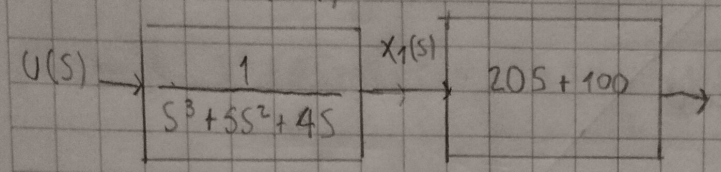
Ej 12.1

$G(S) = \frac{20(S+5)}{S(S+1)(S+4)}$ $\quad S(S+1)(S+4) = S^3+5S^2+4S$



$U(S) = X_1(S)(S^3+5S^2+4S) \quad \mathcal{L}^{-1}\} \quad U = \dddot{X}_1 + 5\ddot{X}_1 + 4\dot{X}_1$

V. estado $X_2 = \dot{X}_1 \quad X_3 = \dot{X}_2 = \ddot{X}_1 \quad \dot{X}_3 = \dddot{X}_1$

Reemplazando $U = \dot{X}_3 + 5X_3 + 4X_2 \quad \underline{\dot{X}_3 = -4X_2 - 5X_3 + U}$ ①

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -4 & -5 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$Y(S) = X_1(S)(20S+100) \quad \mathcal{L}^{-1}\} \quad y = 20\dot{X}_1 + 100X_1$

Reemplazando $y = 20X_2 + 100X_1$

$$y = \begin{bmatrix} 100 & 20 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}$$

Se pide que el sistema tenga $\begin{cases} OS\% = 9.5\% \\ t_s = 0,74\ s \end{cases}$

Se tiene que $OS\% = e^{-(\zeta\pi/\sqrt{1-\zeta^2})} \cdot 100$

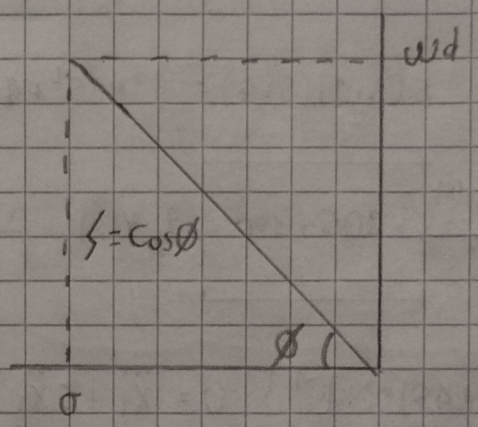
entonces $\frac{9,5}{100} = e^{-(\zeta\pi/\sqrt{1-\zeta^2})} \quad \ln(0,095) = -(\zeta\pi/\sqrt{1-\zeta^2})$

$-2,3539 = -\zeta\pi/\sqrt{1-\zeta^2} \quad 1-\zeta^2 = (\zeta\pi/2,3539)^2 \quad 1 = \zeta^2\left(1+(\pi/2,3539)^2\right)$

$$\zeta = \frac{1}{\sqrt{1+(\pi/2,3539)^2}} = \underline{0,5996}$$

Tenemos

$S = \sigma + j\omega_d = \zeta\omega_n + j\omega_d$



$\phi = \cos^{-1}(\zeta) = \cos^{-1}(0,5996)$

$\phi = 53,16°$

tenemos $T_s = \frac{4}{\sigma}$ $\sigma = 4/T_s = 5,4054$

$\sigma = \zeta\omega_n$ y $\omega_d = \omega_n\sqrt{1-\zeta^2}$ $\omega_n = \sigma/\zeta = 5,4054/0,5996 = 9,015\ rad/s$

$\omega_d = 9,015\sqrt{1-(0,5996)^2} = 7,2147\ rad/s$

tenemos $\omega_d = \sigma \cdot \tan\phi = 5,4054 \cdot \tan(53,16°) = 7,215\ rad/s$

} igual para ambas partes

$S = 5,4054 + j \cdot 7,2147$