GBM6700E – Fall 2025 Practical Assignment 1
Calibration of a radiographic system

Polytechnique Montréal Biomedical Engineering program

General objective

Calibration is an essential step in any 3D reconstruction process starting from 2D images. It allows to determine, thanks to an object of known geometry (calibration object), the geometrical transformation that links the 3D coordinates of a point in space and the coordinates of its projection in an X-ray. In this practical assignment, we will be interested in performing implicit calibration using the Direct Linear Transform (DLT) algorithm. We will evaluate different configurations of the calibration object, then we will study the effect of object identification errors on the radiographs, as well as the effect of 3D object measurement errors.
At Sainte Justine Hospital, the clinical evaluation of scoliosis uses a 3D reconstruction of the patient's spine, ribs and pelvis. These reconstructions are obtained by stereo-radiography. For this purpose, two X-rays of the patient, under two different incidences, are acquired in standing position.

1 Calibration under ideal conditions (5 points)

In this lab, we will consider the 3D reconstruction of the spine only (vertebrae T1 to L5), from two radiographs: the postero-anterior view (PA0) and the lateral view (LAT). The calibration object used consists of 55 radio-opaque beads distributed in two plexiglass plates, called A and B. The patient stands between the two plates. The configuration of the Xray system is shown in 1.

The beads of plate A are divided into 6 rows and 5 columns and are named A_i_j where i is the column number and j is the row number. On plate B, the beads are distributed into 5 rows and 5 columns and are named B_i_j where again i is the column and j is the row. The bead B_3_3 serves as the origin of the global 3D reference frame. The X axis of the reference frame is oriented towards the front of the patient, the Y axis is oriented towards the left of the patient and the Z axis towards the head of the patient. The positions of the beads in the global reference frame are known, as they were measured with a three-dimensional coordinate measuring machine.

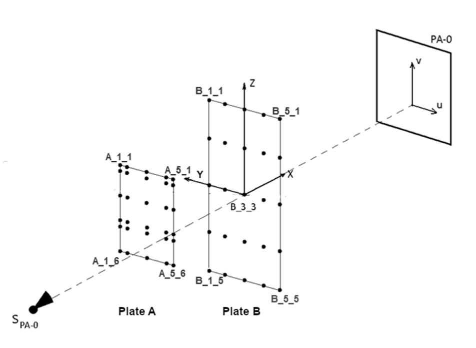

Figure 1: Configuration of the radiographic system (PA0 view)

In order to calibrate the LAT view separately from the PA0 view (and then be able to reconstruct 3D objects), an artificial rotation of 90 degrees around the vertical axis is applied to plates A and B, to create two virtual plates, called C and D. In reality, during the acquisition, the patient is placed on a positioning device and rotated 90 degrees between the two X-ray acquisitions; the actual plates do not move. Thus, plates C and D (with 3D coordinates different from those of A and B) correspond to the projections of the beads in the LAT view. Figure 2 shows the configuration of the four plates, with the virtual plates C and D in red.

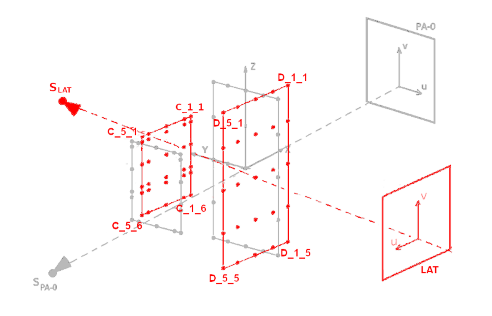

Figure 2: Configuration of the radiographic system (LAT view)

Once the patient's xrays have been acquired, a radiology technician is responsible for identifying several specific points in each image:

- the projections of the calibration beads (those that are visible)
- the projections of 6 anatomical landmarks per vertebra

All of these 2D coordinates are expressed in local reference frames linked to each of the radiographs, then saved. These 2D coordinate frames are centered on the projection of bead B_3_3 (for the PA0 view) or D_3_3 (for the LAT view); the u axis is oriented towards the right of the radiograph and the v axis towards the top of the radiograph. In the files provided to you, all points are labeled and therefore matched between the two radiographs.
Data files provided: Calib_Beads2D.mat, Calib_Beads3D.mat and Vertebrae2D.mat
All these files are processed automatically using the provided library, **available here** https://github.com/vtrno/PyGBM6700E/.
The resulting data structure for the calibration beads is a dictionnary with two keys :

