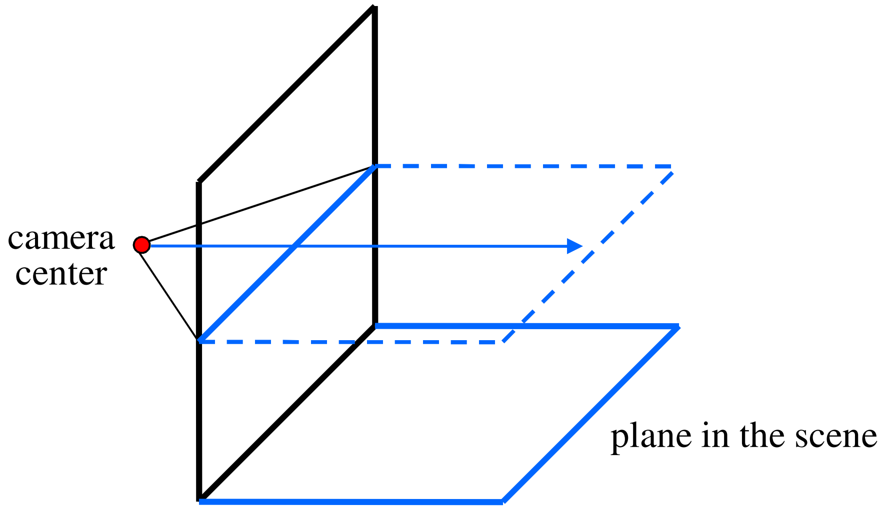
- Each direction in space has its own vanishing point
- Consider 2 directions on a plane (ground plane)
- All vanishing points of lines parallel to the ground plane are located on a “vanishing line” (horizon line).



Slide by Steve Seitz

Vanishing lines of planes

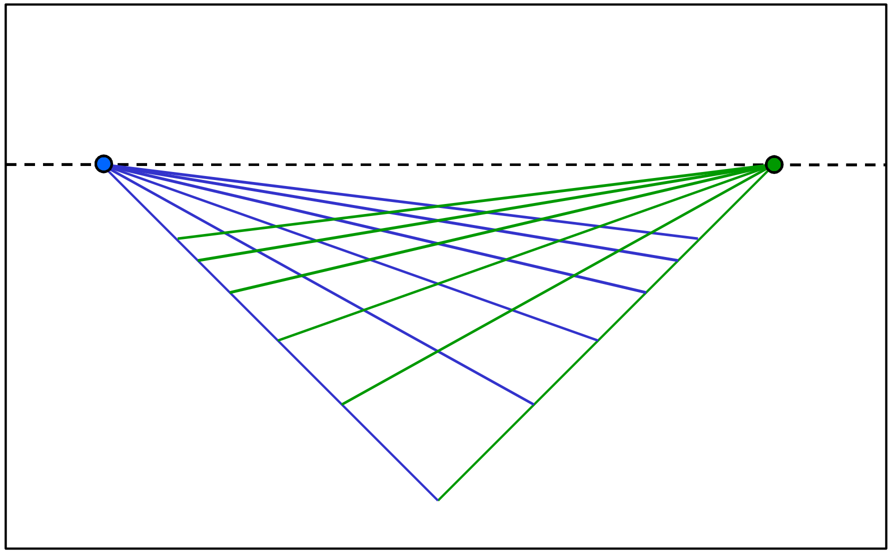


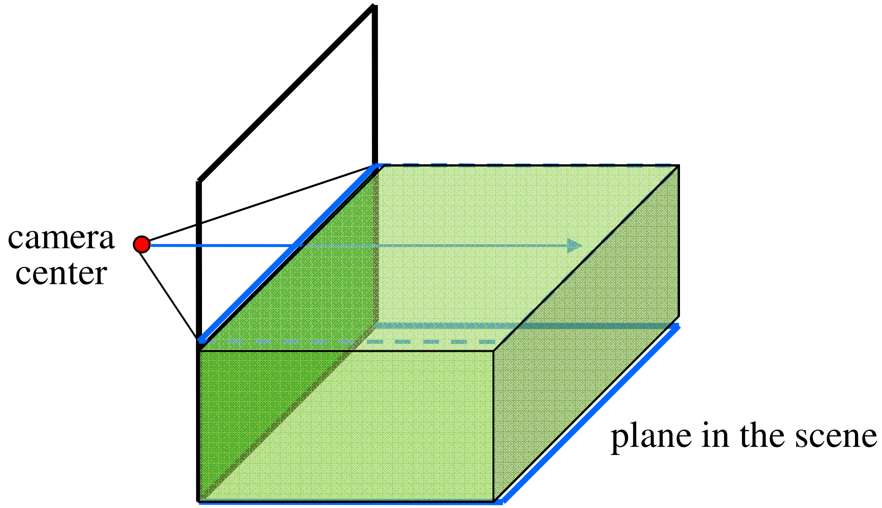


- *Horizon*: vanishing line of the ground plane
  - All points at the same height as the camera project to the horizon
  - Points higher (resp. lower) than the camera project above (resp. below) the horizon
  - Provides way of comparing height of objects

Slide by Steve Seitz

Vanishing lines of planes





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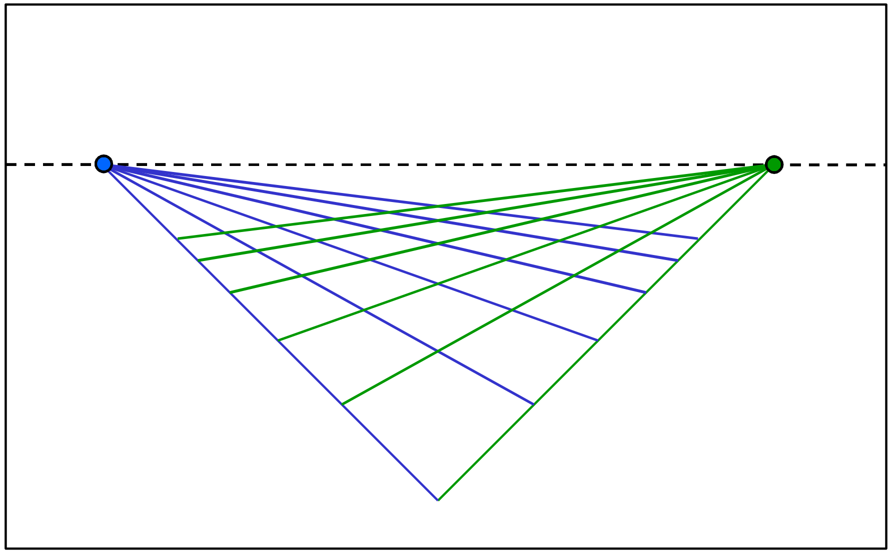
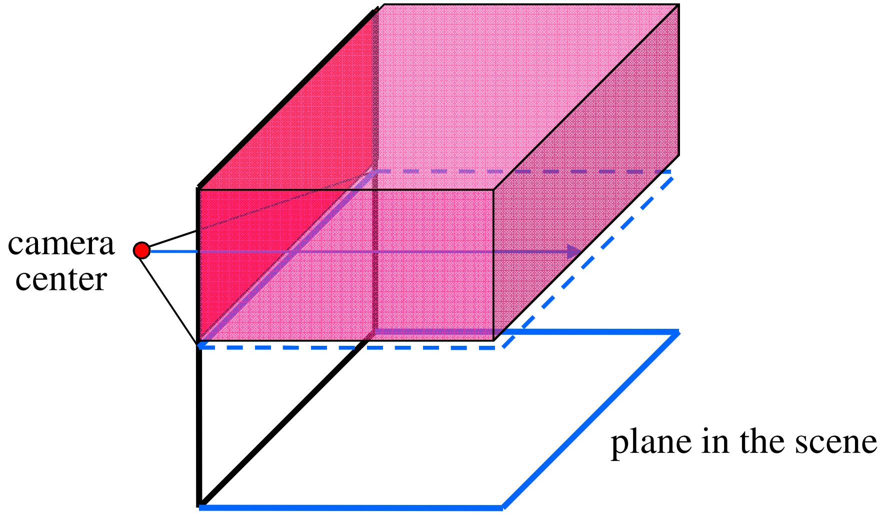
Vanishing lines of planes





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Slide by Steve Seitz

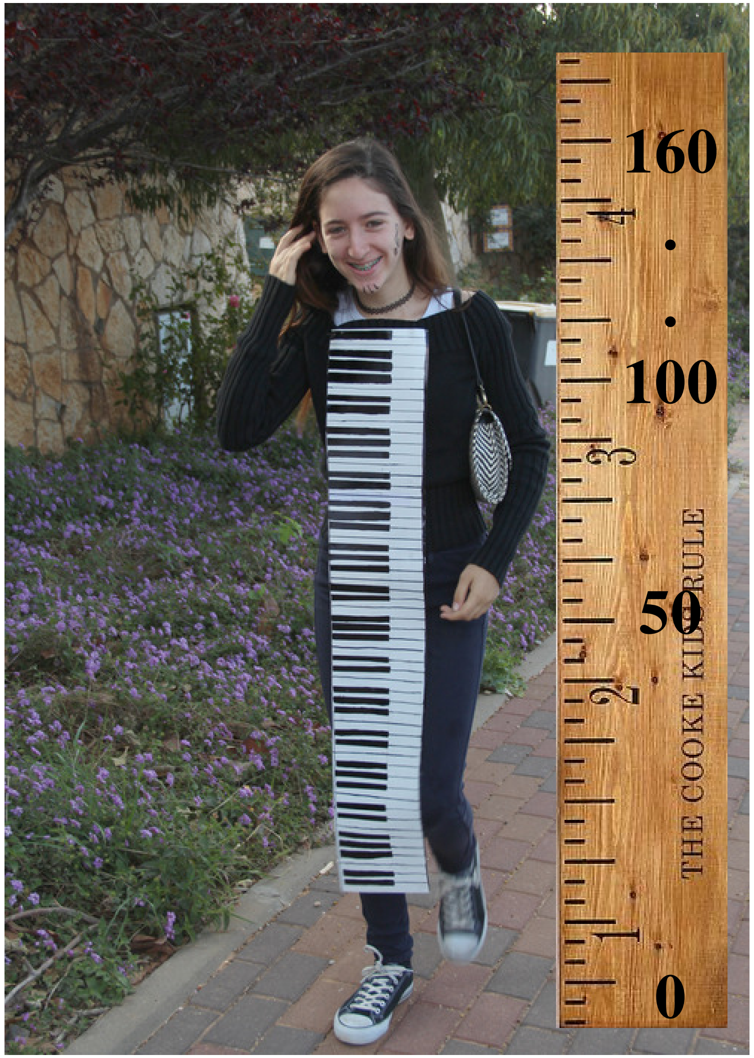
The Horizon

Is the parachutist higher or lower than the person taking this picture?

Slide by Steve Seitz

Measuring Height in Images

מה הגובה?



כרמל =
1.60 מטר

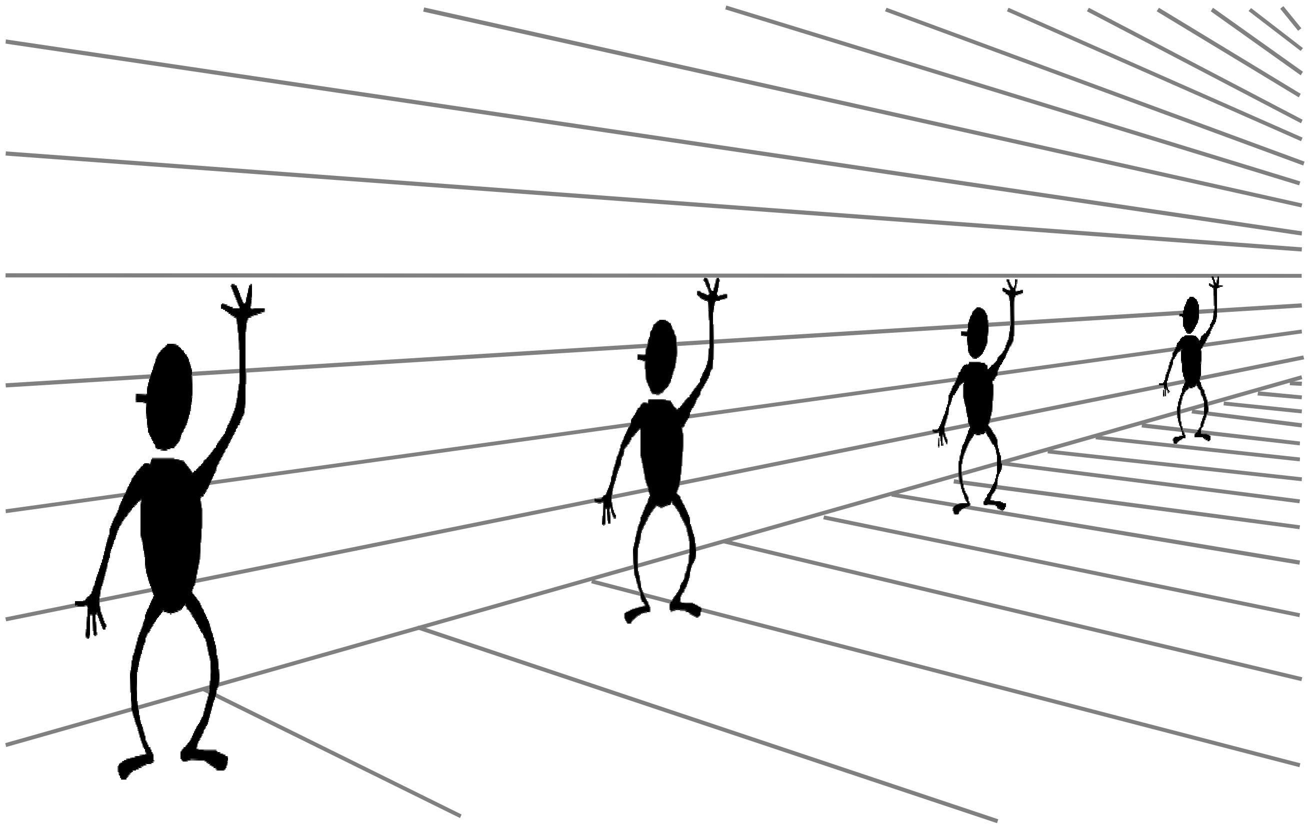
כרמל =
1.60 מטר

מיקה =
1.68 מטר

כרמל =
1.60 מטר

נוני =
? מטר

Perspective cues



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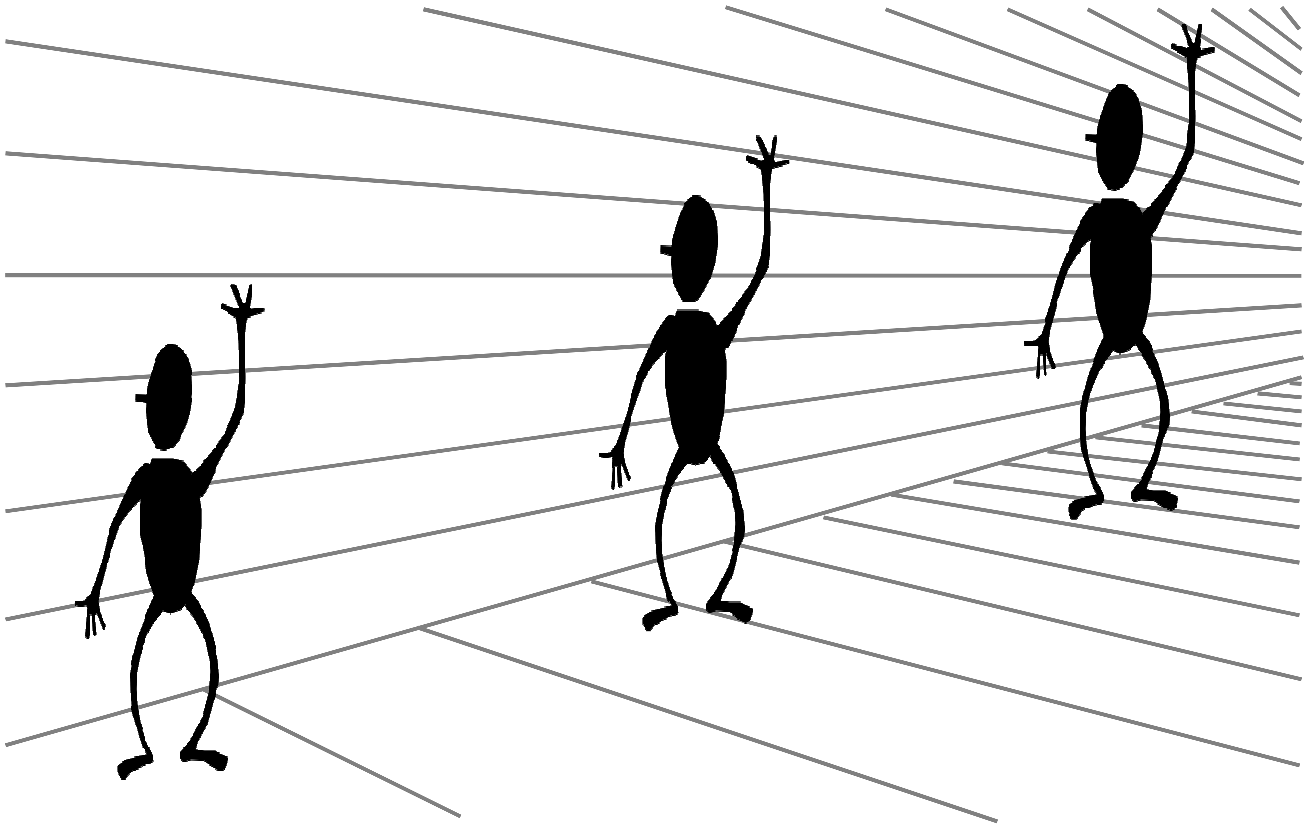
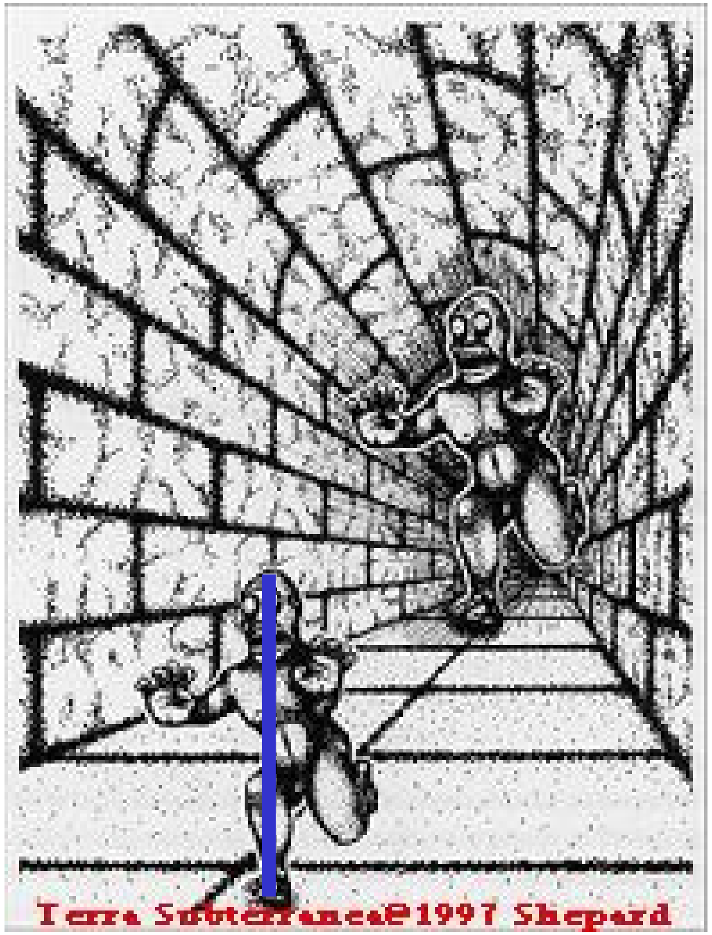
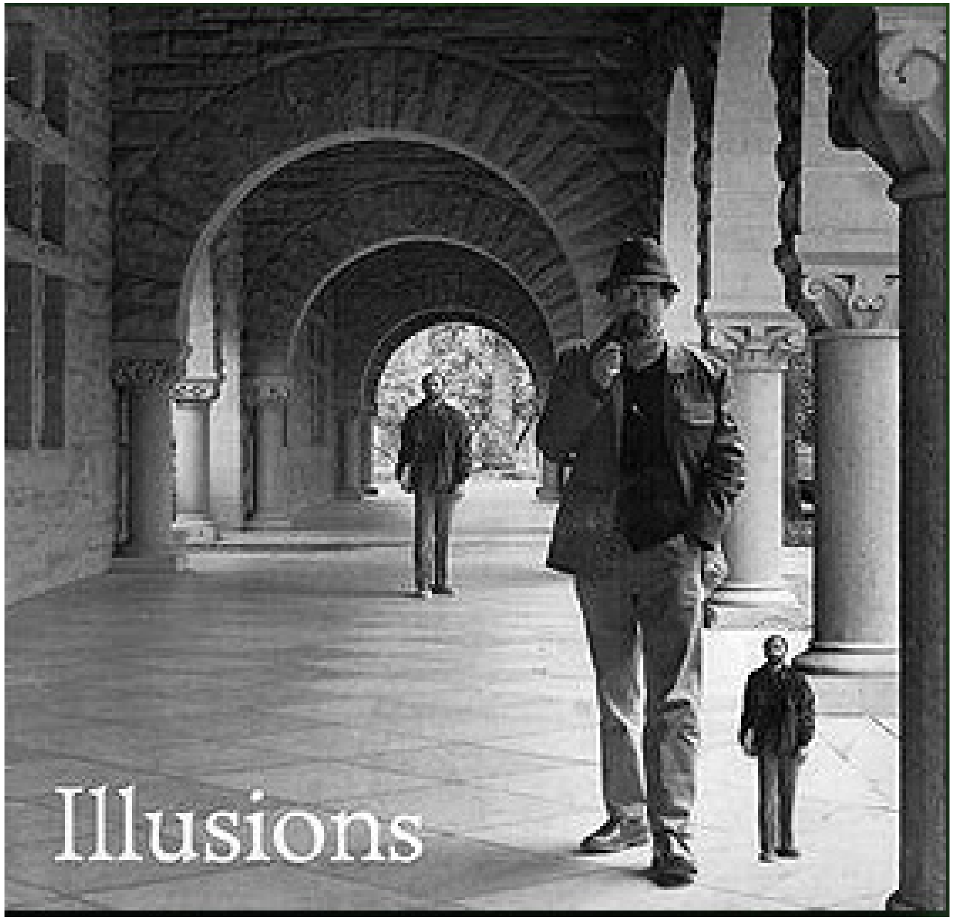
Perspective cues



