Estado Sólido Taller #3: Juan Diego Figueroa Hernández

Capítulo 6

1.) Como los electrones son fermiones, entonces

$$U = \int_0^\infty \frac{1}{e^{(\varepsilon-\mu)/k_BT}+1} \cdot \varepsilon D(\varepsilon)\, d\varepsilon,$$ considerando que $T = 0$ Kelvin y $D(\varepsilon) = \frac{dN}{d\varepsilon}$, se tiene:

$$U = \int_0^{\varepsilon_f} \varepsilon \cdot \frac{dN}{d\varepsilon}\, d\varepsilon = \int_0^{N(\varepsilon_f)} \varepsilon\, dN = \int_0^{N(\varepsilon_f)} \frac{\hbar^2}{2m}\left(\frac{3\pi^2}{V}\cdot N\right)^{2/3} dN$$

$$U = \frac{\hbar^2}{2m} \cdot \frac{3}{5}\left(\frac{3\pi^2}{V}\right)^{2/3} N^{5/3} = \frac{3}{5} N \varepsilon_f$$

2.) a) $$P = -\frac{\partial U}{\partial V} = -\frac{\partial}{\partial V}\left(\frac{\hbar^2}{2m} \cdot \frac{3}{5} \cdot \left(\frac{3\pi^2}{V}\right)^{2/3} \cdot N^{5/3}\right)$$

$$P = -\frac{\partial}{\partial V}\left(Q \cdot V^{-2/3}\right) = \frac{2}{3} Q \cdot V^{-5/3} = \left(\frac{2}{3} Q \cdot V^{-2/3}\right) V^{-1}$$

$$P = \frac{2}{3}\frac{U}{V}$$

(b) $B = -V\frac{\partial P}{\partial V} = -V\frac{\partial}{\partial V}\left(\frac{5}{3}QV^{-5/3}\right)$

$B = V \times \frac{5}{3} \times \frac{5}{3} Q V^{-7/3} = \frac{5}{3}\left(\frac{5}{3}QV^{-5/3}\right)$

$B = \frac{5}{3}P = \frac{5}{3}\left(\frac{2}{3}\frac{U}{V}\right) = \frac{10}{9}\frac{U}{V}$

(c) $B = \frac{10}{9}\frac{U}{V} = \frac{10}{9}\frac{3}{5}\frac{N\varepsilon_F}{V} = \frac{2}{3}\frac{N\varepsilon_F}{V}$

$B = 1{,}97 \times 10^{22}\left[\frac{eV}{cm^3}\right]$

3) $N = \int_0^\infty D(\varepsilon) f(\varepsilon)\, d\varepsilon = \int_0^\infty \frac{m}{\pi\hbar^2} \times \frac{1}{e^{\beta(\varepsilon-\mu)}+1}\, d\varepsilon$

$N = \frac{m}{\pi\hbar^2}\int_0^\infty \frac{d\varepsilon}{e^{\beta(\varepsilon-\mu)}+1} = \frac{m}{\pi\hbar^2} \times \left(-\frac{\ln\left(e^{-\beta(\varepsilon-\mu)}+1\right)}{\beta}\right)\Big|_0^\infty$

$N = \frac{m}{\pi\hbar^2\beta}\ln\left(e^{\beta\mu}+1\right) \Rightarrow \frac{\pi\hbar^2 N\beta}{m} = \ln\left(e^{\beta\mu}+1\right)$

$\therefore e^{\frac{\pi\hbar^2 N\beta}{m}} = e^{\beta\mu}+1 \Rightarrow \beta\mu = \ln\left(e^{\frac{\pi\hbar^2 N\beta}{m}} - 1\right)$

$\mu = k_B T \ln\left(e^{\frac{\pi\hbar^2 N}{k_B T m}} - 1\right)$

4. (a) Suponiendo que el sol se compone solo de H y He y $m_p \approx m_n$, entonces:

$N = \frac{M_\odot \cdot 0{,}75}{m_p} + \frac{M_\odot \times 0{,}25 \times 2}{4\, m_p}$

$N = 0{,}875\,\frac{M_\odot}{m_p} \simeq 0{,}104 \times 10^{57}$ electrones

reemplazando en ε_F, con $R = 2 \times 10^7\,[m]$

$V = \frac{4}{3}\pi R^3 = 3{,}35 \times 10^{22}\,[m^3]$

$\therefore \; \varepsilon_F = \frac{\hbar^2}{2m_e}\left(\frac{3\pi^2 \cdot 0{,}104 \times 10^{57}}{3{,}35 \times 10^{22}}\right)^{2/3} \approx 34\,[\text{KeV}]$

(b) De la ε_F sabemos que $K_F = \left(\frac{3\pi^2 N_e}{V}\right)^{1/3}$ en el límite ultrarrelativista

$\varepsilon_F = \hbar K c = \hbar c\left(\frac{3\pi^2 N_e}{V}\right)^{1/3} \sim \hbar c\left(\frac{N_e}{V}\right)^{1/3}$

(c) Finalmente, en el caso no relativista

$R = 10\,[\text{Km}] \rightarrow V = 4{,}2 \times 10^{42}\,[m^3]$

$\varepsilon_F = 1{,}5 \times 10^{5}\,[\text{MeV}]$

Sin embargo, en el caso relativista $\varepsilon_F = \hbar K c$

$\varepsilon_F = 1{,}3 \times 10^{2}\,[\text{MeV}]$, así que debería considerarse el caso relativista

