Applying successive linear programming for stochastic short-term hydropower optimization

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ABSTRACT

We present a model for operational stochastic short-term hydropower scheduling, taking into account the uncertainty in future prices and inflow, and illustrate how the benefits of using a stochastic rather than a deterministic model can be quantified. The solution method is based on stochastic successive linear programming. The proposed method is tested against the solution of the true non-linear problem in a principal setting. We demonstrate that the applied methodology is a first-order approximation to a formal correct head-of-water optimization and achieve good results in tests. How the concept of stochastic successive linear programming has been implemented in a prototype software for operational short-term hydropower scheduling is also presented, and the model's ability is demonstrated through case studies from Norwegian power industry. From these studies, improvements occurred in terms of the objective function value and decreased risk of spill from reservoirs.

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1. Introduction

In Norway, power supply has traditionally been almost 100% hydropower. Hydropower optimization is challenging, and the main reason is that decisions are coupled in time; the optimization problem includes state-variables such as reservoir levels and stochastic, climate dependent variables where the most important is inflow. Therefore, the full multi-dimensional optimization problem is decomposed into sub-problems. Typically a long-, a medium- and a short-term sub-problem is formulated, where each problem is solved by dedicated solution techniques [1], as illustrated in Fig. 1. This paper presents a model for the short-term optimization of hydropower based on stochastic successive linear programming and illustrates through case studies that the proposed methodology may give improved decision support to producers acting under price and inflow uncertainty, compared to using a deterministic model.

Models for short-term hydropower scheduling have typically focused on the constraints that are important to get feasible or close to feasible generation schedules for the period where the schedules are to be implemented. This period may be different for different systems, and in the Nordic region the watercourses are typically scheduled for the next operating day according to the daily clearing of the day-ahead market. The day-ahead market is the most important market place for power in the Nordic countries with 84% market share in 2013 [2]. Short-term scheduling must consider many constraints due to complex cascaded watercourses, concession rules, shared ownership, multipurpose use, etc., but also technical constraints like startup costs, generation ramping and various market constraints. In Scandinavia, uncertainty in variables such as inflow and market prices are handled by frequent reapplication of models with updated input parameters (also called rolling horizon), or by adding safety constraints that limit the characteristics of optimization models to produce too smart schedules for the hydropower system. The cost of such uncertainty imposed constraints is calculated from sensitivity analyses or based on specific and practical system experience. This works fine as long as some flexibility is available in the hydropower system or in the different markets. The sequential structure of power markets creates needs and opportunities for rescheduling. The volumes offered in the day-ahead spot market reflects expected production for each producer, but these volumes can be adjusted in the intra-day market, Elbas, as new information is revealed over time. In 2013, the total traded volume in the Elspot day-ahead market at Nord Pool was 349 TWh, while the volume traded on Elbas was 4.2 TWh [2]. Producers may also participate in the real-time balancing market for regulating power. In the regulating power market producers bid to sell additional power or buy back power from the market

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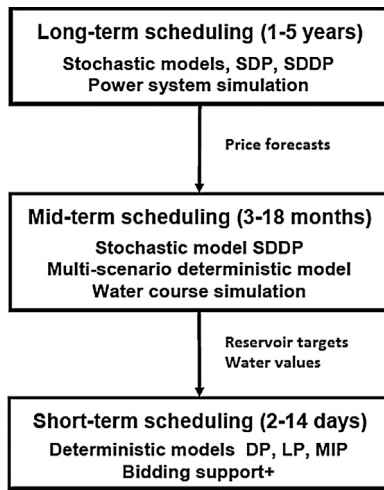
Fig. 1. Hydropower scheduling sub-problems.

in order to maintain the instantaneous balance between supply and demand. Fig. 2 gives an overview of bid and operating hours for the different markets under present organization of markets in Norway.

The behavior of the markets is expected to become much more volatile due to the transition toward more renewable power production in the energy systems. The European Union Renewable Energy Directive 2009/28/EC [3], that was implemented in 2010, define binding targets for 20% renewable contribution to total energy demand by 2020. From January 2012 a joint certificate system was implemented in the Nordic market to ensure development of 26.4 TWh of new renewable energy toward 2020 [4]. To reach European and Nordic targets, intermittent production such as wind- and solar power will play a major role. As a consequence of the increased variability of inter-nordic balancing and regulating power and new cables that are planned from the Nordic system to the rest of Europe, Nordic power prices may also become more volatile in the future. Hydropower producers might then have to use more of the capacity toward the intra-day market or the balancing markets. To optimize production in the future, hydropower

utilities must therefore schedule the watercourses in such a way that obligations in several different markets can be honored. The challenge is to maintain flexibility for fast changes in generation levels without increased spillage or efficiency loss, and to decide what part of the capacity to use in what market. This task calls for an explicit representation of the uncertainty of price and resource availability. Continued operation with multiple re-runs or manual rules for maintaining system flexibility is difficult when the boundary conditions are constantly changing, in which case the safety limits should become an integrated part of the operational decisions.

1.1. Short-term hydropower scheduling

The challenge of short-term planning is to handle non-linear and non-convex elements together with state-dependencies. Non-linearities are present almost everywhere in hydropower modeling, in efficiency curves, reservoir curves, losses and so on. Examples of non-convex elements are minimum generation and spill descriptions. State-dependency also occur several places; water flow through gates and hydraulic connections [5], but regarding overall hydropower efficiency the state-dependency in turbine curves are the most important. Efficiency of hydro turbines depends on head and head depends on reservoirs levels but also discharge dependent losses above and sometimes below the turbine. The head, or pressure height, the coming hour(s) depends on the decision that the operator is making this particular hour. This makes it impossible to build an exact efficiency curve for the turbines for coming hours and in a two-week perspective errors might be large. Different techniques are in use for handling this issue. One method is to apply successive linear programming (SLP) [1,6,7]. This method is implemented in SHOP (Short-term Hydro Optimization Program) [7], which is applied by most large producers in Scandinavia. A large effort has been put into the development of this general hydro-scheduling model so that it includes many details important for Scandinavian watercourses.

Another approach is found in [8]. There the non-linear three-dimensional relationship between the head, the water discharged and the power generated is handled by discretization of a family of non-linear curves. This leads to a mixed-integer linear model that
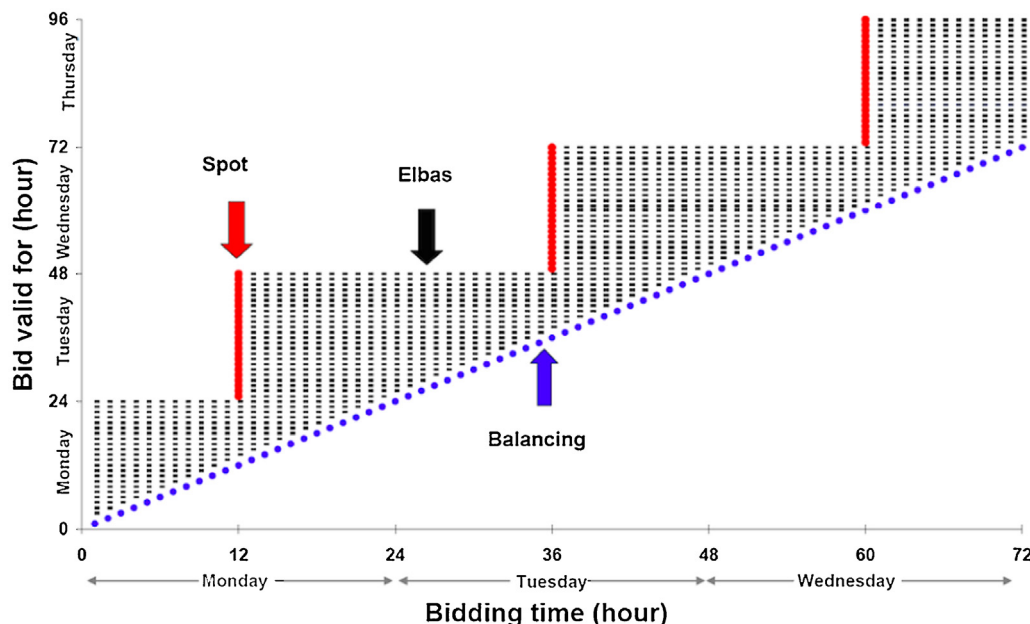

Fig. 2. Hydropower dispatch in the Nordic markets.

uses binary variables to represent which curve is used according to the levels of stored water in the reservoirs. Ref. [9] presents a non-linear model for head-dependent hydro systems and shows that the non-linear formulation outperforms a similar linear model that neglects head-dependency.

The models described in [5–9] are deterministic. In the future, hydropower utilities must schedule the watercourses in such a way that flexibility for fast changes in generation levels is obtained without loss of water or efficiency. In other words, it is a need for an explicit representation of the uncertainty of price and resource availability. Application of a stochastic version of SLP, stochastic successive linear programming, SSLP, for this purpose is described in [10]. Other approaches with higher focus on the stochastic representation and less on hydropower details has also been investigated, for instance in [11,12]. More detailed models based on SSLP are reported in [13,14]. In [13] the increased revenue from using a stochastic model varied within [0; 16%] and in [10] it was 1.3%. Both results are excellent and could in monetary terms defend huge investments both in terms of development of a software model and implementation in the hydropower utilities. In this paper we investigate whether the successive linear programming method can be extended to a stochastic setting and if this new model can reproduce these significant results based on real data.

