PHYSICAL INFORMED NEURAL NETWORKS FOR ASTROPHYSICAL AND COSMOLOGICAL DATA PROCESSING

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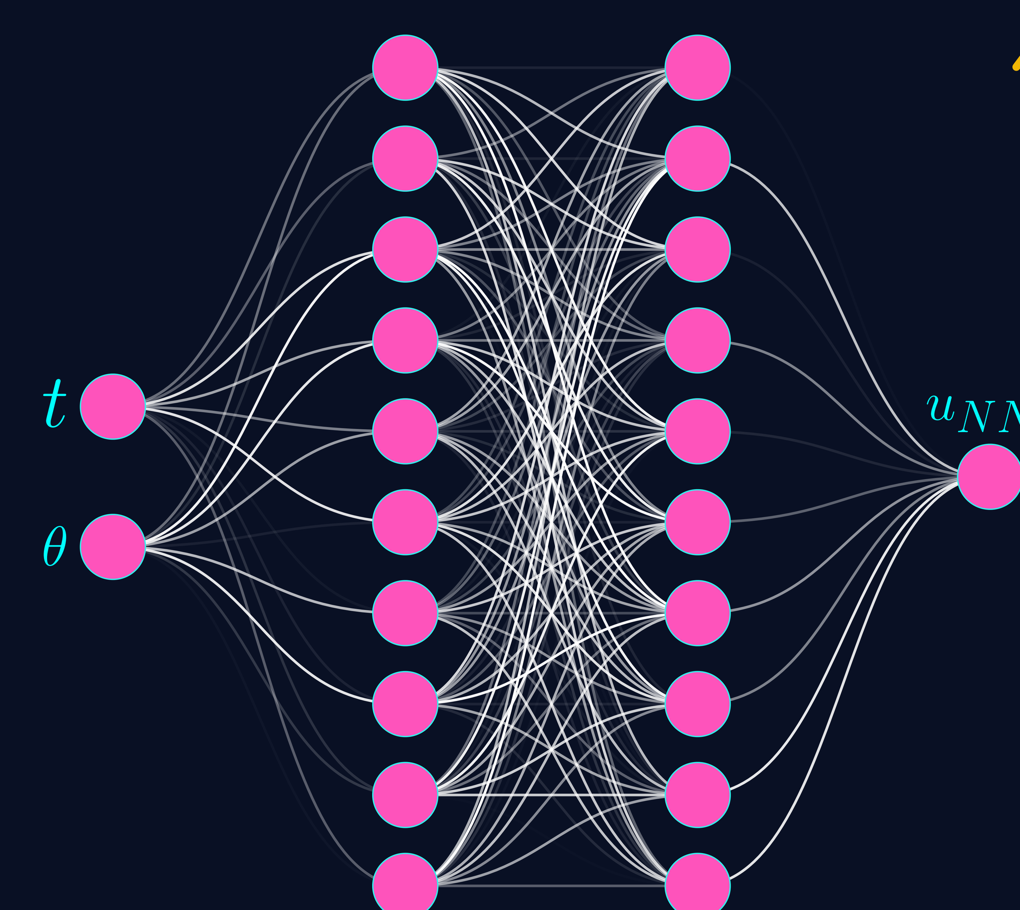
01 INTRODUCTION

Neural networks are universal function approximators [1]. In their simplest form, their operation involves receiving a training dataset **{X,Y}**, generating an output, and adjusting its intrinsic parameters to minimize a **cost function**, which measures how close the output is to the set **Y**. **PINNs** are a paradigm of neural networks that do not require a training dataset [2][5]; instead, they can be provided with a mathematical description of the phenomenon to be modeled. To be precise, a system of differential equations is given, which is included within the **cost function**. If the system has to satisfy initial or boundary conditions, a re-parametrization of the network output is performed to enforce compliance with that conditions. Even more, an **integral** can also be written as a differential equation, allowing PINNs to approximate solutions for both systems of **differential equations and primitives of integrals**.