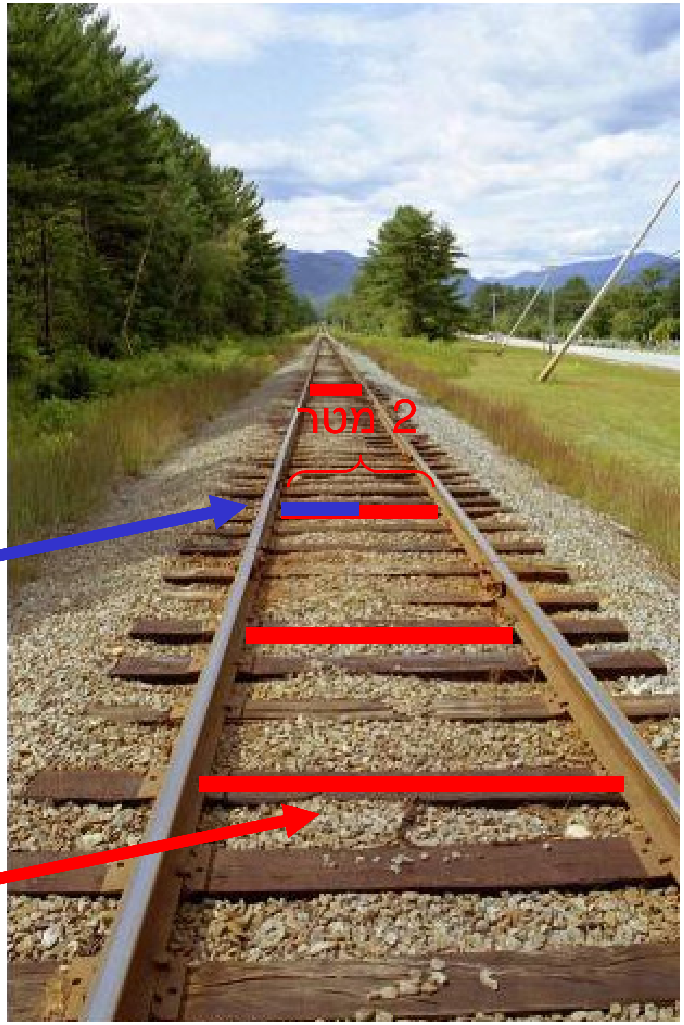
Slide by Steve Seitz

Fun with vanishing points



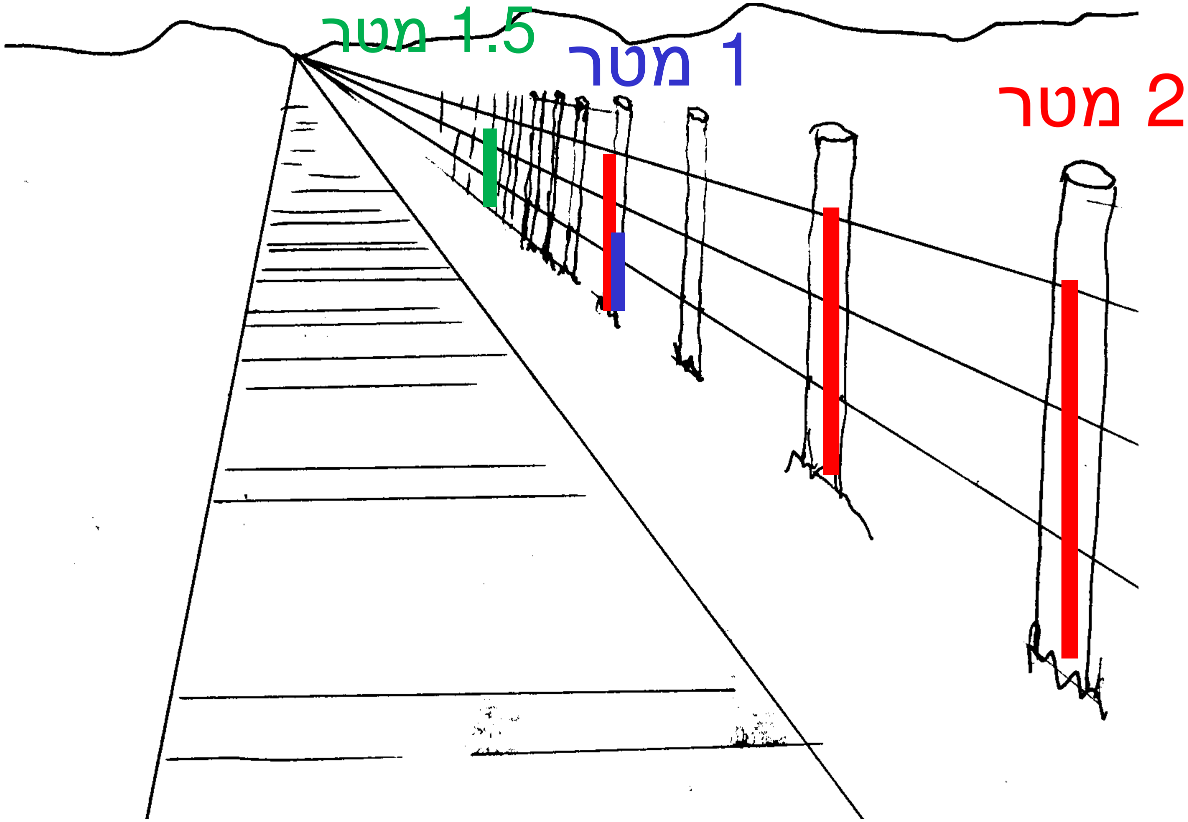


Vanishing points - נקודת מגוז



? מטר

2 מטר


1.5 מטר
1 מטר
2 מטר

מה הגובה של דבש?

נתון: גובה מרצפה לתקרה (חץ הצהוב) הוא 2.40 ס"מ

פתרון: מה הגובה של דבש?

נתון: גובה מרצפה לתקרה (חץ הצהוב) הוא 2.40 ס"מ

8 יחידות

6 יחידות

גובה של דבש הוא 6/8
מהגובה של התקרה.

6/8 * 240 ס"מ = 180 ס"מ

איפה להדביק את פליסיטי?

נתון: גובה מרצפה לתקרה (חץ הצהוב) הוא 2.40 ס"מ
גובה של פליסיטי הוא 60 ס"מ

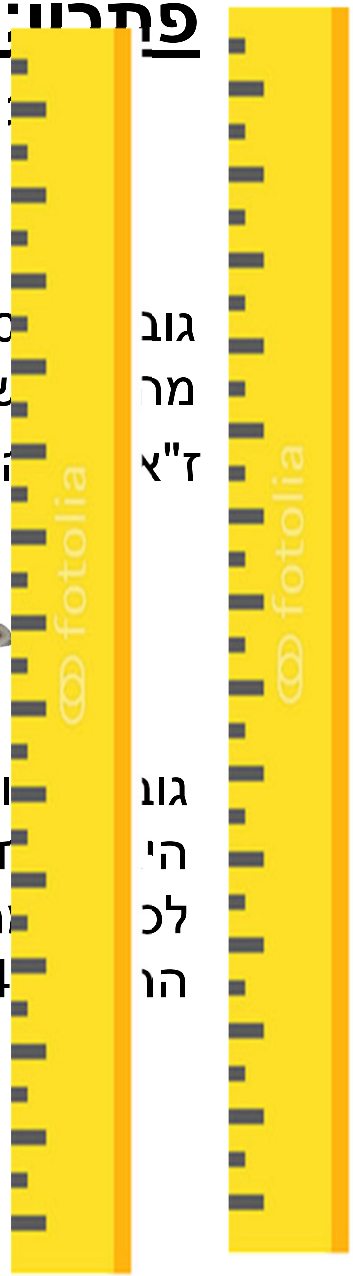
?

פ... איפה להדביק את פליסיטי?

: גובה מרצפה לתקרה (חץ הצהוב) הוא 2.40 ס"מ
גובה של פליסיטי הוא 60 ס"מ

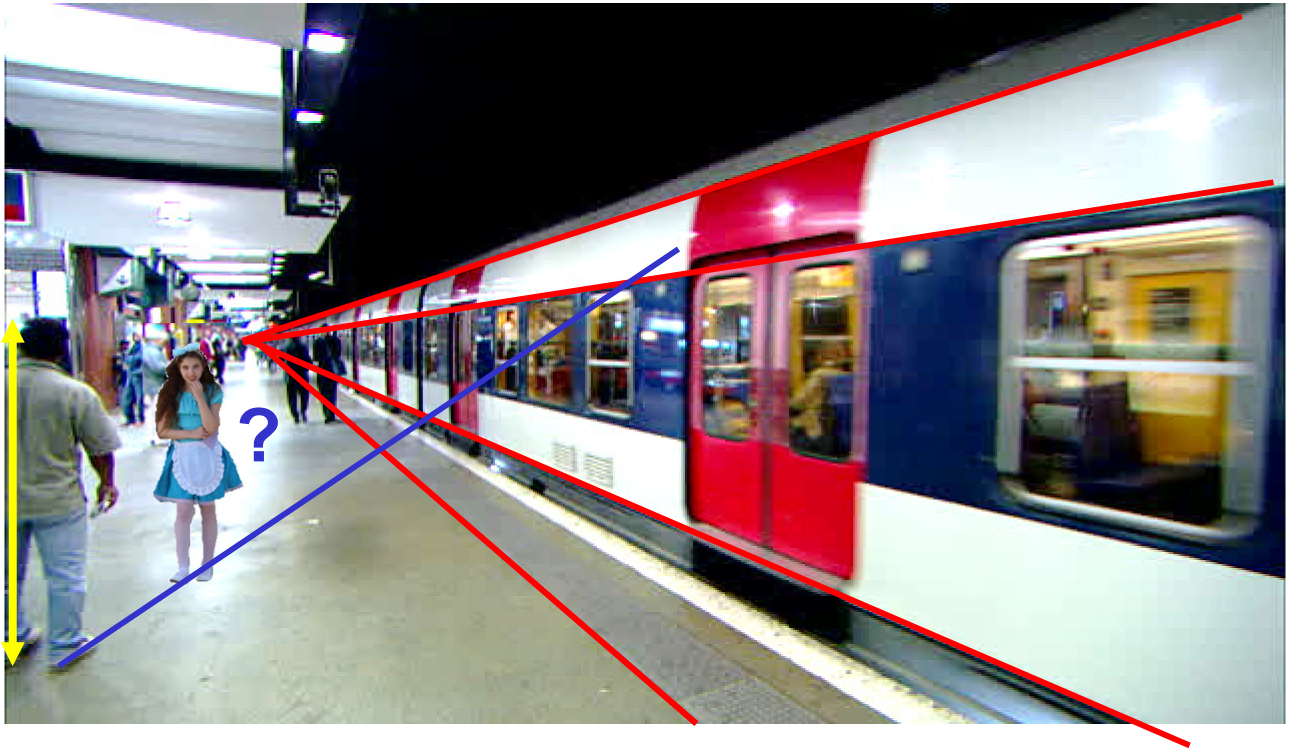
גוב סיטי הוא 60/240
מר של התקרה.
ז"א הגובה של התקרה.

גו ונת פליסיטי
הי חידה
לכ ה איפה שגובה
הו 4 יחידות



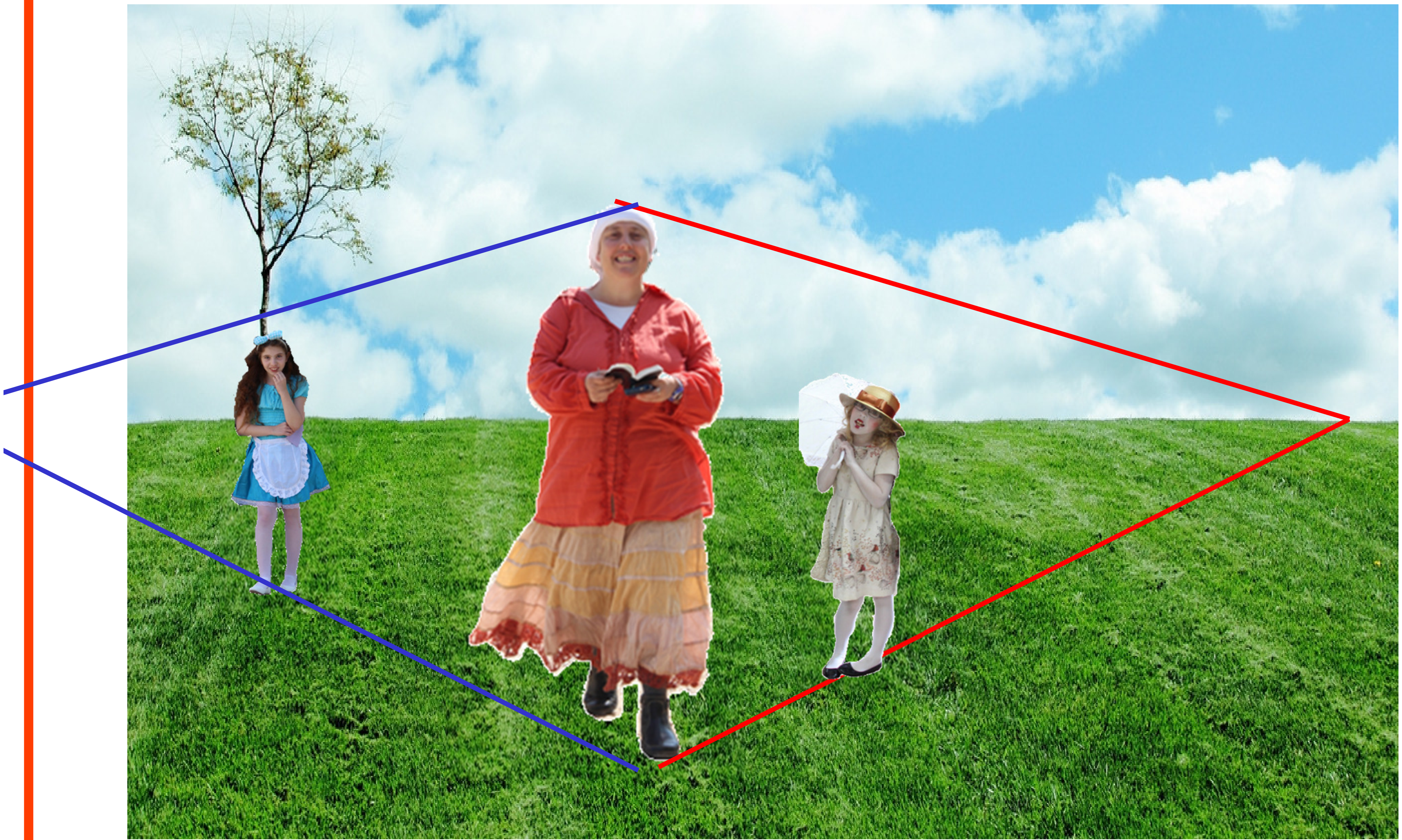
<u>פתרון:</u> מי הכי גבוה? (בשורה השמאלית)

מה הגובה של אליס ?

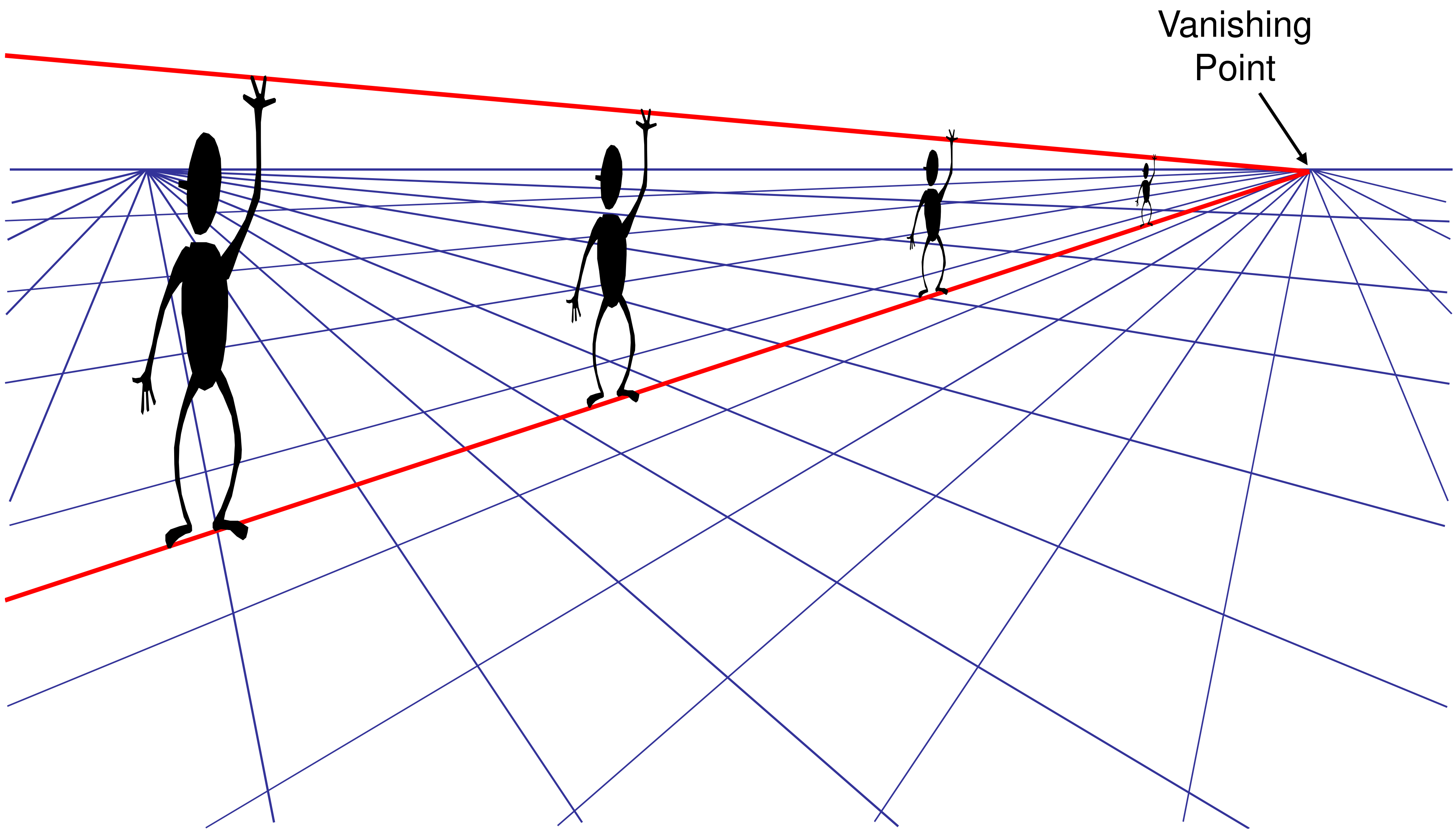


180 ס"מ

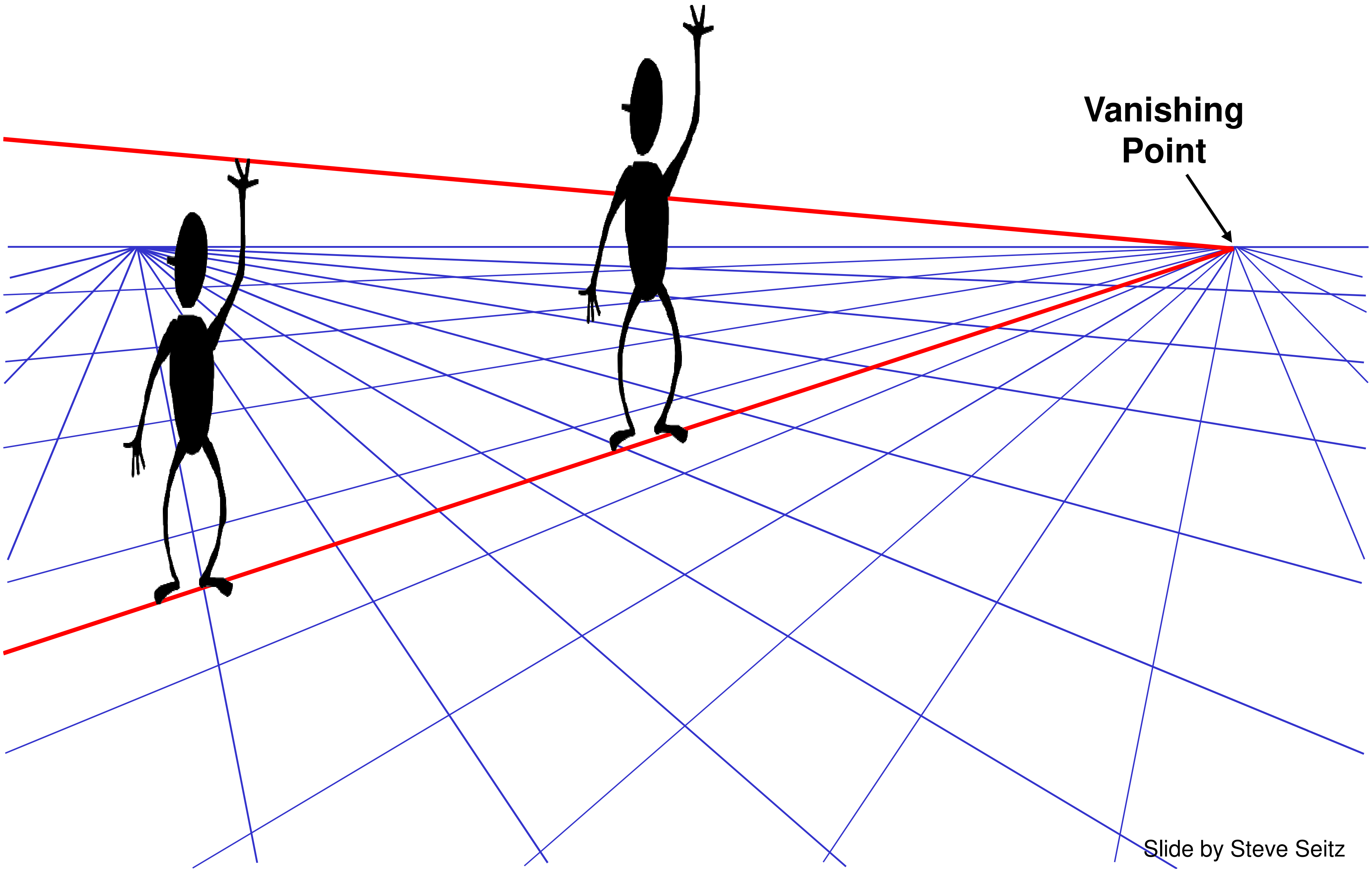
מה הגובה של הללי? של אליס? גובה של פסית הוא 160 ס"מ