The reminder of the paper is organized as follows. In Section 2 we first describe how uncertainty in input variables is specified in the model. Section 3 describes experience with a preliminary prototype implemented in GAMS. This model was used for the initial investigation of convergence qualities, and to test and verify the implemented methodology for head-of-water optimization. The pure linear model formulation is presented and we describe how the non-linear problem elements are handled by an iterative solution procedure. The implemented head-of-water optimization is derived formally. We also formulate and solve a corresponding non-linear problem formulation. In Section 4 we explain how we have implemented uncertain variables in a prototype tool for short-term hydropower scheduling, SHARM, and also the method's capability to become rich enough in detail and fast enough for operational use. Section 5 explains a method for evaluating the solutions from stochastic and deterministic models based on the scenario tree. This method is then deployed to three case studies in Section 6. Final conclusions are provided in Section 7.

2. Modeling uncertainty

The most important candidates for stochastic variables in a short-term hydropower scheduling model are inflow to one or more reservoirs and electricity prices. The electricity market actually consists of several markets including the day-ahead spot market, intra-day and balancing markets as shown in Fig. 2. All markets contribute to the uncertainty when scheduling future generation, and should ideally be taken into account. In the current implementation of the SHARM model however, only the day-ahead spot market is considered, and the spot electricity prices are the only stochastic prices in the model. Section 6 shows plots of typical forecast distributions of price and inflow for a 1 week horizon.

The hydropower scheduling problem is specified as a stochastic mixed-integer linear program, formulated as a deterministic equivalent. Within this formulation, the stochastic variables are specified in the form of a scenario tree. An example of a simple scenario tree is illustrated in Fig. 3. Each node of the tree contains possible future realizations for each of the stochastic variables, and the branches are assigned transition probabilities conditionally on the preceding node. Each root-to-leaf path of the tree constitutes one scenario, such that by scaling the transition probabilities for the branches between the latter two stages to sum to 1, these probabilities equal
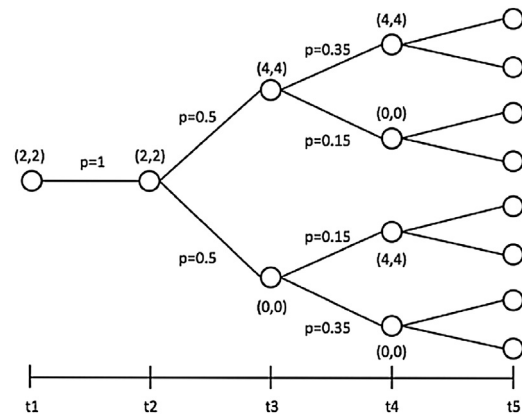

Fig. 3. An example scenario tree: a two-dimensional, five-stage tree with binary branching.

the scenario probabilities. In the example in Fig. 3 the planning period consists of five time steps (stages), the branching factor is two, and there are two stochastic variables, price and inflow. The values of these variables are given in parentheses above/below each node of the tree, so that at the single node at t1, both price and inflow is 2 energy units. The branch probabilities are 0.5 for the transition from stage t2 to t3, and 0.35 or 0.15 for the branches from stage t3 to t4.

In the example in Fig. 3 the tree branches at each stage. For a model with an hourly time resolution and a planning horizon of 1–2 weeks, the number of nodes will rapidly become computationally intractable. As an alternative, branching can be restricted to a limited number of stages. For example, as spot prices are settled for 24 h at a time, branching every 24 h seems reasonable for the case of stochastic prices. Further, assuming that a 24-h ahead deterministic inflow forecast is reliable enough might be a reasonable approximation. In addition to limiting the number of branching points, some form of tree reduction might be needed.

Several methods have been proposed for scenario tree generation and reduction, see e.g. [16] for an overview. In the case studies in Section 6 scenario trees are generated from the set of potentially multi-dimensional scenarios for inflow and price using the tree-generation algorithm in [17–19]. Inflow and price scenarios can be generated by stochastic simulation from a statistical model fitted to historical data. Inflow scenarios for one or more reservoirs can also be based on ensemble prognoses from a physical model like the HBV model [20] or a distributed model such as the ENKI-model [21]. For the former, post-processing is needed to adjust for bias and under-dispersion in practice [22]. However, for the case studies we use unadjusted ensemble forecasts for inflow. Scenario trees that adequately describe the joint forecast distribution of inflow and price might pose problems in terms of the size of the tree. Generation of the joint forecast distributions and multi-dimensional scenarios for inflow and price, as well as scenario tree generation, are problems in their own right, and for simplicity we look at price and inflow uncertainty separately in the case studies of Section 6.

3. Testing aspects of the SHARM model

To study some important aspects of the stochastic hydropower scheduling model implemented in SHARM, numerical simulations based on a simplified representation of the full SHARM model has been conducted using GAMS [23]. The simplified model is sufficient to illustrate some main features of SHARM. We first describe a pure linear stochastic model. However, since the problem actually is non-linear, this simple formulation is not sufficient. Therefore, we show how the linear model is updated in an iterative process to

Table 1
Symbols for pure linear model.

Symbol	Explanation
Sets and indices	
Set I, index i	Nodes in the stochastic tree
Set I^{end}, index i	End nodes, $i_{end} \in I$
Set J, index j	Linear segments for the end-value function
Set M, index m	Units. A unit is a reservoir plus generator(s) directly below the reservoir.
Variables	
e_{ij}	Use of jth segment in function for value of stored water at the end of the planning horizon, $e_{ij} \leq L_j^{max}$
s_{im}	Spillage from reservoir
w_i	Value of stored water at end of planning horizon
x_{im}	Reservoir storage, $X_m^{max} \geq x_{im}$
x_{im}^p	Reservoir storage at the end of the time-step represented by I's parent node
y_{im}	Water used for power generation, $y_{im} \leq Y_m^{max}$
y_{im}^{best}	Use of water at best efficency, $y_{im} \leq Y_m^{best}$
y_{im}^{rest}	Use of water in excess of best efficiency
z_{im}	Generated electricity
Parameters	
E_{im}^{best}	Best efficiency for generator
E_{im}^{rest}	Above best-point efficiency for generator
X_m^{start}	Reservoir storage at start of planning period
X_m^{max}	Maximum reservoir storage
V_{im}^{prod}	Water-course identification parameter, $V_{im}^{prod} \in \{0, 1\}$
L_j^{max}	Length of each time step in marginal value function for stored water at the end of the planning horizon
Q_i	Probability for node I
P_i	Spot price in node I
V_{im}	Inflow to reservoir I
Y_m^{max}	Generator capacity for use of water for generator
Y_m^{best}	Maximum use of water at best efficiency
W_j	Marginal value of jth segment in end-value function for stored water at the end of the planning horizon

account for non-linearities. The model is also modified to account for head-of-water optimization and to give an incentive for convergence of the iterative process. Finally we present a non-linear model formulation that is solved numerically to verify results from the described iterative procedure.

3.1. A pure linear model

The linear model formulation is provided in Eqs. (1)–(6). The model has hourly time-resolution, and the typical planning period is 1–2 weeks. All symbols for the linear model are explained in Table 1. Parameters and sets are capitalized, while variables are not. Superscripts are part of symbol names, while subscripts are indices. In cases of parametric bounds for variables, these constraints are specified as a part of the definition in the list of symbols in Table 1. All variables are positive variables by definition unless otherwise stated.

$$\max \sum_{i \in I} Q_i P_i \sum_m z_{im} + \sum_{i \in I^{End}} Q_i w_i \tag{1}$$

$$y_{im} = y_{im}^{best} + y_{im}^{rest} \tag{2}$$

$$z_{im} = y_{im}^{best} * E_{im}^{best} + y_{im}^{rest} * E_{im}^{rest} \tag{3}$$

$$x_{im} = x_{im}^p + V_{im} - y_{im} - s_{im} + \sum_{n \in M \setminus \{m\}} V_{nm}^{prod} y_{in} \tag{4}$$

$$w_i = \sum_{j \in J} W_j e_{ij} \tag{5}$$

$$\sum_{j \in J} e_{ij} = \sum_m x_{im} \frac{E_{im}^{best}}{\max_{k \in J}\{E_{km}^{best}\}} \tag{6}$$

The problem is formulated as a linear programming problem, where the objective given by Eq. (1), is maximized subject to the constraints in Eqs. (2)–(6) and the parametric constraints for variables given in Table 1. The objective function is the expected income from hydropower production, plus the value of stored water at the end of the planning period. The efficiency of hydropower is increasing in an interval from zero to the best-point production. To avoid non-convexities, we apply a formulation where there are only two steps on the efficiency curve. The efficiency is reduced for production above best-point. The use of water is therefore divided into two variables corresponding to water-use up to best-point and above best-point, cf. Eq. (2). Total electricity production is the sum of used water multiplied with the corresponding energy conversion factor, cf. Eq. (3). The amount of water at the end of each time-step represented by a node, is given by the amount of water passed through from the previous time-step for this branch of the stochastic tree, plus inflow and production-water from upstream units in the river-system, minus the amount of water used for production and possible spillage, cf. Eq. (4). The value of stored water at the end of the planning horizon is defined as a step-wise function, cf. Eq. (5). If more water is stored, the marginal value of water declines. The total amount of water available is the efficiency-weighted sum of amounts in all reservoirs, cf. Eq. (6).

