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Parcial 02
Señales y Sistemas

1. Encuentre la expresión del espectro de Fourier (forma exponencial y trigonométrica) para la señal $x(t) = |A\sin(2\pi F_0 t)|^2$ con $t \in [-\frac{1}{2F_0}, \frac{1}{2F_0}]$, con $A, F_0 \in \mathbb{R}$

Sol. Se tiene que:

$$x(t) = |A\sin(2\pi F_0(t))|^2 = A^2\sin^2(2\pi F_0 t)$$

• $\sin^2(\theta) = \dfrac{1-\cos(2\theta)}{2}$; $T = t_f - t_i = \dfrac{1}{2F_0} - \left(-\dfrac{1}{2F_0}\right) = \dfrac{1}{F_0}$

$$\therefore X(t) = A^2\left(\frac{1-\cos(4\pi F_0 t)}{2}\right) = \frac{A^2}{2} - \frac{A^2}{2}\cos(4\pi F_0 t)$$

• Por serie trigonométrica:

$$x(t) = a_0 + \sum_{n=-N}^{N} a_n\cos(n w_0 t) + b_n\sin(n w_0 t),$$

donde:

$$a_0 = c_0 = \frac{1}{t_f - t_i}\int_{t_i}^{t_f} x(t)\,dt$$

y :

$$a_n = \frac{2}{t_f - t_i}\int_{t_i}^{t_f} x(t)\cos(n w_0 t)\,dt$$

$$b_n = \frac{2}{t_f - t_i}\int_{t_i}^{t_f} x(t)\sin(n w_0 t)\,dt$$

$$c_n = \frac{a_n - j b_n}{2}$$

$$\therefore \int_{-T/2}^{T/2}\cos(n w_0 t)\,dt = \frac{T}{2}$$,

$$a_n = \frac{\langle x(t), \cos(n w_0 t)\rangle}{\|\cos(n w_0 t)\|^2}$$

$$b_n = \frac{\langle x(t), \cos(n w_0 t)\rangle}{T/2}$$

$$x(t) = \frac{A^2}{2} - \frac{A^2}{2}\cos(2w_0 t) = a_0 + a_2\cos(2w_0 t)$$

$a_0 = \frac{A^2}{2}$; $a_2 = -\frac{A^2}{2}$ $c_0 = a_0$; $c_2 = -A^2$; $c_1 = A^2$