5.) $\varepsilon_F = \frac{\hbar^2}{2m}\left(\frac{3\pi^2 N}{V}\right)^{2/3}$, $\frac{N}{V} = \frac{\rho}{m} = n \equiv$ concentración

$\varepsilon_F = \frac{\hbar^2}{2\times(3\times 1{,}67\times 10^{-27})} \times \left(\frac{3\pi^2 \times 1{,}62 \times 10^{-28}}{3\times 1{,}67\times 10^{-27}}\right)^{2/3}$

$\varepsilon_F \approx 4{,}2 \times 10^{-4}\,[\text{eV}]$, $T_F = \frac{\varepsilon_F}{K_B} = 4{,}8\,[K]$

6.) Al realizar transformada de Fourier a la expresión inicial se tendrá:

$m\dot{V}(t) + \frac{V(t)}{\tau} = -eE(t)$

$(-i\omega)\, m V(\omega) + \frac{m V(\omega)}{\tau} = -eE(\omega)$

$-i\omega\tau m V(\omega) + m V(\omega) = -\tau e E(\omega)$

$\therefore \; V(\omega) = -\frac{E(\omega) \times \tau\, e/m}{1 - i\omega\tau}$

$$J = -neV(\omega) = \sigma E(\omega) \; ; \; V(\omega) = \frac{-\sigma}{ne} E(\omega)$$

$$= \frac{-\tau e/m}{1 - i\omega\tau}$$

$$\Rightarrow \frac{\sigma}{ne} = \frac{\tau e/m}{1 - i\omega\tau} \, , \; \sigma = \frac{ne^2\tau}{m}\left(\frac{1}{\omega\tau - i}\right) \cdot \left(\frac{\omega\tau + i}{\omega\tau + i}\right)$$

$$\sigma = \frac{ne^2\tau}{m}\left(\frac{\omega\tau + i}{\omega^2\tau^2 + 1}\right) = \sigma_0\left(\frac{1 + i\omega\tau}{1 + \omega^2\tau^2}\right)$$

7. a) De nuevo realizando trasformada de fourier se tiene:

$$m\left(-i\omega V_x + \frac{V_x}{\tau}\right) = -e\left(E_x + \frac{BV_y}{c}\right)$$

$$m\left(-i\omega V_y + \frac{V_y}{\tau}\right) = -e\left(E_y - \frac{BV_x}{c}\right)$$

Usando la frecuencia ciclotrónica $\omega_c = \frac{eB}{mc}$ y expresando matricialmente:

$$\begin{pmatrix} 1 - i\omega\tau & \omega_c\tau \\ -\omega_c\tau & 1 - i\omega\tau \end{pmatrix}\begin{pmatrix} V_x \\ V_y \end{pmatrix} = \frac{-e\tau}{m}\begin{pmatrix} E_x \\ E_y \end{pmatrix}$$

$$W\bar{V} = -\frac{e\tau}{m}\bar{E}$$

$$\bar{J} = -ne\bar{V} = \frac{ne^2\tau}{m} W^{-1}\bar{E}$$ y usando $\omega_p^2 = \frac{4\pi ne^2}{m}$ (correspondiente a la frecuencia plasmática)

$$\bar{J} = \frac{\omega_p^2\tau/4\pi}{(1 - i\omega\tau)^2 + (\omega_c\tau)^2}\begin{pmatrix} 1 - i\omega\tau & -\omega_c\tau \\ \omega_c\tau & 1 - i\omega\tau \end{pmatrix}\begin{pmatrix} E_x \\ E_y \end{pmatrix}$$

Tomando $\omega\tau \ll 1$, $\omega \gg \omega_c$,

$$\bar{J} = \frac{\omega_p^2 \tau}{4\pi(\omega\tau)^2}\begin{pmatrix} i\omega\tau & \omega_c\tau \\ -\omega_c\tau & i\omega\tau \end{pmatrix}\begin{pmatrix} E_x \\ E_y \end{pmatrix}$$

Se sabe que $\bar{J} = \bar{\bar{\sigma}}\bar{E}$, por lo tanto, al comparar:

$$\bar{\bar{\sigma}} = \begin{pmatrix} \frac{i\omega_p^2}{4\pi\omega} & \frac{\omega_p^2\omega_c}{4\pi\omega^2} \\ \frac{-\omega_p^2\omega_c}{4\pi\omega^2} & \frac{i\omega_p^2}{4\pi\omega} \end{pmatrix} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix}$$

donde $\sigma_{xx} = \sigma_{yy} \wedge \sigma_{xy} = -\sigma_{xy}$

(b) $\bar{\bar{\varepsilon}} = 1 + \frac{i4\pi}{\omega}\bar{\bar{\sigma}} \Rightarrow \varepsilon_{ij} = \delta_{ij} + \frac{4\pi i \sigma_{ij}}{\omega}$