3.2. Iterative updating and avoiding flip–flop

The model defined by Eqs. (1)–(6) is a linear model that is straightforward to solve. However, in reality, a higher head of water will add energy to each m³ of utilized water. This effect is important when scheduling buffer reservoirs and must be implemented in the model. Unfortunately, the model becomes non-linear if efficiencies are defined as variables, cf. Eq. (3). Therefore, we apply an iterative approach. In each iteration, efficiencies are updated in accordance with the reservoir level in the previous round, cf. Eqs. (7) and (8). Additional symbols are explained in Table 2. For simplicity, the applied formulation implicitly assumes that reservoirs have flat bottoms and straight walls.

$$E_{im}^{best,new} = E_{im}^{min} + (E_{im}^{max} - E_{im}^{min}) \frac{x_{im}}{X_m^{max}} \tag{7}$$

$$E_{im}^{rest,new} = E_{im}^{rest} \frac{E_{im}^{best,new}}{E_{im}^{best}} \tag{8}$$

Solving the model in Eqs. (1)–(6) iteratively, while applying the updates in Eqs. (7) and (8), typically gives flip–flops between different solutions. None of the solutions are actually consistent since the

Table 2
Symbols for updating pure linear model, and non-linear model.

Symbol	Explanation
Variables	
Δx_{im}	Altered reservoir level compared to previous iteration
p_{im}	Penalty for altered reservoir level
z_{im}^{extra}	Artificial generation – an incentive that implements head of water optimization in a linear model. Free variable
E_{im}^{best}	Best efficiency for generator
E_{im}^{rest}	Above best-point efficiency for generator
Parameters	
E_{im}^{min}	Best-point efficiency when reservoir is empty
E_{im}^{max}	Best-point efficiency when reservoir is full
$E_{im}^{best,new}$	Updated value for best-point efficiency
E_{im}^{rest}	Updated value for efficiency above best-point
x_{im}	Reservoir level in previous iteration in linear model
T^{pen}	Penalty parameter for deviations from previous solution
z_{im}	Electricity generated in previous solution of linear model
S_m	Above best-point efficiency as relative value compared with best-point efficiency

efficiencies that correspond to the solution of the model will be different from the assumed efficiencies when solving it. On the other hand, the objective function is in many cases rather flat close to the optimum. Thus, for our cases, a small penalty for deviation from the previous-round solution, cf. Eqs. (9) and (10), was sufficient to provide convergence.

$$\Delta x_{im} \geq \begin{cases} x_{im} - X_{im} \\ X_{im} - x_{im} \end{cases} \tag{9}$$

$$p_{im} = T^{pen} \Delta x_{im} \tag{10}$$

Deviation from the previous round is measured by Eq. (9), while the penalty for altered reservoir level in Eq. (10) is included as a negative term in the objective function. However, the penalty is exactly zero when the algorithm converges, defined by $x_{im} = X_{im}$.

3.3. Head of water optimization

The iterative approach gives a solution of the linear model so that the assumed efficiencies that enters into the model are consistent with the corresponding evaluated efficiencies post the optimization. Still, this does not produce a true optimal solution for the problem. The reason is that efficiencies are only updated between each iteration, while the linear optimization model does not take into account that an altered reservoir level will give a different efficiency. If the efficiency is treated as a variable, the model becomes non-linear and different solution methods must be applied. However, in the iterative approach, it is possible to formulate a linear term that actually is a first-order approximation to the missing mechanism in the linear model. This term can be derived by evaluating how much higher production would be due to higher efficiencies if the reservoir level had been slightly higher, while all other variables are assumed constant and equal to the previous solution in the iterative loop. We evaluate this term by differentiation of Eq. (3) with respect to the reservoir level, and applying Eqs. (7) and (8). In Appendix A we show that this gives:

$$z_{im}^{extra} = \frac{(E_{im}^{\max} - E_{im}^{\min})E_{im}^{best}Z_{im}}{X_m^{\max}}(x_{im} - X_{im}) \tag{11}$$

This linear term is included in the objective function so that the efficiency-gain of increased reservoir levels are accounted for in the linear model formulation, while at the same time the term vanishes when the algorithm converges ($x_{im} = X_{im}$). The new objective function for the linear model is:

$$\max \sum_{i \in I} Q_i P_i \sum_{m \in M} (z_{im} + z_{im}^{extra} - p_{im}) + \sum_{i \in I^{End}} Q_i w_i - \sum_{i \in I, m \in M} p_{im} \tag{12}$$

The successive linear model maximizes the objective function in Eq. (12) subject to Eqs. (2)–(6), (9)–(11), and parametric constraints for the variables. Between each iteration efficiencies are updated in accordance with Eqs. (7) and (8).

3.4. Non-linear model

For large numerical systems, it is typically harder to solve a non-linear model using a non-linear solver than solving a linear system using linear solver. However, for the first simple tests, it is possible to define and solve the problem both as a linear and non-linear problem. In this way we have tested the above described iterative approach for a linear model against the solution when applying a non-linear model formulation. In the non-linear model, efficiencies are variables instead of parameters:

$$e_{im}^{best} = E_{im}^{\min} + (E_{im}^{\max} - E_{im}^{\min})\frac{x_{im}}{X_{im}^{\max}} \tag{13}$$
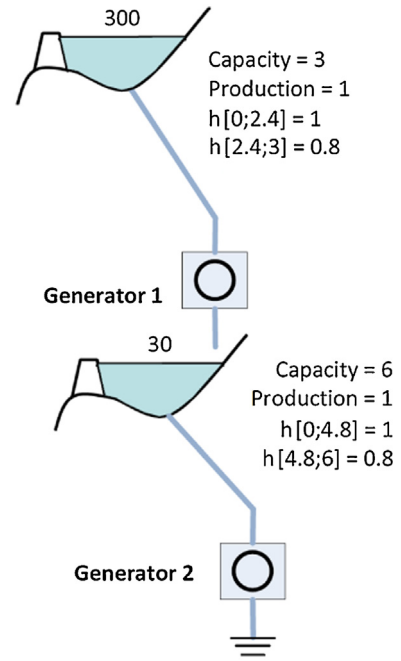

Fig. 4. Example system for GAMS test: two coupled hydropower stations.

$$e_{im}^{rest} = S_m e_{im}^{best} \tag{14}$$

$$z_{im} = y_{im}^{best} e_{im}^{best} + y_{im}^{rest} e_{im}^{rest} \tag{15}$$

In Eq. (14) the relative efficiency for the above best-point production is set in accordance with the corresponding relative efficiency applied in the linear model. The produced amount is now given by Eq. (15). The non-linear model maximizes Eq. (1) subject to Eqs. (2), (4)–(6) and (13)–(15).

3.5. Test settings

We define a system consisting of two coupled hydro-power stations in the same water course, cf. Fig. 4. Above each station/generator there is a reservoir. The uppermost reservoir is ten times larger than the lower reservoir (300 versus 30). Additional inflow in a full reservoir gives spillage to a different water system, and is hence a pure loss to the producer. The inflows to the two reservoirs are perfectly correlated, with 50% probability for each of the two outcomes 0 and 4 in each of the five periods in the planning period, except for the first period where the inflow is 2, cf. Fig. 3. In addition, the lower reservoir receives production water from the above station. The capacity, measured in water units, is 3 for the upper generator and 6 for the lower generator. The highest possible efficiency is 1 for both generators (full reservoir and best-point production up to 80% of capacity). For additional water-use above best-point production the efficiency is reduced by a factor of 20%.

The marginal value of stored water at the end of the planning period takes 10 different values that approximates a linear declining function between 2.24 and 0.8. The same function is used for both reservoirs. The spot price is 2.

3.6. Test results

Fig. 5 shows objective values in different iterations for three simulations with the linear model. The black curve shows a flip–flop between two different solutions when no penalty for deviation is included. The red curve shows that the model converges after 6 iterations when the penalty for previous-round iteration is included.
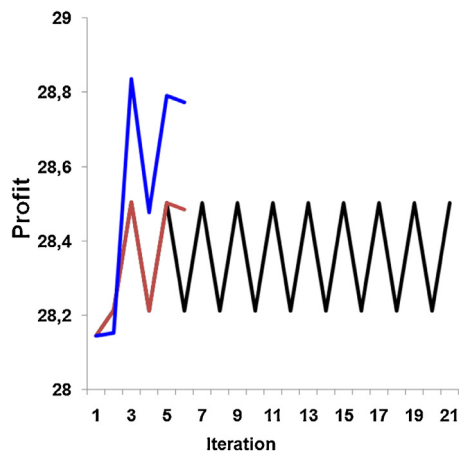
Fig. 5. Objective function with and without penalty for deviation and head-of-water optimization. (For interpretation of the references to color in the text, the reader is referred to the web version of this article.)