$$F\left(t, u, \frac{du(t,\theta)}{dt}, \frac{d^2u(t,\theta)}{dt^2}, \ldots, \frac{d^nu(t,\theta)}{dt^n}\right) = 0,\ u|_{\partial\Omega} = u_{i,0},\ u(t_0) = u_0$$

$$\tilde{u}_{NN}(t,\theta) = u_{NN}(\theta)|_{\partial\Omega} + \left(1 - e^{(t-t_0)}\right)\sum_i \left(1 - e^{\theta_i - \theta_{i,0}}\right) u_{NN}(t,\theta)$$



cost function

$$C = F(\tilde{u}_{NN}, t, \theta)^2$$

Network parameters are updated until C reaches its minimum.

METHODOLOGY

The performance of this type of neural networks, two well-known equations in cosmology and astrophysics were addressed: the Poisson equation (5,6) and cosmological distances (1,2,3). The Poisson equation is important to describe the distribution of mass and the gravitational field in systems such as galaxies, the formation of cosmic structures or gravitational lenses [3]. Cosmological distances are crucial for understanding the scale and expansion of the universe. The integral (4) does not have an analytical solution; therefore, a numerical method should be applied [4].

COSMOLOGICAL DISTANCES

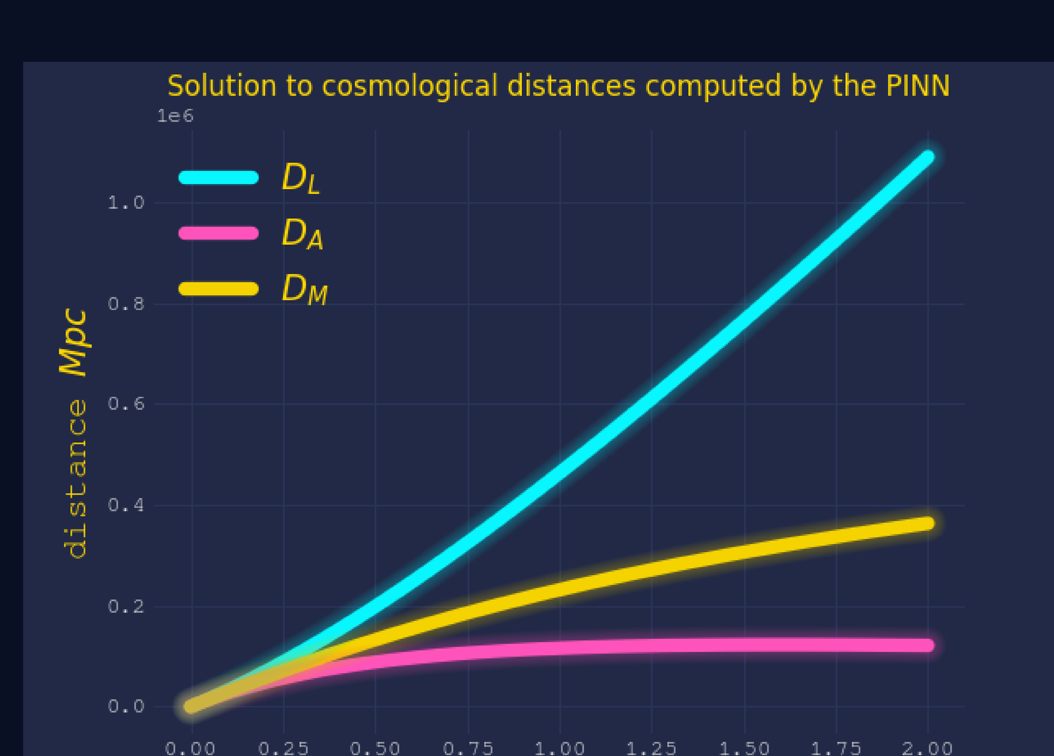
For this case, the PINN was employed to solve equation (4), thus obtaining the derived expressions for cosmological distances.