Considerando que $\vec{K} = k\hat{z}$, de la ec. de onda para $\bar{E}$ puede obtenerse la relación de dispersión por medio de la transformada de Fourier

$$\nabla^2 E = \frac{\varepsilon}{c^2}\frac{\partial^2 E}{\partial t^2}$$

$$\therefore\ c^2k^2E_x = \omega^2\left[E_x\left(1 + \frac{4\pi i}{\omega}\sigma_{xx}\right) + E_y \cdot \frac{4\pi i}{\omega}\sigma_{xy}\right]$$

$$c^2k^2E_y = \omega^2\left[E_x \cdot \frac{4\pi i}{\omega}\sigma_{xy} + E_y\left(1 + \frac{4\pi i}{\omega}\sigma_{yy}\right)\right]$$

$$\begin{pmatrix} c^2k^2 - \omega^2 + \omega_p^2 & -\frac{i\omega_p^2\omega_c}{\omega} \\ \frac{i\omega_p^2\omega_c}{\omega} & c^2k^2 - \omega^2 + \omega_p^2 \end{pmatrix}\begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Para tener sol no triviales requerimos que la matriz no tenga inversa, así que su determinante ha de ser 0

$$(c^2k^2 - \omega^2 + \omega_p^2)^2 - \left(\frac{\omega_p^2\omega_c}{\omega}\right)^2 = 0$$

$$\therefore\ c^2k^2 = \omega^2 - \omega_p^2 \pm \frac{\omega_p^2\omega_c}{\omega}$$

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8) a) Del primer inciso $U = \frac{3}{5} N \varepsilon_F$

$\rightarrow \frac{U}{N} = \frac{3}{5} \varepsilon_F \rightarrow \bar{U} = \frac{U}{N} = \frac{3}{5} \frac{\hbar^2}{2m} \left(\frac{3\pi^2 N}{V} \right)^{2/3}$

tomando $V = \frac{4}{3} \pi r_0^3$, puesto que es una energía por e^- entonces $N = 1$, además, $r_s = \frac{r_0}{a_H}$

$r_0 = r_s \times a_H = \frac{r_s \hbar^2}{q e^2 m}$

$$\therefore \bar{U} = \frac{3}{5} \frac{\hbar^2}{2m} \left(3\pi^2 \cdot \frac{3}{4\pi \left(\frac{r_s \hbar^2}{q e^2 m} \right)^3} \right)^{2/3}$$

$$\bar{U} = \frac{3}{5} \frac{\hbar^2}{2m} \left(3\pi^2 \cdot \frac{3}{4\pi \left[\frac{r_s \hbar^2}{q e^2 m} \right]^3} \right)^{2/3}$$

$$\bar{U} = \frac{3}{5} \frac{\hbar^2}{2m} \left(\frac{9\pi}{4} \right)^{2/3} \left(\frac{q e^2 m}{r_s \hbar} \right)^{2/3}$$

$$\bar{U} = \frac{3}{5} \left(\frac{9\pi}{4} \right)^{2/3} \frac{m e^4}{2\hbar^2} \cdot \frac{1}{r_s^2} \approx \frac{2{,}21}{r_s^2} \left(\frac{m e^4}{2\hbar^2} \right)$$

$$\bar{U} \approx \frac{2{,}21}{r_s^2} \, [Ry]$$

b) $U = \int_0^r D(v) f_{FD} \cdot \varepsilon(v) \, dV$, con $f_{FD} = \begin{cases} \sim 1, & r \leq r_0 \\ \sim 0, & r > r_0 \end{cases}$

en coordenadas esféricas se tendría:

$$U = -\int n \frac{k q e^2}{r} \cdot r^2 dr \cdot 4\pi = -\int \frac{1}{\frac{4}{3}\pi r_0^3} \cdot k q e^2 r \, dr \cdot 4\pi$$

$$U = -\frac{3 q e^2 k}{r_0^3} \cdot \int_0^{r_0} r \, dr = -\frac{3 q e^2 k}{r_0^3} \frac{r_0^2}{2} = -\frac{3 q e^2 k}{2 r_0}$$

en Rydbergs se tiene $r_0 = r_s a_H = r_s \cdot \frac{\hbar^2 e^2}{m}$

$$\therefore\ U = \frac{-3 q_e^2 K}{2 r_0 \left(\frac{\hbar^2}{q_e^2 m}\right)} = -\frac{3}{2}\,\frac{q_e^4 K m}{r_s \hbar^2} = \frac{-3K}{2 r_s}$$

(c) De manera similar al punto (b), pero considerando que es una autointeracción,

$$n(r) = \frac{1}{\frac{4}{3}\pi r^3};$$

$$U = \frac{1}{2}\int_0^{r_0} \frac{-1}{\frac{4}{3}\pi r^3} \times \frac{K q_e^2}{r}\, r^2\, dr \times 4\pi$$

$$U = -\frac{3}{2} K \int^{r_0} q_e^2 \frac{1}{r^2} = \frac{3}{2}\,\frac{K q_e^2}{r_0} \rightarrow \frac{3}{2 r_s}$$

(d) $U = -\frac{1{,}80}{r_s} + \frac{2{,}21}{r_s^2}$, $\dot{U} = 0 = \frac{1{,}80}{r_s^2} - \frac{4{,}42}{r_s^3} = 0$