The blue curve shows how the converged solution is improved when head-of-water optimization is included.

Fig. 6 shows simulation results for 10 different initial reservoir levels in the two reservoirs, ranging from empty reservoirs (0%) to full reservoirs (100%). For each reservoir level, 10 different starting points has been applied for the non-linear model. The figure shows average results for the linear model with and without head-of-water optimization (blue and green columns respectively), and for the non-linear model (red column).

One important finding here is that the applied head-of-water optimization improves the model solution. Depending on the initial reservoir-level, the objective function was improved by up to 1.5% in our cases. The variability of this result for the 10 different starting points for each reservoir level was small. Apparently, the improvement is largest when reservoirs are about half-full, probably because the flexibility in the system is largest in these circumstances. Another major finding is that the applied iterative approach that includes head-of-water optimization, finds a solution that is very close to the optimal solution as calculated by the non-linear model. On average, the objective function is only 0.02% higher in the non-linear model. Therefore, this test clearly indicates that the first-order approximation for the head-of-water optimization in the iterative linear modeling approach provides good results.

However, these results do not prove that the methodology necessary will provide good results on all possible cases, nor do they prove convergence. Experience from the SHOP model confirms that successive linear programming shows adequate convergence for a large range of system topologies and states, and the authors believe that similar convergence properties will hold in the stochastic setting.

4. The SHARM model

After implementation and testing in GAMS, a prototype model was implemented in C++ using experience from implementation of the SHOP model [7]. The name of the prototype is SHARM (Short-term Hydro Application with Risk Modeling). The aim of this more detailed implementation of the concept described in Section 3, was to be able to perform preliminary tests in the hydropower utilities. This means that the level of detail in the prototype needed to be rich enough to describe all important features – at least for simple watercourses – to enable a realistic comparison of results with the currently used deterministic short-term model.

In the prototype implementation that was used for testing in the utilities we allowed for uncertainty in input parameters for inflow and price. As the model first of all is for use in single watercourses it is only possible to model one stochastic price, but the stochastic properties for inflow may be different for each reservoir. This results in a multidimensional description of the uncertainty and the scenario tree can potentially become very large. In this initial approach we assumed that inflow and prices are uncorrelated. The SHARM model does not rely on any particular method for generating input inflow and price scenarios, as long as these can be converted into a scenario tree. Thus, the model can be combined with an external forecast module. The hydropower utilities for which the model was tested have different approaches to generation of inflow and price forecasts.

The model is similar to the model already tested in GAMS and to [13] but with some differences. First of all the model uses iterations and a new iteration is an incremental description of the previous iteration. Second, the model uses a two mode approach for handling turbine minimum generation without using binary variables. The concept of using an incremental description serves two purposes: speed and accuracy. Firstly, solving an incremental model were the next LP or MIP model is build as a Δ from the solution of the previous iteration is like warm starting the solver. As the iterations go fourth more and more variables will remain
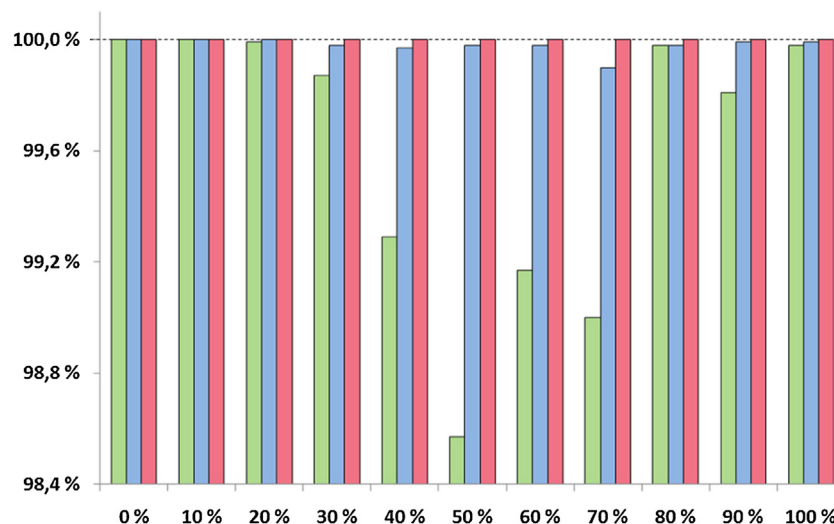

Fig. 6. Objective function for different initial reservoir fillings. (For interpretation of the references to color in the text, the reader is referred to the web version of this article.)
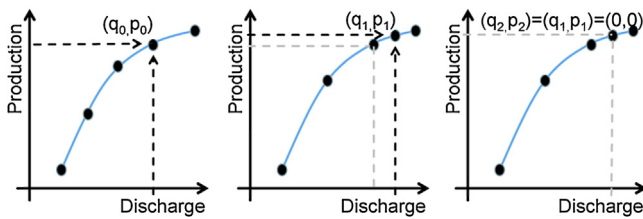
Fig. 7. Shifting linearization points in incremental model building.

unchanged which means that they will remain zero. This can speed up the calculation. Secondly, the number of segments in linearized curves typically defines the accuracy of the produced result. An example is a 300 MW unit where minimum generation is 50 MW. If it is known that this unit is running the efficiency curve might be convex as such. Using 5 segments then leads to 50 MW steps in the output curve. This is a large step when closing in on maximum generation where efficiency typically drops fast. By building incrementally the set point x_0 of the LP-model changes with the iterations and so does the segmentation of the linearization. This means that the result comes close to the result of a quadratic solution method. In practice this means that the incremental solution can provide what the hydropower owner considers a marginal balance between market prices and unit efficiency. The principle of incremental model building is shown in Fig. 7. At the same time this helps limit the traditional problem of flip–flop between solutions when using successive linear programming, as seen in Fig. 5.

In many aspects the minimum generation is important also in hydropower scheduling. When the turbines run at low load, cavitation may occur, and this typically limits minimum generation not to zero but to 30–40% of nominal loading. This may vary strongly from turbine to turbine and with pressure height above the turbine. This constraint could easily prove important regarding other operational constraints. This is handled by using binary variables

to model minimum generation often combined with modeling of startup costs. Although algorithms such as CPLEX has improved tremendously, the dimensionality of modeling thousands of binaries is a limit. This is still a practical limit in larger watercourses with many cascaded plants, and solving a multidimensional scenario tree using MIP for the same watercourse is not yet possible. In the prototype we have adapted a solution where mixed integer programming can be applied for the first part of the tree while the sub-trees further out in the tree will have to do with a heuristic where unit commitment decisions are taken based on the calculated strategy for running the watercourse following a fixed number of iterations. When unit commitment decisions are fixed, turbine minimum and inter-plant topology and losses can be included explicitly in the efficiency curves in the linear model, and new iterations can be used so that all system constraints are respected.

Based on the description above, the concept of the SHARM prototype is illustrated in Fig. 8. First, the model finds unit commitment (UC) plans for all hydropower units and all time steps (left part). Then, with locked UC, the model is solved with detailed efficiency curves which enables quadratic adaptation in the linear model, resulting in detailed and accurate discharge plans for turbines, gates, etc. This is the same principle of successive linear programming as is used in the operational SHOP model described in [6,7] and the method hence has a proven applicability for optimization of short-term hydropower scheduling. In the case studies in Section 6 we do not experience any flip–flop between solutions and 3 iterations in UC mode followed by 3 iterations of the detailed model yields a convergent solution.

