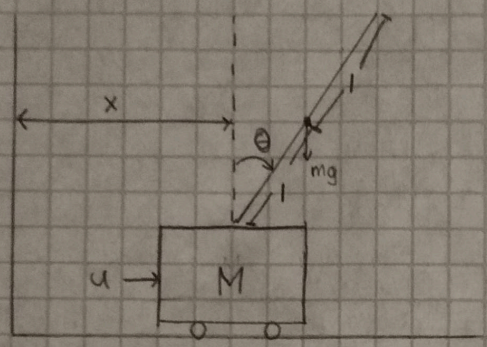
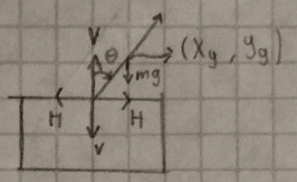
Pendulo Invertido





$X_g = X + l\,sen\theta$
$y_g = l\cos\theta$

M. rotacional $I\ddot{\theta} = Vl\,sen\theta - Hl\cos\theta$ ① en X para m $m\frac{d^2}{dt^2}(X + l\,sen\theta) = H$ ②

en y para m $m\frac{d^2(l\cos\theta)}{dt^2} = V - mg$ ③ carro en X $u = M\ddot{X} + H$ ④

Linealización angulos pequeños $sen\theta \approx \theta$ y $\cos\theta \approx 1$

De ① $I\ddot{\theta} = Vl\theta - Hl$ ⑤ De ② $m(\ddot{X} + l\ddot{\theta}) = H$ ⑥ De ③ $V = mg$ ⑦ $u = M\ddot{X} + H$ ④

De ④ y ⑥ $u - M\ddot{X} = m\ddot{X} + ml\ddot{\theta}$ $u = (M+m)\ddot{X} + ml\ddot{\theta}$ ⑧

De ⑤, ⑥ y ⑦ $I\ddot{\theta} = mgl\theta - lm\ddot{X} - ml^2\ddot{\theta}$ $\ddot{\theta}(I+ml^2) + \ddot{X}lm = mgl\theta$ ⑨

Despejando $\ddot{X}$ $\ddot{X} = \frac{u - ml\ddot{\theta}}{m+M}$ y $\ddot{X} = \frac{mgl\theta - \ddot{\theta}(I+ml^2)}{lm}$

Reemplazando $lmu - (lm)^2\ddot{\theta} = (m+M)mgl\theta - \ddot{\theta}(m+M)(I+ml^2)$

$lmu - (lm)^2\ddot{\theta} = (m+M)mgl\theta - (ml)^2\ddot{\theta} - Mml^2\ddot{\theta} - (m+M)I\ddot{\theta}$

$lmu - (m+M)mgl\theta = -\ddot{\theta}(Mml^2 + (m+M)I)$

$\ddot{\theta} = \frac{(m+M)mgl\theta - lmu}{Mml^2 + I(m+M)}$ ⑩

Despejando $\ddot{\theta}$ $\ddot{\theta} = \frac{u - (m+M)\ddot{X}}{ml}$ y $\ddot{\theta} = \frac{mgl\theta - \ddot{X}lm}{I + ml^2}$

Reemplazando $(I+ml^2)(u - (m+M)\ddot{X}) = (ml)^2(g\theta - \ddot{X})$

$u(I+ml^2) - \ddot{X}((ml)^2 + Mml^2 + I(m+M)) = (ml)^2 g\theta - \ddot{X}(ml)^2$

$u(I+ml^2) - \ddot{X}(Mml^2 + I(m+M)) = (ml)^2 g\theta$

$\ddot{X} = \frac{-(ml)^2 g\theta + u(I+ml^2)}{Mml^2 + I(m+M)}$ ⑪

tenemos $q_1 = X$ $q_2 = \dot{X} = \dot{q_1}$ $q_3 = \theta$ $q_4 = \dot{\theta} = \dot{q_3}$ $y_1 = X = q_1$

$\dot{q_2} = \ddot{X}$ $\dot{q_4} = \ddot{\theta}$ $y_2 = \theta = q_3$

$\dot{q_4} = \frac{(m+M)mgl q_3 - lmu}{Mml^2 + I(m+M)}$ ⑫ $\dot{q_2} = \frac{-(ml)^2 g q_3 + u(I+ml^2)}{Mml^2 + I(m+M)}$ ⑬

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -(ml)^2 g / Mml^2 + I(m+M) & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & (m+M)mgl / Mml^2 + I(m+M) & 0 \end{bmatrix} + \begin{bmatrix} 0 \\ I + ml^2 / Mml^2 + I(m+M) \\ 0 \\ -lm / Mml^2 + I(m+M) \end{bmatrix} u$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}$$