$\therefore\ r_s = 2{,}45$

9.) Caso estático, por lo tanto:

$$\frac{m V_x}{\tau} + \frac{e B V_y}{c} = -e E_x$$

$$\frac{m V_y}{\tau} - \frac{e B V_x}{c} = -e E_y$$

$$\frac{m V_z}{\tau} = -e E_z$$

tomando $w_c = \frac{eB}{mc}$ y en forma matricial:

$$\begin{pmatrix} 1 & w_c\tau & 0 \\ -w_c\tau & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix} = \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} \times \left[\frac{-e\tau}{m}\right]$$

$$\therefore\ \begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix} = \frac{-e\tau}{m\left[1+[w_c\tau]^2\right]}\begin{pmatrix} 1 & -w_c\tau & 0 \\ w_c\tau & 1 & 0 \\ 0 & 0 & 1+[w_c\tau]^2 \end{pmatrix}\begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}$$

tomando $\vec{J} = -ne\vec{v}$ y $\sigma_0 = ne^2\tau/m$

$$\begin{pmatrix} j_x \\ j_y \\ j_z \end{pmatrix} = \frac{\sigma_0}{1+(\omega_c\tau)^2}\begin{pmatrix} 1 & -\omega_c\tau & 0 \\ \omega_c\tau & 1 & 0 \\ 0 & 0 & 1+\omega_c^2\tau^2 \end{pmatrix}\begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}$$

usando $\omega_c\tau \gg 1$ (alto B)

$$\begin{pmatrix} j_x \\ j_y \\ j_z \end{pmatrix} = \frac{\sigma_0}{\omega_c^2\tau^2}\begin{pmatrix} 0 & -\omega_c\tau & 0 \\ \omega_c\tau & 0 & 0 \\ 0 & 0 & (\omega_c\tau)^2 \end{pmatrix}\begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}$$

Considerando que $\bar{J} = \bar{\bar{\sigma}}\bar{E}$ se tiene:

$$\sigma_{xx} = \frac{\sigma_0}{1+(\omega_c\tau)^2} \sim \frac{\sigma_0}{\omega_c\tau} + \frac{\sigma_0}{(\omega_c\tau)^2} \to 0$$

$$\sigma_{yx} = -\sigma_{xy} = \frac{\sigma_0\omega_c\tau}{1+(\omega_c\tau)^2} \sim \frac{\sigma_0}{\omega_c\tau} = \frac{nec}{B}$$

b) $R \approx \dfrac{mv_F}{nd^2e^2} = \dfrac{mv_F}{nl^2e^2} = \dfrac{p}{nl^2e^2} = \dfrac{h/\lambda}{nl^2e^2}$

$R \approx \dfrac{h/l}{nl^2e^2} = \dfrac{h}{l^3ne^2}$ y pero $n = \dfrac{1}{l^3}$

$\therefore \; R \approx \dfrac{h}{e^2}$

Capítulo 7.

1. ⓐ $$\frac{E_{esquina}}{E_{centro}} = \frac{\frac{\hbar^2}{2m}\left[\left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{a}\right)^2\right] \to K^2_{esquina}}{\frac{\hbar^2}{2m}\left(\frac{\pi}{a}\right)^2 \to K^2_{centro}} = 2$$

ⓑ Como se apreció α depende del cociente $\frac{K^2_{esquina}}{K^2_{centro}} = \alpha$

$$\therefore \quad \alpha = \frac{\left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{a}\right)^2}{\left(\frac{\pi}{a}\right)^2} = 3$$

ⓒ Si la energía en todo punto fuera igual, $\alpha = 1$, la celda ya estaría energéticamente llena y el metal sería aislante

2. Para observar la dirección [111], el número de onda se expresa de la forma

$$\vec{K} = \vec{K}' + \vec{G}, \quad \vec{K} = \underbrace{\begin{pmatrix}1\\1\\1\end{pmatrix}\frac{\pi}{a}\alpha}_{\text{dirección}} + \underbrace{\frac{2\pi}{a}\begin{pmatrix}n_1\\n_2\\n_3\end{pmatrix}}_{\text{traslación en la BZ}}, \quad K < 1$$

Entonces la relación de dispersión será:

$$E = \frac{\hbar^2 K^2}{2m} = \frac{\hbar^2}{2m}\left[K_x^2 + K_y^2 + K_z^2\right]$$

$$E = \frac{\hbar^2}{2m}\left[\left(\frac{\pi}{a}\alpha + \frac{2\pi n_1}{a}\right)^2 + \left(\frac{\pi}{a}\alpha + \frac{2\pi n_2}{a}\right)^2 + \left(\frac{\pi}{a}\alpha + \frac{2\pi n_3}{a}\right)^2\right]$$

$$E = \frac{\hbar^2\pi^2}{2ma^2}\left[(\alpha+2n_1)^2 + (\alpha+2n_2)^2 + (\alpha+2n_3)^2\right]$$

$$E = \frac{\hbar^2\pi^2}{2ma^2}\left[3\alpha^2 + 4\alpha(n_1+n_2+n_3) + 4(n_1^2+n_2^2+n_3^2)\right]$$