The functionality in the SHARM model does not include all functionality of the SHOP model. Hydraulic connections that are important in many Norwegian water courses have so far not been included, but the description in [5] can be included here as well. Start-up cost is included as described in [6]. Functionality such
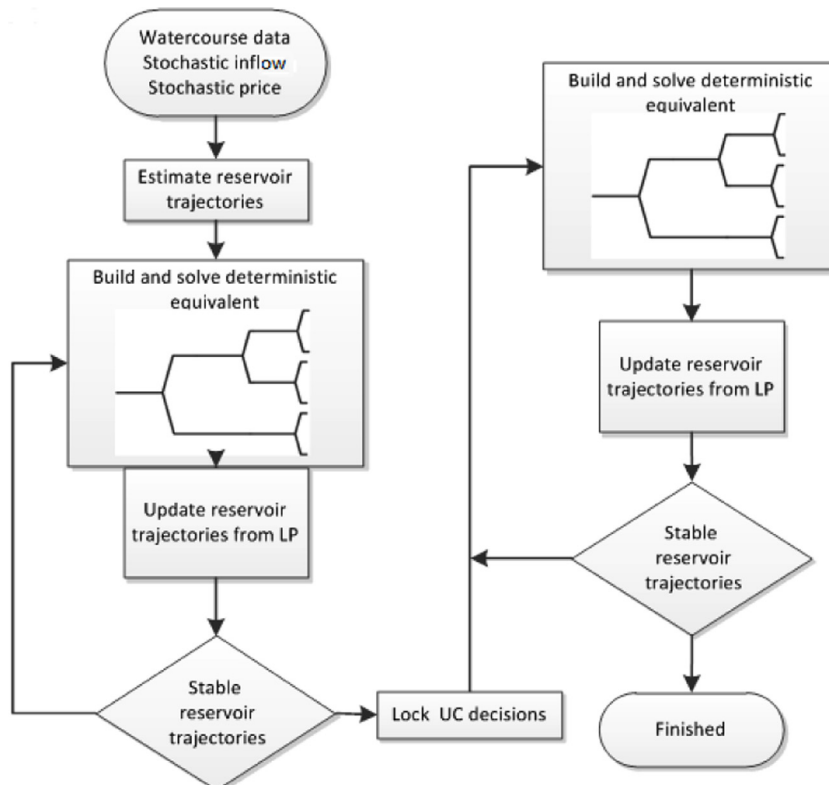

Fig. 8. Outline for the incremental SSLP method.

as primary and secondary reserve that is important in an operational setting [15] has not been included. The SHARM model also lacks time delay and static wave propagation that are important in longer watercourses. We have experience regarding how this can be included and also how it may impact on convergence and calculation time in SLP. In principle all these elements can be implemented also in the SSLP method but this have not yet been tested fully.

In an operational setting, the computation time of the stochastic model cannot exceed certain limits due to routines in the hydropower companies and time constraints in the market. For the system in case A in Section 6 the computational time is 165.75 s for the stochastic model with a 16-scenario, 7-day tree as input, whereas it is 10.23 s for SHOP when average values for price and inflow is used as input. For the smaller case in case study C, SHARM uses 20.41 s compared to SHOP's 1.64 s. The computation time of both the deterministic and stochastic model are dependent on the time horizon and the topology and size of the system under study. Some systems are inherently more complex than others, and some combinations of initial reservoir levels also give the system more flexibility, and hence it may be more challenging to find the optimal solution. The computational time of the stochastic model is also dependent on the size of the scenario tree. In [15] a simulation study indicated that a reduction in tree size of up to 22% does not affect the value of the optimal solution by more than 0.02%, and hence different methods for scenario tree reduction may be used to lower the computational time. In terms of operational use of the model, it will be a trade-off between the longer computational time of stochastic modelling against the manual analysis and multiple re-runs of a deterministic model.

5. Comparing stochastic and deterministic models in SHARM

To quantify the benefits of the stochastic over the deterministic model, the expected profit from operating the system according to the optimal decisions from each of the two models should be calculated with respect to the same forecast distribution for the uncertain variables. The decision variables include scheduled production on each generator and discharge in each gate and bypass gate. In [13] results from a deterministic and a set of stochastic models are compared by evaluating all models with respect to a large set of new simulation scenarios. The models are thus compared by using a different set of scenarios than those used to generate the scenario tree, preventing the stochastic model to optimally adapt to the scenarios used for comparison of the models [12]. In addition, the approach does not allow the deterministic model to be updated as new information becomes available. In practice, decisions are rescheduled at regular intervals based on the most recent information on prices and inflows. For example, in the case of a 24-h market, as the NordPool market, the scheduled decisions are typically updated daily, after each spot market clearing. We therefore take an alternative approach for comparing the two models, where the profit from the deterministic decisions is evaluated with respect to the scenario tree used for the stochastic model, using a step-by-step approach as described below. By taking this approach we try to quantify the performance of the stochastic model over the updated deterministic strategy, if we assume that the scenario tree is very close to the true forecast distribution. In contrast, the approach of [13] compares how stochastic and deterministic approaches compare on average with respect to a new set of simulation scenarios assumed to be the truth. Our selected approach implies that the expected profit from the stochastic model will always be evaluated to be equal to or higher than that from the deterministic model, as illustrated in Fig. 9.
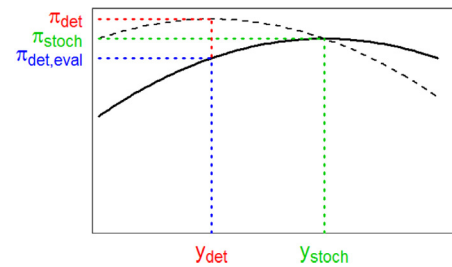

Fig. 9. Expected profit from the stochastic model (full line) and the deterministic model (dashed line) for a one-dimensional decision variable. The values π_{stoch} and π_{det} are the expected profits for the optimal decisions y_{stoch} and y_{det}, and $\pi_{det,eval}$ is the expected profit for decision y_{det} evaluated with respect to the scenario tree.

The expected profit from operating the system according to the optimal decisions from the deterministic model is evaluated by a step-by-step procedure similar to that presented in [15]. Optimal first-stage decisions are computed for a set of deterministic sub-problems at each branching point of the tree. A branching point is a point in time when at least one branching occurs. The tree in Fig. 10 has branching points at t1, t2 and t3. The approach is illustrated in Fig. 11.

In Step 1 the optimal solution to the deterministic problem for the whole planning period is found. The deterministic forecast is computed as the point-wise probability weighted mean. The three remaining steps correspond to the branching points t1, t2, and t3. At each branching point one deterministic sub-problem is specified for each successor branch. All sub-problems at each branching point are solved simultaneously by a single run of the SSLP algorithm, keeping decision variables for time points prior to the branching point fixed to optimal values from previous steps. In Step 2 there are two successor branches at t1, leading to two deterministic sub-problems. For example, branch 2 in Step 2 equals branch 2 in the full tree between t1 and t2, the weighted mean of branches 4 and 5 between t2 and t3, and the weighted mean of branches 8-11 from t3 and onwards. The sub-problems for branches 2 and 3 are solved simultaneously by running the SSLP algorithm using the scenario tree consisting of the three branches 1, 2, and 3 in Step 2, keeping the decision variables for branch 1 fixed to the optimal values from Step 1. Since no path from time t1 and onwards share the same branches, the results should be similar to the ones obtained by solving each of the two deterministic sub-problems one at a time. At Step 3 there are four deterministic sub-problems. In the single SSLP run at this step, decision variables at branch 1 are kept fixed to the solution from Step 1, and at branches 2 and 3 the values are fixed to the solutions from Step 2.

The final step (Step 4) corresponds to solving the stochastic problem for the full scenario tree, but fixing all decisions prior to branching point t3 to values obtained from previous steps. This step is equivalent to evaluating the objective function for the deterministic decisions, but using the forecast distribution defined by the full
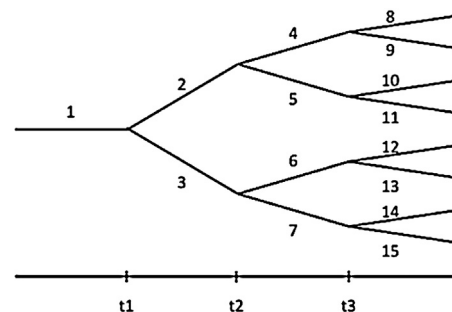

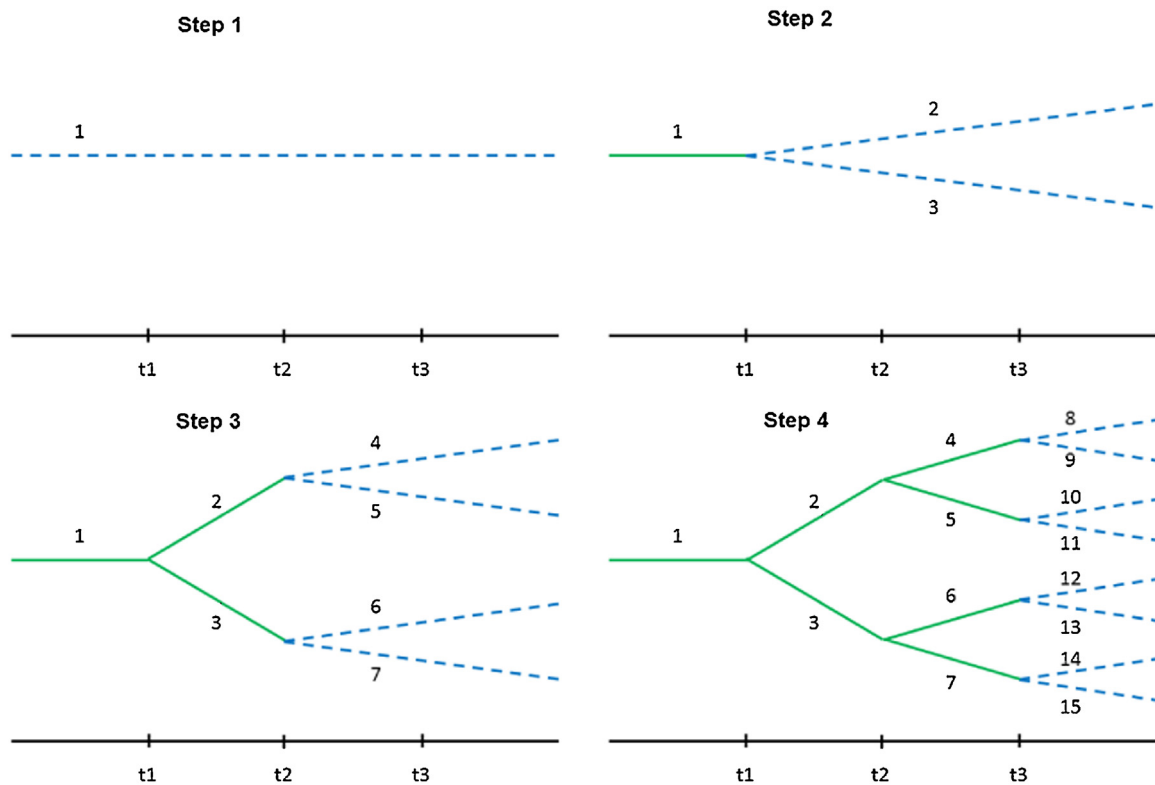**Fig. 10.** Full tree with branching at t1, t2 and t3.