Comparing heights



Comparing heights



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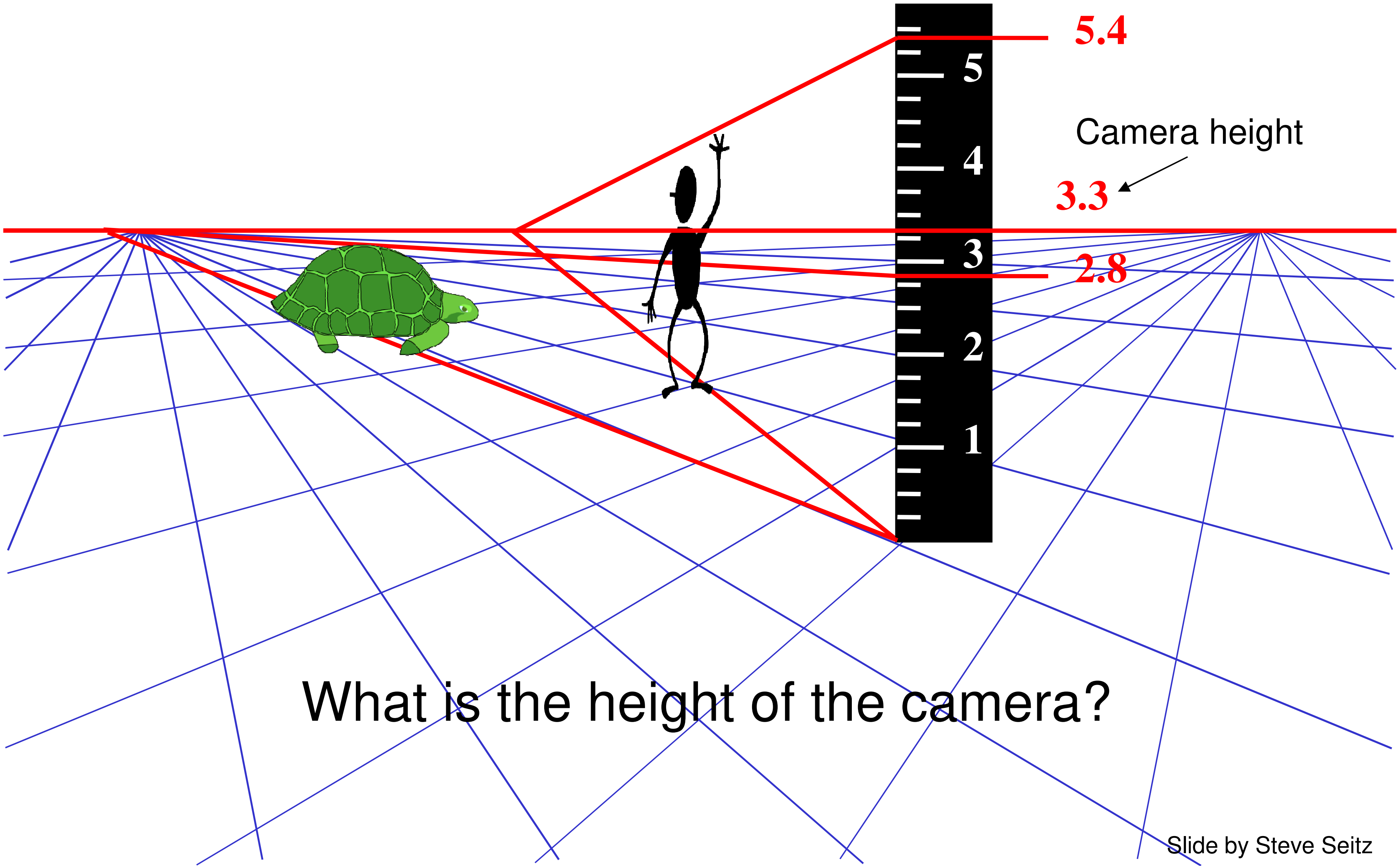
Measuring height



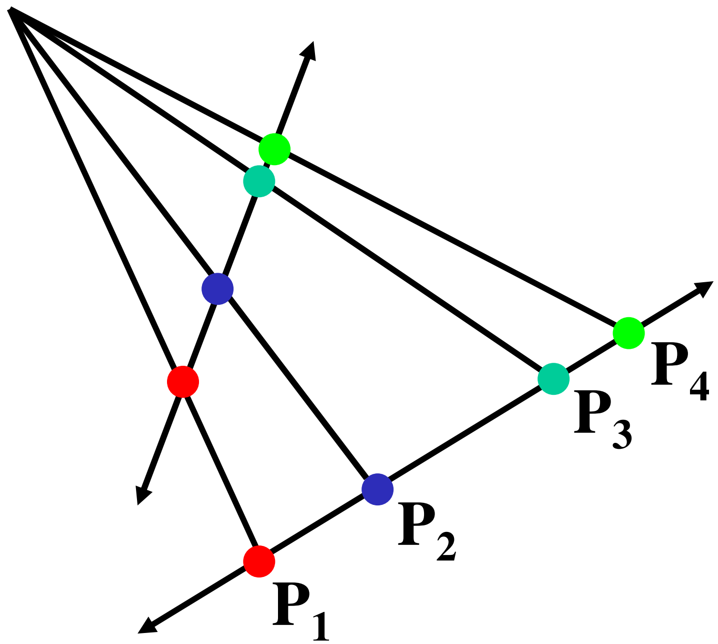
What is the height of the camera?

Slide by Steve Seitz

The cross-ratio

- A *projective invariant*: quantity that does not change under projective transformations (including perspective projection)

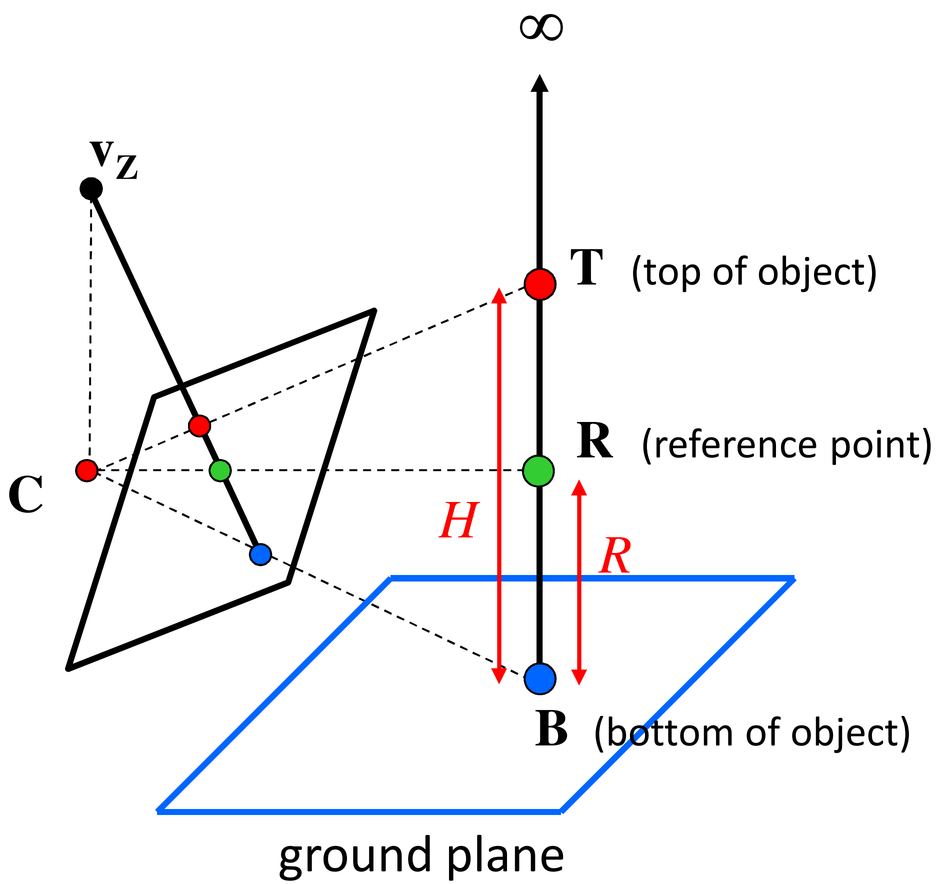
- The cross-ratio of four points:



$$\frac{\|\mathbf{P}_3 - \mathbf{P}_1\| \, \|\mathbf{P}_4 - \mathbf{P}_2\|}{\|\mathbf{P}_3 - \mathbf{P}_2\| \, \|\mathbf{P}_4 - \mathbf{P}_1\|}$$

Note: Can choose any order of the points

Measuring height



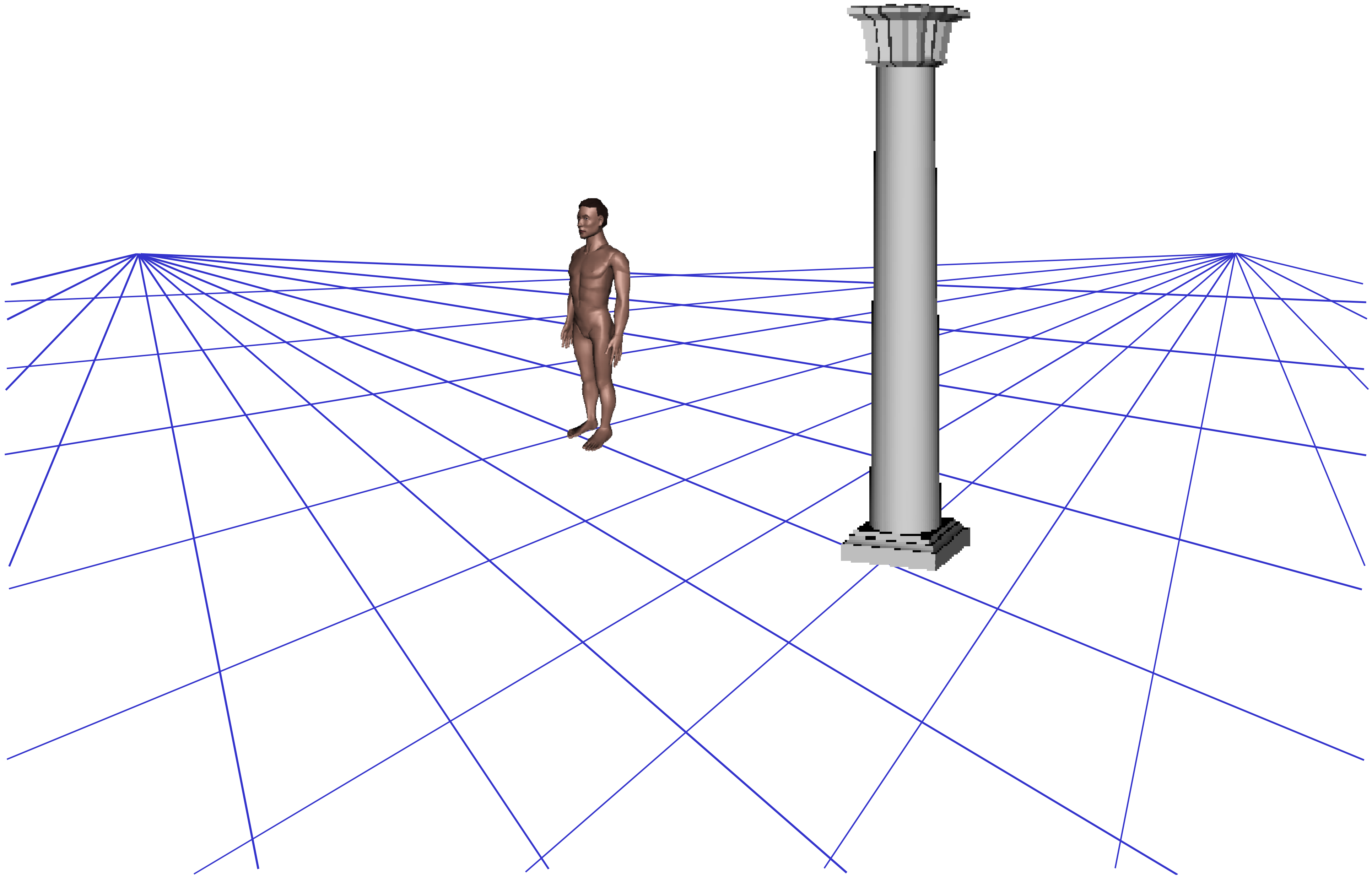
$$\frac{\|\mathbf{T}-\mathbf{B}\|\,\|\infty-\mathbf{R}\|}{\|\mathbf{R}-\mathbf{B}\|\,\|\infty-\mathbf{T}\|} = \frac{H}{R}$$

scene cross ratio

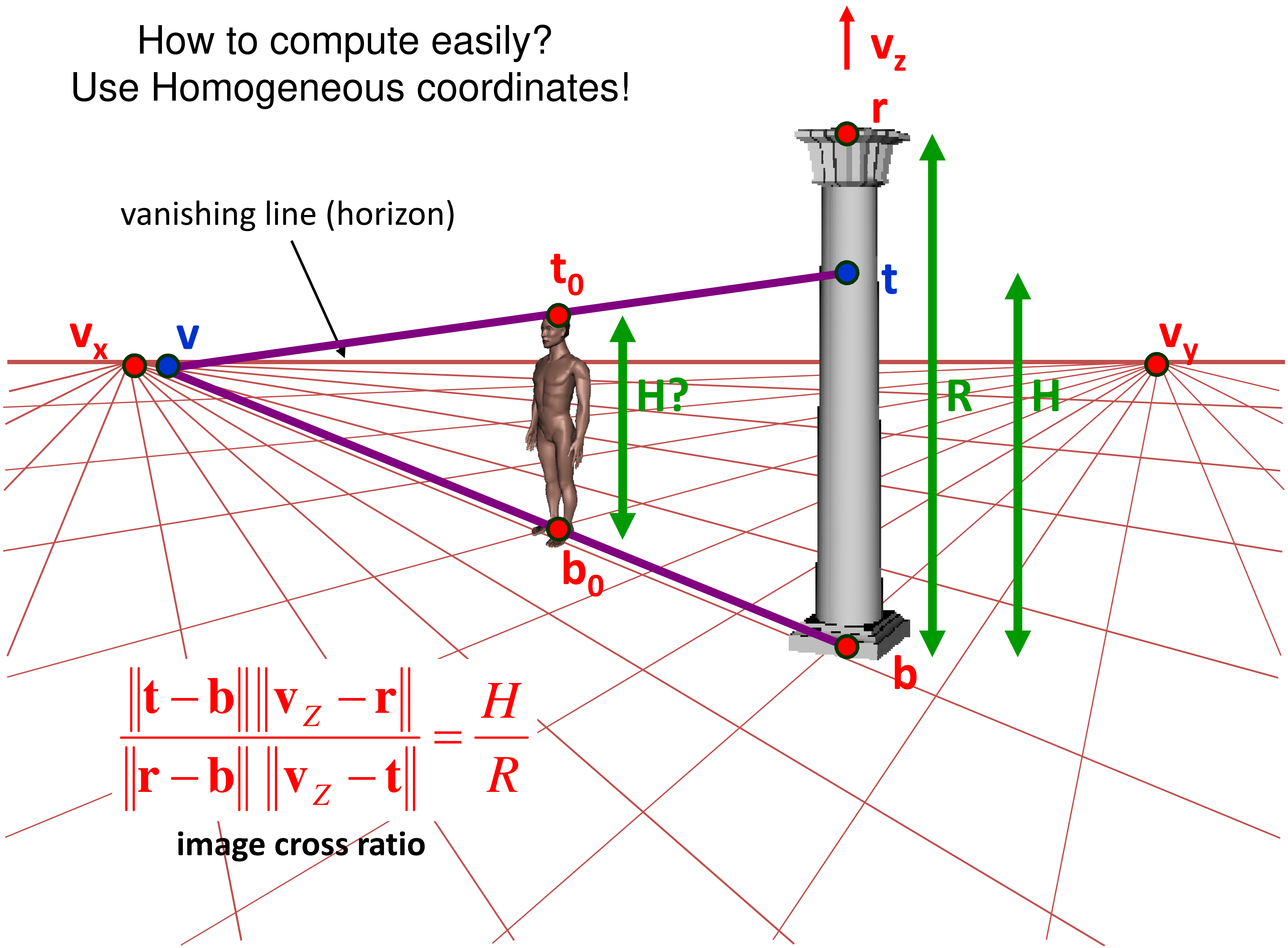
$$\frac{\|\mathbf{t}-\mathbf{b}\|\,\|\mathbf{v}_Z-\mathbf{r}\|}{\|\mathbf{r}-\mathbf{b}\|\,\|\mathbf{v}_Z-\mathbf{t}\|} = \frac{H}{R}$$

image cross ratio

Measuring height without a ruler – Image View



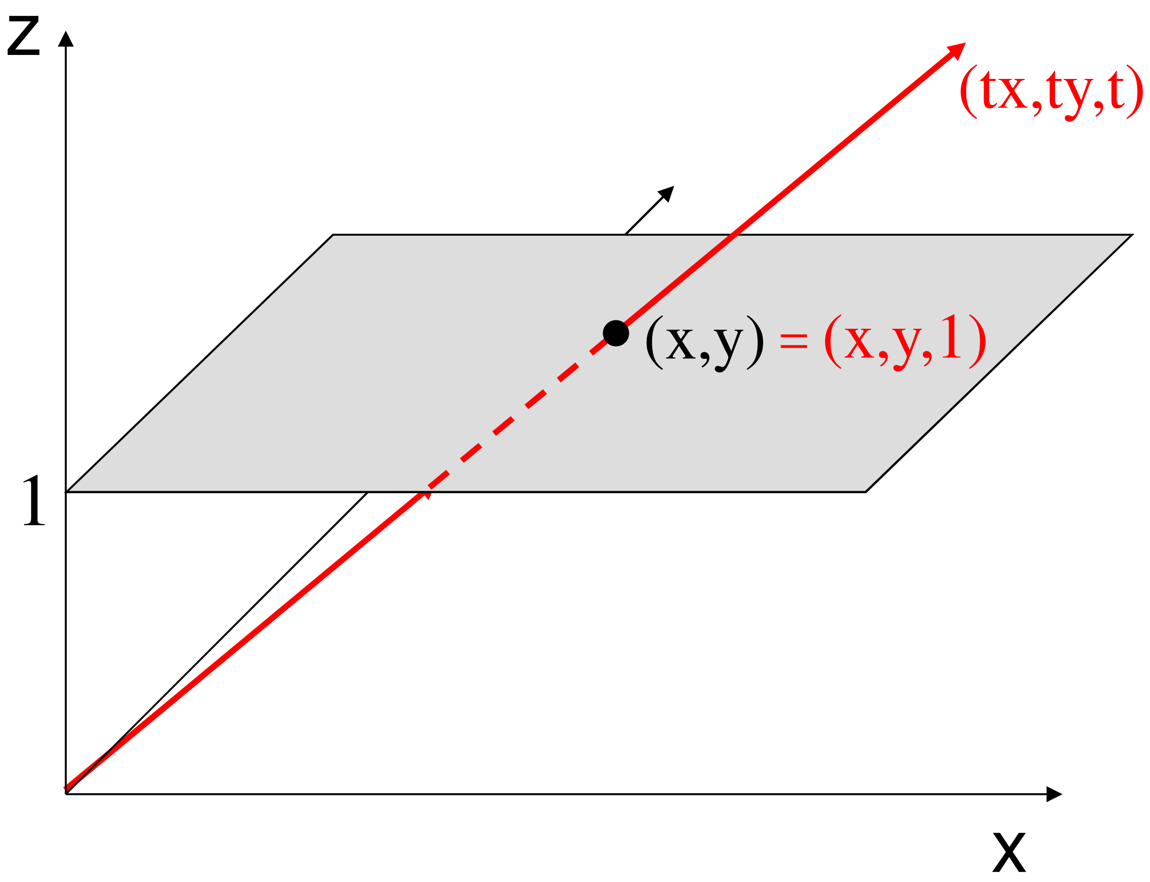
How to compute easily?
Use Homogeneous coordinates!



