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Parcial 1: Señales y sistemas

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a) $d^2(x_1, x_2) = P_{x_1 - x_2} = \lim_{T\to\infty} \frac{1}{T}\int_T |x_1(t) - x_2(t)|^2 dt$

$x_1(t) = Ae^{jw_0 t}$

$x_2(t) = Be^{j5w_0 t}$

$w = \frac{2\pi}{T}$

$\underbrace{A \quad B}_{\text{Constantes}}$

$= \lim_{T\to\infty} \frac{1}{T}\int_T \left| |Ae^{jw_0 t}| - |Be^{j5w_0 t}| \right|^2 dt$

$= \lim_{T\to\infty} \frac{1}{T}\int_T \left| Ae^{jw_0 t} - Be^{j5w_0 t} \right|^2 dt$

Diferencia de cuadrados.
$(a-b)^2 = a^2 - 2ab + b^2$

$= \lim_{T\to\infty} \frac{1}{T}\int_T \left(A^2 e^{2jw_0 t} - 2AB e^{j5w_0 t} e^{jw_0 t} + B^2 e^{j10w_0 t} dt\right)$

$= \lim_{T\to\infty} \frac{1}{T}\int_T A^2 e^{j2w_0 t} dt - 2\int_T AB e^{j6w_0 t} dt + \int_T B^2 e^{j10w_0 t} dt$

- Primera Integral:

sustitución

$u = j2w_0 t$

$t = \frac{u}{j2w_0}$

$dt = \frac{du}{j2w_0}$

$A^2 \int_T \frac{e^u}{j2w_0} du = \frac{A^2 e^u}{j2w_0} = \left.\frac{A^2 e^{j2w_0 t}}{j2w_0}\right|_T$

- Segunda Integral:

sustitución

$u = j6wt$ $\quad dt = \frac{du}{j6w_0}$

$t = \frac{u}{j6w_0}$

$2AB\int_T \frac{e^u}{j6w_0} du = \frac{2AB e^u}{j6w_0} = \frac{2AB e^{j6w_0 t}}{j6w_0 t}$

$= \left.\frac{AB e^{j6w_0 t}}{j3w_0 t.}\right|_T$

• Tercera integral:

Sustitución:

$u = j10w_0t$

$t = \frac{u}{j10w_0}$

$dt = \frac{du}{j10w_0}$

$$= B^2 \int_T \frac{e^u}{j10w_0}\, du = \frac{B^2 e^u}{j10w_0} = \frac{B^2 e^{j10w_0t}}{j10w_0t}\Bigg|_T$$

$$= \lim_{T\to\infty} \frac{1}{T}\left(\frac{A^2 \cdot e^{j2w_0t}}{j2w_0t} - \frac{2ABe^{j6w_0t}}{j6w_0t} + \frac{B^2 e^{j10w_0t}}{j10w_0t}\right)\Bigg|_T$$

$$= \lim_{T\to\infty} \frac{1}{\cancel{T}}\left(\frac{A^2 e^{j2\cdot\left(\frac{2\pi}{T}\right)\cdot T}}{j2\cdot\left(\frac{2\pi}{T}\right)} - \frac{\cancel{2}ABe^{j6\left(\frac{2\pi}{T}\right)T}}{j6\frac{2\pi}{\cancel{T}}} + \frac{B^2 e^{j10\left(\frac{2\pi}{T}\right)T}}{j10\left(\frac{2\pi}{\cancel{T}}\right)}\right)$$

$$= \cancel{\lim_{T\to\infty}}\left(\frac{A^2 e^{j4\pi}}{j4\pi} - \frac{ABe^{j12\pi}}{j6\pi} + \frac{B^2 e^{j20\pi}}{j20\pi}\right)$$

Respuesta: Distancia entre las dos señales:

$$\boxed{\frac{A^2 e^{j4\pi}}{j4\pi} - \frac{ABe^{j12\pi}}{j6\pi} + \frac{B^2 e^{j20\pi}}{j20\pi}}$$

2. $x(t) = 3\cos(1000\pi t) + 5\sin(2000\pi t) + 10\cos(11000\pi t)$

para discretizar se reemplaza: $t = n/F_S$ con $F_S = 5\,KHz$

con $n \in \mathbb{Z}$

$$x[n/F_S] = 3\cos\left[1000\pi \frac{n}{F_S}\right] + 5\sin\left[2000\pi \frac{n}{F_S}\right] + 10\cos\left[11000\pi \frac{n}{F_S}\right]$$

$$x[n/F_S] : 3\cos\left[1000\pi \frac{n}{5000}\right] + 5\sin\left[2000\pi \frac{n}{5000}\right] + 10\cos\left[11000 \frac{\pi}{5000}\right]$$

$$x[n] = 3\cos\left[\frac{\pi n}{5}\right] + 5\sin\left[\frac{2\pi n}{5}\right] + 10\cos\left[\frac{11\pi n}{5}\right]$$

$$\Omega_3 = \frac{11\pi}{5} > \pi \longrightarrow \text{copia}$$

Se halla la frecuencia original:

$$\Omega_{or} = \Omega_3 - 2\pi$$

$$\Omega_{or} = \frac{11\pi}{5} - 2\pi$$

$$\Omega_{or} = \frac{\pi}{5}$$

Se reemplaza en la función discretizada:

$$x[n] = 3\cos\left[\frac{\pi n}{5}\right] + 5\sin\left[\frac{2\pi n}{5}\right] + 10\cos\left[\frac{\pi n}{5}\right]$$

Sumando términos semejantes se obtiene como respuesta:

R/= $\boxed{x[n] = 13\cos\left[\frac{\pi n}{5}\right] + 5\sin\left[\frac{2\pi n}{5}\right]}$