Fig. 11. Subtrees used in the evaluation method.

scenario tree. The optimal objective for the final step is therefore the sought for value for the expected profit for the deterministic model. In the case studies of Section 6, we refer to the strategy obtained from this process as the determinitic strategy or the deterministic solution.

A drawback of the evaluation method described above is that the branching factor for the sub-problems is reduced as the tree is traversed. In addition, the length of the remaining planning period decreases as we move from one branching point to the next. The future will become less and less uncertain as the tree is traversed, favoring the deterministic model. On the other hand, the same scenarios are used for optimization and performance evaluation of the stochastic model, potentially favoring the stochastic model. What we do is to quantify the reduction in solution quality by using a deterministic model, if we assume that the stochastic tree represents the "truth". The rolling horizon approach presented in e.g. [12], where new sub-trees and corresponding deterministic forecasts are generated for each branching node of the tree, is an alternative that accounts for these short-comings of the proposed approach.

6. Case studies

In this section we show three examples to illustrate the potential benefit of the stochastic model. Case A will exhibit a situation where several reservoirs and power plants are located along the same river system, thus giving the producer some flexibility for generation scheduling under normal operating conditions. We look at a situation where the reservoir levels are high and volatile inflows are expected in the coming week. Case B investigates price uncertainty for the same reservoir system. Case C is devoted to a special and difficult situation where a single reservoir must handle potentially very large inflows. In the examples where inflow is stochastic and price is deterministic, we illustrate how the stochastic model sees a risk of high inflows later in the period and therefore reduce

the reservoir volume by producing more in order to avoid spill of water. When price is stochastic, we focus on the allocation of production to the highest priced hours. All case studies and corresponding data have been made available to us by hydropower producers participating in the industry testing of SHARM.

6.1. Case A: Flexible hydro system, inflow uncertainty

In this example, SHARM is used for a real hydropower system consisting of six reservoirs and six plants. The system is illustrated in Fig. 12. The inflow to all reservoirs is assumed to be stochastic, while the spot price is deterministic and increasing with a daily pattern throughout the planning period, as shown in Fig. 13. The
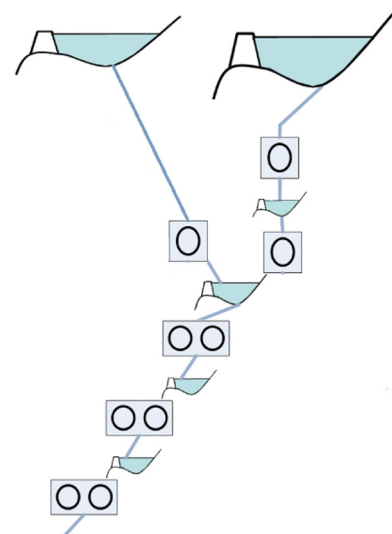

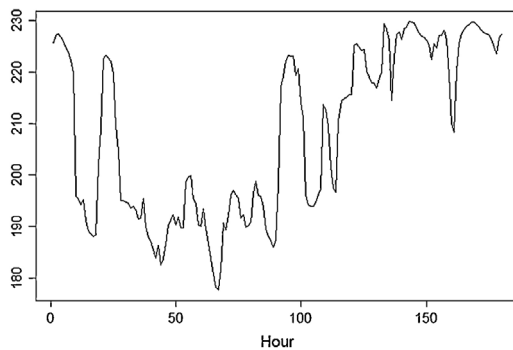**Fig. 12.** System topology for case A.

Fig. 13. Deterministic price forecast for case A.

prices are obtained from the price forecast used by the hydropower producer in their daily operation scheduling

The multi-dimensional scenario tree for inflow is generated by applying the scenario tree generation approach in [17] to a set of ensemble forecasts for the inflow, generated by the HBV model [20]. The 51 scenarios for each reservoir are assigned equal probabilities. This is a questionable assumption, and it is also a well-known fact [22, and references therein] that hydrological ensemble forecasts generated from meteorological ensembles tend to underestimate the uncertainty. However, the decision to use the ensemble scenarios without any additional processing was taken in cooperation with system operators at the power companies and therefore deemed valid for our test cases. It is further assumed that the inflow profiles for the two most downstream reservoirs are equal, so that the inflows differ only by a constant. This leaves us with a five-dimensional scenario tree. The structure of the total scenario tree and the inflow values for one of the reservoirs are shown in Figs. 14 and 15. The future value of water is given as a water value function, so that the water value decreases linearly by increasing reservoir level at the end of the planning period. The mixed-integer problem of the first iteration in the successive linear programming procedure has 165 502 variables and 39 638 constraints, and is solved in about 20 s. Solving the problem with 3 iterations in UC-mode and 3 iterations with locked UC as explained in Section 4 takes a total of 165.75 s on an Intel Core 3 GHz processor with 8 GB RAM.

Using the evaluation method described in Section 5, the objective function is 0.39% higher for the stochastic solution than for the deterministic solution. The break-down of the objective function into income form power sold, end reservoir value, start-up costs and penalties is given in Table 3. Income is higher for the stochastic model due to higher volumes produced during the optimization period. As a consequence, the end reservoir storage is lower. The
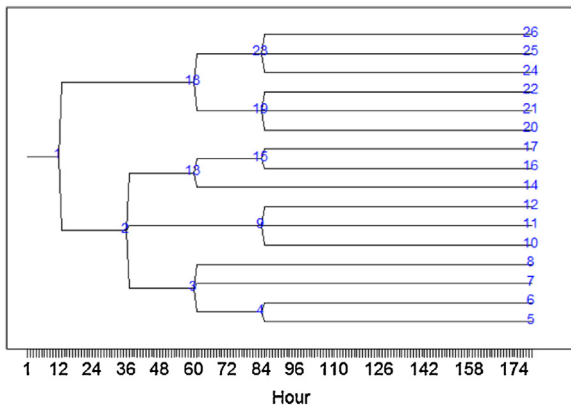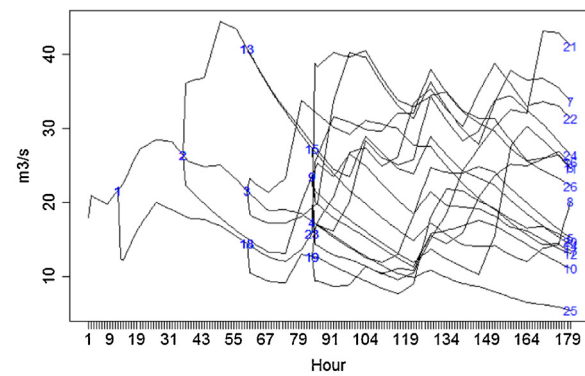

Fig. 15. Actual inflow scenarios for one of the reservoirs in case A.

Table 3
Results for case A. Numbers given relative to the stochastic objective function.

	Stochastic	Deterministic
Income	9.43	7.54
End value	990.80	991.78
Penalties	−0.22	−3.18
Start-up costs	−0.002	−0.005
Total	1000	996.12

balance of producing today versus saving water for later is more shifted toward producing today for the stochastic model than for the deterministic approach. The differences can be explained by the fact the deterministic model do not see the same risk of spill as is represented by the scenario tree in the stochastic model. Seeing the low prices at the start of the period, the deterministic model chooses a low production level in order to save water for more favorable prices later in the period or after the end of the time horizon. This can be seen by comparing Figs. 13 and 16 which shows the prices over the optimization time horizon and the average production in the deterministic and stochastic model. Saving water for later is a viable strategy when the potential for large inflows is not accounted for, but it is exactly in such situations that the strategy fails, as is evident by the larger amount of spill from the deterministic model. In fact, there is 60% more spill from the deterministic than the stochastic model, 34 Mm^3 versus 14 Mm^3, which leads to the higher penalties shown in Table 3. The stochastic model, on the other hand, chooses to produce at higher levels even for less favorable prices in order to avoid spillage. Any losses due to producing at lower prices are offset by the gain from avoided spillage, and hence the stochastic model provides a better strategy in a setting where future inflows are uncertain. The reservoir storage level in the most downstream reservoir is shown in Fig. 17, indicating that the bolder strategy chosen by the deterministic model leads to spill
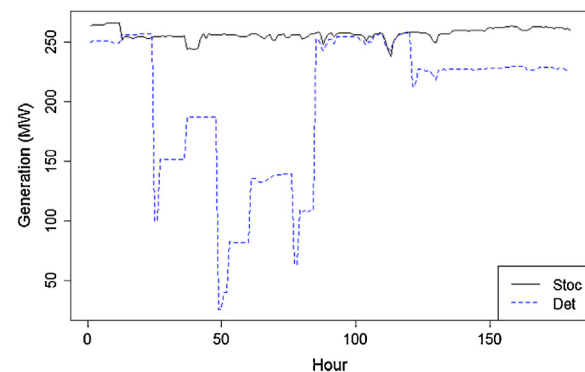

Fig. 14. Scenario tree for inflow in case A.