$$\frac{\|\mathbf{t}-\mathbf{b}\|\,\|\mathbf{v}_Z-\mathbf{r}\|}{\|\mathbf{r}-\mathbf{b}\|\,\|\mathbf{v}_Z-\mathbf{t}\|}=\frac{H}{R}$$

image cross ratio

Homogeneous Coordinates

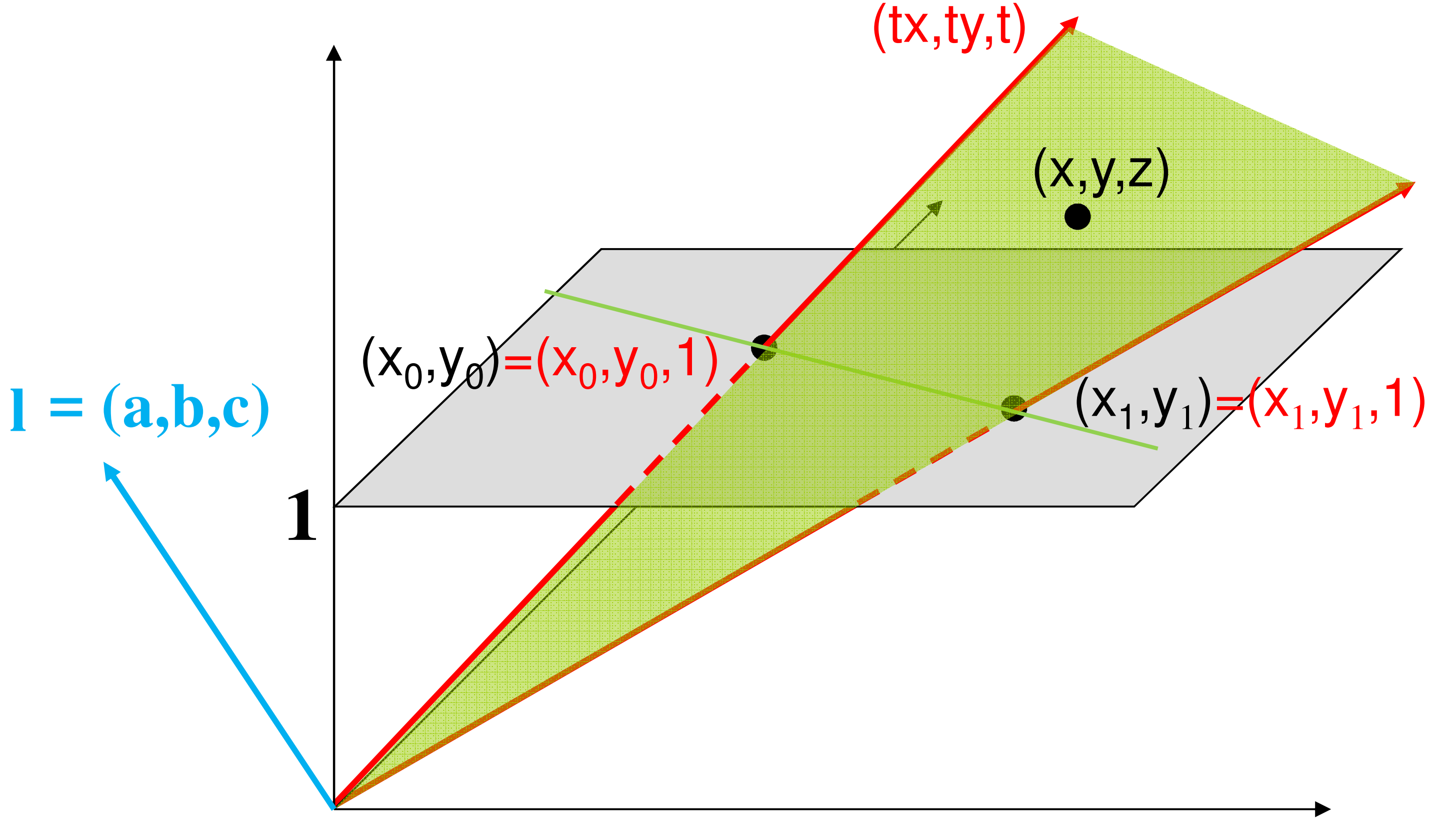


$$(x, y) \rightarrow (x, y, 1) \equiv (tx, ty, t)$$

$$(X, Y, W) \rightarrow \left(\frac{X}{W}, \frac{Y}{W}\right) = (\mathrm{x}, \mathrm{y})$$

Projective Lines

What does a line in the image correspond to in projective space?



A line is a *plane* of rays (x,y,z) through origin satisfying:

$\langle(a,b,c),(x,y,z)\rangle \longrightarrow ax + by + cz = 0$

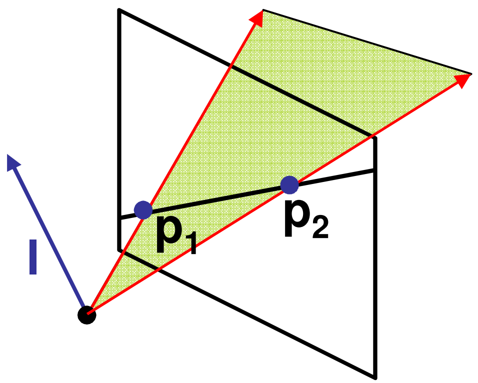
Projective Lines

A line is a *plane* of rays (x,y,z) through origin satisfying:

$$ax + by + cz = 0$$

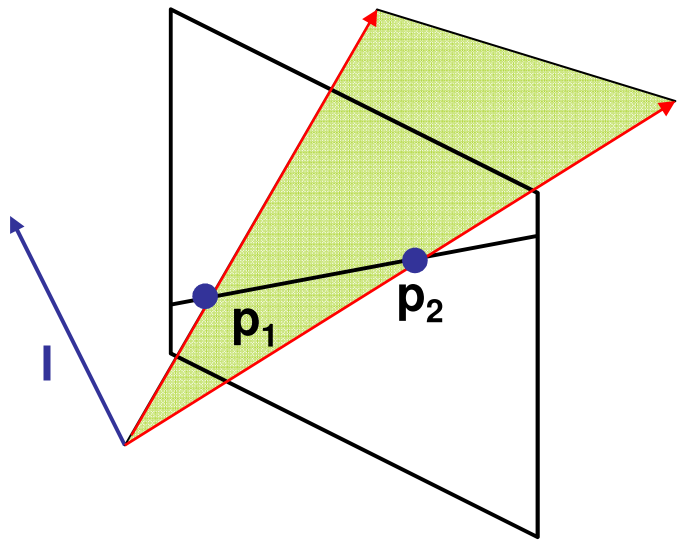
In vector notation:

$$\begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$



$$\mathbf{l}^T \mathbf{p} = \mathbf{0}$$

Point and line duality



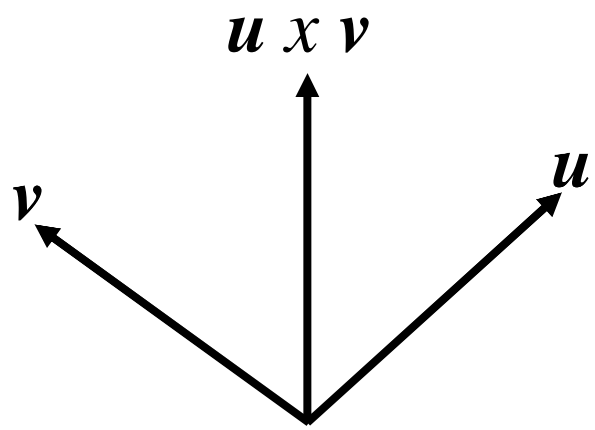
What is the line $\mathbf{l}$ spanned by rays $\mathbf{p}_1$ and $\mathbf{p}_2$?

- $\mathbf{l}$ is $\perp$ to $\mathbf{p}_1$ and $\mathbf{p}_2$ $\Rightarrow$ $\boxed{\mathbf{l} = \mathbf{p}_1 \times \mathbf{p}_2}$
- $\mathbf{l}$ is the plane normal

Cross Product Reminder

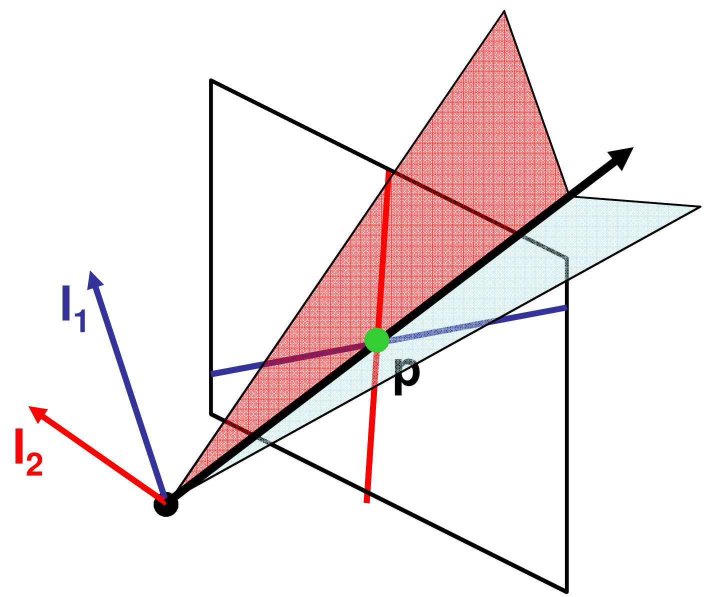
$\boldsymbol{u} = (u_x, u_y, u_z)$

$\boldsymbol{v} = (v_x, v_y, v_z)$



$$\boldsymbol{u} \; x \; \boldsymbol{v} = [\ (u_y v_z - u_z v_y),\ (u_z v_x - u_x v_z),\ (u_x v_y - u_y v_x)\ ]$$

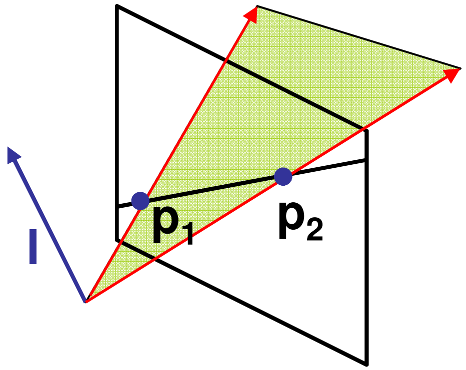
Point and line duality

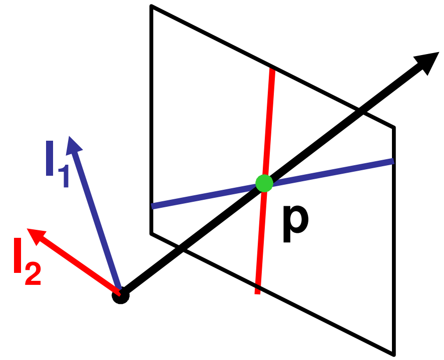


What is the intersection of two lines $\mathbf{l}_1$ and $\mathbf{l}_2$?

$$\mathbf{p} \text{ is } \perp \text{ to } \mathbf{l}_1 \text{ and } \mathbf{l}_2 \;\Rightarrow\; \boxed{\mathbf{p} = \mathbf{l}_1 \times \mathbf{l}_2}$$

Point and line duality





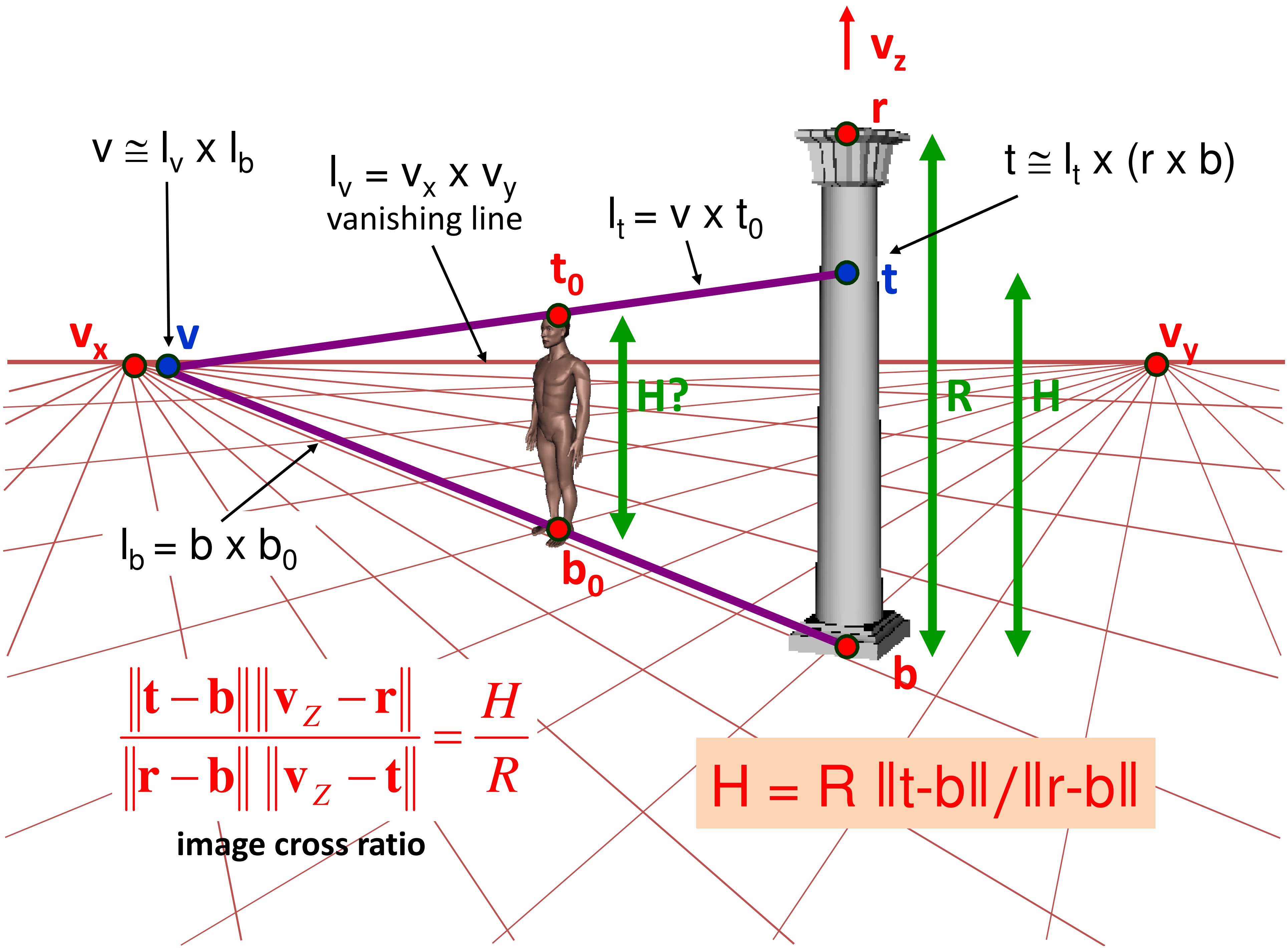
The line $\mathbf{l}$ spanned by rays $\mathbf{p}_1$ and $\mathbf{p}_2$

$$\mathbf{l} = \mathbf{p}_1 \times \mathbf{p}_2$$

The intersection of two lines $\mathbf{l}_1$ and $\mathbf{l}_2$:

$$\mathbf{p} = \mathbf{l}_1 \times \mathbf{l}_2$$

Points and lines are *dual* in projective space

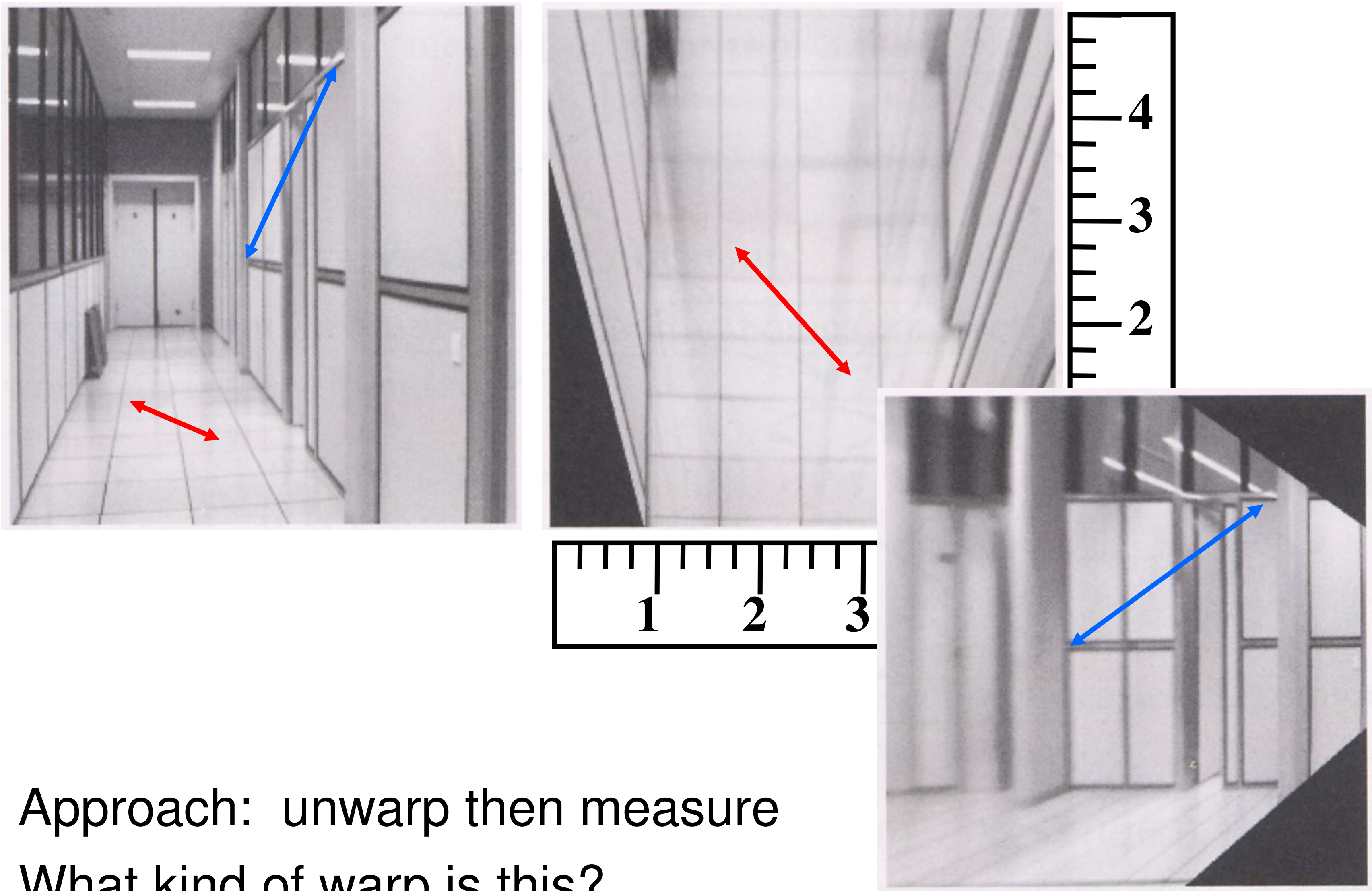


$$\frac{\|\mathbf{t}-\mathbf{b}\|\,\|\mathbf{v}_Z-\mathbf{r}\|}{\|\mathbf{r}-\mathbf{b}\|\,\|\mathbf{v}_Z-\mathbf{t}\|}=\frac{H}{R}$$