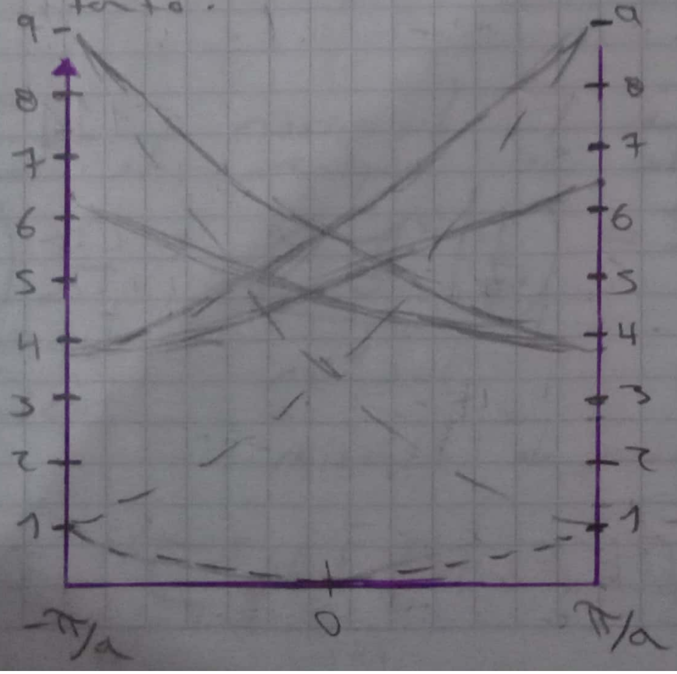
Variando n_i y considerando degeneraciones con $E_0 = \frac{\hbar^2\pi^2}{2ma^2}$

(n_1, n_2, n_3)	E/E_0
$(0,0,0)$	$3\alpha^2$
$(1\bar{1}\bar{1}),(\bar{1}1\bar{1}),(\bar{1},\bar{1},1)$	$3\alpha^2-4\alpha+12$
$(00\bar{2}),(0\bar{2}0),(\bar{2}00)$	$3\alpha^2-8\alpha+16$
$(002),(020),(200)$	$3\alpha^2+8\alpha+16$
(111)	$3\alpha^2+12\alpha+12$

Variando n_i y considerando degeneraciones con $E_0 = \frac{\hbar^2\pi^2}{2ma^2}$

(n_1, n_2, n_3)	E/E_0
$(0,0,0)$	$3\alpha^2$
$(1\bar{1}\bar{1}),(\bar{1}1\bar{1}),(\bar{1},\bar{1},1)$	$3\alpha^2 - 4\alpha + 12$
$(00\bar{2}),(0\bar{2}0),(\bar{2}00)$	$3\alpha^2 - 8\alpha + 16$
$(002),(020),(200)$	$3\alpha^2 + 8\alpha + 16$
(111)	$3\alpha^2 + 12\alpha + 12$
$(\bar{1}\bar{1}\bar{1})$	$3\alpha^2 - 12\alpha + 12$

Por lo tanto:



3.) (a) $$\frac{P\,\mathrm{Sen}(k'a)}{k'a} + \cos(k'a) = \cos(ka)$$

para $k=0$, $$\frac{P\,\mathrm{Sen}(k'a)}{k'a} + \cos(k'a) = 1$$

En la aproximación de ángulos pequeños se tiene:

$$P\left(1 - \frac{(k'a)^2}{6}\right) + \left(1 - \frac{(k'a)^2}{2}\right) = 1$$

$$P - \frac{(k'a)^2}{2}\left(1 + P/3\right) = 0$$

con $P \ll 1 \Rightarrow$ $$k'^2 = \frac{2P}{a^2(1+P/3)} \sim \frac{2P}{a^2}$$

$$\varepsilon = \frac{\hbar^2 k'^2}{2m} \sim \frac{\hbar^2}{2m}\cdot\frac{2P}{a^2} = \frac{\hbar^2 P}{ma^2}$$

(b) si $k' = \pi/a \Rightarrow \cos(k'a) = -1$

$$\therefore \frac{P}{ka}\,\mathrm{Sen}(ka) + \cos(ka) = -1$$

$$\Rightarrow P\,\mathrm{Sen}(ka/2)\cos(ka/2) + \left(1 - 2\,\mathrm{Sen}^2(ka/2)\right) = -1$$

con $p \ll 1 \Rightarrow k^2 = \frac{2p}{a^2(1+p/3)} \sim \frac{2p}{a^2}$

$$\varepsilon = \frac{\hbar^2 k^2}{2m} \sim \frac{\hbar^2}{2m} \cdot \frac{2p}{a^2} = \frac{\hbar^2 p}{ma^2}$$

(b) Si $k' = \pi/a \Rightarrow \cos(k'a) = -1$

$$\therefore \frac{P}{ka}\operatorname{Sen}(ka) + \cos(ka) = -1$$

$$\frac{2P}{ka}\operatorname{Sen}(ka/2)\cos(ka/2) + (1 - 2\operatorname{Sen}^2(ka/2)) = -1$$

$$\frac{2P}{ka}\operatorname{Sen}\left(\frac{ka}{2}\right)\cos\left(\frac{ka}{2}\right) = -2\left(1 - \operatorname{Sen}^2\left(\frac{ka}{2}\right)\right)$$

$$\frac{2P}{ka}\operatorname{Sen}\left(\frac{ka}{2}\right)\cos\left(\frac{ka}{2}\right) = -2\cos^2\left(\frac{ka}{2}\right)$$

$$\frac{2P}{ka}\operatorname{Sen}\left(\frac{ka}{2}\right) = -2\cos\left(\frac{ka}{2}\right)$$

$$\tan\left(\frac{ka}{2}\right) = -\frac{ka}{P}$$

Si $P \ll 1 \Rightarrow -\frac{ka}{P} \to -\infty$, entonces $\tan\left(\frac{ka}{2}\right) \to -\infty$