Fig. 16. Average production in stochastic and deterministic models for case A.
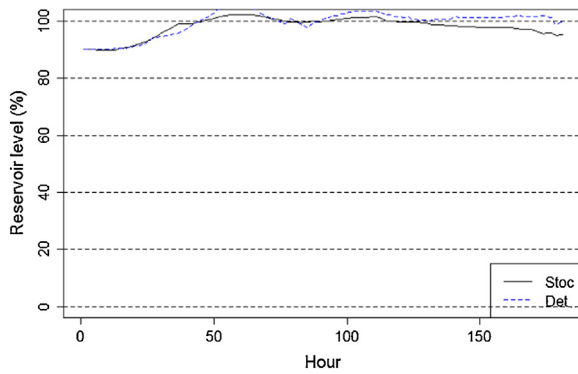
Fig. 17. Average reservoir level for the most downstream reservoir in case A.

Fig. 19. Actual price scenarios used in case B.

while the stochastic model keeps the reservoir within its bounds for a larger set of hours.

The net gain of 0.39% is taken to be a representative estimate of the potential gain of using SHARM in full/empty conditions rather than the current practice of reapplication of a deterministic model, as the case presented above is based on a realistic physical situation and applies information on inflow that is already available to production schedulers at the utilities. Even small gains are interesting to producers as the models for short-term scheduling are applied every day for a large portfolio of reservoir systems.

6.2. Case B: Flexible hydro system, price uncertainty

This case study investigates the same reservoir system as in case A illustrated in Fig. 12, but with the price represented as uncertain variables and the inflow assumed to be deterministic. The inflow is taken as the step-wise probability weighted average of the scenario tree for inflow used in case A. The price is represented by applying the scenario tree generation method of [20] to a set of forecasted market prices obtained from a market analysis company, SKM Market Predictor AS [24], which develops and supplies price forecasts to the power industry. The 60 price scenarios are based on different combinations of fundamental events that affect the market price, such as electricity consumption or transmission to or from connected areas. Prior to the scenario tree generation, all scenarios are given the same probability. This may be a questionable assumption, as the probability of the individual scenarios depends on the probability of the underlying events. Assessment of these probabilities is beyond the scope of this paper. The structure of the scenario tree for price is shown in Fig. 18, and the actual values are shown in Fig. 19. The scenario tree has been generated using a 24-h branching period to reflect the daily clearing of the day-ahead electricity market. The mixed-integer problem of the first iteration
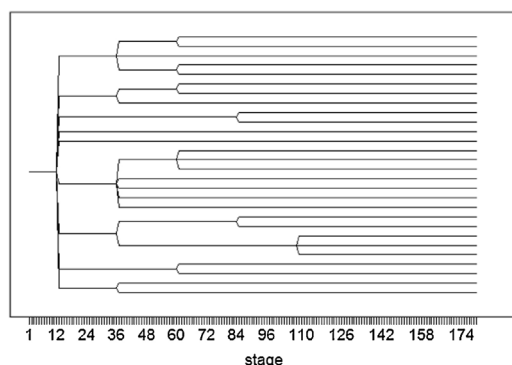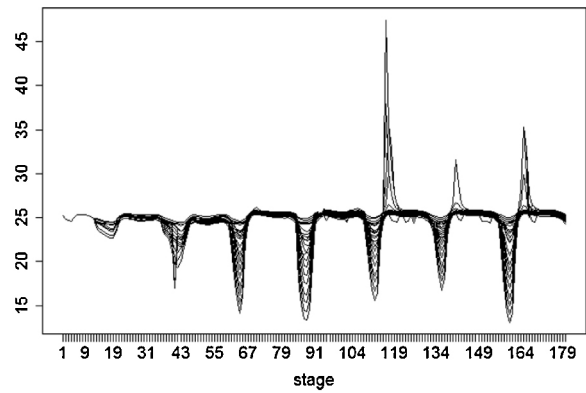
in the successive linear programming procedure has 337 702 variables and 81 542 constraints, and is solved in about 20 s. Solving the problem with 3 iterations in UC-mode and 3 iterations with locked UC as explained in Section 4 takes a total of 186.37 s.

The initial reservoir storage is set at a lower level of 40%, in order to investigate the ability of the two models to select only the high-price hours for production. If the reservoir level is higher, there is enough water in the system to produce in most hours and the gain from allocating production to the highest priced hours diminishes. When reservoir levels are high, the main emphasis is to reduce spill, whereas when reservoir levels are low, optimal allocation of the water as a scarce resource is more important. Looking at the price profiles in Fig. 19, it is evident that the main issue in this particular case is to allocate production such that the nightly dips in price are avoided and enough water is maintained in the system to be able to exploit the potential price peaks toward the end of the period.

Table 4 gives the result for the objective functions of the stochastic and deterministic method. The stochastic method is in total 0.01% better than the deterministic method, which may not be a significant result. This is due to the large contribution of the end reservoir value to the objective function. The numbers for revenues from sales tells a different story: the improvement in revenue is 2.71%. Fig. 20, which shows the average production in the two models, shows that the solutions are quite similar as the production volumes follow the price profile closely. The stochastic model is to a larger extent able to avoid production in the low price hours, and the stochastic model in total produces less than the deterministic model. The deterministic model is actually more able to exploit the first and highest price peak, and allocates slightly more production to this hour than the stochastic model. For the two remaining peaks, the stochastic model allocates more production. In terms of obtained average prices, calculated as the total revenue from sales divided by total production, the stochastic model performs better: obtaining an average price of 25.00 €/MWh against the deterministic model's 24.95 €/MWh, which yields an improvement of 0.19%.

The results for price as uncertain variable indicates a seemingly insignificant gain when measured for the total objective function value. However, in terms of revenues the results indicate that the stochastic model gives a strategy where production to a larger

Fig. 18. Scenario tree for price in case B.

Table 4
Results for case B. Numbers given relative to the stochastic objective function.

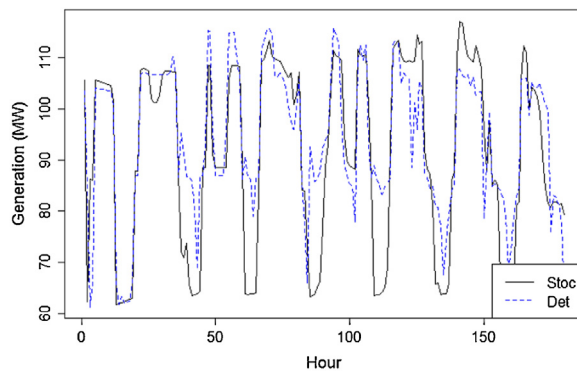|	Stochastic	Deterministic
Income	0.29	0.28
End value	999.71	999.60
Penalties	−0.0	−0.0
Start-up costs	−0.004	−0.005
Total	1000	999.88

Fig. 20. Average production for the stochastic and the deterministic model in case B.

Fig. 22. Actual inflow scenarios for the reservoir in case C.

degree is allocated to high-price hours than in the deterministic strategy. This yields a higher obtained average price, which will benefit producers in the short-term perspective.

6.3. Case C: Less flexible hydro system

The system under study in case C consists of one reservoir and one plant. The reservoir is mostly surrounded by mountains which make it hard to predict when actual inflow peaks will occur. Peak inflow can be higher than the nominal production discharge at the plant which is about 20 m^3/s, reducing the flexibility of the system. In this case study, the reservoir level is as high as 80% at the start of the 1 week optimization period, and high inflows are expected. A scenario tree for inflow is generated by using the scenario tree generation method to a set of 51 ensemble scenarios for inflow. The structure of the scenario tree and the actual values for inflow are shown in Figs. 21 and 22.

Price is kept deterministic and is taken as the price forecast used by the hydropower producer in their daily operation scheduling. The value of water is represented as a linear function of the reservoir levels at the end of the planning period. The water value is low in comparison to the forecasted prices to reflect the high water level in the system and the potential for spill. The mixed-integer problem of the first iteration in the successive linear programming procedure has 14 572 variables and 3976 constraints, and is solved in less than 1 s. Solving the problem with 3 iterations in UC-mode and 3 iterations with locked UC as explained in Section 4 takes a total of 20.41 s.

