

$$U_n = U_{n-1} + \Delta t \frac{dU_{n-1}}{dt}$$

$$\frac{dU_{n-1}}{dt} = \alpha U_{n-1}$$

$$U_n = U_{n-1} + \Delta t \alpha U_{n-1}$$

$$U_n = U_{n-1} (1 + \Delta t \alpha)$$

Demostración
por inducción

$$U_0 = U_0$$

$$U_1 = U_0 (1 + \Delta t \alpha)$$

$$U_2 = [U_0 (1 + \Delta t \alpha)] (1 + \Delta t \alpha)$$

$$U_2 = U_0 (1 + \Delta t \alpha)^2$$

$$U_k = U_0 (1 + \Delta t \alpha)^k$$

Ahora bien, si:

$(1 + \Delta t \alpha) < 0$ el sistema oscilará

$$\Delta t \alpha < -1$$

$$\Delta t > -\frac{1}{\alpha}$$