- *2d* : containing a table with attributes *view, bead, x_2d, y_2d*
- *3d* : containing a table with attributes *view, bead, x_2d, y_2d, z_3d*

The *Spine* class allows automated processing of the vertbrae data in different views. Refer to the documentation for more information.
Question 1.1 (2 points): Starting from the 3D coordinates of the calibration beads and the coordinates of their projections in the X-rays, find the 11 DLT (Li) parameters for each of the projections, namely PA0 and LAT (see Appendix A)).
Question 1.2 (1 point): From the 11 DLT parameters obtained previously, for each of the projections PA0 and LAT, find the intrinsic and extrinsic parameters of the radiographic setup (see Appendix B). The intrinsic parameters are defined as:

- the coordinates (u0, v0) of the projection of the source in the local reference frame linked to the image;
- the focal lengths (cu, cv);

and the extrinsic parameters as:

- the coordinates (X0, Y0, Z0) of the source in the global reference frame;
- the rotation matrix between the local and global reference frames.

Question 1.3 (2 points): Now that the calibration has been performed, proceed with the 3D reconstruction of the T1 to L5 vertebrae, more precisely the 6 anatomical landmarks per vertebra. For the rest of the assignment, this reconstruction will be considered as the ideal reconstruction. The reconstructions that will be obtained in the following sections will be evaluated in relation to this benchmark reconstruction.
Bonus (1 point): Determine whether the values obtained in Question 1.2 for the system parameters make physical sense. Refer to Figure 1 above. To obtain the focal lengths in millimeters, consider that the pixels are squares of side 0.4 mm in length.

2 Effect of calibration object configuration on the reconstruction (5 points)

In this section, we will evaluate different calibration objects, first by varying the number of calibration beads, and then by varying the configuration of these beads.
To quantify the differences between these reconstructions and the ideal reconstruction (obtained previously), we introduce the root mean square (RMS) of the errors:

$$RMS_m = \sqrt{\sum_{i=1}^{N} \frac{(X_{m,i} - X^0_{m,i})^2}{N}}$$

where

- m is the given coordinate axis. Thus we talk about the error along the axis X, Y or Z;
- N is the number of points to reconstruct in 3D;
- $X_{m,i}$ is the coordinate m of point i in the reconstruction to evaluate;
- $X^0_{m,i}$ is the coordinate m of point i in the ideal (benchmark) reconstruction.

The 3D RMS error can be calculated this way:

$$RMS_{3D} = \sqrt{\sum_{i=1}^{N} \frac{\|(X_i - X^0_i)\|^2}{N}}$$

where

- N is the number of points to reconstruct in 3D;
- X_i are the coordinates of point i in the reconstruction to evaluate;
- X^0_i are the coordinates of point i in the ideal (benchmark) reconstruction.

NOTE 1: To make a calibration object with a given number of beads N, you must consider that the beads of plate A in PA0 correspond to those in the same rows and columns of plate C in LAT; the same correspondence applies between plates B and D. So, if we consider a set of N beads on plates A and B in the PA0 view, we take the same N beads on plates C and D in the LAT view.
NOTE 2: for the center of gravity of a calibration object, take the average of the coordinates of the beads on the 4 plates constituting the calibration object.
These notes apply to all the next questions.
Question 2.1 (2 points): Gradually reduce the number of calibration beads, and evaluate the X, Y and Z errors and the total error each time. What is the minimum number of beads to consider? Plot the error curves as a function of the number of calibration beads; discuss your results.
Question 2.2 (1,5 points): By setting the number of calibration beads to eight, vary their spatial

configuration and evaluate the errors. We provide you with 4 different 8-beads subsets. Vizualize the different provided subsets and associate them to their labels : *small volume, medium volume, large volume, coplanar*.
Try changing the calibration volume enclosed by the beads and/or consider only beads from a single plate to make a calibration object, and evaluate the effects on the reconstruction errors. Discuss your results.
Question 2.3 (1,5 points): Select a small calibration volume, corresponding to a small calibration object, and evaluate the extrapolation errors at each point of the 3D spine reconstruction. Plot the error curves per point as a function of the distance of each point to the centre of gravity of the calibration object. Discuss your results.

