2. $x(t) = 3\cos(1000\pi t) + 5\sin(2000\pi t) + 10\cos(11000\pi t)$

para discretizar se reemplaza: $t = n/F_S$ con $F_S = 5\,KHz$

con $n \in \mathbb{Z}$

$$x[n/F_S] = 3\cos\left[1000\pi \frac{n}{F_S}\right] + 5\sin\left[2000\pi \frac{n}{F_S}\right] + 10\cos\left[11000\pi \frac{n}{F_S}\right]$$

$$x[n/F_S] = 3\cos\left[1000\pi \frac{n}{5000}\right] + 5\sin\left[2000\pi \frac{n}{5000}\right] + 10\cos\left[11000\frac{\pi}{5000}\right]$$

$$x[n] = 3\cos\left[\frac{\pi n}{5}\right] + 5\sin\left[\frac{2\pi n}{5}\right] + 10\cos\left[\frac{11\pi n}{5}\right]$$

$$\Omega_3 = \frac{11\pi}{5} > \pi \longrightarrow \text{copia}$$

Se halla la frecuencia original:

$$\Omega_{or} = \Omega_3 - 2\pi$$

$$\Omega_{or} = \frac{11\pi}{5} - 2\pi$$

$$\Omega_{or} = \frac{\pi}{5}$$

Se reemplaza en la función discretizada:

$$x[n] = 3\cos\left[\frac{\pi n}{5}\right] + 5\sin\left[\frac{2\pi n}{5}\right] + 10\cos\left[\frac{\pi n}{5}\right]$$

Sumando términos semejantes se obtiene como respuesta:

$$\boxed{R/= \quad x[n] = 13\cos\left[\frac{\pi n}{5}\right] + 5\sin\left[\frac{2\pi n}{5}\right]}$$

$$d^2(x_1,x_2) = P_{x_1} - P_{x_2} = \lim_{T\to\infty} \frac{1}{T}\int_T |x_1(t) - x_2(t)|^2 dt$$

Donde:

$$x_1(t) = Ae^{jw_0 t} \quad \wedge \quad x_2(t) = Be^{j5w_0 t} \quad ; \quad \text{con} \quad w_0 = \frac{2\pi}{T}$$

$$\rightarrow d^2(x_1,x_2) = \bar{P}_{x_1 - x_2} = \lim_{T\to\infty} \frac{1}{T}\int_T |x_1(t) - x_2(t)|^2 dt$$

$$= \frac{1}{T}\left[\int_T |x_1(t)|^2 dt - 2\int_T x_1(t) - x_2(t) + \int_T |x_2(t)|^2 dt\right]$$

$$= \bar{P}_{x_1} - \frac{2}{T}\int_T x_1(t) \circ x_2(t)\, dt + \bar{P}_{x_2}$$

Entonces:

- $$\bar{P}_{x_1} = \frac{1}{T}\int_0^T |x_1(t)|^2 dt = \frac{1}{T}\int_0^T |Ae^{jw_0 t}|^2 dt = \frac{A^2}{T}\int_0^T (e^{jw_0 t})(e^{jw_0 t})^* dt$$

$$= \frac{A^2}{T}\int_0^T (e^{jw_0 t})(e^{-jw_0 t}) dt = \frac{A^2}{T}\int_0^T (e^{jw_0 t - jw_0 t}) dt = \frac{A^2}{T}\int_0^T (e^0) dt$$

$$= \frac{A^2}{T}\int_0^T 1\, dt = \frac{A^2}{T}\left[t\Big|_0^T\right] = \frac{A^2}{T}[T-0] = \frac{A^2}{T}[T] = A^2$$

- $$\bar{P}_{x_2} = \frac{1}{T}\int_0^T |x_2(t)|^2 dt = \frac{1}{T}\int_0^T |Be^{j5w_0 t}|^2 dt = \frac{B^2}{T}\int_0^T |e^{j5w_0 t}|^2 dt$$

$$= \frac{B^2}{T}\int_0^T (e^{10jw_0 t})(e^{10jw_0 t})^* dt = \frac{B^2}{T}\int_0^T (e^{10jw_0 t})(e^{-10jw_0 t}) dt$$

$$= \frac{B^2}{T}\int_0^T e^{10jw_0 t - 10jw_0 t} dt = \frac{B^2}{T}\int_0^T e^0 dt = \frac{B^2}{T}\int_0^T dt = \frac{B^2}{T}\left(t\Big|_0^T\right)$$

$$= \frac{B^2}{T}(T-0) = \frac{B^2}{T}(T) = B^2$$

- $$\bar{P}_{x_1 - x_2} = \frac{1}{T}\int_0^T -2|x_1(t) \circ x_2(t)|^2 dt = \frac{-2}{T}\int_0^T (Ae^{jw_0 t})(Be^{j5w_0 t}) dt$$

$$= \frac{-2AB}{T}\int_0^T Ae^{jw_0 t} \cdot Be^{5jw_0 t} dt = -\frac{2AB}{T}\int_0^T e^{6jw_0 t} dt$$

. se Integra: $\quad u = 6j\,w_o t \quad ; \quad w_o = \frac{2\pi}{T}$

$$u = 6j\frac{2\pi}{T}t \rightarrow du = \frac{12j\pi}{T}dt \quad ; \quad dt = \frac{T}{12j\pi}du$$

$$\frac{-2AB}{T}\int_0^T e^u \frac{T}{12j\pi}du = \frac{-2AB}{T}\frac{T}{12j\pi}\int_0^T e^u du$$

$$= \frac{-AB}{6j\pi}\left[e^{T(6)\frac{2\pi}{T})} - e^{0(6)\frac{2\pi}{T})}\right] = \frac{-AB}{6j\pi}(e^{12j\pi} - 1)$$

$$e^{12j\pi} = \cos(12\pi) + j\,\mathrm{sen}(12\pi) = 1 + 0 = 1$$

Entonces: $\frac{AB}{6j\pi}(1-1) = \frac{AB}{6j\pi}(0) = 0$.

$$\therefore\ d^2(x_1, x_2) = A^2 + B^2$$