$$D_M = \frac{c(1+z)}{H_0}\int_0^z \frac{dz'}{E(z')} \cdots (1)$$

$$D_L = \frac{c(1+z)}{H_0}\int_0^z \frac{dz'}{E(z')} \cdots (2)$$

$$D_A = \frac{c}{H_0(1+z)}\int_0^z \frac{dz'}{E(z')} \cdots (3)$$

$$u(z) = \int_0^z \frac{dz'}{H(z')} \implies \frac{du}{dz} = \frac{1}{H(z)} \cdots (4)$$



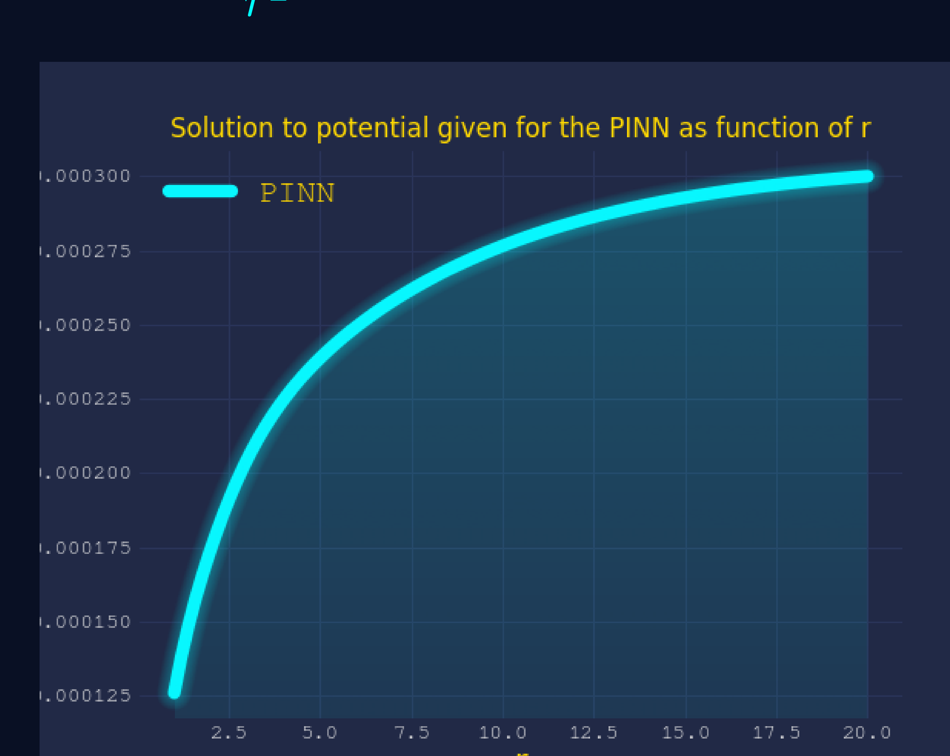
POISSON EQUATION

Here, the simplest case was considered, assuming that the system varies only radially and that its density distribution is given by the conditions described in (7). The PINN solved equation (6).

$$\nabla\Phi = 4\pi G\rho(r) \cdots (5)$$

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d}{dr}\Phi\right) = 4\pi G\rho(r) \cdots (6)$$

$$\rho(r) = \frac{\rho_0}{r^2}, \Phi'(1) = 1, \Phi(1) = 0 \cdots (7)$$



RESULTS AND CONCLUSION

The PINNs were successful in solving the provided systems of equations, with an **MSE error of 0.61** for the three cosmological distances and **7.2e-4** for the Poisson equation. A performance test against the *scipy.integrate* library resulted in an average **reduction in computation time of 76.9%**. These networks address the need for large amounts of data to be trained. However, they tend to be less accurate and more challenging to converge than fully connected networks.

76.9% faster!!

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