3 Effect of digitization errors on the reconstruction (3,5 points)

The projections of the calibration beads in the radiographs (circular in shape) are about 10 pixels in diameter. Since the identification of these small projected beads in the X-rays is done manually, the digitization process is inevitably subject to errors.
Question 3.1 (1,5 points): First, consider the entire set of calibration beads. Add Gaussian noise centered on 0 to the 2D coordinates of the calibration beads projections, to simulate digitizing errors. Then, evaluate the errors on the 3D spine reconstruction. Plot the error curves as a function of the variance of the added noise.
Question 3.2 (2 points): Carry out the same steps as in Question 3.1, but now considering only eight calibration beads. Compare the results obtained here with those of Question 3.1. Discuss your results.

4 Effect of measurement errors on the calibration object (3,5 points)

As mentioned earlier, the 3D coordinates of the calibration beads were previously measured with a coordinate measuring machine. However, errors on these coordinates remain. This is due, among other things, to the fact that the two plates are independent, and that the beads are spheres 2 mm in diameter.
Question 4.1 (1,5 points): Consider the entire set of calibration beads. Add Gaussian noise centered on 0° to the values of the 3D coordinates of the calibration beads, to simulate measurement errors. Then, evaluate the errors on the 3D spine reconstruction. Plot the error curves as a function of the variance of the added noise.
Question 4.2 (2 points): Simulate the worst-case scenario, by considering both noise on the measurements of the 3D bead coordinates and noise on the 2D bead digitizations in the X-rays. Compare the effects of the two noise sources (2D and 3D) on the reconstruction. What are their relative contributions of the two types of noise on the overall reconstruction errors? Discuss your results.

Appendix A

The DLT algorithm shows how to transform the collinearity relation into a simple linear form with 11 implicit parameters:

$$\begin{cases} u_i = \dfrac{L_1 X_i + L_2 Y_i + L_3 Z_i + L_4}{L_9 X_i + L_{10} Y_i + L_{11} Z_i + 1} \\ v_i = \dfrac{L_5 X_i + L_6 Y_i + L_7 Z_i + L_8}{L_9 X_i + L_{10} Y_i + L_{11} Z_i + 1} \end{cases}$$

where

- (u_i, v_i) are the 2D coordinates of the projection of point P_i in the image, expressed in the local coordinate frame linked to the image;
- (X_i, Y_i, Z_i) are the 3D coordinates of point P_i, expressed in the global reference frame;
- $[L_1 \dots L_{11}]$ are the 11 DLT parameters.

Appendix B

Since calibration using the DLT algorithm is implicit, the intrinsic and extrinsic parameters of the system are not obtained directly. However, by identifying the relationship between the DLT parameters and the explicit calibration formula, we can calculate the explicit parameters as follows:

$$\begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix} = \begin{bmatrix} L_1 & L_2 & L_3 \\ L_5 & L_6 & L_7 \\ L_9 & L_{10} & L_{11} \end{bmatrix}^{-1} \begin{bmatrix} -L_4 \\ -L_8 \\ -1 \end{bmatrix}$$

$$d = \frac{-1}{\sqrt{L_9^2 + L_{10}^2 + L_{11}^2}}$$

$$u_0 = (L_1 \times L_9 + L_2 \times L_{10} + L_3 \times L_{11}) \times d^2$$

$$v_0 = (L_5 \times L_9 + L_6 \times L_{10} + L_7 \times L_{11}) \times d^2$$

$$c_u = \sqrt{d^2 \times \left((u_0 L_9 - L_1)^2 + (u_0 L_{10} - L_2)^2 + (u_0 L_{11} - L_3)^2\right)}$$

$$c_v = \sqrt{d^2 \times \left((v_0 L_9 - L_5)^2 + (v_0 L_{10} - L_6)^2 + (v_0 L_{11} - L_7)^2\right)}$$

$$R = \begin{bmatrix} \frac{d}{c_u}(u_0 L_9 - L_1) & \frac{d}{c_u}(u_0 L_{10} - L_2) & \frac{d}{c_u}(u_0 L_{11} - L_3) \\ \frac{d}{c_v}(v_0 L_9 - L_5) & \frac{d}{c_v}(v_0 L_{10} - L_6) & \frac{d}{c_v}(v_0 L_{11} - L_7) \\ L_9 d & L_{10} d & L_{11} d \end{bmatrix}$$