Table 5 shows the objective function value results for the stochastic and the deterministic models. The objective function is 8.9% better for the stochastic solution. This is again due to the fact
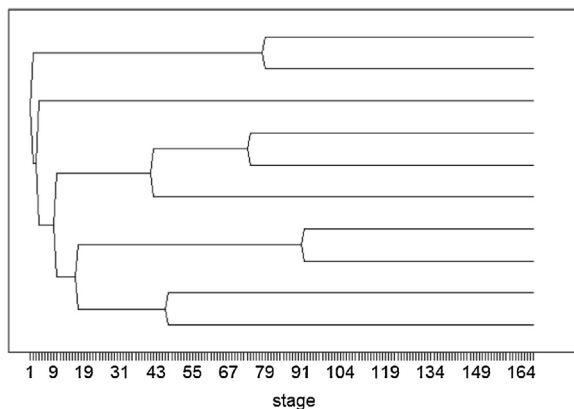
Table 5
Results for case C. Numbers given relative to the stochastic objective function.

	Stochastic	Deterministic
Income	688.37	522.39
End value	312.29	402.73
Penalties	−0.87	−10.66
Start-up costs	−0.20	−3.51
Total	1000	910.95

that the deterministic model chooses a low production level in the start of the planning horizon, failing to recognize the risk of spill if reservoir levels are allowed to rise prior to the potentially very high inflows later in the period. This can be seen from Fig. 23 which shows the reservoir storage level in the three scenarios with highest amounts of spill, and Fig. 24 which show the point-wise weighted average production in the stochastic and deterministic models.

Looking at Fig. 23 it is evident that the deterministic strategy of low (or variable) production in the start of the period fails and leads to spill if the inflow later on turns out to be high. The stochastic model is able to avoid spill to a far greater extent by choosing to produce at high levels from the start. Spill occurs in both models, but the amount is decreased by almost 90% in the stochastic model, 0.09 Mm^3, versus 0.7 Mm^3 in the deterministic strategy. The large gain obtained by the stochastic model in this particular case may not be obtainable on an average basis, but it illustrates that accounting for risk and making more robust production schedules can be very valuable in some situations.
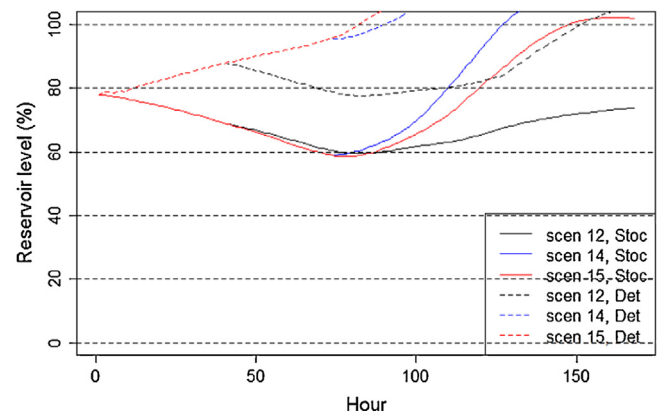

Fig. 21. The structure of the scenario tree for case C.

Fig. 23. The reservoir storage level in the three scenarios with highest spill in case C.
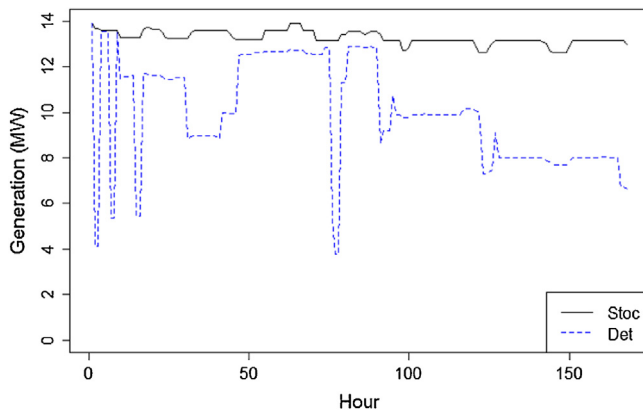
Fig. 24. The average production in the stochastic and deterministic model in case C.

7. Summary and discussion

7.1. Summary of work and testing

In this paper we have presented the concept of stochastic short-term hydropower scheduling implemented in the SHARM model, and illustrated the performance of the model by examples. We have also illustrated how non-linearities can be handled in a in linear model for stochastic short-term hydropower scheduling. The improvement in terms of objective function value and decreased risk of spill by the stochastic model is demonstrated in case studies, as well as the model's ability to allocate production to hours with higher prices.

An elementary representation of the SHARM model is also formulated in the paper. For this model, we demonstrate that the applied methodology is a first-order approximation to a formal correct head-of-water optimization. Simulation results for three model formulations are compared: An iterative solution method with and without head-of-water optimization, and the solution of the non-linear model. In our cases, the head-of-water optimization improves simulation results by approximately 1.5%. The non-linear model gives a slightly better result (0.02%).

7.2. Discussion

Even though we have shown that the implemented method is a first-order approximation for a head-of-water optimization, we have not provided a formal proof that could guarantee that the methodology will provide almost optimal solutions for all cases. The test cases for the simplified version of the SHARM model indicate that the implemented method is well functioning. On average, it provides a solution that is very close to the solution of the corresponding non-linear problem formulation. The relative performance we have documented compared to the solution of the non-linear problem is however valid only for those cases we have analyzed. We have not demonstrated that the properties of the non-linear problem formulation guarantee that the solution of the non-linear model must be a global optimum. However, we have not identified any non-convexities in the optimization problem in the simple model.

When we compare the results from using the stochastic strategy to the deterministic strategy on the basis of the scenario tree, we find that the stochastic model gives more robust results. The three examples shown in the case studies are representative of what we have found in testing together with the hydropower industry. There are, however, some limitations to our chosen method of evaluation. The branching factor (number of branches at each branching point)

is reduced as the tree is traversed. The length of the remaining time horizon is also reduced by traversing the tree. This will reduce the accuracy of the stochastic model and may hence decrease the value of stochastic modelling. On the other hand, taking the stochastic tree as the "truth" clearly benefits the stochastic approach over the use of mean values in the deterministic method. Ideally, evaluation should be based on rolling time horizons for both methods, where the process of updating input and reapplication of the models often seen in hydropower companies could be modeled to a larger degree.

In addition, we have only showed cases where either inflow or price is uncertain. It is difficult to determine which of price and inflow is more important to consider as a stochastic parameter. The results from our case studies indicate that inflow is more important since larger gains are obtained by considering inflow uncertainty than by considering price. This is however not a general result, and we expect the relative importance of price and inflow to depend on the system under study as well as time of year. In the future, prices are expected to become more volatile due to larger share of intermittent renewables, and this will further increase the value of considering uncertain prices. Having only one uncertain variable does not fully utilize the potential of the stochastic model in its current implementation where both parameters can be described as uncertain. We expect the combination of inflow and price uncertainty to further increase the added value of stochastic modelling.

7.3. Further work

A joint market formulation that accounts for the different electricity markets could be promising. The installed capacity for any given producer is a constraining factor, even in cases where there is large flexibility for the amount for available energy in the system. How to develop consistent strategies for bidding into sequential electricity markets could be an area of application for the stochastic short-term model. In addition, inflow and prices are often correlated. Methods for representing correlated scenarios for inflow and price are therefore of interest, and may further contribute to the application of the stochastic model in practice. In a multiple market setting, prices in additional electricity market must also be adequately modeled. The SHARM model will be further tested by the industry, both regarding cost benefits and other application areas.

Appendix A.

In the following we derive how the produced amount will be increased if the head of water increases because of a higher reservoir level. We calculate this by differentiation of Eq. (3) with respect on the reservoir level. In this calculation, we treat the solution for

all variables as constants, given from the previous solution of the model. Thus, this is a first-order approximation:

We define:

$$dz = \left(y_{im}^{best} \frac{dE_{im}^{best}}{dx_{im}} + y_{im}^{rest} \frac{dE_{im}^{rest}}{dx_{im}} \right) dx_{im}$$

$$= (E_{im}^{max} - E_{im}^{min}) \frac{y_{im}^{best}}{X_m^{max}} \left(1 + \frac{y_{im}^{rest} E_{im}^{rest}}{E_{im}^{best}} \right) dx_{im}$$

$$= \frac{E_{im}^{max} - E_{im}^{min}}{X_m^{max}} \left(y_{im}^{best} + y_{im}^{rest} \frac{E_{im}^{rest}}{E_{im}^{best}} \right) dx_{im}$$

$$= \frac{(E_{im}^{max} - E_{im}^{min}) E_{im}^{best} Z_{im}}{X_m^{max}} dx_{im} \tag{A.1}$$

which gives Eq. (11).

$$z_{im}^{extra} \equiv dz_{im} \tag{A.2}$$

$$dx_{im} \equiv (x_{im} - X_{im}) \tag{A.3}$$

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