$\therefore \frac{ka}{2} \to -\frac{\pi}{2}$ o $k \to -\frac{\pi}{a}$

luego $E = \frac{\hbar^2}{2m} k^2 = \frac{\hbar^2}{2m} \frac{\pi^2}{a^2}$

Para el gap sea $E_g = E(\delta) - E(-\delta)$

$E_g = \frac{\hbar^2}{2ma^2}\left[(k+\delta)^2 - (k-\delta)^2 \right]$

$E_g = \frac{2\hbar^2 k\delta}{ma^2} = \frac{2\hbar^2\pi\delta}{ma^2}$ y al reemplazar en la e.c de Kronig penney se tiene:

$$\frac{P \operatorname{Sen}(k'a)}{k'a} + \cos(k'a) = -1$$

Tomamos $k'a << 1$:

$\frac{P \operatorname{Sen}(\pi+\delta)}{\pi+\delta} + \cos(\pi+\delta) = -1$

$\frac{-P\delta}{\pi+\delta} - \left(1 - \frac{\delta^2}{2}\right) = -1$, $-\frac{P\delta}{\pi+\delta} + \frac{\delta^2}{2} = 0$

Como $\delta << 1 \Rightarrow \frac{-P}{\pi} + \frac{\delta}{2} = 0 \Rightarrow \delta = \frac{2P}{\pi}$

$\therefore E_g = \frac{2\hbar^2\pi}{ma^2} \times \frac{2P}{\pi} = \frac{4\hbar^2 P}{ma^2}$

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$$\frac{p\sin(\pi+\delta)}{\pi+\delta} + \cos(\pi+\delta) = -1$$

$$\frac{-p\,\delta}{\pi+\delta} - \left(1 - \frac{\delta^2}{2}\right) = -1 \quad , \quad -\frac{p\,\delta}{\pi+\delta} + \frac{\delta^2}{2} = 0$$

como $\delta \ll 1 \Rightarrow -\frac{p}{\pi} + \frac{\delta}{2} = 0 \Rightarrow \delta = \frac{2p}{\pi}$

$$\therefore E_g = \frac{2\hbar^2 \pi}{ma^2} \times \frac{2p}{\pi} = \frac{4\hbar^2 p}{ma^2}$$

4.) a) El diamante tiene dos átomos por base ubicados en $R_1 = (0, 0, 0)$ $\wedge$ $R_2 = (1/4, 1/4, 1/4)$ con $U = U_{R_1} + U_{R_2}$

de manera tal que en una serie de fourier se tiene

$$U_0 \sum_i U_{0i} = U_0\left(e^{i\vec{G}\cdot r_1} + e^{i\vec{G}\cdot r_2}\right)$$

$$= U_0\left(1 + e^{i\left(\vec{G}\cdot\frac{a}{4}(1,1,1)\right)}\right)$$

si $U = 0$ $\wedge$ $U_0 \neq 0$ $\Rightarrow e^{i\vec{G}\cdot\frac{a}{4}(1,1,1)} = -1$

$\vec{G}\cdot\frac{a}{4}(1,1,1) = \pi \Rightarrow \vec{G} = \frac{4\pi}{a}\hat{n} = 2\lambda$

b) Usando la ecuación central con $\mathbb{K} = \pm\frac{\vec{\sigma}}{2}$ ($\vec{\sigma}$ implica dos celdas) se conocen soluciones a $(\lambda - \varepsilon)^2 = U^2$, luego $\lambda = \varepsilon \pm U_g \rightarrow \varepsilon = \lambda \mp U_g$, reemplazando $\lambda = \frac{\hbar^2}{2m} k^2 = \frac{\hbar^2}{2m} \frac{\sigma^2}{4} = \frac{(\hbar\sigma)^2}{8m}$, el gap de energía corresponde a:

$\Delta\varepsilon = (\lambda + U_g) - (\lambda - U_g) = 2U_g$, reemplazando:

$\Delta\varepsilon = E_g = 2U_{1g}(1 + e^{i\vec{\sigma}\cdot\mathbb{r}\frac{\pi}{4}})$, para $E_g = 0$ se requiere $\sigma \cdot \mathbb{r}_F = 0 \rightarrow \vec{\sigma} \perp r_F$

5.) Sea $k = \mathrm{Re}\{k\} + i\,\mathrm{Im}(k)$ tal que $\mathrm{Im}(k) \ll \mathrm{Re}(k)$, en el centro $\frac{\hbar^2 [\mathrm{Re}(k)]^2}{2m} = \varepsilon$, por lo tanto:

$$\lambda = \frac{\hbar^2}{2m}(a + ib)^2 = \frac{\hbar^2}{2m}((a^2 - b^2) + i2ab)$$