image cross ratio

$$H = R\,\|t-b\| / \|r-b\|$$

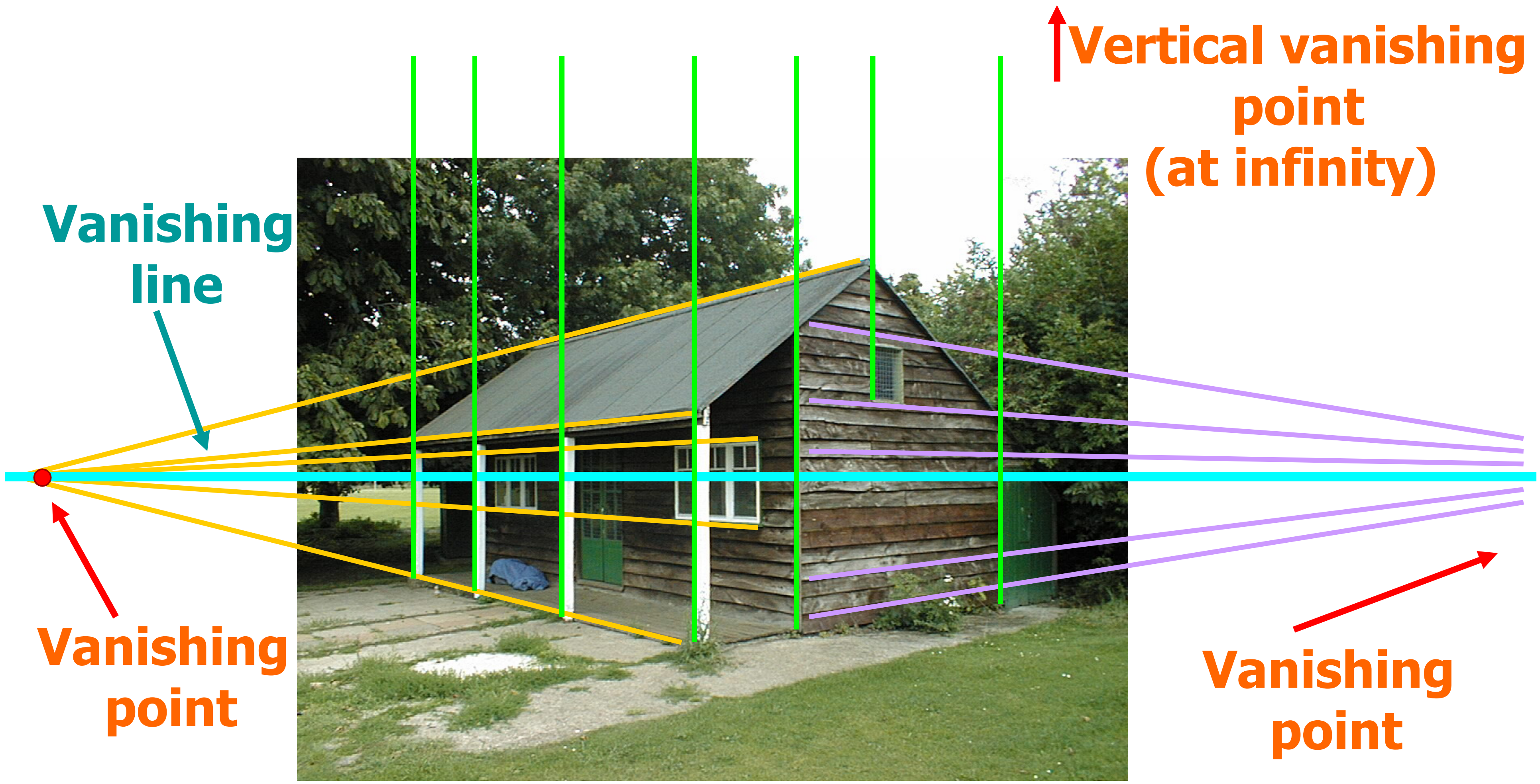
Measurements on planes



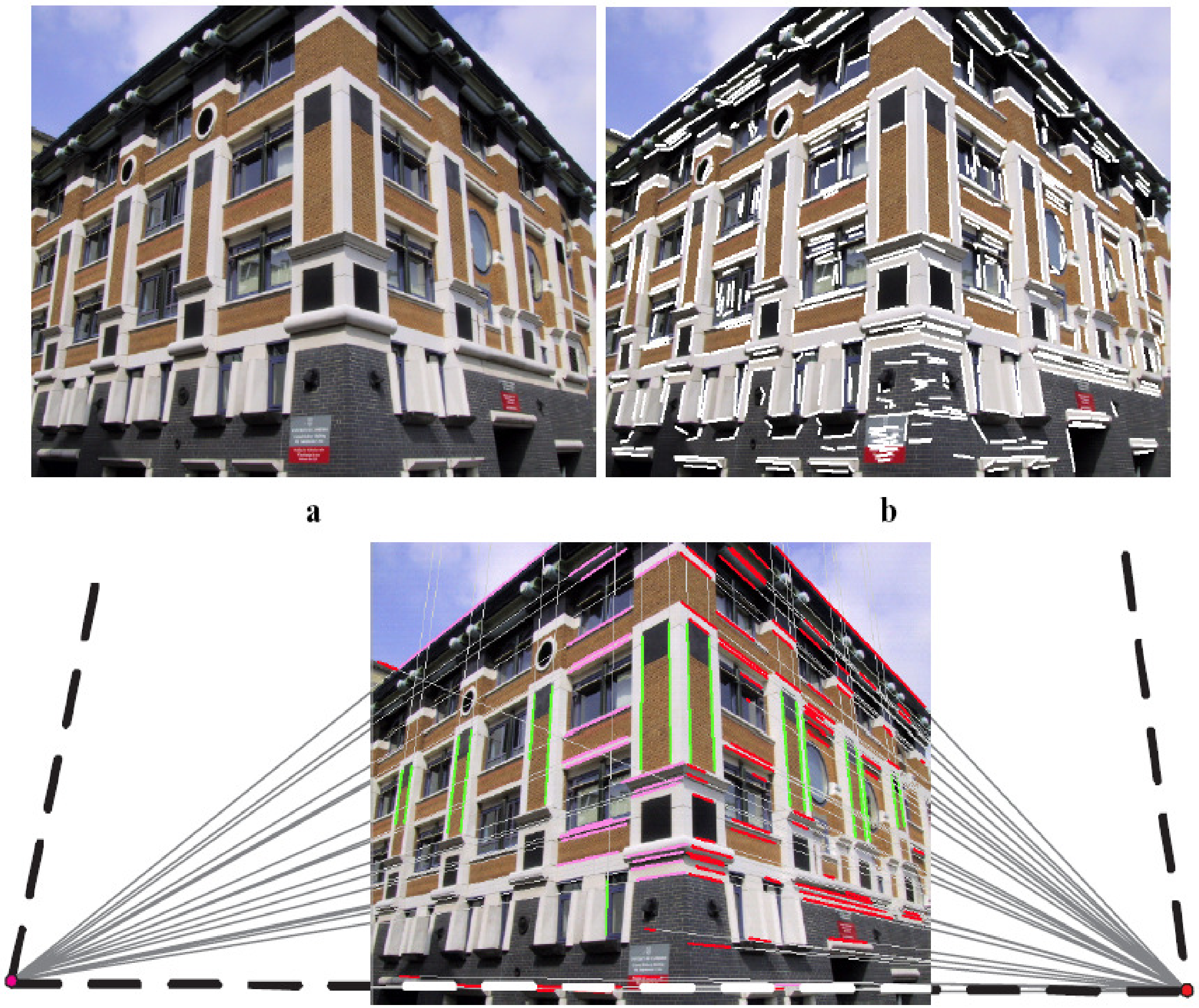
Approach: unwarp then measure

What kind of warp is this?

Computing Vanishing Line

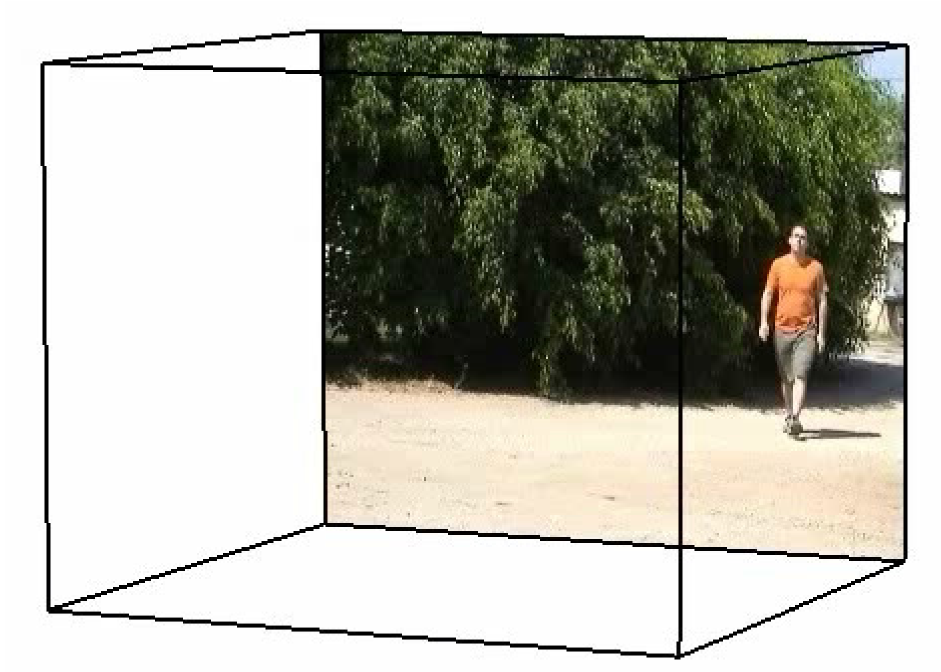


Automatic Computing Vanishing Line



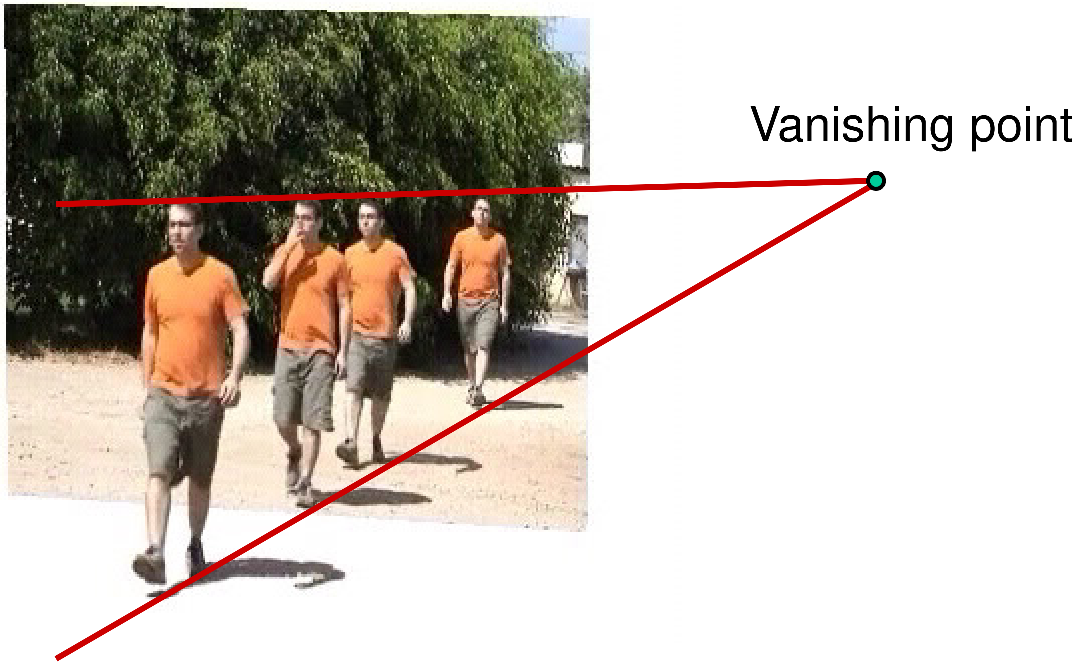
Vanishing Line from Video

Assumption: The motion is planar.



from Anat Axelrod

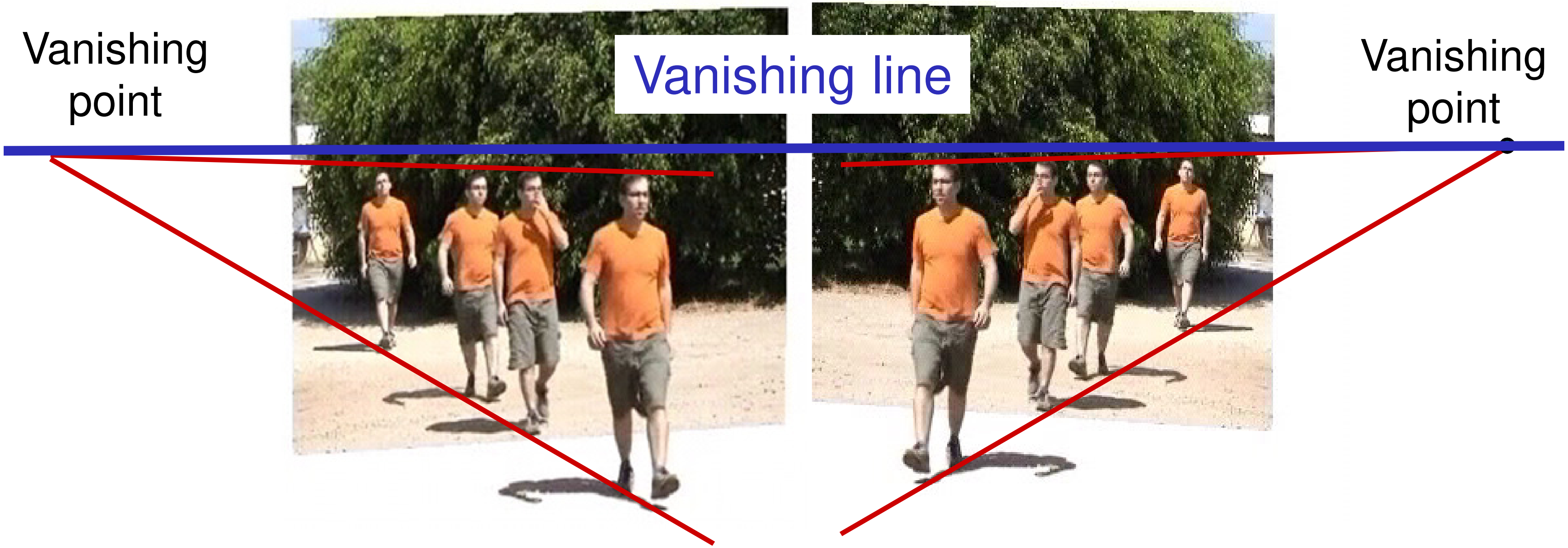
Vanishing Line from Video



Key Poses

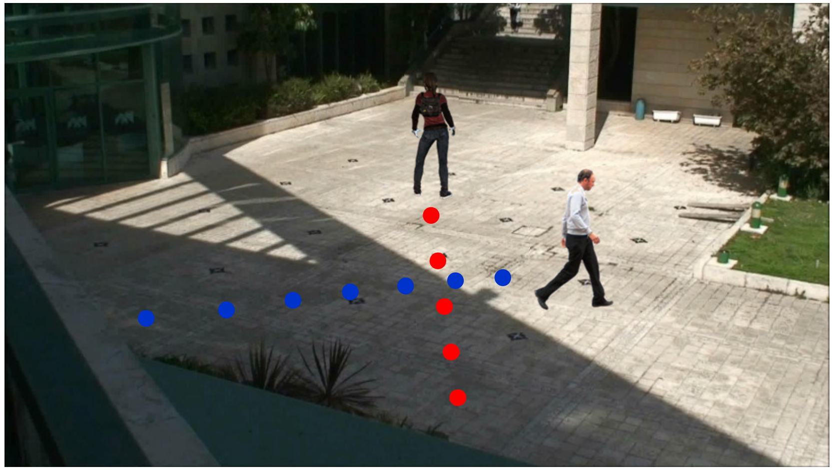
from Anat Axelrod

Vanishing Line from Video

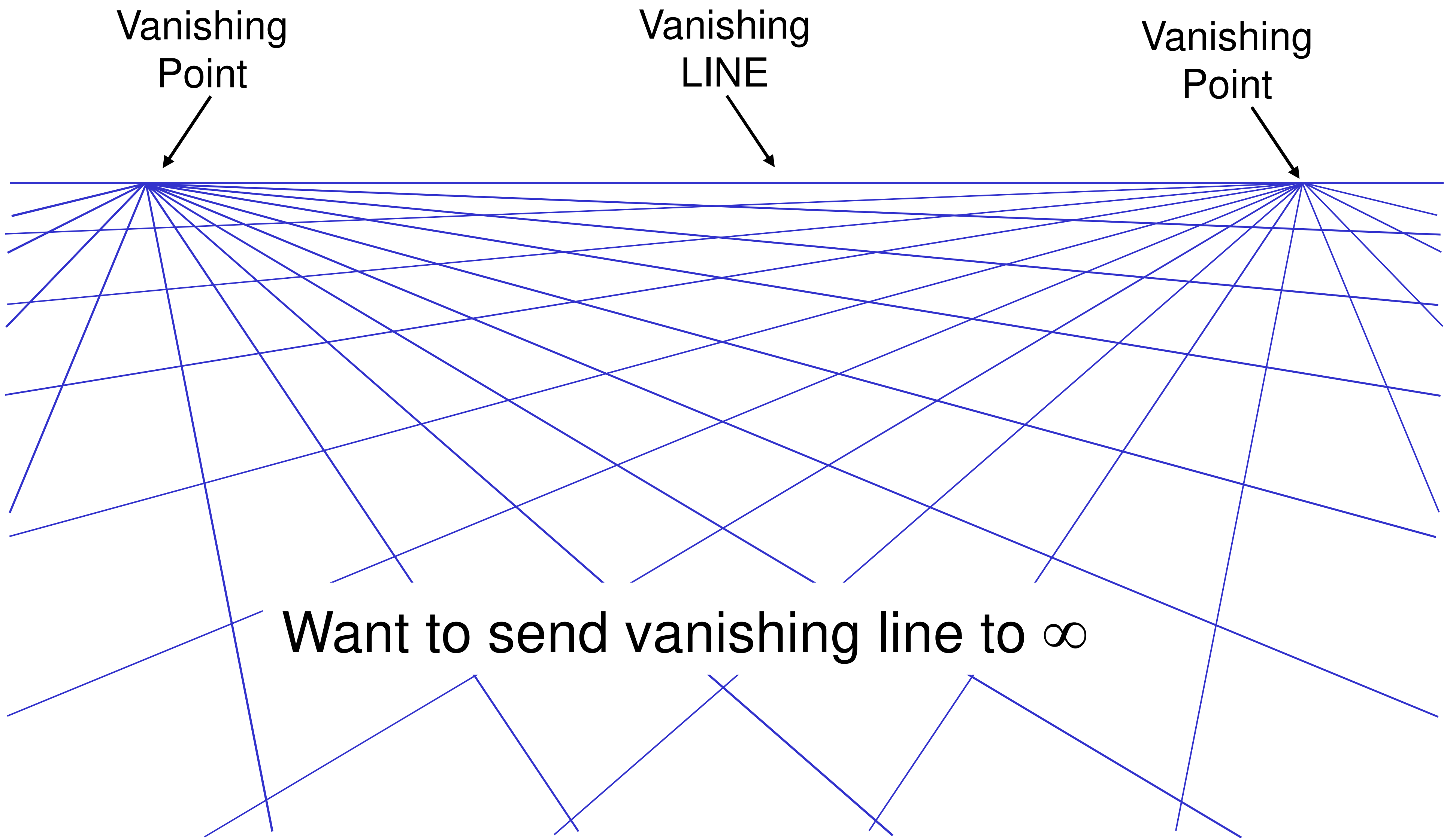


from Anat Axelrod

Rectification from Vanishing Points



Rectification from Vanishing Points



Want to send vanishing line to ∞

Rectification from Vanishing Points

Want to send vanishing line to ∞

$\mathbf{l}_\infty$ = Vanishing line $\mathbf{l}_\infty$ = (a,b,c)

Homography that sends $\mathbf{l}_\infty$ to ∞ :

$$H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & c \end{bmatrix}$$

$(x,y,z) \in \mathbf{l}_\infty \longrightarrow ax + by + cz = 0$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & c \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ y \\ 0 \end{bmatrix}$$

Point in ∞ (homogeneous coordinates)

Rectification from Vanishing Points

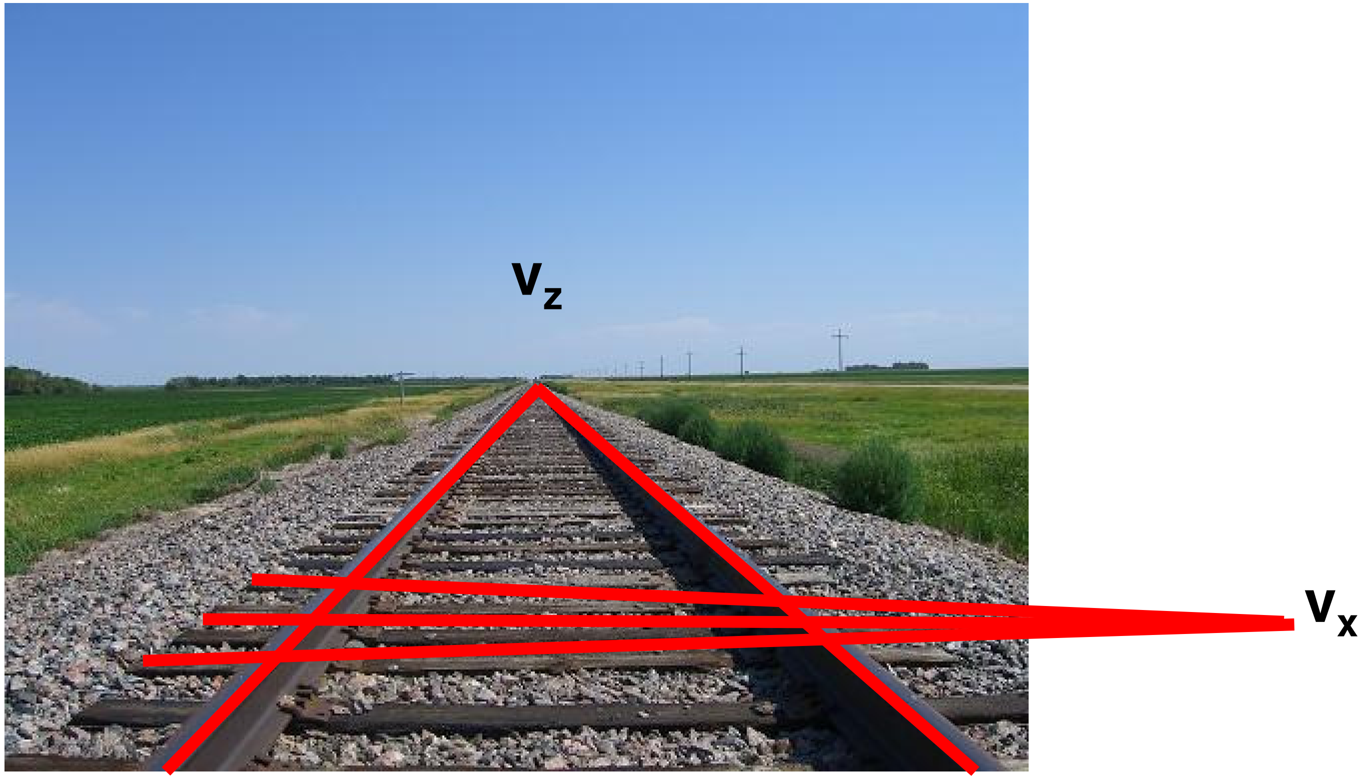
Want to send vanishing line to ∞

Note: for any affine transformation **A** : $A = \begin{bmatrix} a & b & e \\ c & d & f \\ 0 & 0 & 1 \end{bmatrix}$

