

$$G(s) = \frac{9}{s^2 + 2s + 9}$$

$$\omega_n = 3 \omega_n^2 \quad s^2 + 2\zeta\omega_n s + \omega_n^2$$

$$\omega_n = 3 \quad 2\zeta\omega_n = 2$$

$$\zeta = 1/3$$

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Sobamortiguado

Con entrada  $\Gamma \frac{1}{s}$

$$Y(s) = \frac{9}{(s^2 + 2s + 9)(s)}$$

$$S = \frac{-2 \pm \sqrt{4 - (4 \cdot 9)}}{2} = \frac{-2 \pm \sqrt{-32}}{2} = \frac{-2 \pm 4\sqrt{2}j}{2}$$

$$S = -1 \pm 2\sqrt{2}j$$

$$S_1 = -1 + 2\sqrt{2}j$$

$$2\sqrt{2} \approx 2.83$$

$$S_2 = -1 - 2\sqrt{2}j$$

$$Y(s) = \frac{9}{(s+1+2\sqrt{2}j)(s+1-2\sqrt{2}j)(s)} = \frac{A}{s+1+2\sqrt{2}j} + \frac{B}{s+1-2\sqrt{2}j} + \frac{C}{s}$$

$$A(s+1-2\sqrt{2}j) + B(s+1+2\sqrt{2}j) + C(s+1+2\sqrt{2}j)(s+1-2\sqrt{2}j) = 9$$

$$Si \quad S = -1 + 2\sqrt{2}j \quad B(4\sqrt{2}j)(-1+2\sqrt{2}j) = 9 \quad B(-4\sqrt{2}j - 16) = 9$$

$$B = \frac{-9}{-16 + 4\sqrt{2}j} = \frac{-9(16 - 4\sqrt{2}j)}{256 + 32} = \frac{-144 + 36\sqrt{2}j}{288} = -\frac{1}{2} + \frac{\sqrt{2}j}{8}$$

$$Si \quad S = -1 - 2\sqrt{2}j \quad A(-4\sqrt{2}j)(-1-2\sqrt{2}j) = 9 \quad A(4\sqrt{2}j)(1+2\sqrt{2}j) = 9$$

$$A(4\sqrt{2}j - 16) = 9 \quad A = \frac{9}{4\sqrt{2}j - 16} = \frac{-9(16 + 4\sqrt{2}j)}{256 + 32}$$

$$A = \frac{-144 - 36\sqrt{2}j}{288} = -\frac{1}{2} - \frac{\sqrt{2}j}{8}$$

$$Si \quad S = 0 \quad 9C = 9 \quad C = 1$$

$$Y(s) = \frac{-\frac{1}{2} + \frac{\sqrt{2}j}{8}}{s+1+2\sqrt{2}j} + \frac{-\frac{1}{2} - \frac{\sqrt{2}j}{8}}{s+1-2\sqrt{2}j} + \frac{1}{s}$$

$$Y(s) = \frac{-1/2}{s+1+2\sqrt{2}j} - \frac{\sqrt{2}/8j}{s+1+2\sqrt{2}j} - \frac{1/2}{s+1-2\sqrt{2}j} + \frac{\sqrt{2}/8j}{s+1-2\sqrt{2}j} + \frac{1}{s}$$

$$y(t) = (-1/2 - \sqrt{2}/8j) e^{-(1+2\sqrt{2}j)t} + (-1/2 + \sqrt{2}/8j) e^{-(1-2\sqrt{2}j)t} + 1$$

$$e^{-(1+2\sqrt{2}j)t} = e^{-t} \cdot e^{-2\sqrt{2}jt} = e^{-t} (\cos(-2\sqrt{2}t) + j \operatorname{Sen}(-2\sqrt{2}t))$$

$$= e^{-t} (\cos(2\sqrt{2}t) - j \operatorname{Sen}(2\sqrt{2}t))$$

$$e^{-(1-2\sqrt{2}j)t} = e^{-t} \cdot e^{2\sqrt{2}jt} = e^{-t} (\cos(2\sqrt{2}t) + j \operatorname{Sen}(2\sqrt{2}t))$$



$$y(t) = \left(-\frac{1}{2} - \frac{\sqrt{2}}{8}j\right)(e^{-t}[\cos(2\sqrt{2}t) - j\sin(2\sqrt{2}t)]) + \left(-\frac{1}{2} + \frac{\sqrt{2}}{8}j\right)(e^{-t}[\cos(2\sqrt{2}t) + j\sin(2\sqrt{2}t)]) + 1$$

$$y(t) = e^{-t}\cos(2\sqrt{2}t)\left(-\frac{1}{2} - \frac{\sqrt{2}}{8}j - \frac{1}{2} + \frac{\sqrt{2}}{8}j\right) + je^{-t}\sin(2\sqrt{2}t)\left(\frac{1}{2} + \frac{\sqrt{2}}{8}j - \frac{1}{2} + \frac{\sqrt{2}}{8}j\right) + 1$$

$$y(t) = e^{-t}\cos(2\sqrt{2}t)(-1) + je^{-t}\sin(2\sqrt{2}t)\left(\frac{\sqrt{2}}{4}j\right) + 1$$

$$y(t) = -e^{-t}\cos(2\sqrt{2}t) - \frac{\sqrt{2}}{4}e^{-t}\sin(2\sqrt{2}t) + 1$$

$$G(s) = \frac{9}{s^2+9} \quad \underbrace{\omega_n = 3}_{\zeta = 0} \quad \left. \begin{matrix} \zeta \omega_n = 0 \\ \zeta = 0 \end{matrix} \right\} \begin{matrix} \text{Sin amortiguamiento} \\ \text{oscila de forma eterna} \end{matrix}$$

Con entrada  $\square$   $\frac{1}{s}$

$$Y(s) = \frac{9}{(s^2+9)s} \quad s^2+9=0 \quad s^2=-9 \quad s = \sqrt{-9} = \pm 3j$$

$$Y(s) = \frac{9}{(s+3j)(s-3j)s} = \frac{A}{s+3j} + \frac{B}{s-3j} + \frac{C}{s}$$

$$A(s-3j)s + B(s+3j)s + C(s^2+9) = 9$$

$$\text{Si } s=3j \quad B(6j)(3j)=9 \quad B(18 \cdot -1)=9 \quad \underline{B = -1/2}$$

$$\text{Si } s=-3j \quad A(-6j)(-3j)=9 \quad A(-18)=9 \quad \underline{A = -1/2}$$

$$\text{Si } s=0 \quad C(9)=9 \quad \underline{C=1}$$

$$Y(s) = \frac{-1/2}{s+3j} - \frac{1/2}{s-3j} + \frac{1}{s}$$

$$y(t) = -\frac{1}{2}e^{-3jt} - \frac{1}{2}e^{3jt} + 1$$

$$e^{-3jt} = \cos(-3t) + j\sin(-3t) = \cos(3t) - j\sin(3t)$$

$$e^{3jt} = \cos(3t) + j\sin(3t)$$

$$y(t) = -\frac{1}{2}(2\cos(3t)) + 1 = \underline{-\cos(3t) + 1}$$

$$e^{ej} = \cos \theta + j\sin \theta$$