La ecuación central cuyo resultado es $(\lambda^* - \varepsilon)(\lambda - \varepsilon) = U^2$

$$|\lambda|^2 - \lambda^*\varepsilon - \varepsilon\lambda + \varepsilon^2 = U^2$$

$$|\lambda|^2 - \varepsilon(\lambda^* + \lambda) + \varepsilon^2 = U^2$$

$$\frac{\hbar^2}{2m}(a^2 + b^2) - \varepsilon\left(\frac{\hbar^2}{2m}(a^2 - b^2)\right) + \varepsilon^2 = U^2$$

y como $\varepsilon = \frac{\hbar^2}{2m} a^2$, entonces:

$$\frac{\hbar^2}{2m}(a^2 + b^2) - \frac{\hbar^4}{4m^2} a^2 (a^2 - b^2) + \frac{\hbar^4}{4m^2} a^4 = U^2$$

$$\frac{\hbar^2}{2m}(a^2 + b^2) + \frac{\hbar^4}{4m^2} a^2 b^2 = U^2$$

$$\frac{\hbar^2}{2m}\left(\frac{a^2+b^2}{\hbar^2}\right)2m + \frac{\hbar^2}{2m}a^2b^2 = \frac{2mU^2}{\hbar^2}$$

$$\frac{2m(1+b^2/a^2)}{\hbar^2}\frac{\hbar^2}{2m} + \frac{\hbar^2}{2a^2}b^2 = \frac{2mU^2}{\hbar^2a^2} = \frac{2mU^2}{\hbar^2\sigma}\times 4$$

$$\frac{\hbar^2}{2m}\mathrm{Re}(k)^2 \approx \frac{2mU^2}{\hbar^2\sigma}4$$

6.) $\cdot \quad \cdot(\pi/a, \pi/a)$

De $U = -4U\cos\left(\frac{2\pi x}{a}\right)\cos\left(\frac{2\pi y}{a}\right)$ en exponenciales:

$$U = -4U\left(e^{i\frac{2\pi(x+y)}{a}} + e^{i\frac{2\pi(x-y)}{a}} + e^{-i\frac{2\pi(x-y)}{a}} + e^{-\frac{2\pi}{a}(x+y)}\right)$$

Comparamos con la expansión en series de Fourier $U = \sum_j U_g e^{ig_j r}$ por lo cual:

$U_g = -U$

$g_1 = \frac{2\pi}{a}(1,1) \rightarrow$ esquina! $\rightarrow$ único relevante $\rightarrow \vec{g_j} = 2\vec{k}$

$g_2 = \frac{2\pi}{a}(1,-1)$

$g_3 = \frac{2\pi}{a}(-1,1)$

$g_4 = \frac{2\pi}{a}(-1,-1)$

$\therefore \quad (\lambda_k - \varepsilon)C(k) + U_g C(k-g) = 0$

$(\lambda_{k-g} - \varepsilon)C(k-g) + U_{-g}C(k) = 0$

$(\lambda_k - \varepsilon)C(k) + U_g C(-k) = 0$

$(\lambda_k - \varepsilon)C(k) - U_g C(k) = 0$

De nuevo, se lleva a forma matricial y se exige $\det A = 0$, por lo tanto

$$(\lambda_k - \varepsilon)^2 - U^2 = 0 \Rightarrow \varepsilon = \lambda_k \pm U$$

$$\varepsilon_g = (\lambda_k - U) - (\lambda_k + U) = 2U$$

Capítulo 8

1.) ⓐ $E_d = \frac{m_H q_e^4}{8 \varepsilon_0^2 \varepsilon_r^2 h^2} \times \frac{m_e^*}{m_e} \sim 6{,}3 \times 10^{-4}$

ⓑ $a_0 = \underbrace{a_H}_{r_{bohr}} \times \varepsilon_r \times \left(\frac{m_e^*}{m_e}\right)^{-1} \sim 63{,}6 \, [nm]$

ⓒ Suponiendo $N = 1 \Rightarrow n = \frac{N}{V_0} = \frac{N}{\frac{4}{3}\pi a_0^3} = \frac{1}{\frac{4}{3}\pi a_0^3} > 1$

Cuando la concentración de un solo átomo en estado base es mayor que 1

2.) ⓐ $n_0 = 2\left(\frac{m_e k_B T}{2\pi \hbar^2}\right)^{3/2} \simeq 3{,}85 \cdot 10^{13}$

$$n \simeq (n_0 N_d)^{1/2} e^{-\frac{E_d}{2k_B T}} \simeq (3{,}85)^{1/2} e^{e^{\frac{m_e V}{0{,}345 meV}}} = 0{,}464 \times 10^{3} \frac{\text{donores}}{cm^3}$$

a tanto $T \sim 0$

$$n = n_0 [f_{FD}(T=0)] = \frac{n_d}{2} = 0{,}5 \times 10^{3} \frac{\text{donores}}{cm^3}$$

ⓑ $R_H = \frac{1}{n e} \simeq 1356 \times 10^{-6}$

3.) Sabemos del capítulo 6 que:

$$J_e = \sigma_e \begin{pmatrix} 1 & -\omega_c \tau_e & 0 \\ \omega_c \tau_e & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} E$$

Para los huecos se define entonces:

$\sigma_e \to \sigma_h$, $w_0 \to -W_e^*$, $\tau_e \to \tau_h$ ∴

$$\bar{J}_h = \sigma_h \begin{pmatrix} 1 & W_e^* \tau_h & 0 \\ -W_e^* \tau_h & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} E$$

Con $\bar{J}_T = \bar{J}_h + \bar{J}_e$, cuyo tensor de conductividad eléctrica tiene la forma:

$$\begin{pmatrix} \sigma_e + \sigma_h & \sigma_h W_e^* \tau_h - \sigma_e W_e \tau_e \\ -\tau_h W_e^* \sigma_h + \sigma_e w_e \tau_e & \sigma_e + \sigma_h \end{pmatrix}$$