AH also sends sends $\mathbf{l}_\infty$ to ∞

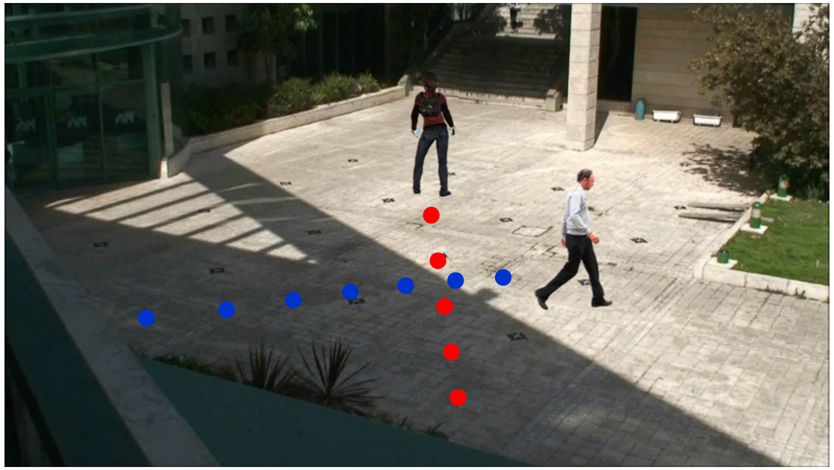
Need additional info for **A** (e.g. orthogonal directions in world)

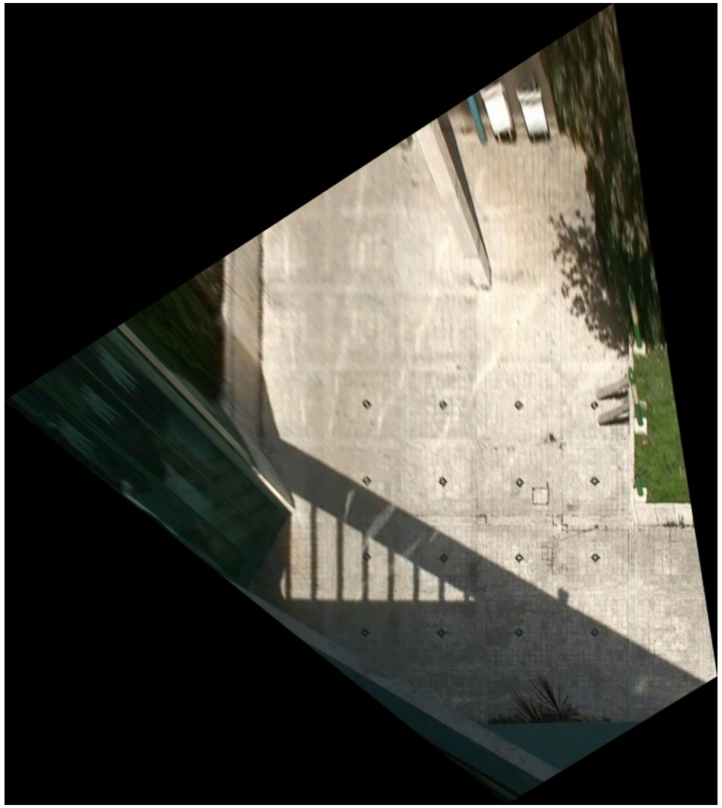
Rectification from Vanishing Points



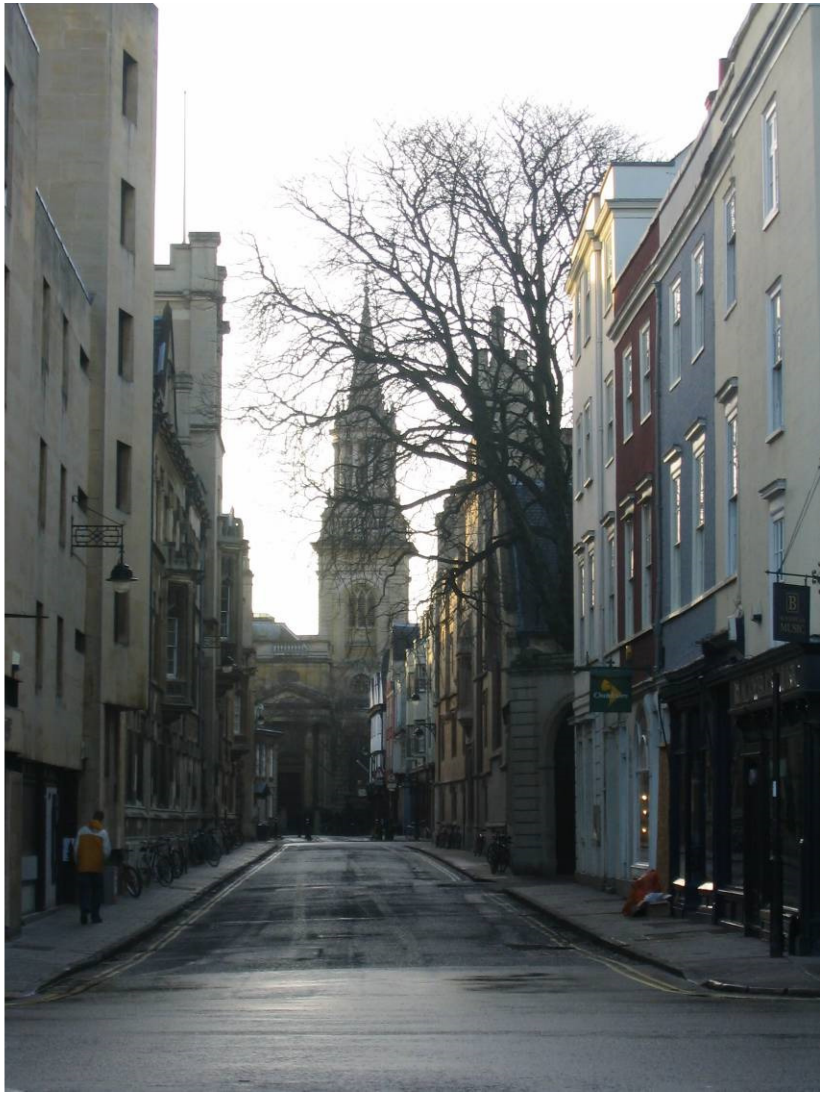
Send to $\mathbf{v}_x$ to (1 0 0) and $\mathbf{v}_z$ to (0 1 0)

Rectification from Vanishing Points





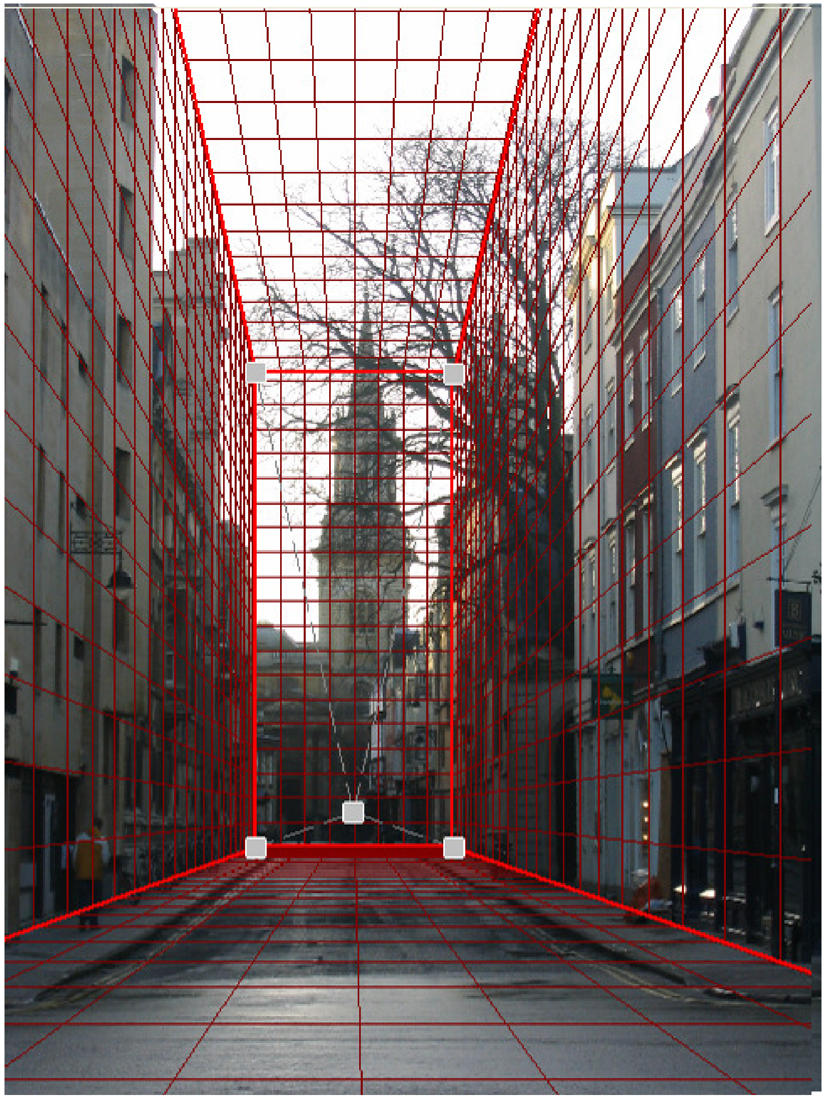
More Fun with Vanishing Points



3D scene
walk through

D. Hoiem, A.A. Efros, and M. Hebert, "Automatic Photo Pop-up", ACM SIGGRAPH 2005

3D scene walk through



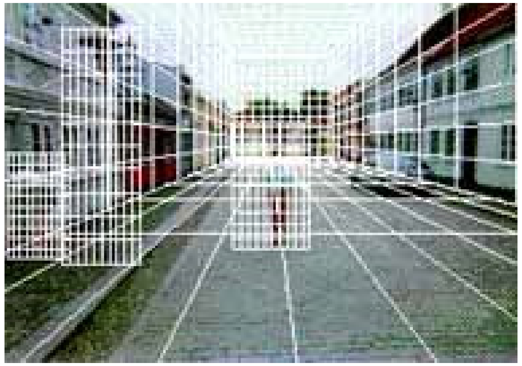
D. Hoiem, A.A. Efros, and M. Hebert, "Automatic Photo Pop-up", ACM SIGGRAPH 2005

“Tour into the Picture”

Y. Horry K. Anjyo, and K. Arai, Proc. SIGGRAPH 1997.

Create a 3D “theatre stage” of five billboards

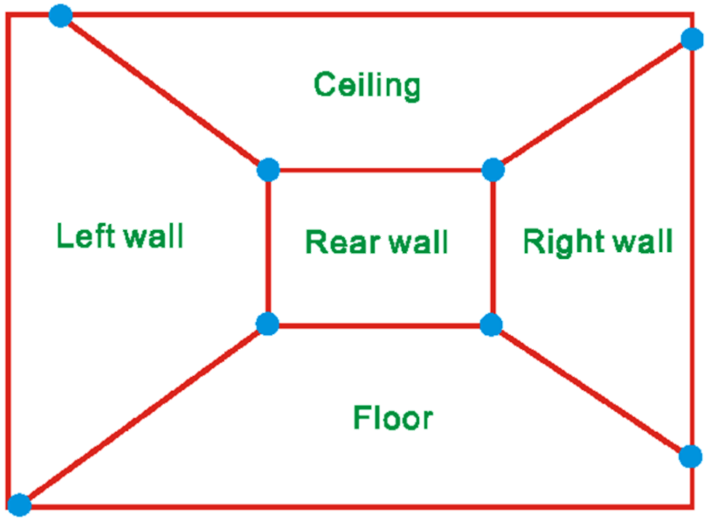
Specify foreground objects through bounding polygons



Use camera transformations to navigate through the scene

The idea

Many scenes (especially paintings), can be represented as an axis-aligned box volume (i.e. a stage)



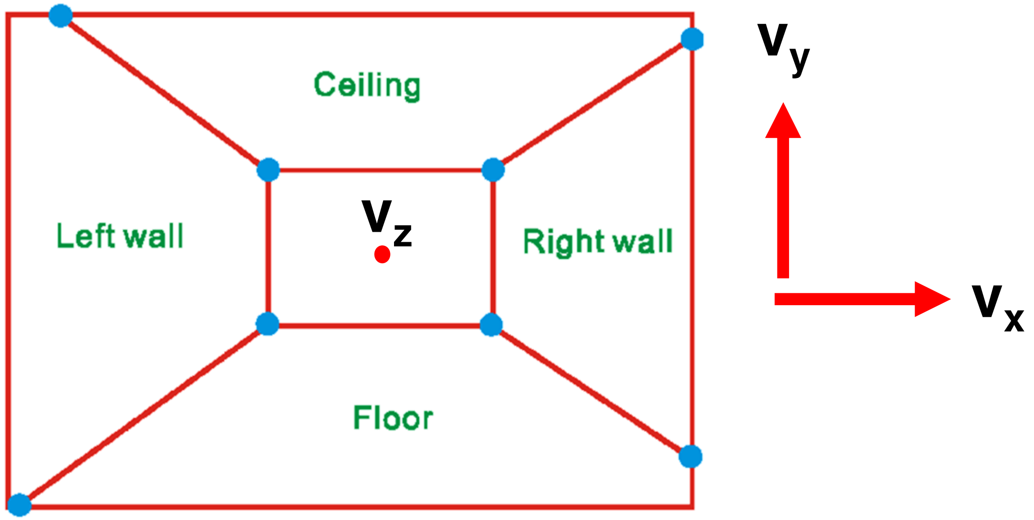
Fronto-parallel view

Key assumptions:

- All walls of volume are orthogonal
- Camera view plane is parallel to rear wall
- Camera up is normal to the floor

The idea

Many scenes (especially paintings), can be represented as an axis-aligned box volume (i.e. a stage)

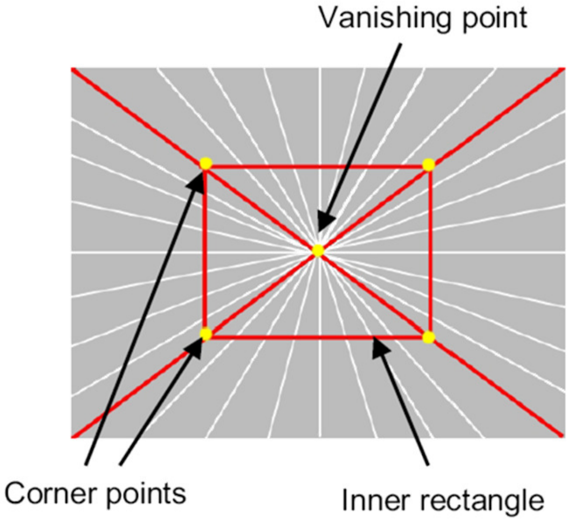


How many vanishing points does the box have?

- Three, but two at infinity
- Single-point perspective

The vanishing point can be used to fit the box to the particular scene.

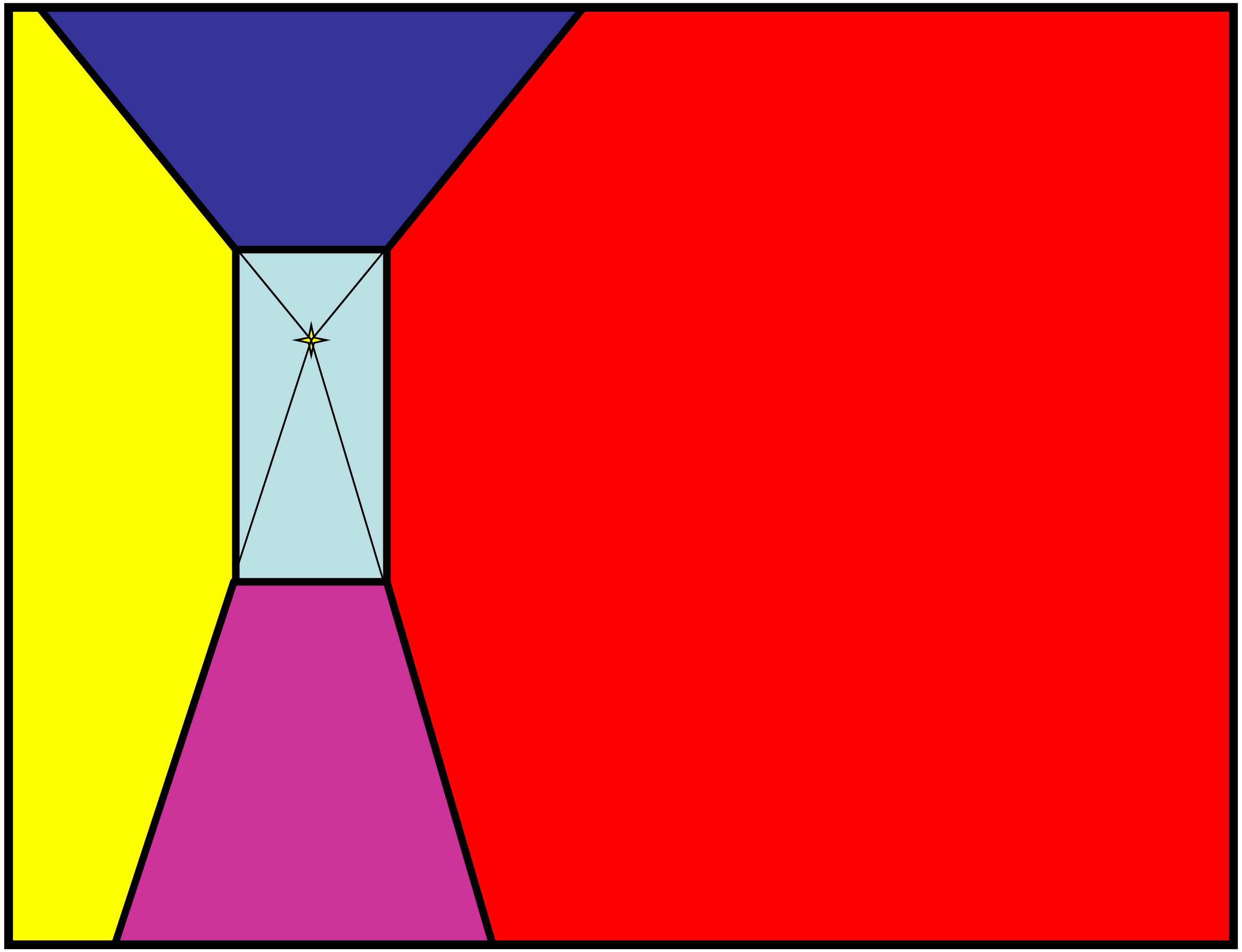
Fitting the box volume



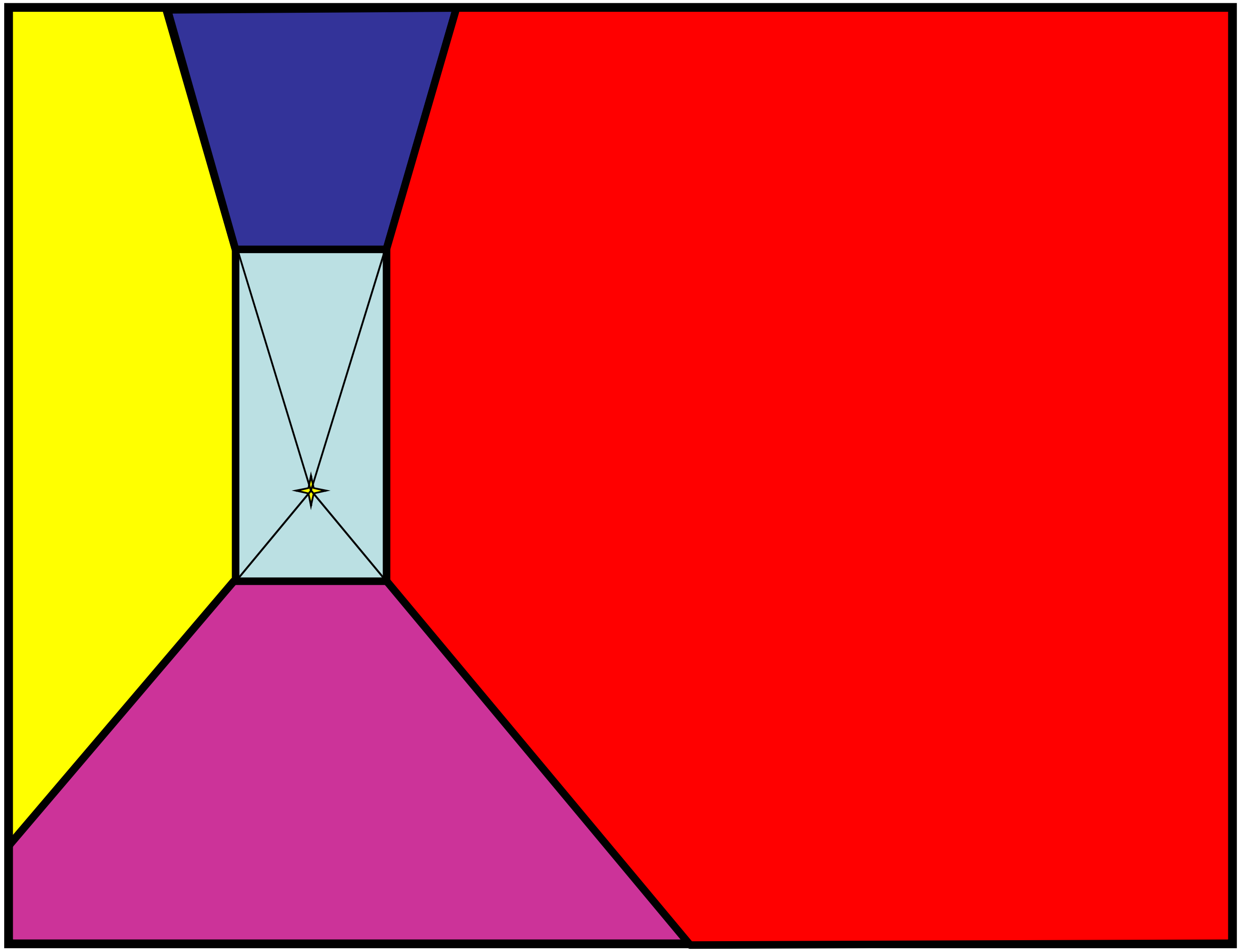
User controls the inner box and the vanishing point placement.

Q: What’s the significance of the vanishing point location?

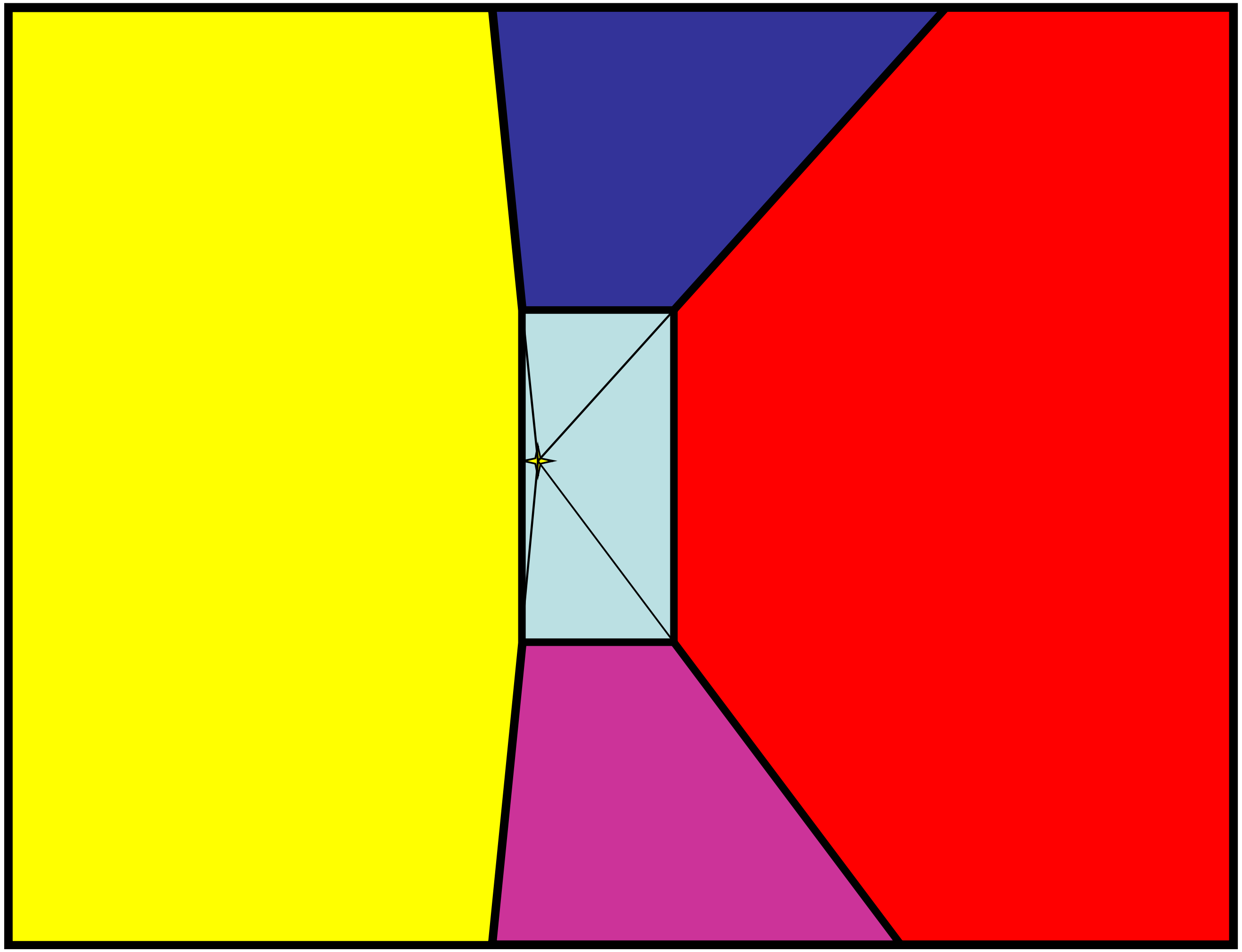
Example of user input: vanishing point and back face of view volume are defined



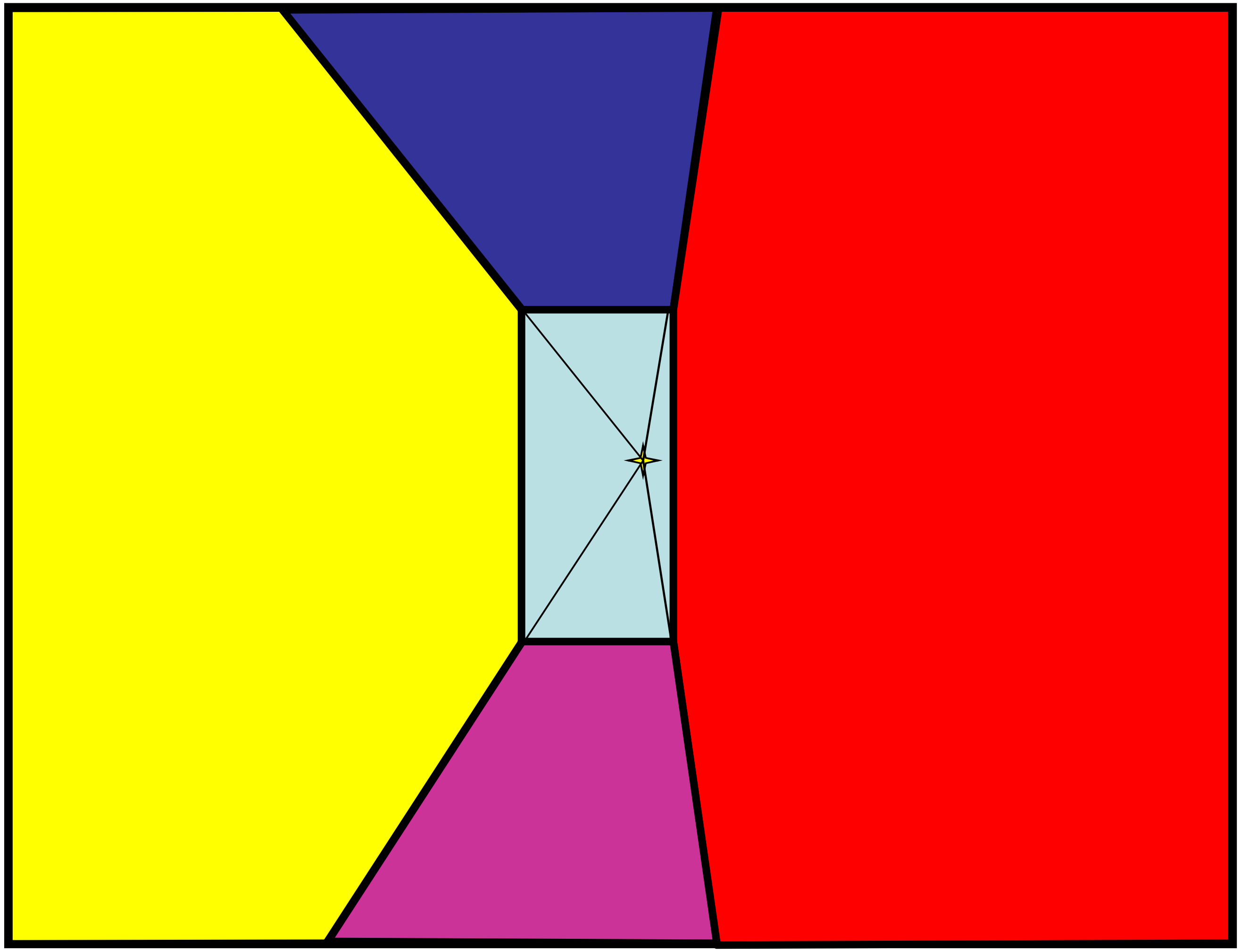
Example of user input: vanishing point and back face of view volume are defined



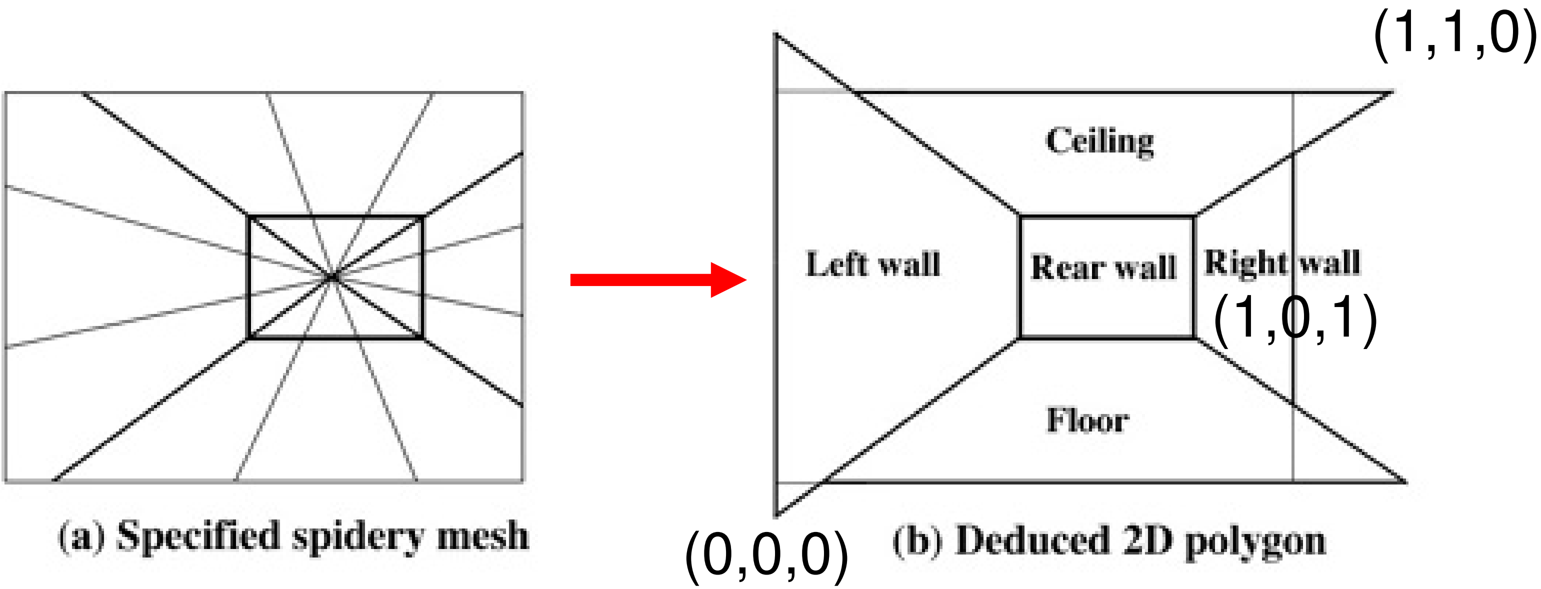
Another example of user input: vanishing point and back face of view volume are defined



Another example of user input: vanishing point and back face of view volume are defined



Building 3D model of the scene

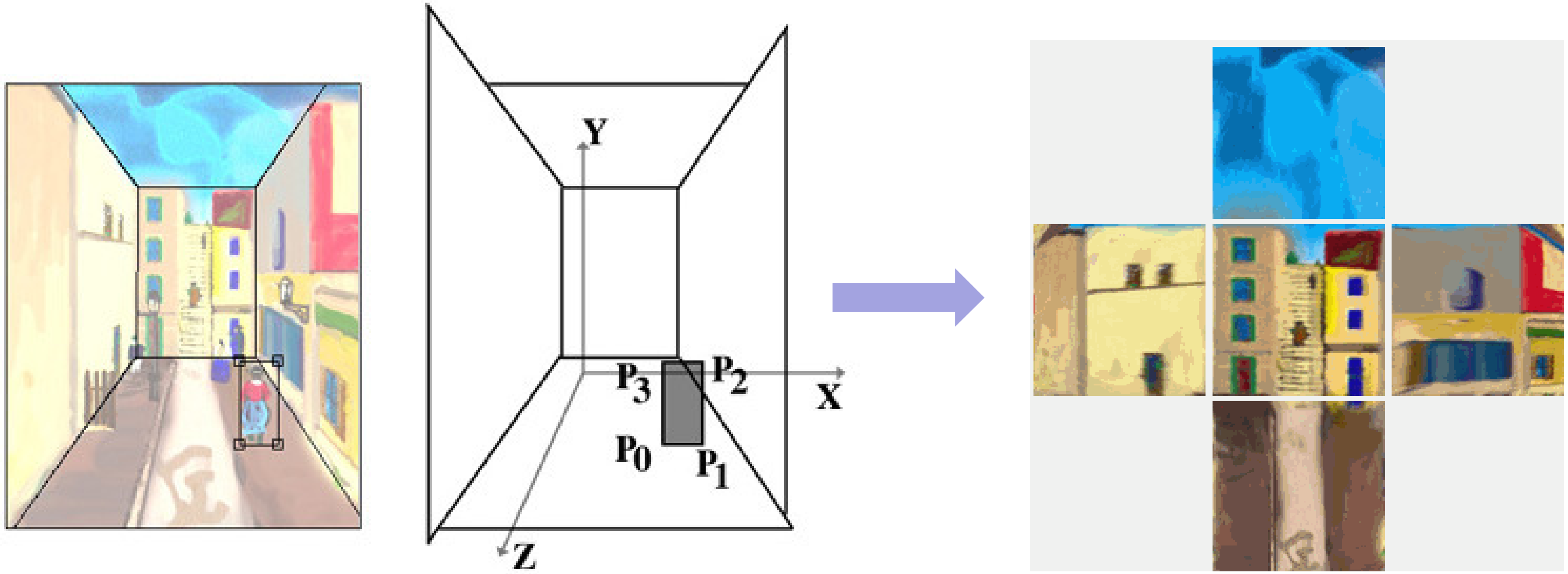


(a) Specified spidery mesh

(b) Deduced 2D polygon

Given vanishing points “invent” 3D box coordinates and map to scene

Building 3D model of the scene



(a) Specifying of a foreground object

(b) Estimating the vertices of the foreground object model

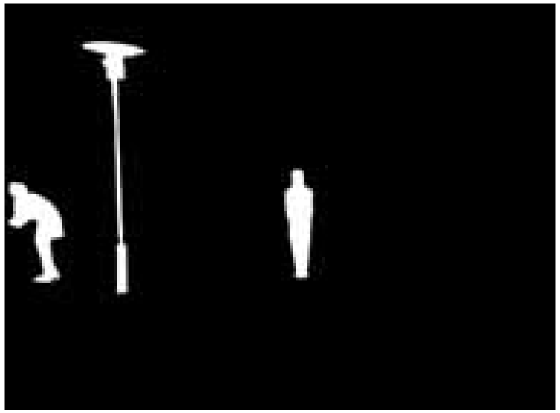
Given vanishing points “invent” 3D box coordinates and map to scene

Image to 3D

Demo

What about Foreground Objects?

Original
image



Foreground
mask

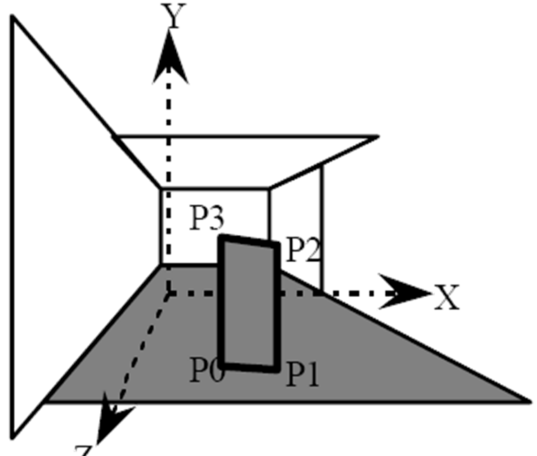
Background
Image

Foreground Objects

- Add vertical rectangles for each foreground object

- Can compute 3D coordinates P0, P1 since they are on known plane. .