Se le quitaron las componentes referentes a z dado que $B = B\hat{k}$

$\Rightarrow E_z = 0 \Rightarrow J_z = 0$.

$$W_0^* = \frac{eB}{m_h} \; ; \; \sigma_h = \frac{ne^2 \tau_h}{m_h} \; ; \; \mu_h = \frac{e \tau_h}{m_h}$$

$$W_e = \frac{-eB}{m_e} \; ; \; \sigma_e = \frac{ne^2 \tau_e}{m_e} \; ; \; \mu_e = \frac{e \tau_e}{m_e}$$

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Se le quita la z a los componentes referentes dado que $B = B\hat{k}$
$\Rightarrow E_z = 0 \Rightarrow J_z = 0$.

$$\omega_h = \frac{eB}{m_h} \; ; \; \sigma_h = \frac{pe^2\tau_h}{m_h} \; ; \; \mu_h = \frac{e\tau_h}{m_h}$$

$$\omega_e = \frac{-eB}{m_e} \; ; \; \sigma_e = \frac{ne^2\tau_e}{m_e} \; ; \; \mu_e = \frac{e\tau_e}{m_e}$$

Operando y reemplazando en el tensor conductividad se tiene:

$$J_T = \begin{pmatrix} e(p\mu_h + n\mu_e) & Be(p\mu_h^2 - n\mu_e^2) \\ Be(n\mu_e^2 - p\mu_h^2) & e(p\mu_h + n\mu_e) \end{pmatrix} E_{xy}$$

Si se considera el efecto Hall en el equilibrio, se tiene $J_y = 0$

$$\therefore \; E_x\sigma_{21} + E_y\sigma_{22} = J_y = 0$$

$$\therefore \; e[Be(n\mu_e^2 - p\mu_h^2)]E_x + [p\mu_h + n\mu_e]E_y = 0$$

$$E_x = E_y\left(-\left(\frac{p\mu_h + n\mu_e}{n\mu_e^2 - p\mu_h^2}\right)\right) = -\frac{A}{C}E_y$$

$\therefore \quad J_x = eE_x A - e\,C E_y\,, \quad E_x = -\frac{A}{C} E_y$

luego $\quad J_x = -\frac{e A^2}{C} E_y - e\,C E_y = -eE_y\left(\frac{A^2}{C} + C\right)$

Así que, el coeficiente de Hall es:

$$R_H = \frac{E_y}{J_x B} = \frac{E_y}{-eE_y\left(\frac{A^2}{C} + C\right)B},$$

Considerando $B^2 \to 0$, entonces:

$$R_H = \frac{-(n M_e^2 - p M_h^2)}{e(p M_h + n M_e)^2}, \text{ si } b = \frac{M_e}{M_h} \text{ entonces:}$$

$$R_H = \frac{p - nb^2}{e(p+nb)^2}$$

4.) Hacemos uso de $\frac{d\vec{k}}{dt} = \frac{e}{\hbar^2} \nabla_k U \times B$

para cada dimensión se tiene que:

$(*)\ \frac{\partial k_x}{\partial t} = -\frac{e k_z B_y}{m_e} = j\omega k_{0x}$

..., entonces.

$$R_H = \frac{-(nM_e^2 - pM_h^2)}{e(pM_h + nM_e)^2}, \quad \text{si } b = \frac{M_e}{M_h} \text{ entonces:}$$

$$R_H = \frac{p - nb^2}{e(p+nb)^2}$$

4.) Hacemos uso de $\frac{d\vec{k}}{dt} = \frac{e}{\hbar^2} \nabla_k U \times B$

para cada dimensión se tiene que:

$$(*)\quad \frac{\partial k_x}{\partial t} = \frac{-e k_z B_y}{m_e} = j\omega k_{ox}$$

$$(**)\quad \frac{\partial k_y}{\partial t} = \frac{e k_z B_x}{m_e} = j\omega k_{oy}$$

$$(***)\quad \frac{\partial k_z}{\partial t} = e\left(\frac{k_x B_y}{m_T} - \frac{k_y B_x}{m_T}\right) = j\omega k_{oz}$$

en $\nabla_k U \times B = \hat{x}\left(\frac{-k_z B_y}{m_e}\right) + \hat{y}\left(\frac{k_z B_x}{m_e}\right) + \hat{z}\left(\frac{k_x B_y}{m_T} - \frac{k_x B_y}{m_T}\right)$

Donde para las anteriores 3 ec.s se proponen ondas planas como solución:

$$\underbrace{\begin{pmatrix} -j\omega & 0 & \mp \frac{eB_y}{m_e} \\ 0 & -j\omega & \pm \frac{eB_x}{m_e} \\ \pm \frac{eB_y}{m_e} & \mp \frac{eB_x}{m_T} & -j\omega \end{pmatrix}}_{Q} \begin{pmatrix} K_0 x \\ K_0 y \\ K_0 z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

De nuevo, para sol no triviales

$$\det Q = 0 \Rightarrow -j\omega\left(-\omega^2 + \frac{e^2 B_x^2}{m_T m_e}\right) \mp \frac{eB_y}{m_e}\left(\pm j\omega \frac{eB_y}{m_T}\right) = 0$$

$$\therefore \omega^2 = \frac{e^2(B_x^2 + B_y^2)}{m_T m_e} = \frac{e^2 |\vec{B}|^2}{m_T m_e}$$