(a) Specifying of a foreground object



(b) Estimating the vertices of the foreground object model

(c) Three foreground object models

The entire process flow diagram

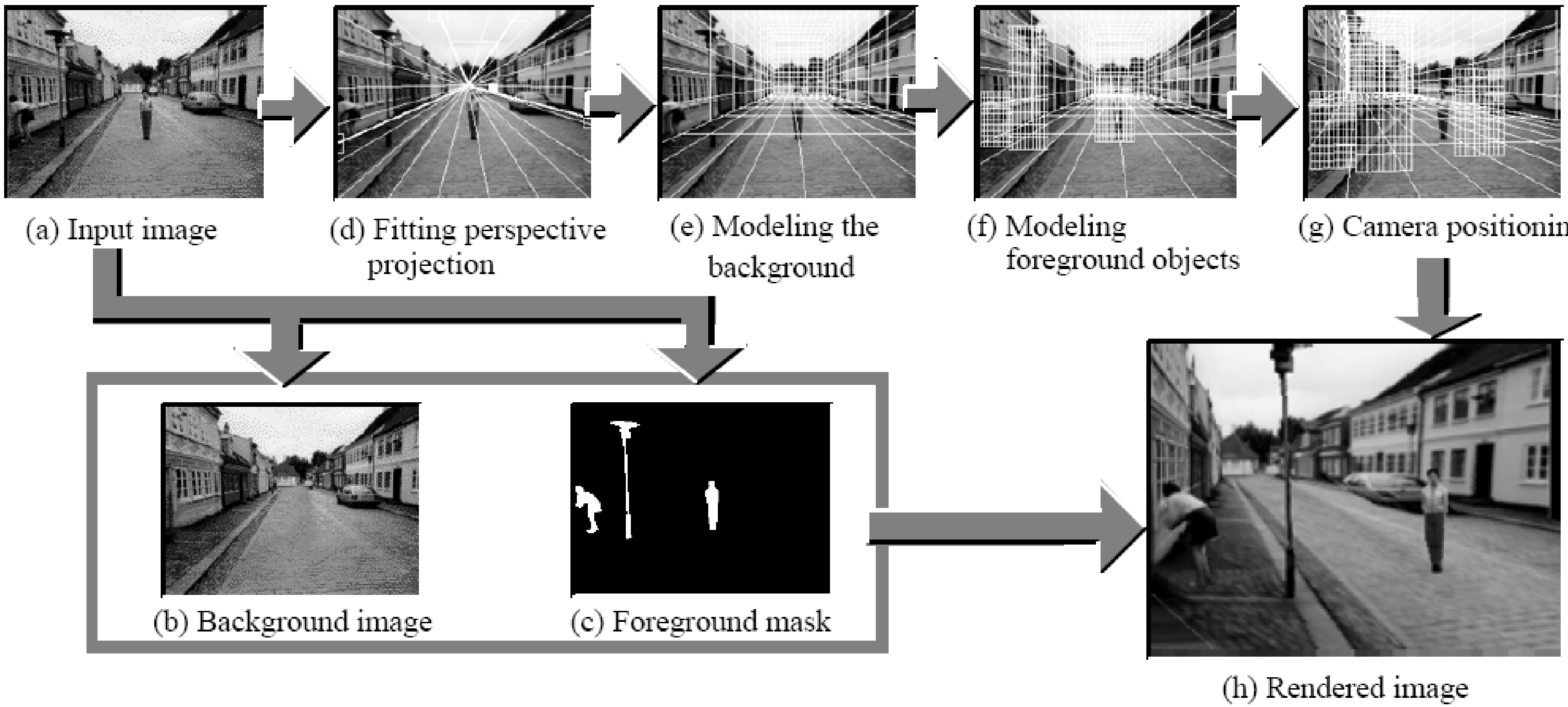
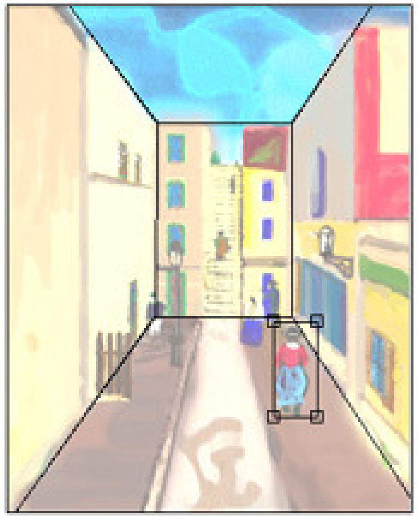


Figure 1. Process flow diagram

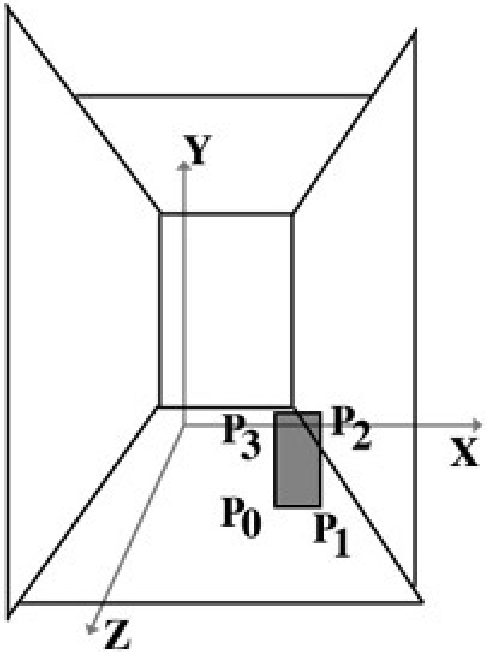
Step 1 – Masking B/F

(a) Input image (b) Background (c) Foreground mask

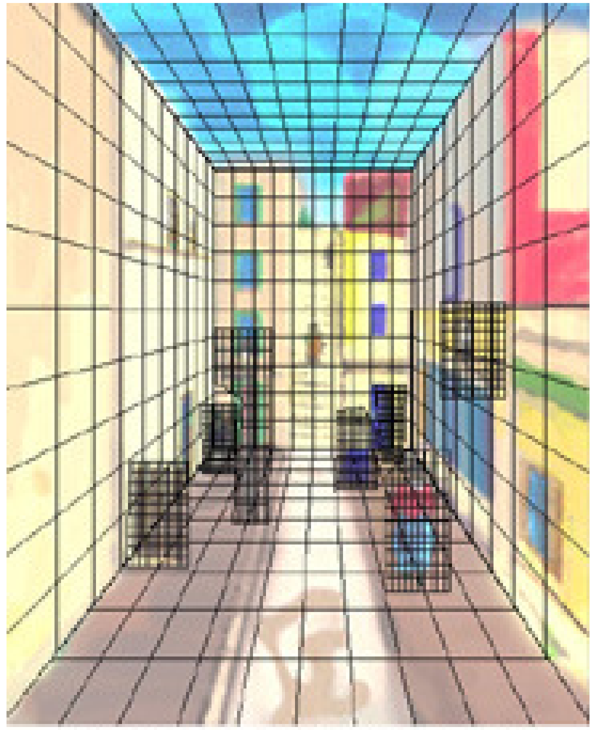
Step 2 – Modeling



(a) Specifying of a foreground object



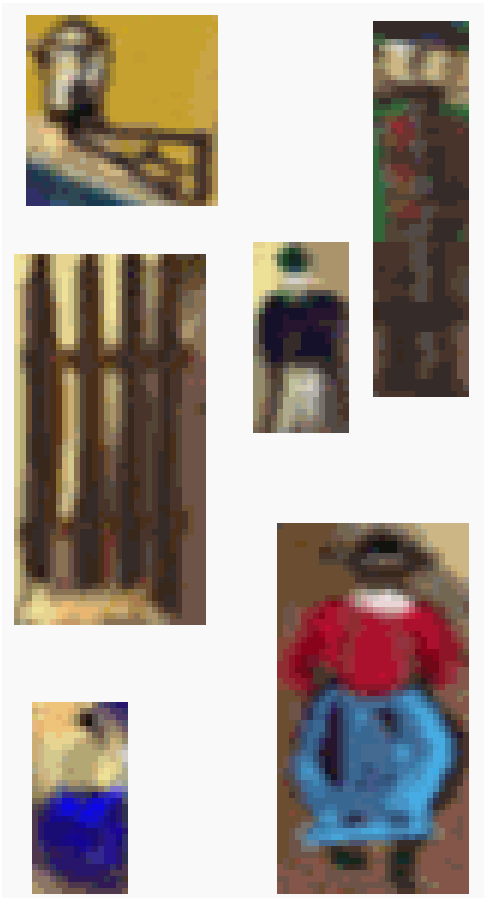
(b) Estimating the vertices of the foreground object model



(c) Foreground object models

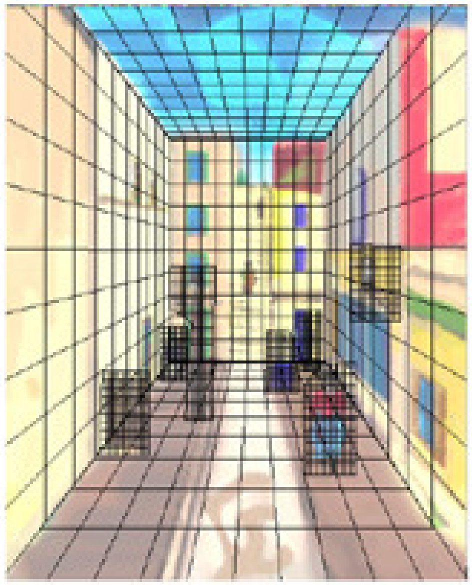
Step 3 – Texture mapping

Background

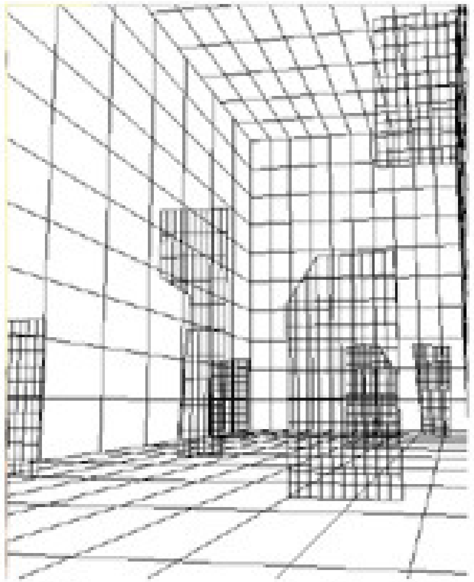


Foreground

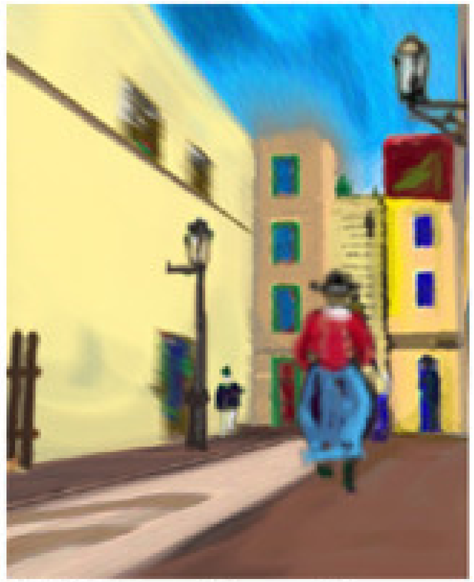
Step 4 – Key Frame Rendering



(f) Foreground model



(g) Camera positioning

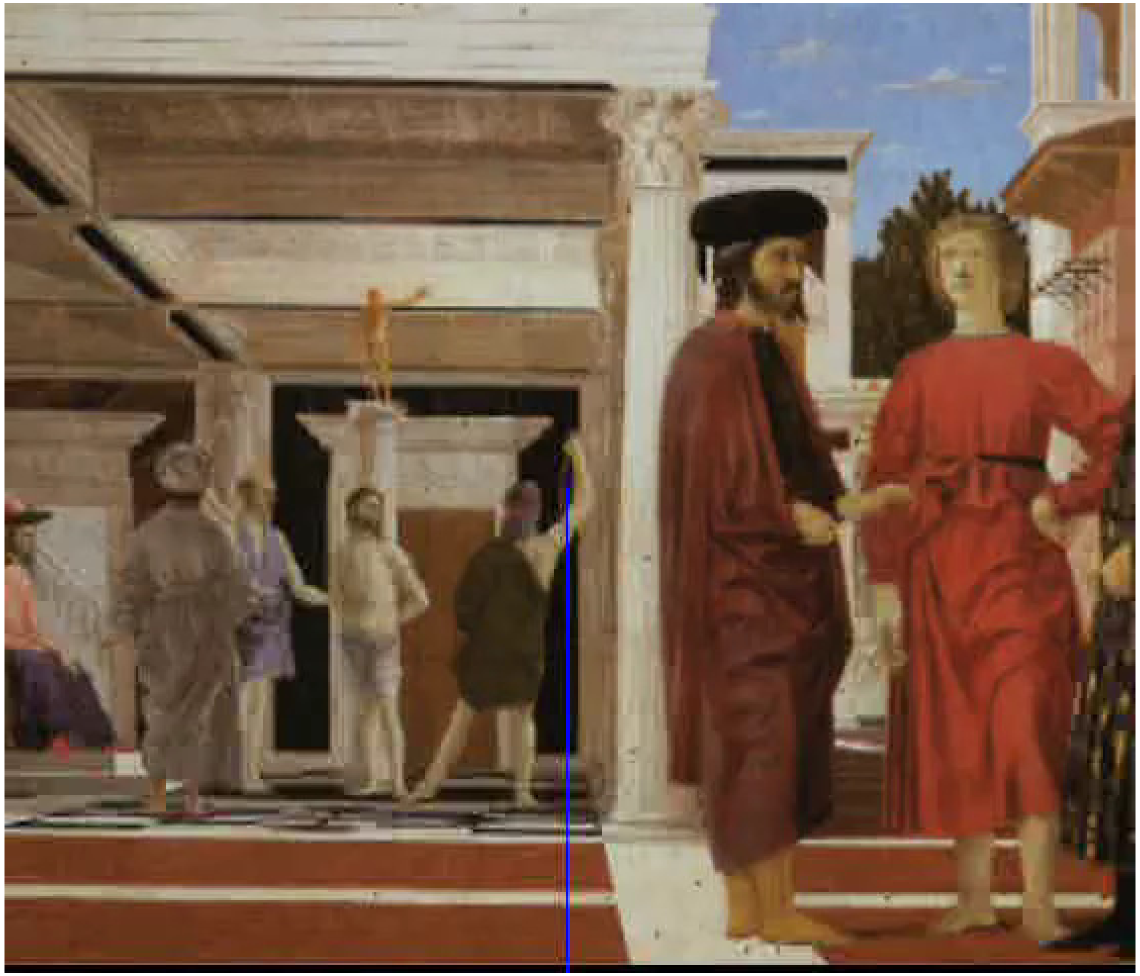


(h) Rendered image

Videos

Videos

Examples



Flagellation by Pietro della Francesca (*1416-92, Italian Renaissance period*)
Animation by Criminisi et al., ICCV 99

Demos\flagella_camera.mpg

Examples

Animation by Criminisi et al., ICCV 99

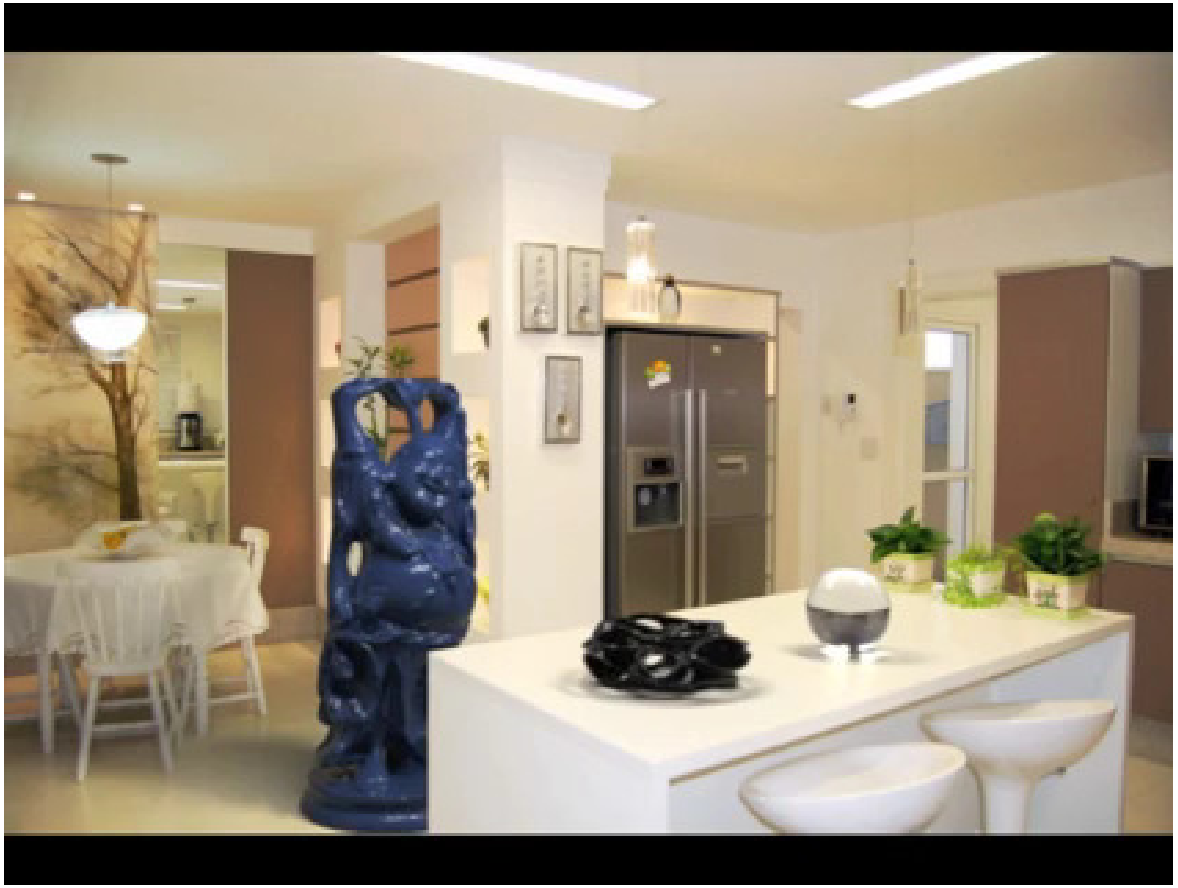
Demos\hutme_camera.mpg

Application: Image editing

Inserting synthetic objects into images:
http://vimeo.com/28962540

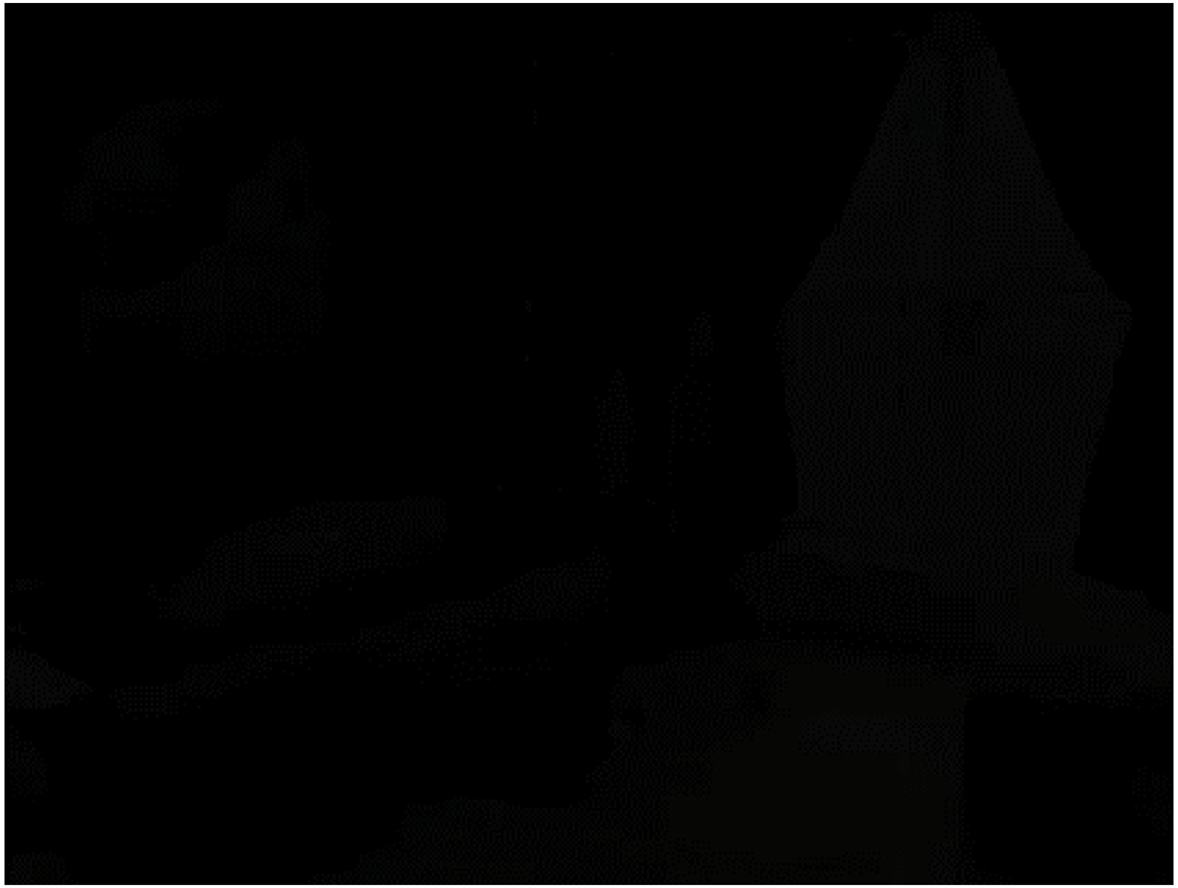
K. Karsch and V. Hedau and D. Forsyth and D. Hoiem, “Rendering Synthetic Objects into Legacy Photographs,” *SIGGRAPH Asia* 2011

Rendering Synthetic Objects into Legacy Photographs



K. Karsch and V. Hedau and D. Forsyth and D. Hoiem, “Rendering Synthetic Objects into Legacy Photographs,” *SIGGRAPH Asia* 2011

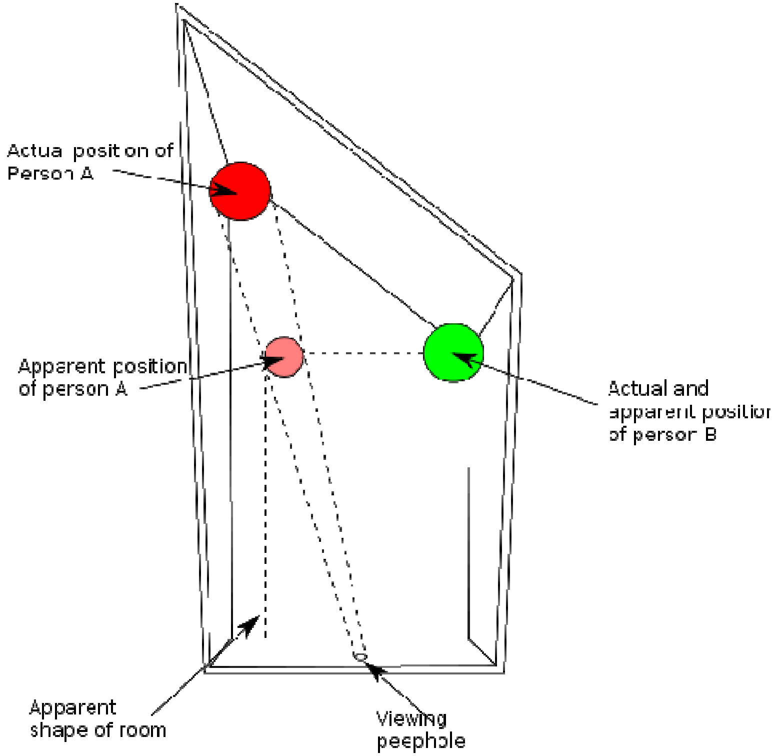
Rendering Synthetic Objects into Legacy Photographs



K. Karsch and V. Hedau and D. Forsyth and D. Hoiem, “Rendering Synthetic Objects into Legacy Photographs,” *SIGGRAPH Asia* 2011

Recovery of 3D structure

Ames Room



http://en.wikipedia.org/wiki/Ames_room

Salt desert south america

Salar de Uyuni

Uyuni Salt Flats


Toddy

מדבר המלח